5.10.) $x_1 = x_0 + s_0$

s_0 determined by $J_f(x_0)\, s_0 = -f(x_0)$

$$J_f(x_0)\, s_0 = \begin{bmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} x_1^2 \\ 2x_1x_2 - 1 \end{bmatrix} \quad -f(x_0)$$

$$x_0 = [0 \;\; 1]^T$$

$$\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$s_0 = [0.5 \; -0.5]^T$$

$$x_1 = x_0 + s_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

5.13.) $J(x) = \begin{bmatrix} 1 & 0 \\ x_2 & x_1 \end{bmatrix}$

newton's method will fail for starting points of the form $x = [0 \;\; \alpha]^T$

α is arbitrary since $J(x)$ is singular at these points.

5.4.)

$f(x) = x^{-1} - y$

$f'(x) = -x^{-2}$

$f(x) = 0$

$$x_{k+1} = x_k - \frac{(x_k^{-1} - y)}{(-x_k^{-2})} = x_k + x_k^2(x_k^{-1} - y) = 2x_k - x_k^2 y$$

no divisions

5.9.)

a.) $J_f(x_k)s_k = \begin{bmatrix} 2x_1 & 2x_2 \\ 2x_1 & -1 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} x_1^2 + x_2^2 - 1 \\ x_1^2 - x_2 \end{bmatrix} = -f(x_k)$

b.) $J_f(x_k)s_k = \begin{bmatrix} 2x_1 + x_2^3 & 3x_1x_2^2 \\ 6x_1x_2 & 3x_1^2 - 3x_2^2 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} x_1^2 + x_1x_2^3 - 9 \\ 3x_1^2x_2 - x_2^3 - 4 \end{bmatrix} = -f(x_k)$

c.) $J_f(x_k)s_k = \begin{bmatrix} 1 - 2x_2 & 1 - 2x_1 \\ 2x_1 - 2 & 2x_2 + 2 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} x_1 + x_2 - 2x_1x_2 \\ x_1^2 + x_2^2 - 2x_1 + 2x_2 + 1 \end{bmatrix} - f(x_k)$

d.) $J_f(x_k)s_k = \begin{bmatrix} 3x_1^2 & -2x_2 \\ 1 + 2x_1x_2 & x_1^2 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} x_1^3 - x_2^2 \\ x_1 + x_1^2x_2 - 2 \end{bmatrix} = -f(x_k)$

e.) $\cancel{2} J_f(x_k)s_k = \begin{bmatrix} 2\cos(x_1) - 5 & -\sin(x_2) \\ -4\sin(x_1) & 2\cos(x_2) - 5 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = -\begin{bmatrix} 2\sin(x_1) + \cos(x_2) - 5x_1 \\ 4\cos(x_1) + 2\sin(x_2) - 5x_2 \end{bmatrix}$

~~Soto~~

Exercises 5.1, 5.2, 5.3, 5.4, 5.9, 5.10, 5.13

5.1.)

A.) $f(x) = x^2 - 2 \qquad x_0 = 1$

$f'(x) = 2x$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 + \frac{1}{2} = 3/2$$

b.) $x_0 = 1, \ x_1 = 2$

Secant method -> $x_2 = x_1 - \dfrac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$

$$x_2 = 2 - \frac{(2(2-1))}{(2-(-1))} = 4/3$$

5.2)

a.) $x_{k+1} = x_k - \dfrac{(x_k^3 - 2x_k - 5)}{3x_k^2 - 2}$

b.) $x_{k+1} = x_k + \dfrac{(e^{-x_k} - x_k)}{(e^{-x_k} + 1)}$

c.) $x_{k+1} = x_k - \dfrac{(x_k \sin(x_k) - 1)}{(x_k \cos(x_k) + \sin(x_k))}$

5.3.)

a.) $f(x) = x^2 - y$ $\qquad f(x) = 0$ is $x_{k+1} = x_k - \dfrac{(x_k^2 - y)}{2x_k}$

$f'(x) = 2x$

b.) must satisfy $4 \cdot 2k = m$

4bits -> $k = \lceil \log_2(24/4) \rceil = 3$ iterations

53 bits -> $k = \lceil \log_2(53/4) \rceil = 4$ iterations