Groups Practice Questions

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1. Let $n = p^2q$, where p and q are distinct prime numbers. How many numbers m satisfy $1 \leq m \leq n$ and gcd (m, n) = 1? Note that gcd (m, n) is the greatest common divisor of m and n.

   (A) p(q – 1)
   (B) pq
   (C) (p2- 1) (q – 1)
   (D) p(p – 1) (q – 1)

2. A binary operation on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is TRUE about ?
   (A) Commutative but not associative
   (B) Both commutative and associative
   (C) Associative but not commutative
   (D) Neither commutative nor associative

3. Let G be a group with 15 elements. Let L be a subgroup of G. It is known that $L \neq G$ and that the size of L is at least 4. The size of L is ____

4. There are two elements x, y in a group $(G,*)$ such that every element in the group can be written as a product of some number of x's and y's in some order. It is known that

   $x * x = y * y = x * y * x * y = y * x * y * x = e$

   where e is the identity element. The maximum number of elements in such a group is ____

5. Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of *G* is ___

6. Let G be a group of 35 elements. Then the largest possible size of a subgroup of G other than G itself is _____

7. Let G be a group of order 6, and H be a subgroup of G such that $1<|H|<6$. Which one of the following options is correct?
   (A) Both G and H are always cyclic
   (B) G may not be cyclic, but H is always cyclic
   (C) G is always cyclic, but H may not be cyclic
   (D) Both G and H may not be cyclic

8. On a set $A = \{a, b, c, d\}$ a binary operation $*$ is defined as given in the following table.

$*$	a	b	c	d
a	a	c	b	d
b	c	b	d	a
c	b	d	a	c
d	d	a	c	b

The relation is

A. Commutative but not associative

B. Neither commutative nor associative

C. Both commutative and associative

D. Associative but not commutative