Language of logic practice questions

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1. Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate

$$P(x) = \neg(x = 1) \wedge \forall y(\exists z(x = y * z) \Rightarrow (y = x) \vee (y = 1))$$

(A) $P(x)$ being true means that x is a prime number

(B) $P(x)$ being true means that x is a number other than 1

(C) $P(x)$ is always true irrespective of the value of x

(D) $P(x)$ being true means that x has exactly two factors other than 1 and x

2. Consider the following logical inferences.

I_1: If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2: If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is **TRUE**?

(A) Both I_1 and I_2 are correct inferences
(B) I_1 is correct but I_2 is not a correct inference
(C) I_1 is not correct but I_2 is a correct inference
(D) Both I_1 and I_2 are not correct inferences

3. What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

(A) $\exists x\ (\text{real}(x) \vee \text{rational}(x))$
(B) $\forall x\ (\text{real}(x) \rightarrow \text{rational}(x))$
(C) $\exists x\ (\text{real}(x) \wedge \text{rational}(x))$
(D) $\exists x\ (\text{rational}(x) \rightarrow \text{real}(x))$

4. What is the logical translation of the following statement?
"None of my friends are perfect"
(A) $\exists x(F(x) \wedge \neg P(x))$
(B) $\exists x(\neg F(x) \wedge P(x))$
(C) $\exists x(\neg F(x) \wedge \neg P(x))$
(D) $\neg \exists x(F(x) \wedge P(x))$

5. Which one of the following is NOT logically equivalent to $\neg \exists x(\forall y(\alpha) \wedge \forall z(\beta))$
(A) $\forall x(\exists z(\neg \beta) \rightarrow \forall y(\alpha))$
(B) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg \alpha))$
(C) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg \beta))$
(D) $\forall x(\exists y(\neg \alpha) \rightarrow \exists z(\neg \beta))$

6. Consider the statement

"Not all that glitters is gold"

Predicate $glitters(x)$ is true if x glitters and predicate $gold(x)$ is true if x is gold. Which one of the following logical formulae represents the above statement?

(A) $\forall x$: $glitters(x) \Rightarrow \neg gold(x)$

(B) $\forall x$: $gold(x) \Rightarrow glitters(x)$

(C) $\exists x$: $gold(x) \wedge \neg glitters(x)$

(D) $\exists x$: $glitters(x) \wedge \neg gold(x)$

7. Which one of the following propositional logic formulas is TRUE when exactly two of p, q, and r are **TRUE**?

(A) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(B) $(\sim (p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(C) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(D) $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

8. Which one of the following Boolean expressions is NOT a tautology?

(A) $((a \longrightarrow b) \wedge (b \longrightarrow c)) \longrightarrow (a \longrightarrow c)$

(B) $(a \leftrightarrow c) \longrightarrow (\sim b \longrightarrow (a \wedge c))$

(C) $(a \wedge b \wedge c) \longrightarrow (c \vee a)$

(D) $a \longrightarrow (b \longrightarrow a)$

9. The CORRECT formula for the sentence, "not all rainy days are cold" is

(A) $\forall$d (Rainy(d) $\wedge$~Cold(d))

(B) $\forall$d (~Rainy(d) $\rightarrow$ Cold(d))

(C) $\exists$d (~Rainy(d) $\rightarrow$ Cold(d))

(D) $\exists$d (Rainy(d) $\wedge$~Cold(d))

10. Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(A) $(\neg p \vee q) \wedge (p \vee \neg q)$ (B) $(\neg p \vee q) \wedge (q \rightarrow p)$

(C) $(\neg p \wedge q) \vee (p \wedge \neg q)$ (D) $(\neg p \wedge \neg q) \vee (p \wedge q)$

11. Consider the following two statements.

$S1$: If a candidate is known to be corrupt, then he will not be elected

$S2$: If a candidate is kind, he will be elected

Which one of the following statements follows from $S1$ and $S2$ as per sound inference rules of logic?

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

12. Which one of the following well formed formulae is a tautology?

(A) $\forall x\, \exists y\, R(x,y) \leftrightarrow\ \exists y\, \forall x\, R(x,y)$

(B) $(\forall x\, [\exists y\, R(x,y) \rightarrow S(x,y)]) \rightarrow\ \forall x \exists y\, S(x,y)$

(C) $[\forall x\, \exists y\, (P(x,y) \rightarrow R(x,y)] \leftrightarrow [\forall x\, \exists y\, (\neg P(x,y) \vee R(x,y)]$

(D) $\forall x\, \forall y\, P(x,y) \rightarrow\ \forall x\, \forall y\, P(y,x)$

13. Consider the following statements relating to the level of poker play of four players P, Q, R and S.

I. P always beats Q

II. R always beats S

III. S loses to P only sometimes

IV. R always loses to Q

Which of the following can be logically inferred from the above statements?

(i) P is likely to beat all the three other players

(ii) S is the absolute worst player in the set

14. Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention with the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

(i) The author believes that computers are not good for us

(ii) Mobile computers and the internet are both intended inventions

(A) (i)

(B) (ii) only

(C) both (i) and (ii)

(D) neither (i) nor (ii)

15. All hill-stations have a lake. Ooty has two lakes.

Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

(i) Ooty is not a hill-station

(ii) No hill-station can have more than one lake.

(A) (i) Only (B) (ii) Only (C) both (i) and (ii) (D) neither (i) nor (ii)

16. Which one of the following well-formed formulae in predicate calculus is NOT valid?

(A) $(\forall x\, p(x) \Rightarrow \forall x\, q(x)) \Rightarrow (\exists x \neg p(x) \vee \forall x\, q(x))$

(B) $(\exists x\, p(x) \vee \exists x\, q(x)) \Rightarrow \exists x\, (p(x) \vee q(x))$

(C) $\exists x\, (p(x) \wedge q(x)) \Rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$

(D) $\forall x\, (p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$

17. The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statements below?

I. $p \Rightarrow q$

II. $q \Rightarrow p$

III. $(\neg q) \vee p$

IV. $(\neg p) \vee q$

(A) I only (B) I and IV only (C) II only (D) II and III only

18. Consider the first-order logic sentence $F: \forall x(\exists y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are *implied* by F?

I. $\exists y(\exists x R(x, y))$

II. $\exists y(\forall x R(x, y))$

III. $\forall y(\exists x R(x, y))$

IV. $\neg \exists x(\forall y \neg R(x, y))$

(A) IV only (B) I and IV only (C) II only (D) II and III only

19. Let p, q, and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is

(A) a tautology.

(B) a contradiction.

(C) always TRUE when p is FALSE.

(D) always TRUE when q is TRUE.