END TERM GUIST, SEPTEMBER 2021 PAPER: IT223 TIME: 3 HOURS

1. i) We usually write numbers in decimal form (or base 10), meaning numbers are composed using 10 different "digits" {0,1,…,9}. Sometimes though it is useful to write numbers hexadecimal or base 16. Now there are 16 distinct digits that can be used to form numbers: {0,1,…,9,A,B,C,D,E,F}. So for example, a 3 digit hexadecimal number might be 2B8.
    a. How many 2-digit hexadecimals are there in which the first digit is E or F? Explain your answer in terms of the additive principle (using either events or sets).
    b. How many 3-digit hexadecimals start with a letter (A-F) and end with a numeral (0-9)? Explain.
    c. How many 3-digit hexadecimals start with a letter (A-F) or end with a numeral (0-9) (or both)? Explain. (6)

1. ii) Use Warshall's algorithm to find the transitive closure of these relations on {1,2,3,4}
{(2,1), (2,3), (3,1),(3,4), (4,1), (4,3)} (10)

1. iii) Find whether the following are totally ordered sets giving proper reasons: (4)
    a. If 'A' is any set of real numbers then the poset [A,<=] is ___
    b. If A = {1,2,3,.....10} then the poset [A,<=] is a ___
    c. If A = { 1,2,6,30,60,300} then [A,/] is ___
    d. If S = {Φ, {a},{b},{a,b}} then [S, ⊆] is ___

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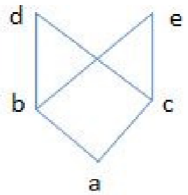
2. i) Let A = R – {3} and B = R – {1}. A function f: A → B is defined by f(x) = (x-2)/(x-3). Which of the following is true? Give proper reasoning. (6)
    a. f is one-one but not onto
    b. f is onto but not one-one
    c. f is a bijection
    d. f is neither one-one nor onto

2. ii) Which of the following functions have inverse defined on their ranges (where the domain is the set of real numbers)? (6)
    a. $f(x) = x^2$
    b. $f(x) = x^3$

2.iii) The poset diagram of a poset P = {a,b,c,d,e} is shown below: (8)



Which of the following statements is not true? Give proper explanation.

a) 'P' is not a lattice
b) The subset {a,b,c,d} of P is a lattice
c) The subset {b,c,d,e} of P is a lattice
d) The subset {a,b,c,e} of P is a lattice

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3. i) Show that √2 is irrational. (4)

3. ii) Prove by mathematical induction that $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 for all positive integers. (7)

3. iii) Which of the following is not logically equivalent to: (7)

$$\neg \exists x(\forall y(\alpha) \wedge \forall z(\beta))$$

(A) $\forall x(\exists z(\neg\beta) \rightarrow \forall y(\alpha))$
(B) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg\alpha))$
(C) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg\beta))$
(D) $\forall x(\exists y(\neg\alpha) \rightarrow \exists z(\neg\beta))$

3.iv) Show that $p \longrightarrow q \equiv (\neg p) \vee q$. (2)

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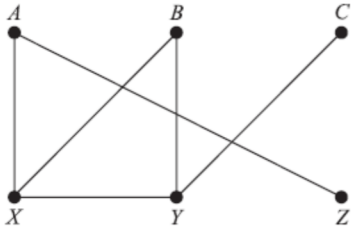
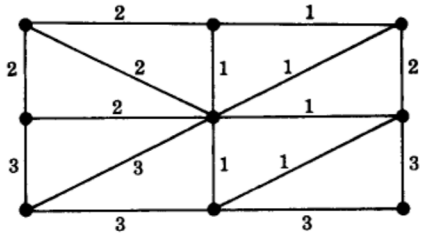
4. i) Let G be the graph in Figure below. (10)



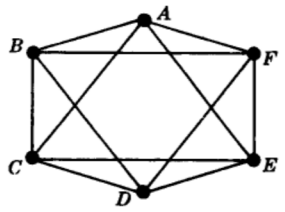
Find:

a) all simple paths from A to C;
b) all cycles;
c) subgraph H generated by $V' = \{B,C,X, Y\}$;
d) $G - Y$;
e) In graph theory, a vertex whose removal makes the graph disconnected is called a cut-point. Find all cut points;

4.ii) Find a minimal spanning tree T for the weighted graph G in Figure below. (7)



4. iii) Draw a planar representation of the following graph: (3)



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5. i) A binary operation on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is TRUE about ? (4)

a. Commutative but not associative

b. Both commutative and associative

c. Associative but not commutative

d. Neither commutative nor associative

5. ii) On a set A={a,b,c,d} a binary operation * is defined as given in the following table. (4)

*	a	b	c	d
a	a	c	b	d
b	c	b	d	a
c	b	d	a	c
d	d	a	c	b

The relation is

a. Commutative but not associative

b. Neither commutative nor associative

c. Both commutative and associative

d. Associative but not commutative

5.iii) Let G be a group of order 6, and H be a subgroup of G such that $1<|H|<6$. Which one of the following options is correct? (4)

a. Both G and H are always cyclic

b. G may not be cyclic, but H is always cyclic

c. G is always cyclic, but H may not be cyclic

d. Both G and H may not be cyclic

5.iv) If G = {1,3,5,7} is a group w.r.t $\otimes_8$ which of the following is not true? (4)

a) The inv of 1 is 1

b) The inv of 3 is 3

c) The inv of 5 is 7

d) The inv of 7 is 7

5.v) Let G = ({0,1,2,3,4,5}, $\oplus_6$) is a group which of the following is/are sub-groups of G? (4)

a) H1 = {1,3}

b) H2 = {1,5}

c) H3 = {0,3}

d) H4 = {0,2,4}

e) H5 = {0,2,3,5}

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END OF QUESTION PAPER