Shadow interpolation on images project

2017

1 problem statement

We are given a sequence of pictures of a diffusive object with various lighting conditions.
We know that in diffusive object, there are 3 basis functions that span all the possible pictures (determined by lighting direction).
And we would like to add shadows to this problem. the problem with shadows is that it can not be represented by a low amount of basis functions.
So we are seeking a smart interpolation for the representation of the missing parts between the images.

2 Simple interpolation

Given 2 pictures of logical shadow representation I_0 and I_1 where $I_i \in \{0,1\}^{H\times W}$
I_i is the shadow representation of image i.
We are seeking a vector mapping

$$\Phi_v(t) : \mathbb{R}^2 \to \mathbb{R}^2$$

$$x \mapsto \Phi_v(t;x)$$

With the constraints

$$\Phi_v(0) = id$$

$$I_1 = I_0 \cdot \Phi_v(1)$$

As an initial idea we will define Φ_v as

$$\Phi_v = id + t \cdot v$$

This is just an initial model for the vector field and it is going to be changed when we advance with the project.

We define our discrete domain as a graph: $D := (\nu, \varepsilon, f)$
Where ν is set of image vertices (pixel corner)
ε as the edges and f is image faces (pixel center)
Using this definition, assuming there are n vertices and m faces, we define

$$\text{Image } I : \nu \to \mathbb{R},\ I \in \mathbb{R}^n$$
$$\text{Vector field velocity } v : f \to \mathbb{R}^2,\ v \in \mathbb{R}^{2m}$$

We want to calculate ϕ_v by minimizing

$$\operatorname*{argmin}_v \|I_1 - I_0 \cdot \Phi_v(1)\|_D^2 = \int_D \|I_1(x) - I_0(x) \cdot \Phi_v(1;x)\|_D^2 dx$$

This is an ill-posed problem, so we define a regularizing term as follows

$$\int_D \langle v, \triangle v \rangle da$$

So our minimization problem is

$$\operatorname*{argmin}_v \frac{1}{2}\|v\|_{D,\triangle} + \frac{1}{2\sigma^2}\|I_1 - I_0 \cdot \Phi_v(1)\|_D^2$$

Calculating the regularization term in matrix form:

$$\int_D \langle v, \triangle v \rangle da \quad = \tag{1}$$

$$\sum_{j \in f} A_f(j) \cdot v(j)^T \cdot \triangle v(j) \quad = \tag{2}$$

$$v^T \cdot A_f \cdot \triangle v \tag{3}$$

Where A_f is a diagonal matrix of vertices areas.

Calculating the data fidelity term in matrix form:

$$\int_D \| I_1 - I_0 \cdot \Phi_v(1) \|_D^2 \; da \tag{4}$$

$$\sum_{i \in v} A_v(i) \cdot (I_1(i) - I_0(i) \cdot \Phi_v(1; i))^2 \tag{5}$$

Assigning $\Phi_v(1; i)$ for every i, we get $\Phi_v(1; i) = id(i) - v(i)$ using bilinear interpolation from faces to vertices

Where $v(i) = \frac{\sum\limits_{j \in N(i)} v(j)}{\sum\limits_{j \in N(i)} 1}$

is the mean value of the 1-ring of vertex i.

$$\sum_{i \in v} A_v(i) \cdot [I_1(i) - I_0(i) \cdot (id(i) - v(i))]^2 = \tag{6}$$

$$\sum_{i \in v} A_v(i) \cdot \left[I_1^2(i) - 2I_0(i)I_1(i)(id(i) - v(i)) + I_0^2(i)(id(i) - v(i))^2 \right] = \tag{7}$$

$$\sum_{i \in v} A_v(i) \cdot \left\{ [I_0^2(i)] \cdot v^2(i) + [2I_0(i)I_1(i) - 2I_0^2(i)] \cdot v(i) + [I_1^2(i) - 2I_0(i)I_1(i) + I_0^2(i)] \right\} = \tag{8}$$

$$A_v \cdot [v^T I_0^T I_0 v + 2v^T I_0^T (I_1 - I_0) + (I_1 - I_0)^2] \tag{9}$$

We got a quadratic form $A_v \cdot [v^T K v - 2V^T f + c]$ where
$K = I_0^T I_0$ is PSD Matrix ($K \geq 0$)
$f = -I_0^T (I_1 - I_0)$
$c = (I_1 - I_0)^T (I_1 - I_0)$

find v^{opt}

$$v^{opt} = \underset{v}{\operatorname{argmin}} \frac{1}{2} v^T A_f \triangle v + \frac{1}{2\sigma^2} (I_1 - I_0 + v \cdot I_0)^T A_v (I_1 - I_0 + v \cdot I_0) \tag{10}$$

derivative:

$$\frac{dE}{dv} = A_f \triangle v + \frac{1}{\sigma^2} I_0^T A_v (I_1 - I_0 + v I_0) \tag{11}$$

Steepest Descent:

$$v^{k+1} = v^k - \alpha \frac{dE}{dv} \tag{12}$$