Problem 1

Since we are given a discrete set of data points and are tasked with finding the least squares polynomial, we can use the linear system

$$\begin{bmatrix} \sum_{i=0}^{m} x_i^{0+0} & \sum_{i=0}^{m} x_i^{0+1} & \cdots & \sum_{i=0}^{m} x_i^{0+n} \\ \vdots & & & \vdots \\ \vdots & \vdots & \sum_{i=0}^{m} x_i^{j+k} & \vdots \\ \vdots & & & \vdots \\ \sum_{i=0}^{m} x_i^{n+0} & \sum_{i=0}^{m} x_i^{n+1} & \cdots & \sum_{i=0}^{m} x_i^{n+n} \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_j \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{m} x_i^{0} y_i \\ \vdots \\ \sum_{i=0}^{m} x_i^{j} y_i \\ \vdots \\ \sum_{i=0}^{m} x_i^{n} y_i \end{bmatrix}$$

to find the coefficients. Since we are using the standard power functions as the set to form the least squares polynomial, the polynomials will be of the form

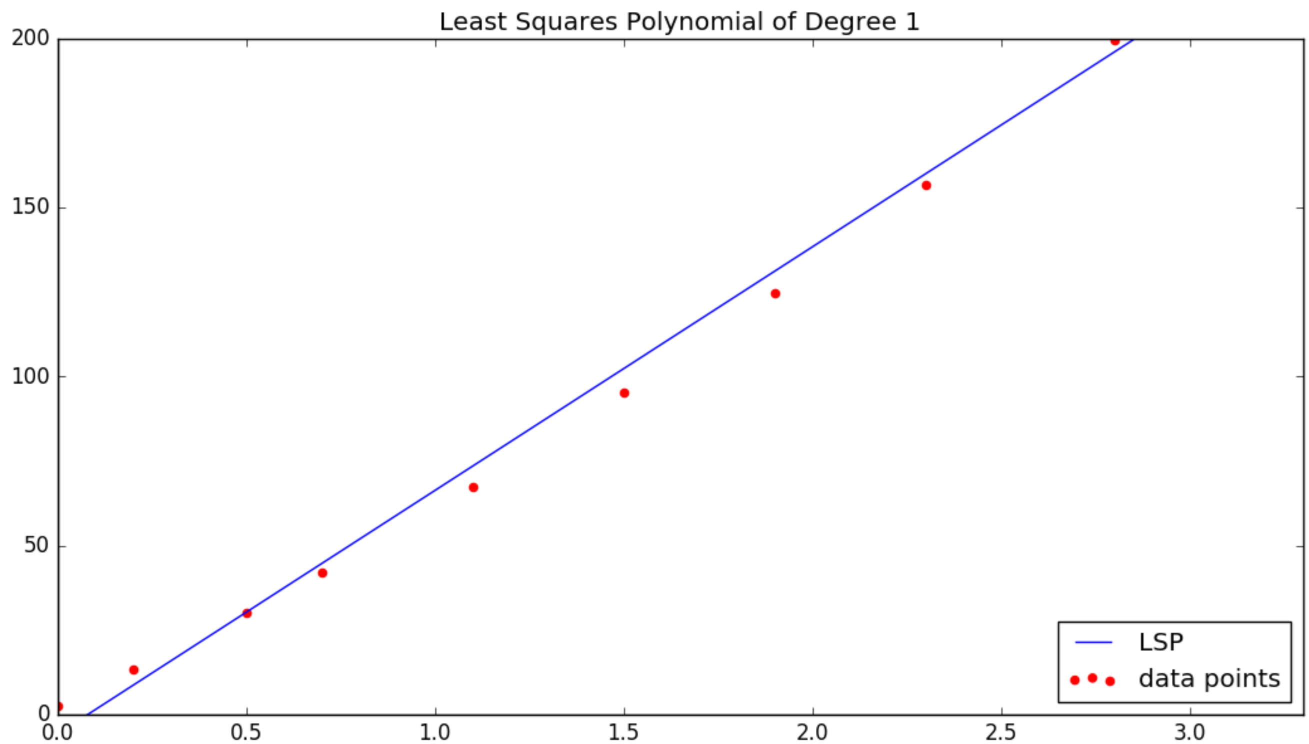
$$a_0 + a_1x + \ldots + a_nx^n$$

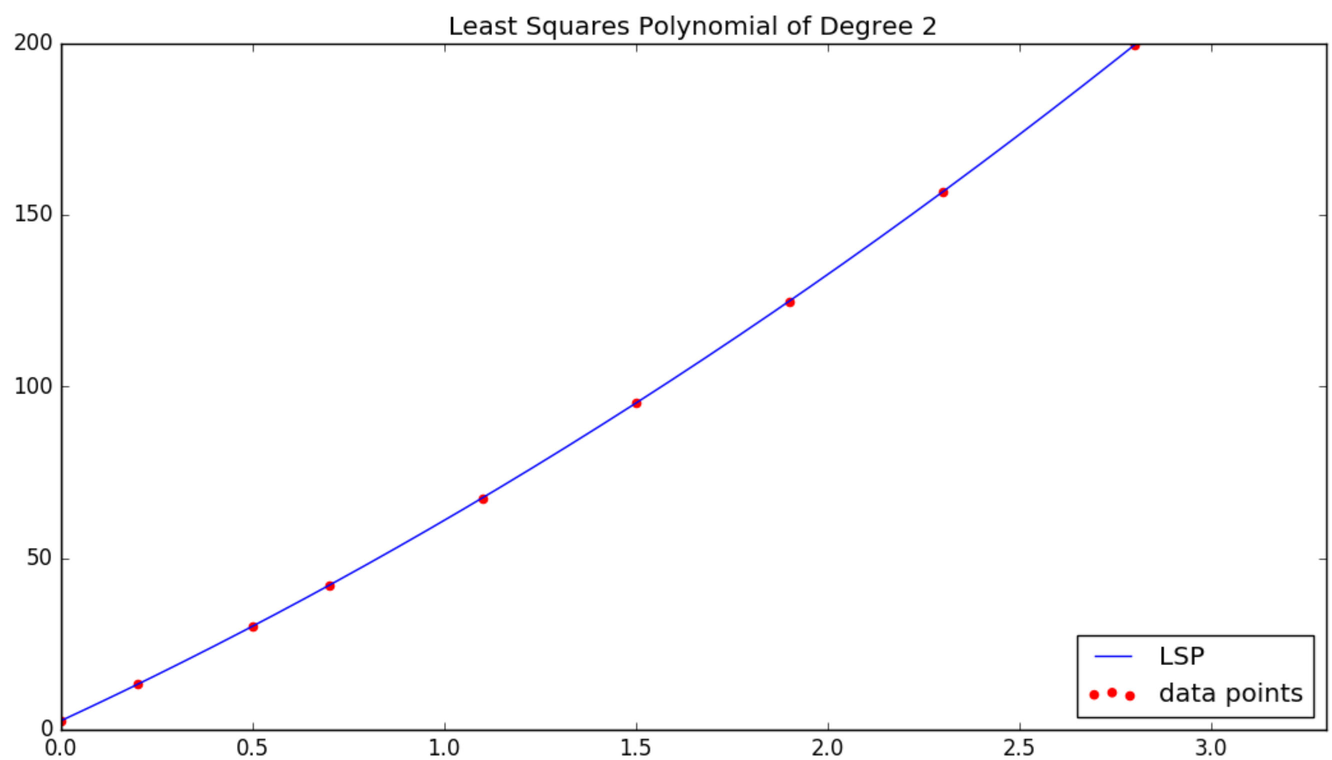
Table 1 shows the results of solving the system with different values of n.

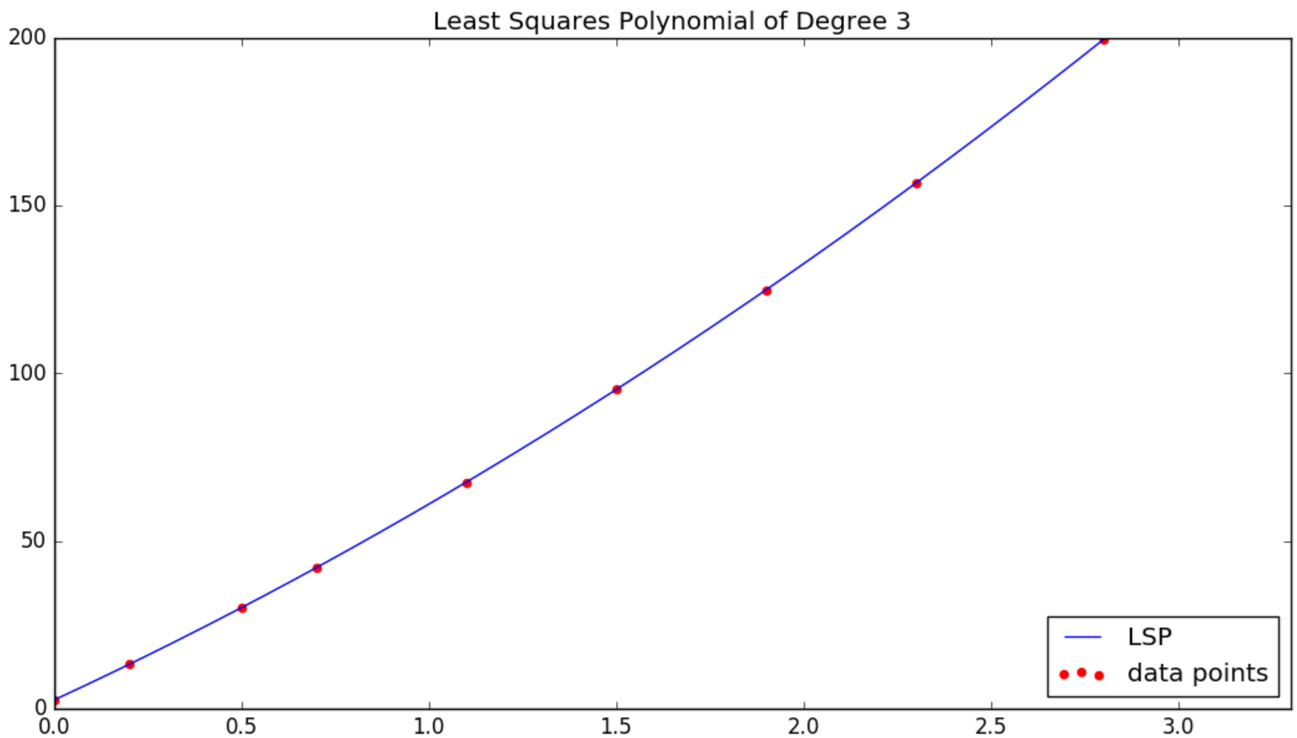
| **Degree** | **Sum of Squared Errors** |
|---|---|
| 1 | 329.013193034 |
| 2 | 0.001442912 |
| 3 | 0.000527341 |
| 4 | 0.0000424597 |

Table 1. The coefficients of the n'th degree least squares polynomial for the given data.

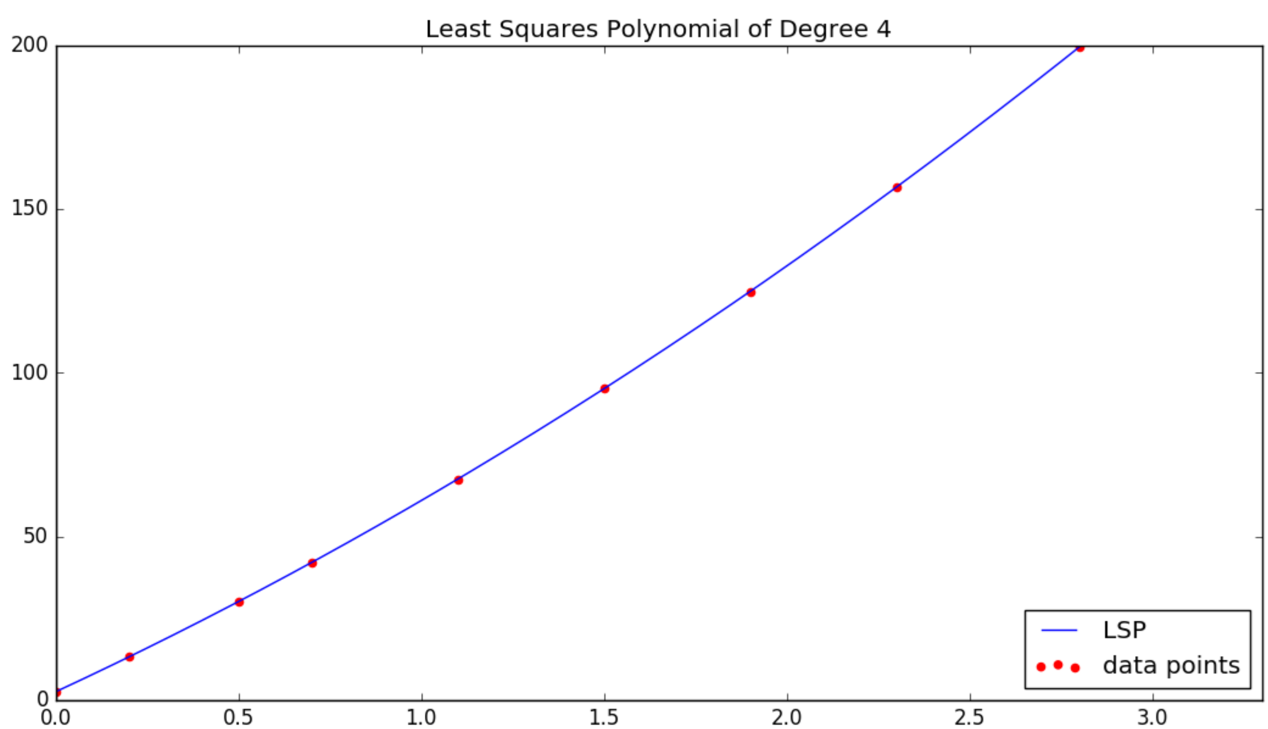
Which polynomial to choose might depend on the problem at hand. Depending on what the data is supposed to represent, we might want to choose a linear or quadratic. Basing our choice on the sum of the squared errors only, the fourth degree polynomial is clearly the best.


Least Squares Polynomial of Degree 1
200
150
100
50
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
LSP
data points


Least Squares Polynomial of Degree 2
200
150
100
50
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
LSP
data points



Least Squares Polynomial of Degree 3
200
150
100
50
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
LSP
data points

Least Squares Polynomial of Degree 4
200
150
100
50
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
LSP
data points