

For more work with the data table estimate $\int_1^{1.8} f(x) dx$ using Romberg integration.

For $h = 0.4$

$$T(h) = T(0.4) = \frac{0.4}{2} [f(1.0) + 2f(1.4) + f(1.8)]$$

$$= 0.2 [1.0 + 2(0.67032005) + 0.44932896]$$

$$= 0.557993812$$

$$T\left(\frac{h}{2}\right) = T(0.2) = \frac{0.2}{2} [f(1.0) + 2f(1.2) + 2f(1.4) + 2f(1.6) + f(1.8)]$$

$$= .1 [1 + 2(0.81873075) + 2(0.67032005) + 2(0.54881164) + 0.44932896]$$

$$= 0.552505384$$

$$T\left(\frac{h}{4}\right) = T(0.1) = \frac{0.1}{2} [f(1.0) + 2f(1.1) + 2f(1.2) + 2f(1.3) + 2f(1.4) + 2f(1.5) + 2f(1.6) + 2f(1.7) + f(1.8)]$$

$$= 0.05 [1 + 2(0.90483742) + 2(0.81873075) + 2(0.74081822) + 2(0.67032005) + 2(0.60653066) + 2(0.54881164) + 2(0.49658530) + 0.44932896]$$

$$= 0.551129852$$

Romberg:

$$T_2(h) = T\left(\frac{h}{2}\right) + \frac{T\left(\frac{h}{2}\right) - T(h)}{2^{2(2-1)} - 1} = 0.552505384 + \frac{0.552505384 - 0.557993812}{3} \\ = 0.552505384 - 0.001829476 = 0.550675908$$

$$T_2\left(\frac{h}{2}\right) = T\left(\frac{h}{4}\right) + \frac{T\left(\frac{h}{4}\right) - T\left(\frac{h}{2}\right)}{2^{2(2-1)} - 1} = 0.551129852 + \frac{0.551129852 - 0.552505384}{3} \\ = 0.551129852 - 0.0004585106 = 0.5506713413$$

$$T_3(h) = T_2\left(\frac{h}{2}\right) + \frac{T_2\left(\frac{h}{2}\right) - T_2(h)}{2^{2(3-1)} - 1} = 0.5506713413 + \frac{0.5506713413 - 0.550675908}{15} \\ = 0.5506713413 - 0.0000003044 = 0.5506710369$$

0.557993812		
0.552505384	0.550675908	
0.551129852	0.5506713413	0.5506710369