

<u>x</u>	<u>x(h)</u>
1.0	1.00000000
1.1	0.90483742
1.2	0.81873075
1.3	0.74081822
1.4	0.67032005
1.5	0.60653066
1.6	0.54881164
1.7	0.49658530
1.8	0.44932896

Table 1

We used the Centered Difference formula with Richardson's extrapolation method on the data in Table 1 to approximate $L'(1.4)$. In matrix form, the approximation is described below:

$$\begin{array}{rcl}
 D_1(h) & & -0.6883388 \\
 D_1(h/2) \quad D_2(h) & = & -0.674797775 \quad -0.6702841 \\
 D_1(h/4) \quad D_2(h/2) \quad D_3(h) & & -0.6714378 \quad -0.670317808333 \quad -0.670320055556
 \end{array}$$

After 3 iterations, we arrive at an estimate for the derivative:

$$L'(1.4) = -0.670320055556$$

The next attempt to estimate the derivative of $L'(1.4)$ involves finding the Lagrange Polynomial for the data, finding its derivative, and then evaluate at the given point.

The Lagrange Polynomial:

$$-19.84x^8 + 228.1x^7 - 1143x^6 + 3257x^5 - 5775x^4 + 6522x^3 - 4581x^2 + 1827x - 315.4$$

The first derivative of the Lagrange Polynomial:

$$-158.7x^7 + 1597x^6 - 6857x^5 + 1.628e+04x^4 - 2.31e+04x^3 + 1.957e+04x^2 - 9161x + 1827$$

The derivative evaluated at 1.4:

$$L'(1.4) = -0.670700864926$$

Comparing the values found for an approximation of the derivative at the point 1.4 to 3 decimal places we see that the Lagrange method matches one of the approximations from the Richardson method. Since we know that approximations become more accurate as we proceed down and to the right, it is safe to assume that the approximation found by the Lagrange method is less accurate than the one found by Richardson's method.

$$L'(1.4) = -0.671$$

$$D_1(h/4) = -0.671$$

$$D_3(h) = -0.670$$

For more work with the data table estimate $\int_1^{1.8} f(x) dx$ using Romberg integration.

For $h = 0.4$

$$T(h) = T(0.4) = \frac{0.4}{2} [f(1.0) + 2f(1.4) + f(1.8)]$$

$$= 0.2 [1.0 + 2(0.67032005) + 0.44932896]$$

$$= 0.557993812$$

$$T\left(\frac{h}{2}\right) = T(0.2) = \frac{0.2}{2} [f(1.0) + 2f(1.2) + 2f(1.4) + 2f(1.6) + f(1.8)]$$

$$= .1 [1 + 2(0.81873075) + 2(0.67032005) + 2(0.54881164) + 0.44932896]$$

$$= 0.552505384$$

$$T\left(\frac{h}{4}\right) = T(0.1) = \frac{0.1}{2} [f(1.0) + 2f(1.1) + 2f(1.2) + 2f(1.3) + 2f(1.4) + 2f(1.5) + 2f(1.6) + 2f(1.7) + f(1.8)]$$

$$= 0.05 [1 + 2(0.90483742) + 2(0.81873075) + 2(0.74081822) + 2(0.67032005) + 2(0.60653066) + 2(0.54881164) + 2(0.49658530) + 0.44932896]$$

$$= 0.551129852$$

Romberg:

$$T_2(h) = T\left(\frac{h}{2}\right) + \frac{T\left(\frac{h}{2}\right) - T(h)}{2^{2(2-1)} - 1} = 0.552505384 + \frac{0.552505384 - 0.557993812}{3} \\ = 0.552505384 - 0.001829476 = 0.550675908$$

$$T_2\left(\frac{h}{2}\right) = T\left(\frac{h}{4}\right) + \frac{T\left(\frac{h}{4}\right) - T\left(\frac{h}{2}\right)}{2^{2(2-1)} - 1} = 0.551129852 + \frac{0.551129852 - 0.552505384}{3} \\ = 0.551129852 - 0.0004585106 = 0.5506713413$$

$$T_3(h) = T_2\left(\frac{h}{2}\right) + \frac{T_2\left(\frac{h}{2}\right) - T_2(h)}{2^{2(3-1)} - 1} = 0.5506713413 + \frac{0.5506713413 - 0.550675908}{15} \\ = 0.5506713413 - 0.0000003044 = 0.5506710369$$

0.557993812		
0.552505384	0.550675908	
0.551129852	0.5506713413	0.5506710369