x	x(h)
1.0	1.00000000
1.1	0.90483742
1.2	0.81873075
1.3	0.74081822
1.4	0.67032005
1.5	0.60653066
1.6	0.54881164
1.7	0.49658530
1.8	0.44932896

Table 1

We used the Centered Difference formula with Richardson's extrapolation method on the data in Table 1 to approximate *L'(1.4).* In matrix form, the approximation is described below:

$$\begin{matrix} D_1(h) & & \\ D_{1(h/2)} & D_2(h) & \\ D_{1(h/4)} & D_2(h/2) & D_3(h) \end{matrix} = \begin{matrix} -0.6883388 & & \\ -0.674797775 & -0.6702841 & \\ -0.6714378 & -0.670317808333 & -0.670320055556 \end{matrix}$$

After 3 iterations, we arrive at an estimate for the derivative:

L'(1.4) = -0.670320055556

The next attempt to estimate the derivative of *L'(1.4)* involves finding the Lagrange Polynomial for the data, finding its derivative, and then evaluate at the given point.

The Lagrange Polynomial:

-19.84 x^8 + 228.1 x^7 - 1143 x^6 + 3257 x^5 - 5775 x^4 + 6522 x^3 - 4581 x^2 + 1827 x - 315.4

The first derivative of the Lagrange Polynomial:

-158.7x^7 + 1597x^6 - 6857x^5 + 1.628e+04x^4 - 2.31e+04x^3 + 1.957e+04x^2 - 9161x + 1827

The derivative evaluated at 1.4:

***L'(1.4)* = -0.670700864926**

Comparing the values found for an approximation of the derivative at the point 1.4 to 3 decimal places we see that the Lagrange method matches one of the approximations from the Richardson method. Since we know that approximations become more accurate as we proceed down and to the right, it is safe to assume that the approximation found by the Lagrange method is less accurate than the one found by Richardson's method. In this case, the extrapolation uses more data to approximate the derivative than does simply differentiating an approximating polynomial.

$L'(1.4) = -0.671$
$D_1(h/4) = -0.671$
$D_3(h) = -0.670$

If we use Richardson extrapolation on the Lagrange interpolant, we might expect a better approximation due to the fact that the interpolant allows us to get smaller step sizes, and thus, better approximations. We would be justified in using the interpolant because of the spacing of the data points and the value we are approximating. Data points that are equally spaced throughout the entire interval is not ideal, but when we cannot change the points, we can still closely approximate values that lie near the center of the interval. *1.4* is exactly in the center, and the nodes are equally spaced, so we can expect the interpolant to accurately approximate *L(1.4)* as well as the values near it that are used in Richardson Extrapolation. We are not approximating *L(x)* for values of *x* near the ends of the interval on which it is defined (in this case, with these data points, Lagrange interpolation would be a poor choice), therefore, we can confidently use Lagrange interpolation. Using the Lagrange interpolant with Richardson extrapolation yields the following table.

Tolerance	**h\`(1.4) with Lagrange interpolation**
1e-2	-0.684416619551
1e-4	-0.671418561644
1e-6	-0.670389442517
1e-8	-0.670324405641
1e-10	-0.670321141566
1e-12	-0.670320121237
1e-14	-0.670320057465
1e-16	-0.670320054279

Table 2. Approximations to by applying Richardson Extrapolation on a Lagrange interpolating polynomial for the data in table 1. Lower tolerances are used to get better approximations.

From this, we can see fantastic agreement with the value approximated by Richardson extrapolation using only the raw data. This is very good news because it shows us what we can get away with when we have a nice data set: first, we do not need a very small initial step size; second, we do not need a fancy interpolant; and third, we do not need many iterations.