

1. For the function $g(x) = e^{-x^2/2}$ use the centered difference formula with $h = 10^{-1}, 10^{-2} \dots 10^{-20}$ to create a table of estimates of $g'(1.4)$. Include a column for the relative error of each estimate.

Using the central difference formula, we were able to create the table of estimates of $g'(1.4)$ below. We can see that an error occurs when h is less than or equal to 10^{-16} because it is causing the central difference formula to divide by a very small number. We can also see that the relative error reaches its lowest point around the fifth iteration when h is 10^{-5} but then the relative error begins to increase again after that.

$$g'(1.4) = -0.52543553839195933364$$

h-value	Estimate	Relative error
10^{-1}	-0.52452445426194760358	0.00173395985509470487
10^{-2}	-0.52542643080942530442	0.00001733339652262963
10^{-3}	-0.52543544731642466417	0.00000017333341202652
10^{-4}	-0.52543553748118920765	0.00000000173336224796
10^{-5}	-0.52543553838713119575	0.00000000000918883010
10^{-6}	-0.52543553832884448695	0.00000000012011910514
10^{-7}	-0.52543553841211121380	0.00000000003835271635
10^{-8}	-0.52543553397121911530	0.00000000841347776335
10^{-9}	-0.52543558393125522343	0.00000008666961513331
10^{-10}	-0.52543552842010399218	0.00000001897826586298
10^{-11}	-0.52543525086434783589	0.00000054721767084446
10^{-12}	-0.52546855755508659058	0.00006284151092693245
10^{-13}	-0.52513549064769904362	0.00057104577505083661
10^{-14}	-0.52458037913538646535	0.00162752458501378501
10^{-15}	-0.58286708792820718372	0.10930275046109579062
10^{-16}	0.0	1.0
10^{-17}	0.0	1.0
10^{-18}	0.0	1.0
10^{-19}	0.0	1.0
10^{-20}	0.0	1.0