**Problem 4**

To find all solutions to the following nonlinear system of equations:

,

we note that any solution will be the intersection points of and . To get the intersection points, we equate the two functions:

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Any solution to the nonlinear system will have values of *x* that are roots to the cubic polynomial . Graphing this polynomial reveals only one real-valued root which is near *x* =0.7. Using Newton's Method with *f(x) = ,* *f’(x) = ,* andan initial value of 0.5, the one and only root is 0.725270085, correct to 9 decimal places. Plugging this into either of the equations of *y* above, we get and *.* These are the only intersection points, and thus, the only solutions to the nonlinear system.

Alternatively, we could have proceeded by writing *y* in terms of *x*, as follows:

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This cubic, as shown above, has *x =* 0.725270085 as its only solution. Plugging this into either equation in the system yields , exactly as before.

Yet another method for finding a solution to the nonlinear system is to use the two-dimensional version of Newton's Method with

and

Starting with an initial guess of , we get the following sequence of approximations.

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Thus, the solution to the system when using the two-dimensional Newton’s Method with an initial guess of *x* = 1 and *y* = 1 is *x* = and *y* = 1.025686791.