|  |  |
| --- | --- |
| **x** | **x(h)** |
| 1.0 | 1.00000000 |
| 1.1 | 0.90483742 |
| 1.2 | 0.81873075 |
| 1.3 | 0.74081822 |
| 1.4 | 0.67032005 |
| 1.5 | 0.60653066 |
| 1.6 | 0.54881164 |
| 1.7 | 0.49658530 |
| 1.8 | 0.44932896 |

*Table 1*

We used the Centered Difference formula with Richardson’s extrapolation method on the data in Table 1 to approximate *L’(1.4).* In matrix form, the approximation is described below:

*D*1(*h*) -0.6883388

*D*1(*h/2) D*2 (*h*) = -0.674797775 -0.6702841

*D*1(*h/4) D*2 (*h/2*) *D*3 (*h*) -0.6714378 -0.670317808333 -0.670320055556

After 3 iterations, we arrive at an estimate for the derivative:

***L’(1.4) = -0.670320055556***

The next attempt to estimate the derivative of *L’(1.4)* involves finding the Lagrange Polynomial for the data, finding its derivative, and then evaluate at the given point.

The Lagrange Polynomial:

***-19.84 x^8 + 228.1 x^7 - 1143 x^6 + 3257 x^5 - 5775 x^4 + 6522 x^3 - 4581 x^2 + 1827 x - 315.4***

The first derivative of the Lagrange Polynomial:

***-158.7x^7 + 1597x^6 - 6857x^5 + 1.628e+04x^4 - 2.31e+04x^3 + 1.957e+04x^2 - 9161x + 1827***

The derivative evaluated at 1.4:

***L’(1.4) =* -0.670700864926**

Comparing the values found for an approximation of the derivative at the point 1.4 to 3 decimal places we see that the Lagrange method matches one of the approximations from the Richardson method. Since we know that approximations become more accurate as we proceed down and to the right, it is safe to assume that the approximation found by the Lagrange method is less accurate than the one found by Richardson’s method.

***L’(1.4) =* -0.671**

***D1(h/4) = -0.671***

***D3(h) = -0.670***

For more work with the data table estimate using Romberg integration.

For h = 0.4

T (h) = T (0.4) = [f (1.0) + 2 f (1.4) + f (1.8)]

= 0.2 [1.0 + 2(0.67032005) + 0.44932896]

= 0.557993812

T () = T (0.2) = [f (1.0) + 2 f (1.2) + 2 f (1.4) + 2 f (1.6) + f (1.8)]

= .1 [1 + 2(0.81873075) + 2(0.67032005) + 2(0.54881164) + 0.44932896]

= 0.552505384

T () = T (0.1) = [f (1.0) + 2 f (1.1) + 2 f (1.2) + 2 f (1.3) + 2 f (1.4)

+ 2 f (1.5) + 2 f (1.6) + 2 f (1.7) + f (1.8)]

= 0.05 [1 + 2(0.90483742) + 2(0.81873075) + 2(0.74081822)

+ 2(0.67032005) + 2(0.60653066) + 2(0.54881164)

+ 2(0.49658530) + 0.44932896]

= 0.551129852

Romberg:

T2 (h) = T () + = 0.552505384 +

= 0.552505384 – 0.001829476 = 0.550675908

T2 () = T () + = 0.551129852 +

= 0.551129852 – 0.0004585106 = 0.5506713413

T3 (h) = T2 () + = 0.5506713413 +

= 0.5506713413 – 0.0000003044 = 0.5506710369

|  |  |  |
| --- | --- | --- |
| 0.557993812 |  |  |
| 0.552505384 | 0.550675908 |  |
| 0.551129852 | 0.5506713413 | 0.5506710369 |