First exercise: Classifying MNIST with MLPs

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1 Introduction

In the following I report about my implementation and application of a multilayer perceptron (MLP) for handwritten digit classification with the MNIST dataset. I briefly describe my problems while implementation but focus more on it's application and an excursion to regularization, because for the former a stricter course was already given.

2 Implementation

The implementation of the MLP was done according to the given framing, wherefore I don't write much about it. The framing was only changed moving code to separate functions or methods to reuse it, adding regularization (see section 4) and storing error rate history for plotting. Difficult was, that the parts of the MLP all depend on each other, such that debugging can only be done by changing parts and checking *does it learn?* or *are the gradients right?* (with gradient check). Probably consequent test driven development with unit tests even for the layer could help, but was not here. As an advantage is the simplicity of the MLP algorithm, such that a lot of parts easily can be manually compared and verified with the text book formulas.

I especially had difficulties with the gradient checker. Even after verifying parts to work equal to `scipy.optimize.check_grad` and slight modification in both the network and checker code, the check failed. Still, the network seems to be correct, because learning works even if the check fails.

3 Application

The overall task with the MLP is to recognize (*classify*) handwritten digits of the *MNIST* dataset with few mistakes. Finding a setup with such a MLP involves changing the meta parameters and verifying the error rate on data we did not train on. Meta parameters can be of the network (*number of layers or hidden units, choice of activation functions*) or the training itself (*learning rate, batch size, choice of optimization method, ...*). Due to the long training time, we used a reduced training set with about 10000 items. Error rate is not the same as with full training set, but usually the tendency is. Still training time is about one to three minutes on a *Intel(R) Core(TM) i7-5600U CPU @ 2.60GHz* wherefore meta parameters could only be tried manually and sparse. Some choices are obvious. For gradient descent optimization is very difficult to find a learning rate for which it learns but not overshoots. The stochastic gradient descent (SGD) in contrast is way more robust even if it introduces the batch size as another meta parameter. More difficult is, that several meta parameters affect the error rate in a coupled fashion. Rules of thumb can help a bit to reduce the search space, e.g. *if we add layers, we should also increase the learning rate* worked for me. This rules were especially important, because a lot of meta parameter combinations did lead to no learning at all (error rate did not descrease much over epochs). To at least find a combination for further parameter tuning we started with parameter combinations found in the exercise or internet [1]. Also first trying meta parameters like the learning rate on logarithmic scale worked pretty well.

The setup Figure 1 we ended up with, validation error converges after 17 epochs on the full MNIST training training dataset (50000 images). The test error on the unseen test dataset is 2%.

4 Regularization

With the high amount of parameters (number of weight values), the MLP is capable to express even complex functions. This can lead to the problem of *overfitting* which means the network learns to express exactly the

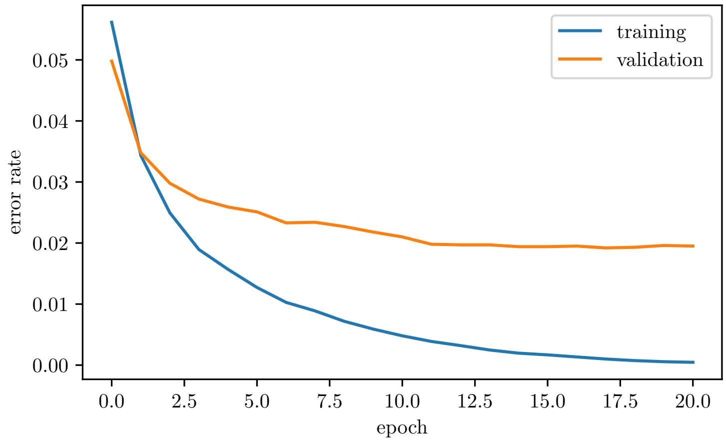
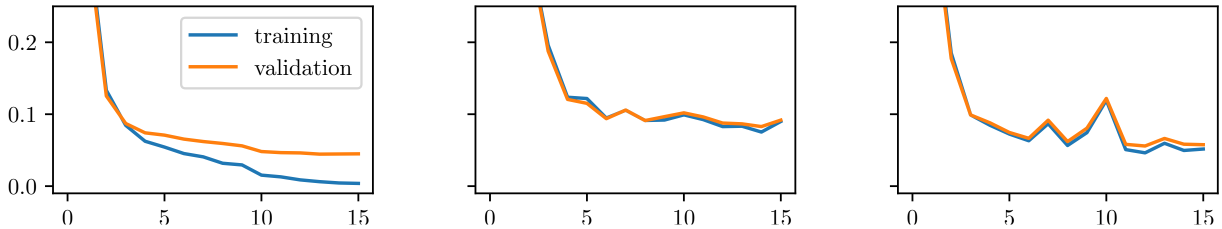


Figure 1: Change of training and validation error while training epochs of MLP on full MNIST dataset. MLP has one hidden layer with 350 units and ReLu activation. Optimization with SGD used a learning rate of 0.02 and batches of 24 images.



(a) none (b) L_1 ($\alpha = 0.0001$) (c) L_2 ($\alpha = 0.005$)

Figure 2: Classification error while SGD training of MLP subset of MNIST without and with regularization. MLP has two hidden layers with each 100 units and ReLu activation.

noisy training data instead of the general underlying function. You see this generalization lack in Figure 1 as the gap between training and validation curves. Regularization is an easy method to prevent overfitting by limiting the values of the parameter vectors. Therefore I implemented L_1 and L_2 regularization according to [2].

L_2 regularization adds the squared sum of weights to the loss. This means a large parameter values (huge square weight values) increase the loss a lot and are therefore avoided. L_1 works similar but with the sum of absolute parameter values. The amount of regularization can be controlled with the meta parameter α, which we did set to $\alpha = \frac{2}{\#\theta}$ in the lecture.

$$L_{L_2}(f_\theta, D) = L(f_\theta, D) + \frac{\alpha}{2}\theta^T\theta, \qquad \theta \leftarrow \theta - \epsilon(\alpha\theta + \Delta_\theta L(f_\theta, D)) \tag{1}$$

$$L_{L_1}(f_\theta, D) = L(f_\theta, D) + \alpha\|\theta\|, \qquad \theta \leftarrow \theta - \epsilon(\alpha\, sign(\theta) + \Delta_\theta L(f_\theta, D)) \tag{2}$$

You see in Figure 2 that the regularization can prevent overfitting like expected, validation and training error are very close. Still also some draws of regularization are visible. Both the train and the validation errors are higher with regularization. Possibly this can be explained, by seeing the regularization update step also as forgetting learned parameters. In Figure 2c epochs 7 and 10 we see another incident, probably also caused by the forgetting, the error rates jump up instead of descreasing.

References

[1] `http://deeplearning.net/tutorial/mlp.html`, last visit 2017/11/06

[2] I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning, sec. 7.1*, (MIT Press, 2016)