Probability and Computing, 2nd Edition

Solutions to Chapter 5: Balls, Bins, and Random Graphs

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5.1

As $(1+1/n)^n$ increases, we find the smallest n to reach the threshold.
$(1+1/n)^n$ first reaches $0.99e$ at $n=50$, and $0.999999e$ at $n=499982$.
Since $(1-1/n)^n$ also increases, we solve in a similar way. $(1-1/n)^n$ first reaches $0.99/e$ at $n=51$ and $0.999999/e$ at $n=499991$.

5.2

Recall the formula used in the birthday paradox: If there are N possibilities, then we solve for the smallest n that satisfies $\prod_{i=1}^{n-1}(1-\frac{i}{N}) \approx \prod_{i=1}^{n-1} e^{-i/n} = e^{-(n-1)n/2N} < 1/2$. Note that we omitted the final approximation to derive exact numerical answers.
Regardless of whether the number of Social Security number digits is 9 or 13, using the last four digits gives $N=10000$ and this gives $n=119$.
In the case where the number of digits is 9 ($N=10^9$), we get $n=37234$.
In the case where the number of digits is 13 ($N=10^{13}$), we get $n=3723298$.

5.3