Exercise2_605

Alvaro Bueno

2/6/2018

Exercise C25

let V be the set $\in C^2$ with the usual vector addition, but with scalar multiplication defined as:

$$\propto \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \propto y \\ \propto x \end{bmatrix}$$

Determine if V is a vector space with these operations.

Proof for Scalar Multiplication Associativity (SMA)

1. $$\propto (\beta U) = (\propto \beta) U$$

2. $$\propto (\beta \begin{bmatrix} x \\ y \end{bmatrix}) = (\propto \beta) \begin{bmatrix} x \\ y \end{bmatrix}$$

3. $$\propto (\begin{bmatrix} \beta y \\ \beta x \end{bmatrix}) = \begin{bmatrix} \propto \beta y \\ \propto \beta x \end{bmatrix}$$

4. $$\begin{bmatrix} \propto \beta x \\ \propto \beta y \end{bmatrix} \neq \begin{bmatrix} \propto \beta y \\ \propto \beta x \end{bmatrix}$$

   As we can see on step 4. there's no equality on the SMA proof, so V is not a vector space.