UNIVERSITAT POLITÈCNICA
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Beam propagation and Fourier optics

POLARIZATION

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Contents

1 Jones Vector of the reflected Beam

Explain how to calculate the Jones vector for the reflected beam.

1.1 Theory

We can use the basic Fresnel equations to solve this problem, getting a complex angle θ_2.

Based on this scheme :

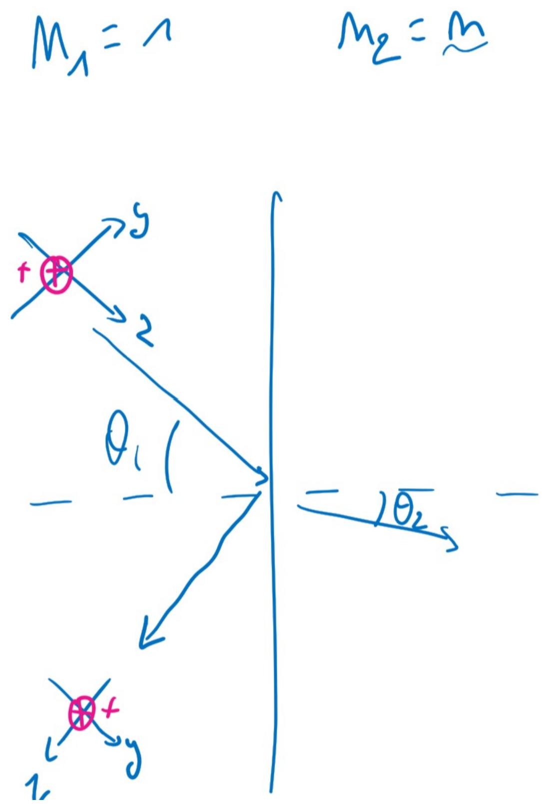

Figure 1: Problem Scheme

We can get the angle θ_2 with fresnel :

$$\sin(\theta_2)\,\tilde{n} = \sin(\theta_1)$$

$$\theta_2 = \text{Arcsin}\left(\frac{\sin(\theta_1)}{\tilde{n}}\right)$$

Based on the references reflection for TE (left) and TM (right) modes (see figures 2 and 3) from (1).

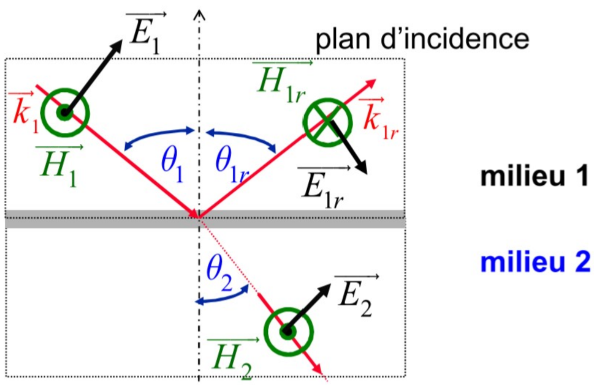

Figure 2: TM

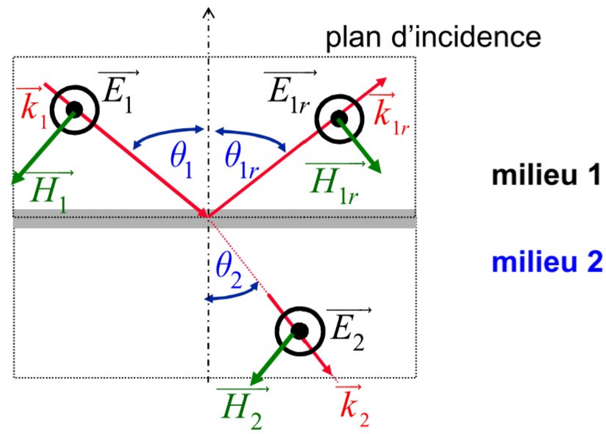

Figure 3: TE

we have the following reflection coefficients (TM on the left and TE on the right) from :

$$E_{1r}^{\parallel} = \frac{n_1\cos(\theta_2) - n_2\cos(\theta_1)}{n_2\cos(\theta_1) + n_1\cos(\theta_2)} \cdot E_1^{\parallel} \qquad \Bigg| \qquad E_{1r}^{\perp} = \frac{n_1\cos(\theta_1) - n_2\cos(\theta_2)}{n_1\cos(\theta_1) + n_2\cos(\theta_2)} \cdot E_1^{\perp}$$

We can then compute the new Jones vectors, Jx corresponding to the TE mode and Jy to the TM mode as follow:

$$J_{x,ref} = \underbrace{\frac{\cos\theta_1 - \tilde{n}\cos\theta_2}{\cos\theta_1 + \tilde{n}\cos\theta_2}}_{\Gamma_x} \cdot J_x$$

$$J_{y,ref} = - \underbrace{\frac{\cos\theta_2 - \tilde{n}\cos\theta_1}{\tilde{n}\cos\theta_1 + \cos\theta_2}}_{\Gamma_y} \cdot J_y$$

Some remarks on this result :

- The Γ parameters are defined as in the matlab code
- The minus sign in front of Γ_y is due to the inversion of the y axis in the figure 1 compared to the reference of figure 2

1.2 Matlab Code

The matlab function used is the following and only apply the theory that has been developed :

```
function [Jxr,Jyr] = Jones_reflection(Jx,Jy,n, theta1)
%   Function to compute the jones vector of a reflected beam of
%   monochromatic light incident with Jones vector (Jx , Jy) on a
%   semi-infinite substrate with complex refractive index n
%   The beam comes from a media with refractive index =1 and with an angle
%   theta1(in degrees) w.r.t the normal

%   Computation of theta2 (complex in case of a complex n)
theta1 = theta1*pi/180 %angle conversion to radians
theta2 = asin(sin(theta1)/n)

%computation of the 2 reflection coefficients
GammaX = (cos(theta1) - n*cos(theta2))/(cos(theta1)+n*cos(theta2));%coefficient for E orthogonal to the incidence plane
GammaY = (cos(theta2) - n*cos(theta1))/(n*cos(theta1)+cos(theta2));%coefficient for E parallel to the incidence plane

%    Computation of the Jones vector components
Jxr = GammaX*Jx;
Jyr = -GammaY*Jy;

end
```

2 ψ and χ parameters of the reflected Beam

Idem to find the ψ and χ parameters of the reflected beam.

2.1 Theory

The Jones vector components being calculated, it is possible to get the ψ and χ parameters of the reflected Beam by just using the conversion equations given in Jones_21-22.pdf (See (2)) :

$$\begin{array}{ll} \tan 2\psi = \frac{2r}{1-r^2}\cos\phi, & r = \frac{a_y}{a_x} = \frac{E_{0y}}{E_{0x}} \\ \sin 2\chi = \frac{2r}{1-r^2}\sin\phi, & \phi = \phi_y - \phi_x \end{array} \tag{1}$$

2.2 Matlab Code

The matlab funtion used is the following :

```
% calc params de polaritzaci  del vector de Jones vJ
% formules pags 29 i 33 de Azzam, noms dels angles segons Photonics, ch. 6
% modified version from the course
function [psi,chi]=param_pol_Photonics(vJ)
    if (vJ(1)==0. && vJ(2)==0.), psi=0; chi=0;nr=0; fi=NaN;
    elseif (vJ(1)==0.),psi=-pi/2;chi=0;r=abs(vJ(2));fi=angle(vJ(2));
    else
        aux=vJ(2)/vJ(1);
        tan2psi=2*real(aux)/(1-(abs(aux))^2);
        psi1=atan(tan2psi)/2;
        psi2=psi1+pi/2;
        psi3=psi1-pi/2;
        if (real(aux)>0 && psi1>0 && psi1>=-pi/2 && psi1<pi/2)psi=psi1; end
        if (real(aux)>0 && psi2>0 && psi2>=-pi/2 && psi2<pi/2) psi=psi2; end
        if (real(aux)>0 && psi3>0 && psi3>=-pi/2 && psi3<pi/2) psi=psi3; end
        if (real(aux)<0 && psi1<0 && psi1>=-pi/2 && psi1<pi/2) psi=psi1; end
        if (real(aux)<0 && psi2<0 && psi2>=-pi/2 && psi2<pi/2) psi=psi2; end
        if (real(aux)<0 && psi3<0 && psi3>=-pi/2 && psi3<pi/2) psi=psi3; end
        if (real(aux)==0 && abs(aux)>1), psi=-pi/2; end
        if (real(aux)==0 && abs(aux)<1), psi=0; end
        if (real(aux)==0 && abs(aux)==1), psi=0; end
        sin2chi=2*imag(aux)/(1+(abs(aux))^2);
        chi=asin(sin2chi)/2;
    end
end
```

```
function [psi_r,chi_r] = Ellipse(Jx,Jy,n,theta1)
    %Compute the psi and chi parameters to caracterize the polarisation
    %after reflection of monochromatic light incident with Jones vector
    %(Jx , Jy) on a semi-infinite substrate with complex refractive index n
    %The beam comes from a media with refractive index =1 and with an angle
    %theta1(in degrees) w.r.t the normal


    %Computation of the Jones vector components after reflection
    [Jxr,Jyr] = Jones_reflection(Jx,Jy,n,theta1);
    vJ = [Jxr , Jyr]

    %Computation of the parameters psi and chi
    [psi_r,chi_r] = param_pol_Photonics(vJ)
end
```

3 Ratio of reflected energy

Idem to find the ratios of reflected energy (with respect to the incident one).

3.1 Theory

The Jones vector components being calculated, it is possible to get the intensity of the incident wave (I_i) and the intensity of the reflected wave (I_r) using Jones_21-22.pdf (See (2)).
As for a plane wave the Energy is proportional to the intensity, the ration of Energy α is given by the ration of intensity :

$$\alpha = \frac{I_r}{I_i} = \frac{|J_{x,r}|^2 + |J_{y,r}|^2}{|J_x|^2 + |J_y|^2}$$

3.2 Matlab Code

The matlab funtion used is the following :

```
function [alpha] = Energy_reflected(Jx,Jy,n,theta1)
    %Compute the percentage of reflected energy
    %after reflection of monochromatic light incident with Jones vector
    %(Jx , Jy) on a semi-infinite substrate with complex refractive index n
    %The beam comes from a media with refractive index =1 and with an angle
    %theta1(in degrees) w.r.t the normal

    %Computation of the Jones vector components after reflection
    [Jxr,Jyr] = Jones_reflection(Jx,Jy,n,theta1);

    %Computation of the percentage of intensity that is reflected
    Ii = (abs(Jx))^2 + (abs(Jy))^2;
    Ir = (abs(Jxr))^2 + (abs(Jyr))^2;
    alpha = Ir/Ii
end
```

4 Values of interest

Use your scripts for the case of incident right circular light and for two substrates: n2 = 1.523 and n2 = 0.45 + 5.47j Show the results for both substrates and for the incidence angles $50°$, $60°$ and $70°$ in a Table of values.

The Jones vector corresponding to a right circular polarization is given by

$$J = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

The Tables with values of interest are given below :

4.1 n2 = 1.523

θ_1 [deg]	(Jx,r ; Jy,r)	(ψ ; χ) [rad]	α
50	(-0.3438 ; 0.0622i)	(0 ; -0.1790)	0.0610
60	(-0.4295 ; - 0.0386i)	(0 ; 0.0896)	0.0930
70	(-0.5560 ; - 0.2034i)	(0 ; 0.3508)	0.1753

4.2 n2 = 0.45 + 5.47j

θ_1 [deg]	(Jx,r ; Jy,r)	(ψ ; χ)[rad]	α
50	(-0.9558 - 0.2241i ; -0.5048 + 0.8127i)	(0.7451 ; -0.6221)	0.9396
60	(-0.9701 - 0.1755i ; -0.6136 + 0.7212i)	(0.7453 ; -0.5217)	0.9342
70	(-0.9829 - 0.1208i ; -0.7786 + 0.5119i)	(0.7455 ; -0.3512)	0.9245

2 comments about this results :

- The complex refraction index corresponds to absorption of energy by the media. This leads to complex reflection coefficients, and then variation of the phase shift between the 2 components of the Jones vector.
  Indeed we see that for a real n, ψ is equal to 0 after reflection for a right circular polarization but it is no longer the case for a complex refraction coefficient.
- We observe an increase of the reflected energy for media with complex n.

References

[1] **LEPL1203_CM4**

[2] **Jones_21-22.pdf**