Beam propagation and Fourier optics

ANISOTROPIC MEDIA

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Contents

1 Output polarization states as a function of β

Calculate the output polarization states as a function of β and represent them in the Poincarée Sphere for several input polarization: linear at $0°$, linear at $90°$, and circular right

1.1 Theory

The Jones Vector at the output of a twisted nematic liquid crystal cell is given by :

$$J_{out} = \mathrm{M}(\beta)\, J_{in} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} X - iY & Z \\ -Z & X + iY \end{pmatrix} J_{in}$$

with

$$\gamma = \sqrt{\alpha^2 + \beta^2} \quad X = \cos\gamma \quad Y = \frac{\beta}{\gamma}\sin\gamma \quad Z = \frac{\alpha}{\gamma}\sin\gamma$$

and α, β the 2 parameters of the twisted nematic liquid crystal cell.

In our case $\alpha = \frac{\pi}{2}$ and the system is considered for $\beta \in [0; 2\pi]$

1.2 Matlab Code

Based on this, the matlab function that compute the Jones Vector at the output is the following :

```
function [Jout] = J_OUT(Jin,beta)
        %Calculates the output polarization state in Jones Vector form
        % as a function of beta (alpha fixed to pi/2)
        %Jout = [Joutx ; Jouty] that could be a complex vector
        %Jin : Jin= [Jx ; Jy] and |Jin|=1
        %beta : birefringence [rad]

%parameters definition
gamma = sqrt((pi/2)^2+beta^2);
X = cos(gamma);
Y = beta/gamma*sin(gamma);
Z = pi/2/gamma*sin(gamma);

%Jones matrix of the twisted nematic liquid crystal cell M definition
A = [0 -1; 1 0];
B = [X-Y*1i Z; -Z X+Y*1i];
M = A*B;

%Output Jones vector computation
Jout = M*Jin;

end
```

The matlab code that represents the polarization states on the Pointcarée sphere is the following :

```
Jin1 = [1 ; 0] % lin pol at 0 deg
Jin2 = [0 ; 1] % lin pol at 90 deg
Jin3 = [1/sqrt(2) ; 1*i/sqrt(2)] %circ right pol

%plot of the Jones vectors on the Point Carre sphere
PoincareSphere();
for beta=0:2*pi/100:2*pi
  %Representation of Jout in the point Carre Sphere
    Jout = J_OUT(Jin3,beta);
    S = JonesToStokes(Jout);
    plot3(S(2),S(3),S(4),'ro','markerfacecolor','r','markersize',14);
end
```

1.3 Point Carée Spheres

The polarization states are represented on the pointcarée sphere for several input polarizations:

1.3.1 Linear at $0°$

Represented in figure 1

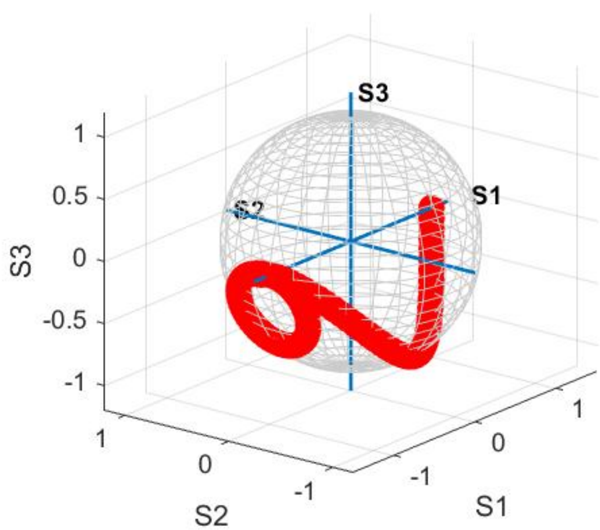

Figure 1: Jout for $\beta \in [0; 2\pi]$ and linear at $0°$ input polarization

1.3.2 Linear at $90°$

Represented in figure 2

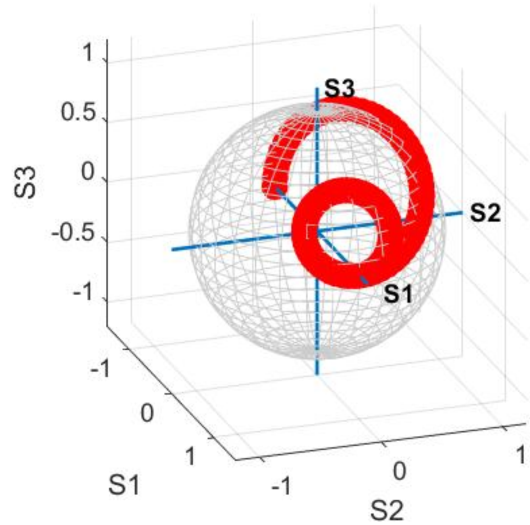

Figure 2: Jout for $\beta \in [0; 2\pi]$ and linear at $90°$ input polarization

1.3.3 Circular Right

Represented in figure 3

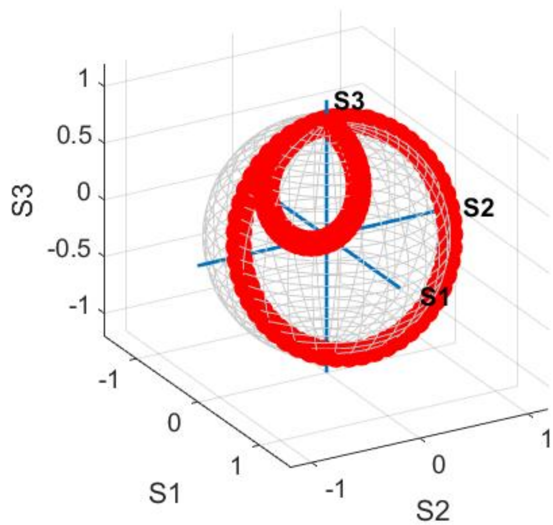

Figure 3: Jout for $\beta \in [0; 2\pi]$ and right circular input polarization

2 Ouput Intensity as a function of β

If at the input we have linear polarization at 0° and at the output we place a polarizer with the transmission axis along the y axis, evaluate the intensity at the output as a function of β

2.1 Theory

The Jones Vector at the output of a twisted nematic liquid crystal cell in serie with a polarizer with the transmission axis along the y axis is given by :

$$J_{out,2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathrm{M}(\beta)\, J_{in}$$

with $\mathrm{M}(\beta)$ defined in the previous section.
As a reminder, in our case $\alpha = \frac{\pi}{2}$ and we will study the system for $\beta \in [0; 2\pi]$.

The intensity I of the Jones Vector $J = \begin{pmatrix} J_x \\ J_y \end{pmatrix}$ can be computed as

$$I = \frac{|J_x|^2 + |J_y|^2}{2\eta}$$

η being the medium impedance, in our case assumed to be $\eta = 377\Omega$

2.2 Matlab Code

The matlab code that computes the intensity at the output of the system is the following :

```
function [intensity] = INTENSITY(beta)
        % Computation of the intensity [W/m^2] if at the input we have linear polarization
        % at 0  and at the output we place a polarizer with the transmission axis
        % along the y axis
        %beta : birefringence [rad]

%(1) Jones Vector computation after linear polarizer

Jout1 = J_OUT([1 ; 0],beta);
Jout2 = [0 0; 0 1]*Jout1;

%(2) Computation of the intensity
eta = 377;
intensity = (abs(Jout2(1))^2 + abs(Jout2(2))^2)/(2*eta);


end
```

The matlab code that plots the intensity at the output of the system for $\beta \in [0; 2\pi]$ is the following :

```
%plot of the intensity for beta between 0 and 2 pi
Intensity = zeros(101);
beta=0:2*pi/100:2*pi;
for  i =1:101
    Intensity(i)= INTENSITY(beta(i));
end

figure
plot(beta,Intensity,'m')

title('Plot 2')
xlabel('beta [rad]')
ylabel('Intensity at the output [W/m^2]')
xlim([0 2*pi])
```

2.3 Plot

Applying this, we get the following plot (figure 4):

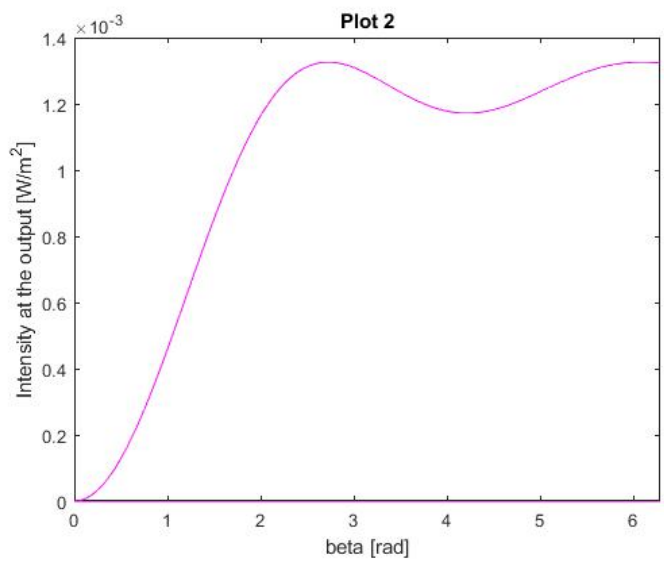

Figure 4: I at the output for $\beta \in [0; 2\pi]$ and linear at 0° input polarization