Beam propagation and Fourier optics

GEOMETRICAL OPTICS

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Contents

1 Function definition

Define a function that, for any object distance, returns the image distance and magnification given by one spherical interface.

The matlab funtion used is the following :

```
function [s_prime,M] = dist_magn(R,s,n,n_prime)
    %Function that compute the image distance in cm and the Magnification
    %INPUTS :
    %   R = the diopter radius [cm]
    %   s = the object distance from the diopterc [cm]
    %   n = the refractive index in the object side
    %   n_prime = the refractive index in the object side

    %   image distance
s_prime = n_prime/((n/s)+((n_prime-n)/R));

    %   Magnification
M = n*s_prime/(n_prime*s);
end
```

The equations were derived using the diopter equations under paraxial assumptions :

$$\frac{n'}{s'} = \frac{n}{s} + \frac{n'-n}{r}$$

$$\Longleftrightarrow s' = \frac{n'}{\frac{n}{s} + \frac{n'-n}{r}}$$

image distance

$$\beta' = \frac{n s'}{n' s}$$

2 Simple diopter

For our sphere, only for objects placed at distances -60<d<0 from the first spherical interface, plot the image distances given by this first interface. Explain the meaning of the asymptotes of the plot. Write a table with the values for d=-60, -30, -15, -10, 0 cm.

2.1 Matlab Code

The matlab code used is the following :

```
x = -60:0.1:0;
y = zeros(601);
for i =1:601
    [y(i),magn] = dist_magn(4,x(i),1,1.33);
end
figure
plot(x,y,'m')
ylim([-60 60])

title('Plot 1')
xlabel('Object position w.r.t. the diopter [cm]')
ylabel('Image position w.r.t. the diopter [cm]')

fprintf('y(-60) = %f \n',y(1));
fprintf('y(-30) = %f \n',y(301));
fprintf('y(-15) = %f \n',y(451));
fprintf('y(-10) = %f \n',y(501));
fprintf('y(0) = %f \n',y(601));
```

This leads to the following plot (Figure 1).

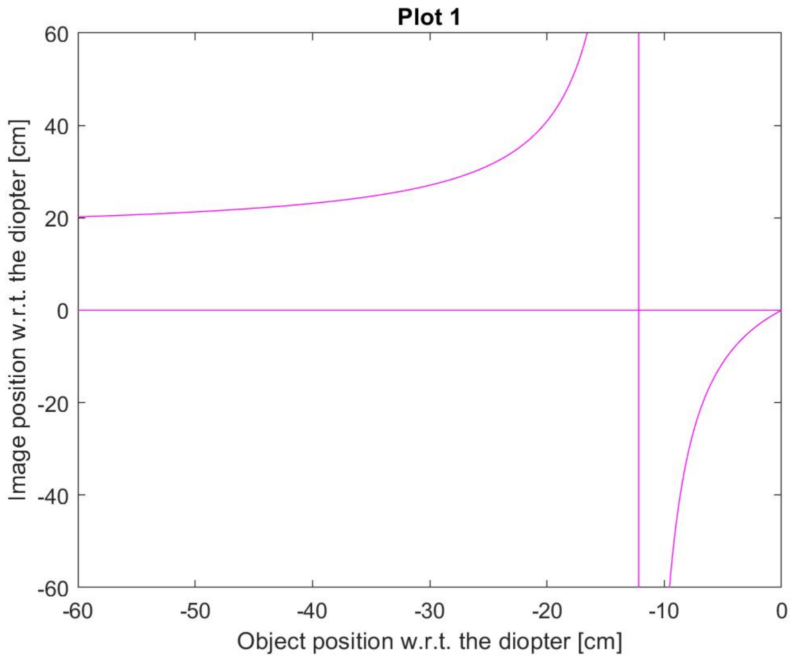


Figure 1

2.2 Asymptotes

We observe 2 asymptots, a vertical and a horizontal one.

2.2.1 Vertical Asymptote

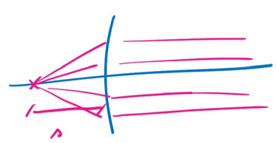
The vertical asymptote corresponds to the focal object distance, leading to parallel rays after the diopter and an infinite distance of the image. The value of the focal object distance can be found using the equations under paraxial assumptions.

We see $\underbrace{\text{focal point}}_{s' = \infty}$ in $s \simeq -12$

$\rightarrow$ Check with equation

$\Downarrow$

$$\frac{n'}{\underbrace{\infty}_{0}} = \frac{n}{s} + \frac{n'-n}{r}$$



$$\rightarrow s = \frac{n}{\frac{n-n'}{r}} = \frac{nR}{n-n'} = \frac{4}{1-1,33} = -12,1212\text{ cm}$$

OK

This value for the vertical asymptote corresponds to the one on the plot.

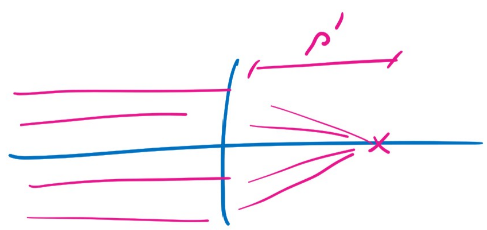
2.2.2 Horizontal Asymptote

The horizontal asymptote corresponds to the focal image distance, leading to parallel rays before the diopter and an infinite distance of the object. The value of the focal image distance can be found using the equations under paraxial assumptions.

H Asymptote in $s' \simeq 16$

$$\to \frac{n'}{s'} = \underbrace{\frac{n}{\infty}}_{0} + \frac{n'-n}{r}$$

$$\Leftrightarrow s' = \frac{n' R}{n'-n} = \frac{1{,}33 \cdot 4}{0{,}33} = 16{,}1212 \text{ cm}$$



This value for the horizontal asymptote corresponds to the one on the plot.

2.3 Table with values of interest

object distance [cm]	image distance [cm]
-60	20.20
-30	27.05
-15	84.00
-10	-76.00
0	0

Table 1: : Image distance for several object distance values

3 Double diopter

For our sphere, plot the final image distances given by the two spherical interfaces (measured from the second interface), for objects placed at distances -60<d<0 from the first spherical interface. Explain the meaning of the asymptotes of the plot. Write a table of the values for d=-60, -10, -5, 0 cm.

3.1 Matlab Code

The matlab code used is the following :

```
%In the left diopter referential
x1 = -60:0.1:0;
y1 = zeros(601);

for i =1:601
    %computation of the first image
    [y1(i),magn] = dist_magn(4,x1(i),1,1.33);
end

%In the right diopter referential
x2 = zeros(601);
y2 = zeros(601);
for i =1:601
    %change of reference
    x2(i) = y1(i)-8;

    %computation of the second image
    [y2(i),magn] = dist_magn(-4,x2(i),1.3,1);
end
figure
plot(x1,y2,'r')
ylim([-60 60])

title('Plot 2')
xlabel('Object position w.r.t. the left diopter [cm]')
ylabel('Image position w.r.t. the right diopter [cm]')

fprintf('y(-60) = %f \n',y2(1));
fprintf('y(-10) = %f \n',y2(501));
fprintf('y(-5) = %f \n',y2(551));
fprintf('y(0) = %f \n',y2(601));


%Image distance for infinite object
x1 = -100000000000;
[y1,magn] = dist_magn(4,x1,1,1.3);
[y_asympt,magn] = dist_magn(-4,y1-8,1.3,1);
fprintf('y_asympt = %f \n',y_asympt);
```

This leads to the following plot (Figure 2).

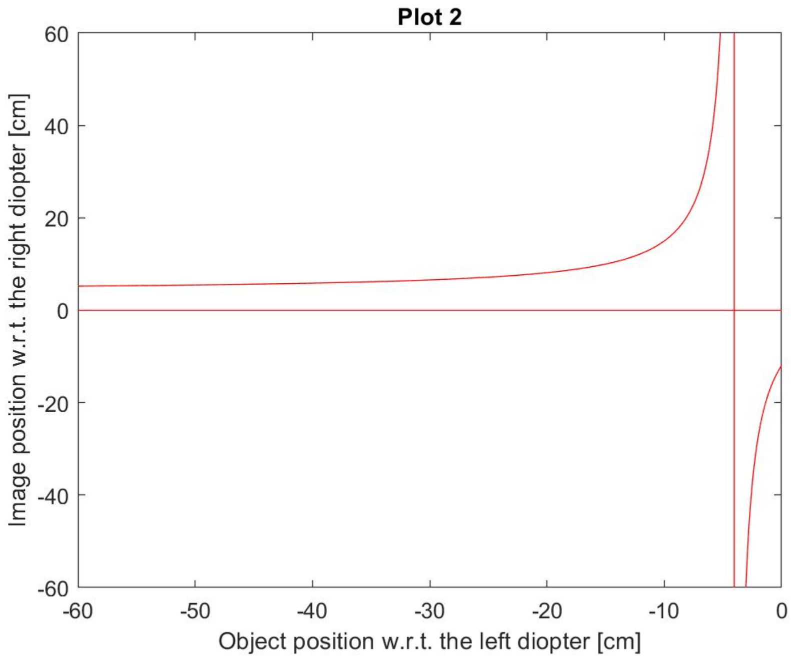
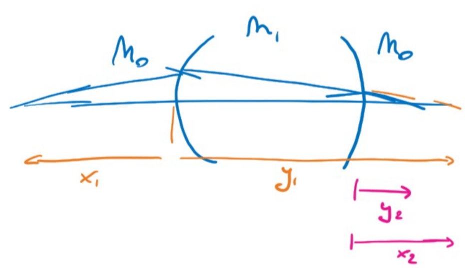


Figure 2

3.2 Asymptotes

We observe 2 asymptots, a vertical and a horizontal one.
The system variables are defined as followed with a sketch of possible values of x_1, y_1, x_2 and y_2:



$$\begin{cases} \dfrac{n_1}{y_1} = \dfrac{n_0}{x_1} + \dfrac{n_1 - n_0}{R_1} \rightarrow \dfrac{n_1}{x_2 + 8} = \dfrac{n_0}{x_1} + \dfrac{n_1 - n_0}{R_1} \\ \dfrac{n_0}{y_2} = \dfrac{n_1}{x_2} + \dfrac{n_0 - n_1}{R_2} \rightarrow x_2 = \dfrac{n_1}{-\dfrac{n_0 - n_1}{R_2} + \dfrac{n_0}{y_2}} \\ y_1 = x_2 + 8 \end{cases}$$

$$y_2(x_1) \rightarrow \frac{n_1}{\dfrac{n_1}{-\dfrac{n_0 - n_1}{R_2} + \dfrac{n_0}{y_2}} + 8} = \frac{n_0}{x_1} + \frac{n_1 - n_0}{R_1}$$

3.2.1 Vertical Asymptote

The vertical asymptote corresponds to the focal object distance, leading to parallel rays after the thick lens and an infinite distance of the image. The value of the focal object distance x_1 can be found using the relation between y_2 and x_1 found previously under paraxial assumptions.

if $y_2 \to \infty$ (V. Asymptote)

$$\frac{1,33}{\underbrace{\frac{1,33}{-\frac{0,33}{4}}+8}_{-0,1638}} = \frac{1}{x_1} + \frac{0,33}{4}$$

$$\Rightarrow \boxed{x_1 = -4,0601 \ [cm]}$$

This value for the vertical asymptote corresponds to the one on the plot.

3.2.2 Horizontal Asymptote

The horizontal asymptote corresponds to the focal image distance, leading to parallel rays on the left side of the thick lens made of 2 diopters and an infinite distance of the object. The value of the focal image distance y_2 can be found using the relation between y_2 and x_1 found previously under paraxial assumptions.

if $x_1 \to \infty$ (H Asympt)

$$\frac{1,33}{-\frac{1,33}{+0,33/4+\frac{1}{y_2}}+8} = \frac{0,33}{4}$$

$$\Leftrightarrow \underbrace{\frac{1,33\cdot 4}{0,33}-8}_{8,1212} = \frac{1,33}{-\frac{0,33}{4}+\frac{1}{y_2}}$$

$$\Leftrightarrow -\frac{0,33}{4}+\frac{1}{y_2} = \frac{1,33}{8,1212} = 0,1638$$

$$\frac{1}{y_2} = 0,2463$$

$$\Rightarrow \boxed{y_2 = 4,060 \ [cm]}$$

This value for the horizontal asymptote corresponds to the one on the plot. and is confirmed with the last part of the matlab code that computes this value for value of x_1 tending to $-\infty$.

3.3 Table with values of interest

object distance [cm]	image distance [cm]
-60	5.22
-10	15.00
-5	73.226
0	-11.94

Table 2: : Image distance for several object distance values

4 Magnification

For our sphere, plot the final magnification given by the two spherical interfaces, for objects placed at distances -60<d<0 from the first interface. Explain the meaning of the asymptotes of the plot. Write a table with the values for d=-60, -15, -5, -2, 0 cm.

4.1 Matlab Code

The matlab code used is the following :

```
%In the left diopter referential
x1 = -60:0.1:0;
magn1 = zeros(601);
y1 = zeros(601);

for i =1:601
    %computation of the first image and magnification
    [y1(i),magn1(i)] = dist_magn(4,x1(i),1,1.33);
end

%In the right diopter referential
x2 = zeros(601);
magn2 = zeros(601);

for i =1:601
    %change of reference
    x2(i) = y1(i)-8;

    %computation of the second Magnification
    [y2,magn2(i)] = dist_magn(-4,x2(i),1.3,1);
end
figure
plot(x1,magn2,'g')
ylim([-10 10])

title('Plot 3')
xlabel('Object position w.r.t. the left diopter [cm]')
ylabel('Total Magnification [\]')

%Calcul of values of interest
fprintf('magn2(-60) = %f \n',magn2(1));
```

```
fprintf('magn2(-15) = %f \n',magn2(451));
fprintf('magn2(-5) = %f \n',magn2(551));
fprintf('magn2(-2) = %f \n',magn2(581));
fprintf('magn2(0) = %f \n',magn2(601));


%Calcul de l'asymptote H
x1 = -100000000000;
[y1,magn] = dist_magn(4,x1,1,1.3);
[y_asympt,magn] = dist_magn(-4,y1-8,1.3,1);
fprintf('magn_asympt = %f \n',magn);
```

This leads to the following plot (Figure 3).

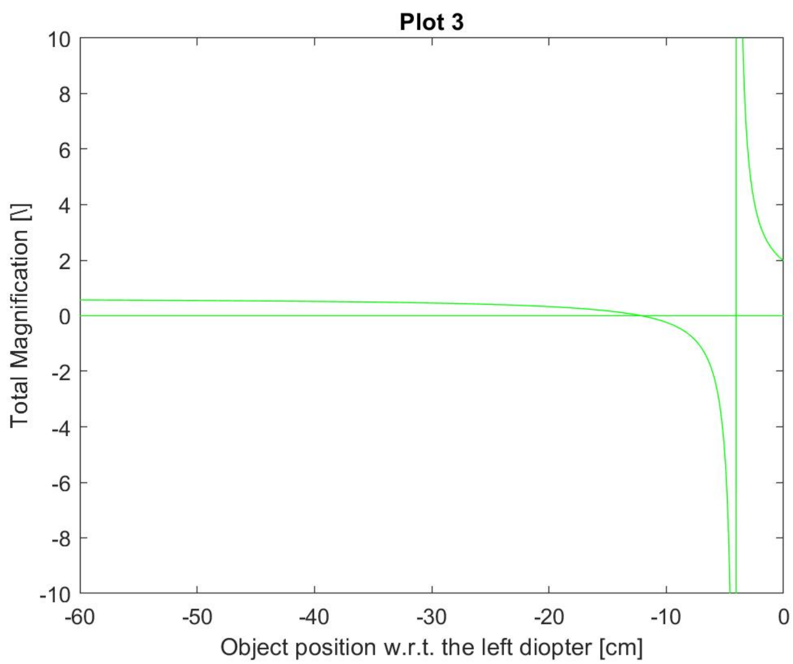


Figure 3

4.2 Asymptotes

We observe 2 asymptots, a vertical and a horizontal one.
The system variables are defined as followed in the previous section with a sketch of possible values of x_1, y_1, x_2 and y_2.

4.2.1 Vertical Asymptote

The vertical asymptote position corresponds focal object distance, leading to parallel rays after the thick lens, an infinite distance of the image and then an infinite magnification. The value of the focal object distance x_1 can be found using the relation between y_2 and x_1 as found in section 3.2.1 under paraxial assumptions.
The vertical asymptote is positioned on $x_1 = -4.06[cm]$

4.2.2 Horizontal Asymptote

The horizontal asymptote corresponds to the magnification when the object is at an infinite distance from the left lens, For this position of the object, we are in a situation such that

- $x_1 = -\infty$
- $y_1 = 16.1212[cm]$ (focal distance of the left diopter)
- $x_2 = y_1 - 8 = 8.1212[cm]$
- $y_2 = 4.0606[cm]$ (found in the section 3.2.2)

. The magnification in this configuration can then be computed as followed :

$$\beta = \frac{n_1 y_2}{n_0 x_2} = 0.665$$

This value correponds to the position of the asymptote showed by the plot and is confirmed with the last part of the matlab code that compute this value for value of x_1 tending to $-\infty$.

4.3 Table with values of interest

object distance [cm]	Magnification [/]
-60	0.569
-15	0.175
-5	-5.041
-2	3.266
0	1.985

Table 3: : Image distance for several object distance values