LELEC2660 - Project - Report 1
Transformer Design

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1 Introduction

This report take place in the design an AC-DC converter made of a **full-bridge single phase diode rectifier** and a **flyback converter** placed in series as shown in Figure 1.

More precisely we will in this first report design the transformer used in this circuit.

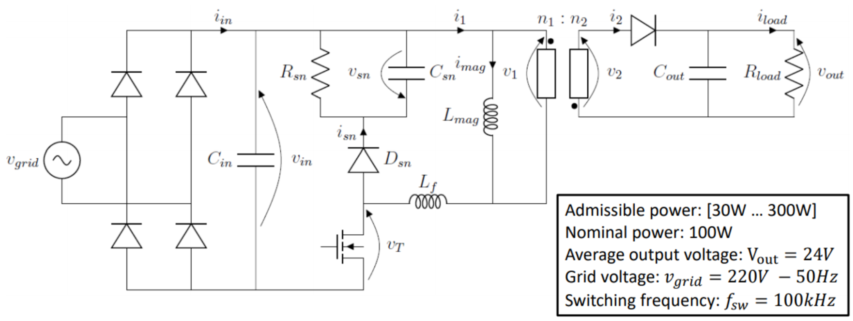


Figure 1: full circuit

2 Quick Circuit Analyse

2.1 Voltage Analysis

We first make some simplification to get the idea how the circuit behave and the links between the averages values. Assuming C_1 is well designed, we can neglect the ripple in vin(t) and consider it is equal to its mean value such that $v_{in}(t) \approx V_{in}$ that we will assume equal to 300[V] (the peak value of $vgrid$ being $\approx 310V$, allowing a ripple on input capacitance of 20V and thus approximately 300V average on voltage of C_{in}). We also consider v_{out} equal to its mean value $V_{out} = 24V$, assuming a capacitance C_{out} large enough. We can thus rewrite the circuit as a simplified circuit as shown in Figure 2 where we have an DC input. Furthermore we will here assume that transistor and diodes are ideals, so we neglect the voltage drop when switched ON.Also we neglect the leakage inductance and thus also neglecting the snubber circuit for the moment.

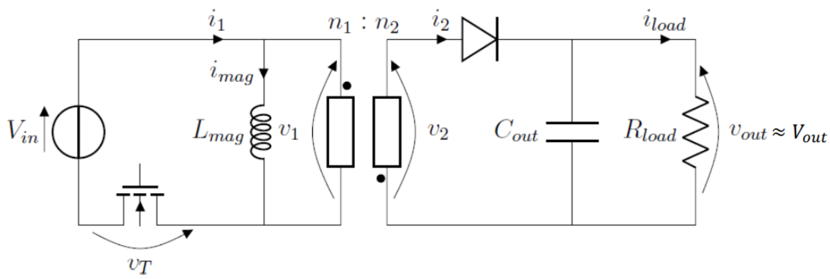


Figure 2: simplified circuit

This circuit has 2 main ways to work:

- When the transistor is passing (during θT each period), the magnetising inductance L_{mag} stores energy and the diode in the secondary is blocking due to the negative voltage to its limits.
- When the transistor is blocking (during $(1-\theta)T$ each period), the diode is conducting and the inductance provide the energy to the secondary.

Assuming we are in steady state with period $T = \frac{1}{f_{sw}}$, it means:

$$i_{mag}(t) = i_{mag}(t+T)$$

<=>

$$i_{mag}(t) - i_{mag}(t+T) = 0 = \frac{1}{L}\int_t^{t+T} v_1 = \frac{1}{L}(\int_t^{t+\theta T} V_{in} + \int_{t+\theta T}^{t+T} -\frac{n1}{n2}V_{out})$$

So the average voltage on L_{mag} must be zero during the period and we can thus, by deriving the above equation and equaling to zero, write:

$$V_{out} = \frac{n_2}{n_1}\frac{\theta}{1-\theta}V_{in} \tag{1}$$

We can observe that in the relation between V_{in} and V_{ou} there is two degree of liberty to adjust: $\frac{n1}{n2}$ winding ration and the duty cycle θ.

2.2 Current Analysis

We have when the transistor conducts :

$$i_{mag} = i_{mag,min} + \frac{V_{in}}{L_{mag}}$$

$$i_1 = i_{mag}$$

$$i_2 = 0$$

When the diode conducts :

$$i_{mag} = i_{mag,min} - \frac{n_1 V_{out}}{n_2 L_{mag}}(t - \theta T_{sw})$$

$$i_1 = 0$$

$$i_2 = \frac{n_1}{n_2} i_{mag}$$

Based on this we find the following expressions for the average values of i_1, i_2 and i_{mag}:

$$i_{mag,avg} = \frac{n_2}{n_1(1-\theta)} i_{load} \tag{2}$$

$$i_{2,avg} = i_{load} \tag{3}$$

$$i_{1,avg} = \theta i_{mag,avg} = \frac{n_2\theta}{n_1(1-\theta)} i_{load} = \frac{V_{out} i_{load}}{V_{in}} \tag{4}$$

The last equation lead us to

$$i_{mag,avg} = \frac{P_{load}}{V_{in}\theta} \tag{5}$$

Concerning the ripple on the magnetising current, we have that :

$$\Delta i_{mag} = i_{mag,max} - i_{mag,min} = i_{mag}(\theta T_{sw}) - i_{mag}(0)$$

$$\Delta i_{mag} = \frac{V_{in}\theta T_{sw}}{L_{mag}} \tag{6}$$

All this reuslts can be seen on the figure 3.

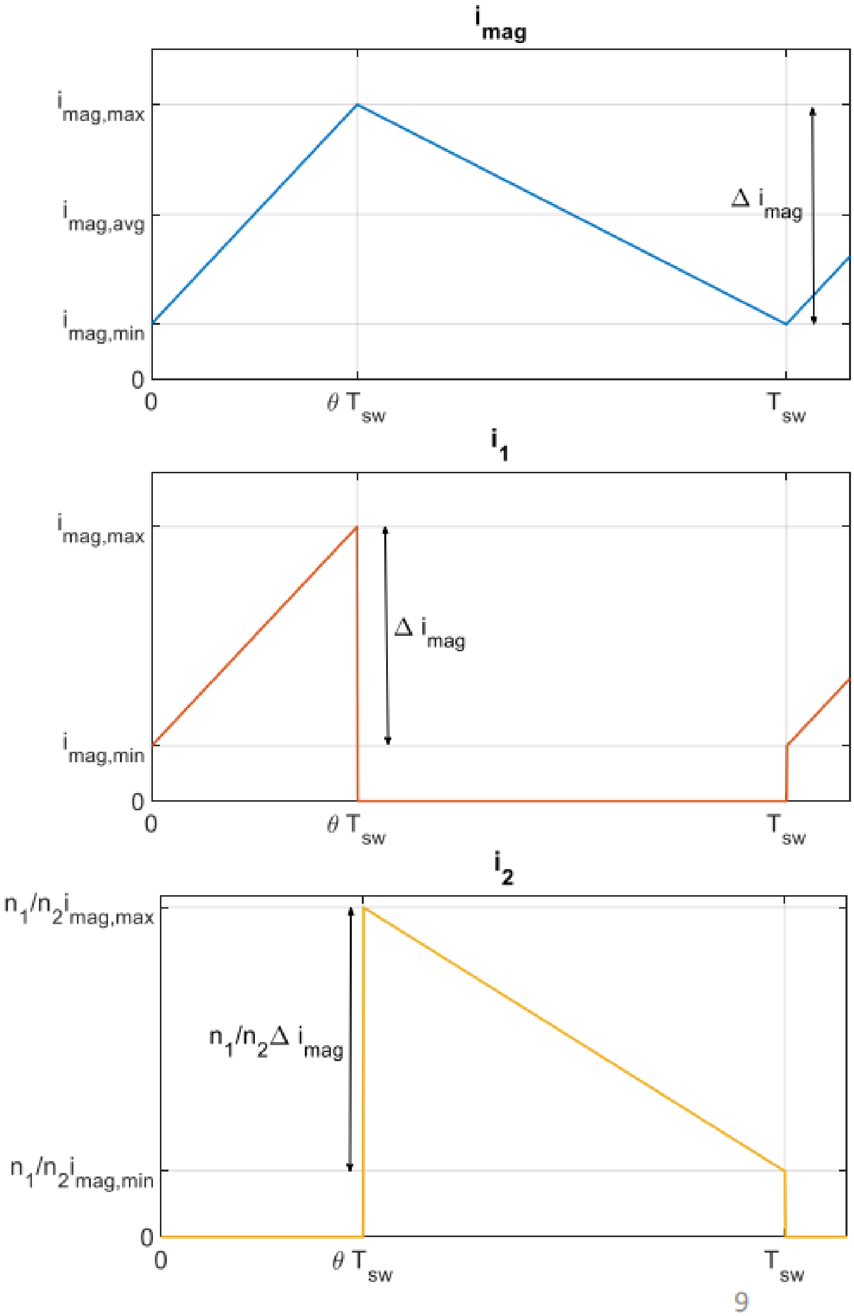


Figure 3: current waveforms

3 Maximum value of θ

First, applying Ampere's law on the magnetic circuit of the transformer and taking into account the magnetising inductance when the transistor does not conduct leads to the following result:

$$n_1(i_{mag} - i_1) = n_2 i_2 \tag{7}$$

In order to understand the effect the maximum value θ_{max}, the capacitance L_f and the snubber circuit have to be taken into account.
Applying basic circuits law, when the transistor stops to conduct there is a current flowing through the leakage inductance L_f, if there was no snubber circuit, in theory the current would decrease instantaneously to zero meaning an infinite negative value of $V_{Lf} = V_{in} + \frac{n_1 V_{out}}{n_2} - v_t$ thus a infinite value of v_t, in practise we get a very short spike of a very high negative V_{Lf} (and thus a very quick decrease of I_{Lf}) and thus a high $v_t > V_{DDS,max}$ that could destroy the transistor. So by adding the snubber circuit, it limits this spike voltage of v_t to $v_{sn} + V_{in}$, the voltage v_{sn} that we approximate to be a constant V_{sn} assuming a large capacitance in the snubber circuit,

thus we have when we open the transistor switch:

$$V_{Lf} = L_f \frac{di_{sn}}{dt} = -V_{sn} + V_1 = -V_{sn} + \frac{n_1 V_{out}}{n_2} < 0$$

as i_1 is decreasing at this time.
It implies that

$$V_{sn} > \frac{n_1 V_{out}}{n_2}$$

But also that

$$V_{sn,max} = \alpha \frac{n_1 V_{out}}{n_2} = \alpha V_{in} \frac{\theta}{1-\theta} \tag{8}$$

by using the previous equations.
The previous introduced α parameter is a security parameter (greater than 1) that we can fix.

With a Kirckoff voltage law we have for the limits voltage values that

$$v_{t,max} = V_{in} + v_{sn,max}$$

That we can rewrite with the equation (8)

$$v_{t,max} = \frac{1 + \theta(\alpha - 1)}{1 - \theta} V_{in} \tag{9}$$

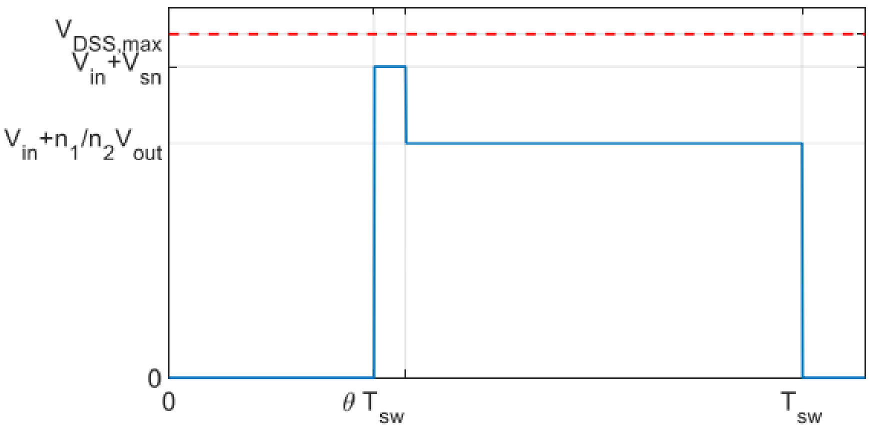


Figure 4: vt(t)

Finally, we introduce a new security factor V_{marg} that permits to ensure a certain safety margin between the maximum operating voltage across the transistor $v_{t,max}$ and it's maximal admissible value $V_{DDS,max}$. Such that

$$v_{t,max} = V_{DDS,max} - V_{marg}$$

by replacing with expression (9) we get

$$\frac{1 + \theta(\alpha - 1)}{1 - \theta} V_{in} = V_{DDS,max} - V_{marg}$$

$$\theta \leq \frac{V_{DDS,max} - V_{marg} - V_{in}}{V_{DDS,max} - V_{marg} + V_{in}(\alpha - 1)} \tag{10}$$

Thus the maximum voltage across the transistor constraint leads to a condition on θ as previously precised.

4 Constraints

A prior study of the circuits and transformer lead us to take some constraints into account when design the transformer.

1. **Winding Area**
The transformer must be shaped such that the wires fit the winding area. That can be mathematicaly expressed as

$$S_1 + S_2 \leq K_f W_a \tag{11}$$

with

- K_f = fill factor that take into account the space between conductors, it is a correction factor inferior to 1 that reduce the total space available because of the losses of the spaces between conductors.
- W_a = winding area.
- S_k = sum of all the sections of the conductors of winding k.

2. **Maximum voltage across the transistor**
The maximum voltage across the transistor constraint leads to a condition on θ (as explained in the next section SECTION) and to a constraint on the rate n_1/n_2.

We get from the previous section that

$$\theta \leq \frac{V_{DDS,max} - V_{marg} - V_{in}}{V_{DDS,max} - V_{marg} + V_{in}(\alpha - 1)} \tag{12}$$

with $V_{DDS,max}$ the maximum voltage that the transistor can support and security factors V_{marg} and α that respectively are fixed to 20[V] and 2 (thoose are values we decide).
By replacing with the value that we found we get $\boxed{\theta_{max} = 0.3548}$ This equation combined with equation (1) and considering that the function $\frac{\theta}{1-\theta}$ is an increasing function from 0 to 1, allow us to write the second condition :

$$\frac{n_1}{n_2} = N \leq \frac{V_{in}\theta_{max}}{V_{out}(1 - \theta_{max})} \tag{13}$$

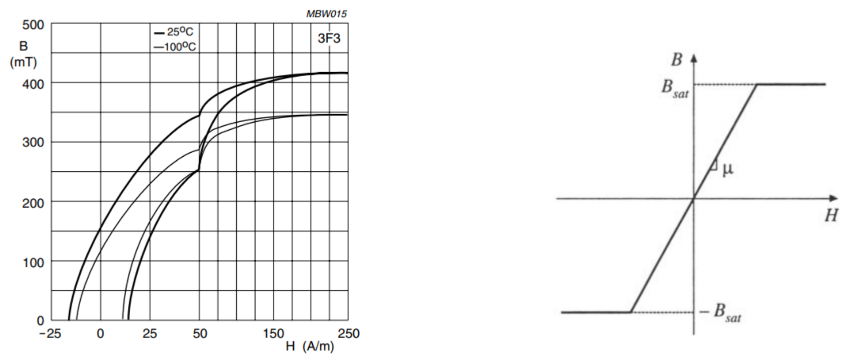
We get $\boxed{N_{max} = 6.875}$

3. **Minimum Magnetising Inductance**
Due to continuous condition : $i_{mag,avg} \geq \frac{\Delta i_{mag}}{2}$ This lead to the following condition on the magnetising inductance:

$$L_{min} = \frac{\theta_{max}^2 V_{in}^2}{2P_{load,min} f_{sw}} \tag{14}$$

As $P_{load,min} = 30[W]$ we get $\boxed{L_{min} = 1.88[mH]}$.

4. **Maximum flux density**
We impose that the maximum flux density must be lower than the saturation level. For this project we will use a ferrite core type 3F3.
We modelise here the relation B(H) in the core as shown in the Figure 6b and the characteristic B-H relation is show in Figure 6a.
As we can see $B_{sat} \approx 300[mT]$ if we take the worst case.

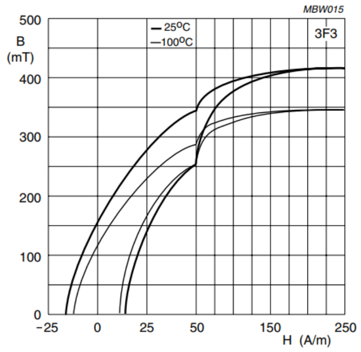


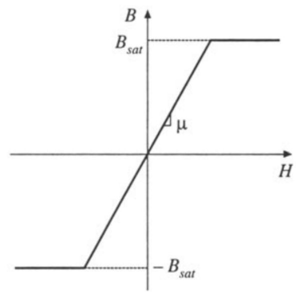
(a) Characteristic B-H curve

(b) Model B-H curve

From the Lenz-Faraday equation : as $emf = \frac{d\Phi}{dt} = n\frac{d\phi}{dt}$ we get

$$\frac{v_1}{n_1} = \frac{-v_2}{n_2} = \frac{d\phi}{dt} = \frac{d(A_{eq} \cdot B)}{dt}$$





(a) Characteristic B-H curve

(b) Model B-H curve

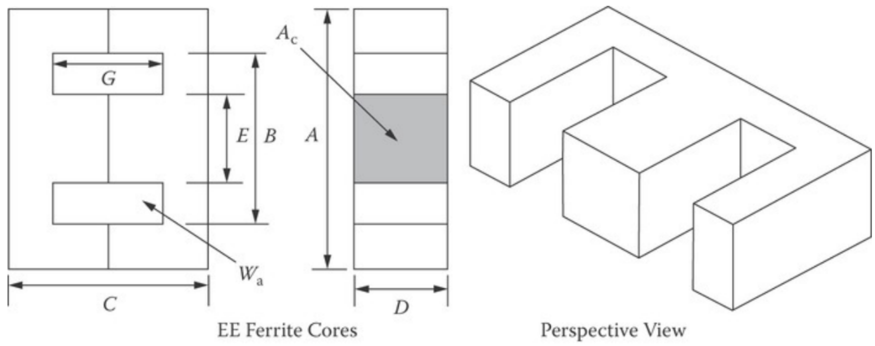


Figure 7: Main shape of the transformer core

with A_eff the area of the core passed by magnetic flux.
The minus sign in front of v_2 comes from the dot convention : As the currents entering the dots create magnetic flux in the same direction, the 2 references of currents defined in figure (1) are of same signs (because they are equivalent referred to the dot) but voltages are of opposite one as the dots are inverted.

We see that for $|H| > \frac{B_{sat}}{\mu}$,increasing the current no longer increase B so $\frac{dB}{dt} = 0$ so by Faraday's law of induction v1 and v2 will also be zero.
So we have to avoid this saturation zone and have $B \leq B_{max}$ We then chose a 25% safety to fix the maximum reachable magnetic field for our design :

$$\boxed{B_{max} = 225[mT]}$$

5 Core parameters

The shape of the core are as shown in the Figure 7.

To calculate the magnetic properties of the soft magnetic core, an equivalent core model is used. The following parameters of this model are given in the datasheet: the equivalent area A_e, the equivalent length (perimeter the center of the ring) L_e and the equivalent volume V_e as shown on in the Figure 8.

6 Computation of L_{mag} based on datasheet

To calculate l_{mag} we use the equivalent ring model, we extracte from the datasheet A_e, L_e and V_e. By applying two coils on this ring with the same convention as in Figure 1 and using the From the Lenz-Faraday equation : as $emf = \frac{d\Phi}{dt} = n\frac{d\phi}{dt}$ we get

$$\frac{v_1}{n_1} = \frac{-v_2}{n_2} = \frac{d\phi}{dt} = \frac{d(A_{eq} \cdot B)}{dt}$$

where we have the magnetic flux $\phi = B * A_e$.
The minus sign in front of v_2 comes from the dot convention used in Figure 1: As the currents entering the dots create magnetic flux in the same direction, the 2 voltages induced with respect to the dots are equal but

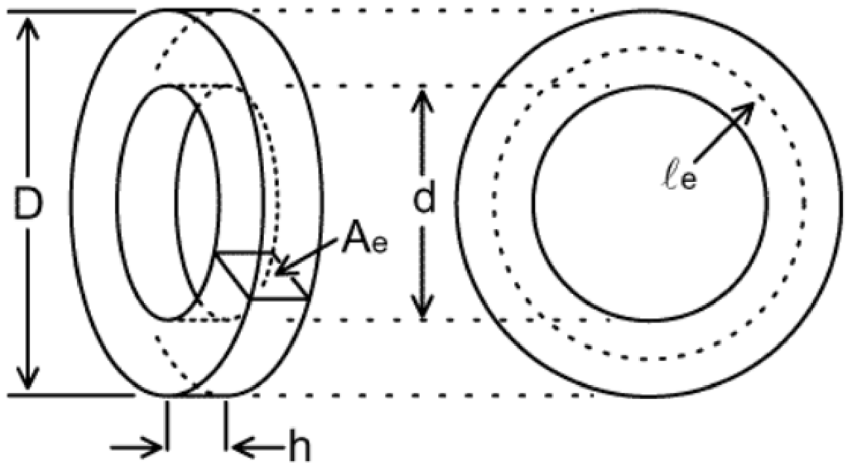


Figure 8: Main shape of the transformer core

as as the conventions voltages used are of opposite side with respect of the dots there is a minus. Now by using Ampere's law to the circle of the center of the ring, we have $\int_\Gamma \vec{H} \cdot \vec{dl} = i_{int}$, assuming the field is constant and concentrated into the core we get : $\int_\Gamma \vec{H} \cdot \vec{dl} = H \cdot l_e$ Furthermore we get

$$i_{int} = n_1 * i_1 + n_2 * i_2$$

By defining i_{mag} so that $n_1 * i_{mag} = n_1 * i_1 + n_2 * i_2$ we get $i_{mag} = i_1 + \frac{n_2}{n_1} * i_2$. So by having $B < B_{max}$ we have in the core $B = \mu * H$ and thus:

$$v_1 = n_1 * A_e * \mu * \frac{d(H)}{dt} = \frac{n_1 * A_e * \mu}{l_e}\frac{di_{int}}{dt} = \frac{n_1 * A_e * \mu}{l_e}\frac{d(n_1 i_1 + n_2 i_2)}{dt} = \underbrace{\frac{A_e \cdot \mu \cdot n_1^2}{l_e}}_{L_{mag}}\frac{d(i_{mag})}{dt}$$

So $L_{mag} = \frac{A_e \cdot \mu \cdot n_1^2}{l_e}$

7 Effect of air gap on L_{mag}

Now if we consider that we remove a part a small portion of the ring and thus having a small air gap of length l_g, by applying Ampere's law we have this time:

$$H_e * (l_e - l_g) + H_g * l_g = n_1 * i_{mag}$$

where H_e and H_g are respectively the magnetic field intensity in the core and in the air gap, as in both area the magnetic flux intensity B is the same and that $H_e = \frac{B}{\mu}$ and $H_g = \frac{B}{\mu_0}$, we obtain:

$$B = \frac{n_1 * i_{mag}}{\frac{l_e - l_g}{\mu} + \frac{l_g}{\mu_0}}$$

So

$$L_{mag} = \frac{A_e \cdot n_1^2}{\frac{l_e - l_g}{\mu} + \frac{l_g}{\mu_0}}$$

We can simply furthermore considering $\mu >> \mu_0$

$$L_{mag} \approx \frac{A_e \cdot \cdot n_1^2}{\frac{l_g}{\mu_0}} = \frac{A_e \cdot \mu_0 \cdot n_1^2}{l_g} \tag{15}$$

and

$$B \approx \frac{\mu_0 \cdot n_1 \cdot i_{mag}}{l_g} < \frac{\mu \cdot n_1 \cdot i_{mag}}{l_e} \text{ (the value without air gap)} \tag{16}$$

So the airgap reduce the inductance and meaning it is possible to have a higher current before the core starts to saturate so having an airgap is an advantage regarding the constraint previously explained $B \leq B_{max} \approx 240[mT]$.

8 final choice and design of transformer

So now we have to choose a transformer and make a design where we choose two parameters, the value of the duty cycle θ and the value of n1 (the value of n2 is directly depending on value of θ and n1, equation 1 can be used to obtain $\frac{n2}{n1}$ in function of θ and then using n1 we have the value of n2). Let's recap the different constraint in terms of equations:

1 $\theta \leq \theta_{max}$ which induces $N = \frac{n1}{n2} \leq N_{max}$

2 $\underbrace{L_{mag}}_{=\frac{A_e \cdot \mu \cdot (n1)^2}{l}} \geq L_{min} = \frac{\theta_{max}^2 V_{in}^2}{2 \cdot P_{load,min} f_{sw}}$

3 $B_{peak} = \frac{\mu \cdot n1}{l} \cdot i_{mag,peak} \leq B_{max} \iff i_{mag,peak} \leq \underbrace{\frac{l}{\mu \cdot n1} B_{max}}_{i_{mag,peak,max}}$

4 $S_1 + S_2 \leq Kf \cdot W_{air}$ with $Kf = 0.5$

In the above formulas if we take a transformer without air gap l=le and $\mu = \mu_r \cdot \mu_0$ and with airgap l=l_g and $\mu = \mu_0$.

So now we have to look for each possible transformer if there is a possible set of n1,θ that satisfy the above equation-constraints. As those equations depend on the length of the air-gap, in the datasheet there is for each transformer several possibles air-gap length available. First we will test if it possible to select one transformer with no air-gap. As in the number 3 constraint $i_{mag,peak,max}$ is inversely proportional to n1, the lower n1 is, the higher we can have a $i_{mag,peak}$, and by deriving the constraint number 2 we obtain:

$$n1 \geq \sqrt{\frac{l_e \cdot \theta_{max}^2 V_{in}^2}{A_e \cdot \mu \cdot 2 \cdot P_{load,min} \cdot f_{sw}}} = n1_{min} \tag{17}$$

Furthermore we also can express $i_{mag,peak}$ in function of θ as:

$$i_{mag,peak} = i_{mag,avg} + \frac{\Delta i_{mag}}{2} = \frac{P_{load,max}}{\theta \cdot V_{in}} + \frac{V_{in} \cdot \theta \cdot T}{2 \cdot L_{mag}} = \frac{P_{load,max}}{\theta \cdot V_{in}} + \frac{V_{in} \cdot \theta \cdot T \cdot l_e}{2 \cdot A_e \cdot \mu \cdot (n1)^2} \tag{18}$$

In this formula we take $P_{load} = P_{load,max}$ to have the highest case value of $i_{mag,peak}$ regarding the load. So now we have to check for $\theta \in [0, \theta_{max}]$ if it is possible to have $i_{mag,peak} \leq i_{mag,peak,max}$ by taking $n1 = n1_{min}$. To do so we will plot $i_{mag,peak}$ in function of θ and see if it goes below $i_{mag,peak,max}$.
First for each transformer without we calculate $i_{mag,peak,max}$ using the values extracted in the datasheets and the equation 17, we obtain the following values:

Transformers	le	Ae	$n1_{min}$	$i_{mag,peak,max}$
E30/15/7	67 mm	60 mm^2	29	207mA
E32/16/9	74 mm	83 mm^2	26	255mA
E42/21/15	97 mm	178 mm^2	21	414mA
E42/21/20	97 mm	233 mm^2	18	482mA
E55/28/21	124 mm	353 mm^2	17	653mA

Table 1: calculation of $i_{mag,peak,max}$ without air-gap for each transformer

In the above Table 1 the first and second column are filled using the datasheet for each type of transformer, the $n1_{min}$ column is calculated using the equation (17) and by taking $\mu = \mu_r * \mu_0$, the value $\mu_r = 2000$ is found in the datasheet 3F3 ferroxcube spec. The last column gives the $i_{mag,peak,max}$ calculated given the equation of constraint number 3 with $l = le$ and by taking $n1 = n1_{min}$ calculated just before.
Now for each transformer we plot the function which gives $i_{mag,peak}$ in function of θ over the interval $[0, \theta_{max}]$, this function is given in equation (18) and we check if it goes below the corresponding $i_{mag,peak,max}$.

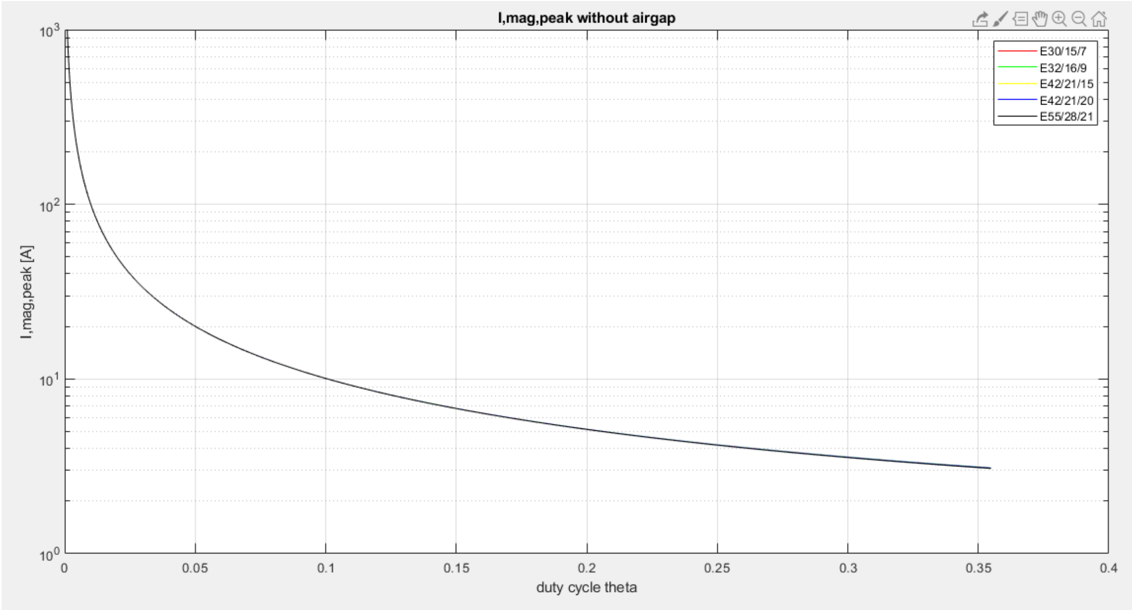


Figure 9: $\imath_{mag,peak}$ without air-gap

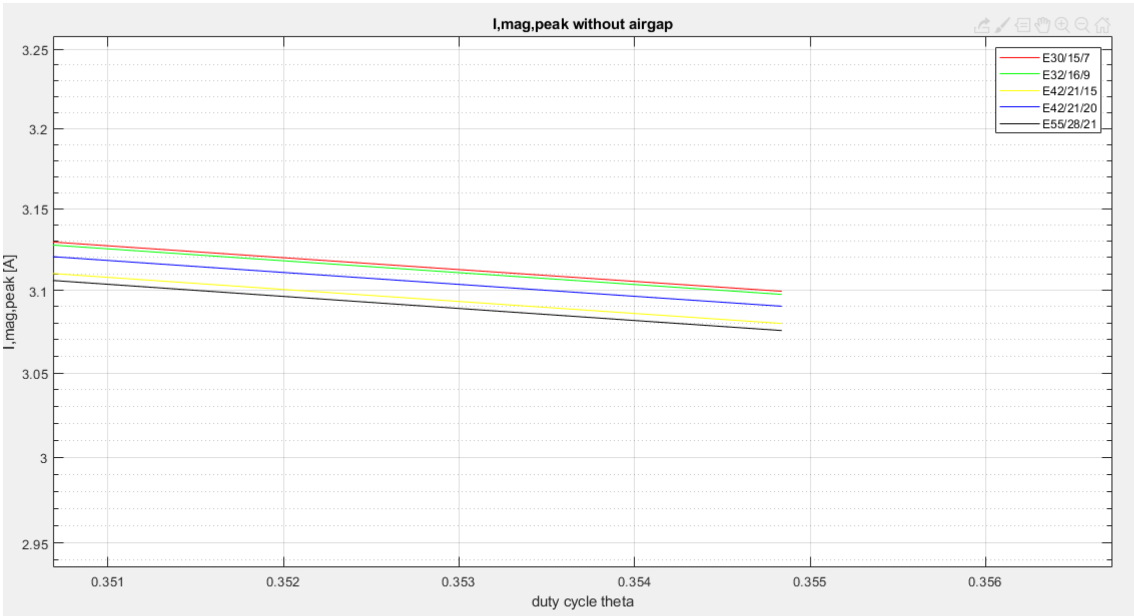


Figure 10: $i_{mag,peak}$ without air-gap zoom

By plotting each $i_{mag,peak}$ we clearly see that for all transformer without air-gap the current is too high (on the curves the minimums are of order 3 A whereas the maximums in the table are lower than 1 A) meaning the core will saturate.

So we have to select a transformer with an air-gap. for each transformer we extract the air-gap lengths availables and for each case we do a similar procedure but now, according to simplifications of equation in equation (15) and (16), we replace le by lg and μ by μ_0 in the formula used to compute. For transformer E30/15/7, we have:

	lg	Ae	$n1_{min}$	$i_{mag,peak,max}$
E30/15/7	100 μm	60 mm^2	51	$351mA$
	180 μm	60 mm^2	68	$474mA$
	240 μm	60 mm^2	78	$551mA$
	330 μm	60 mm^2	91	$649mA$
	580 μm	60 mm^2	121	$858mA$
	1100 μm	60 mm^2	166	$1186mA$

Table 2: calculation of $i_{mag,peak,max}$ with air-gap for E30/15/7

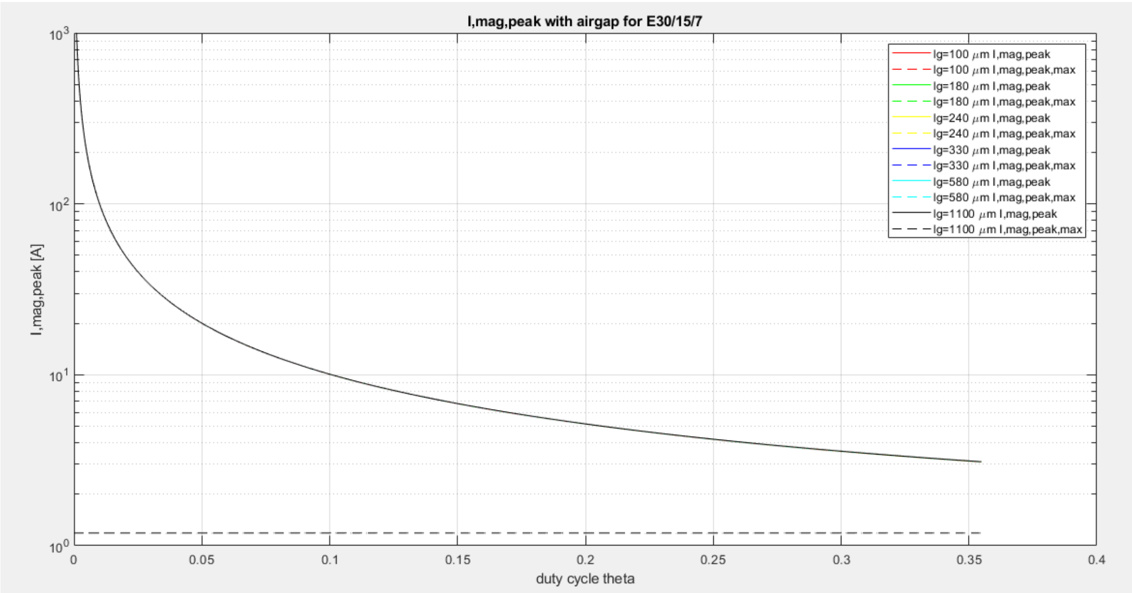


Figure 11: $\imath_{mag,peak}$ with air-gap for E30/15/7

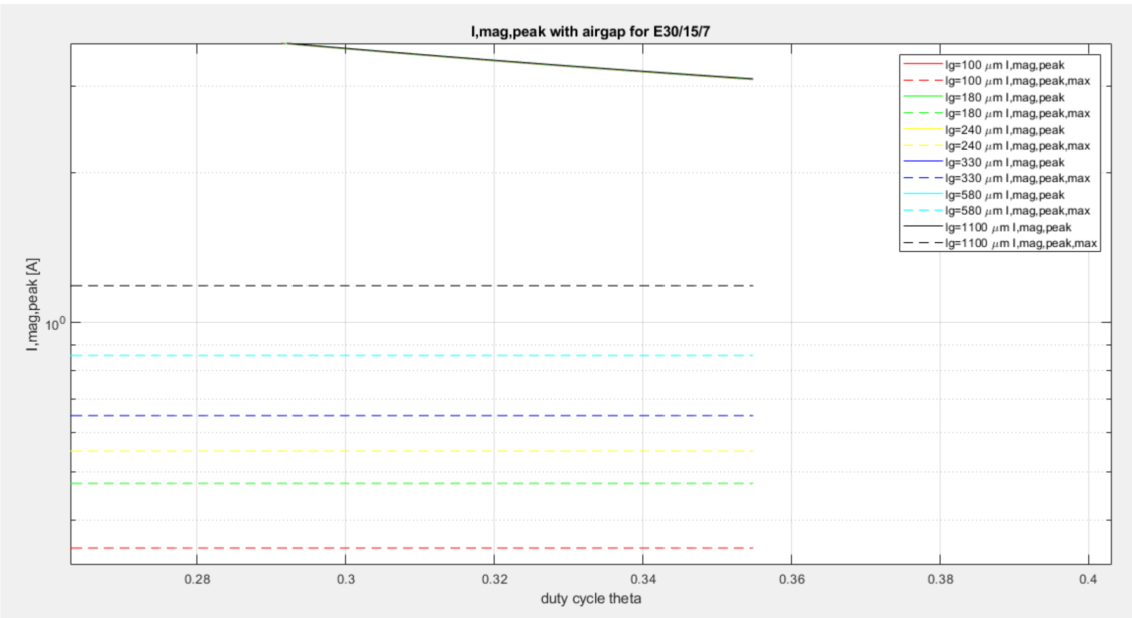


Figure 12: $i_{mag,peak}$ with air-gap for E30/15/7 zoom

We observe that there is still no possible θ to get a current under the maximum, so we eliminate E30/15/7 (the $i_{mag,peak}$ curves are very close to each other, they are almost identical as the black one and appear if we would zoom more and specifically on those curves).

Now we do the same for E32/16/9, we have:

	lg	Ae	$n1_{min}$	$i_{mag,peak,max}$
E32/16/9	150 μm	83 mm^2	53	$0.507A$
	260 μm	83 mm^2	69	$0.675A$
	360 μm	83 mm^2	81	$0.796A$
	480 μm	83 mm^2	94	$0.914A$
	860 μm	83 mm^2	125	$1.232A$
	1600 μm	83 mm^2	171	$1.675A$

Table 3: calculation of $i_{mag,peak,max}$ with air-gap for E32/16/9

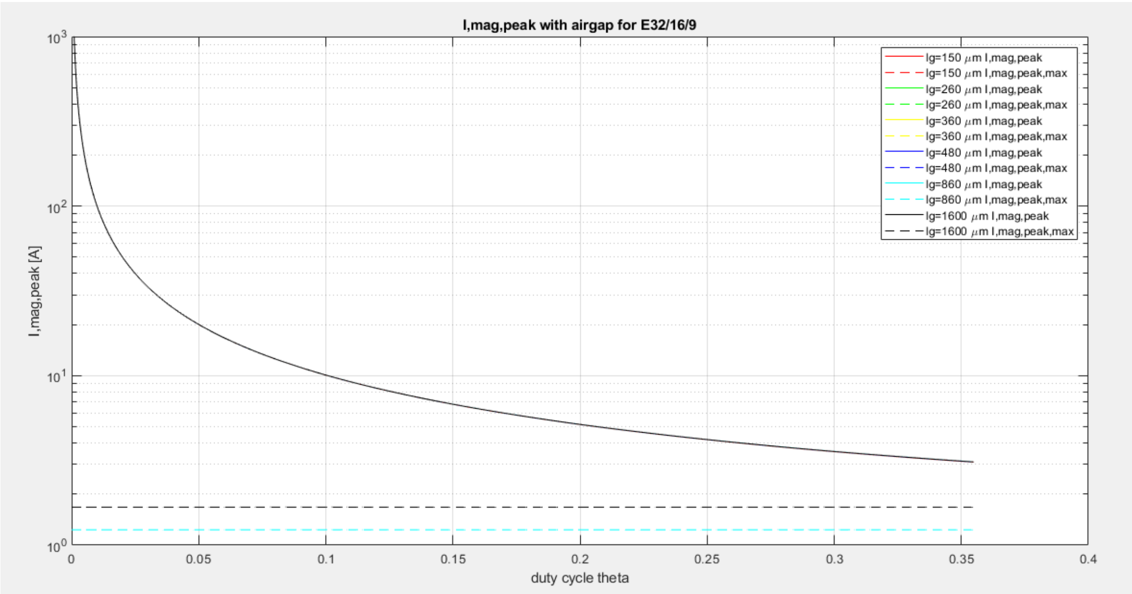


Figure 13: $\imath_{mag,peak}$ with air-gap for E32/16/9

The conclusions is similar as previously, we cannot select E32/16/9 as possible solution. Now we move on E42/21/15, we have:

	lg	Ae	$n1_{min}$	$i_{mag,peak,max}$
E42/21/15	360 μm	178 mm^2	56	$1.151A$
	630 μm	178 mm^2	73	$1.545A$
	850 μm	178 mm^2	85	$1.790A$
	1140 μm	178 mm^2	99	$2.062A$
	2060 μm	178 mm^2	132	$2.794A$
	3960 μm	178 mm^2	183	$3.875A$

Table 4: calculation of $i_{mag,peak,max}$ with air-gap for E42/21/15

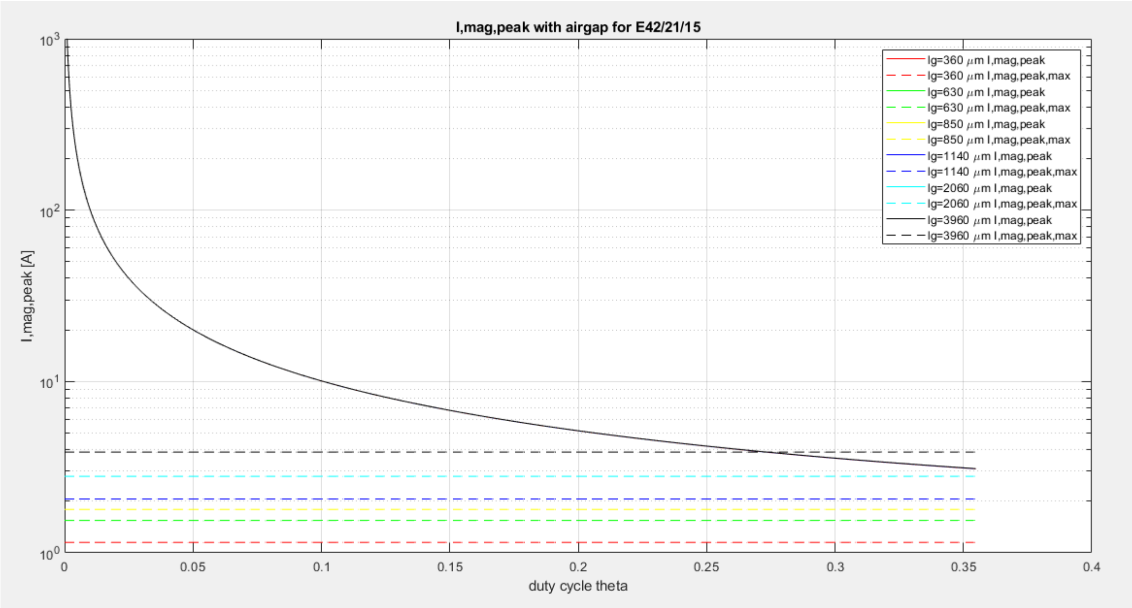


Figure 14: $\imath_{mag,peak}$ with air-gap for E42/21/15

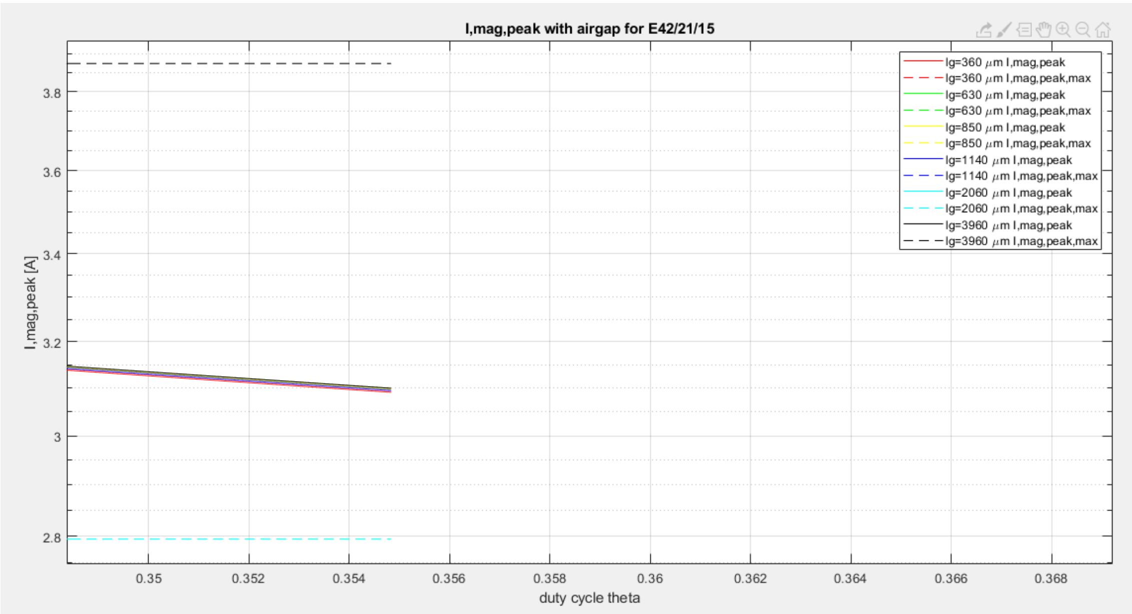


Figure 15: $\imath_{mag,peak}$ with air-gap for E42/21/15 zoom

Now we can see that for $lg = 3960\mu m$ it is possible to have a $i_{mag,peak} < i_{mag,peak,max}$ so this is a candidate for a solution.
Let's now see E42/21/20:

	lg	Ae	$n1_{min}$	$i_{mag,peak,max}$
E42/21/20	490 μm	233 mm^2	57	$1.539A$
	850 μm	233 mm^2	75	$2.029A$
	1160 μm	233 mm^2	87	$2.387A$
	1540 μm	233 mm^2	100	$2.757A$
	2800 μm	233 mm^2	135	$3.714A$
	5320 μm	233 mm^2	186	$5.121A$

Table 5: calculation of $i_{mag,peak,max}$ with air-gap for E42/21/20

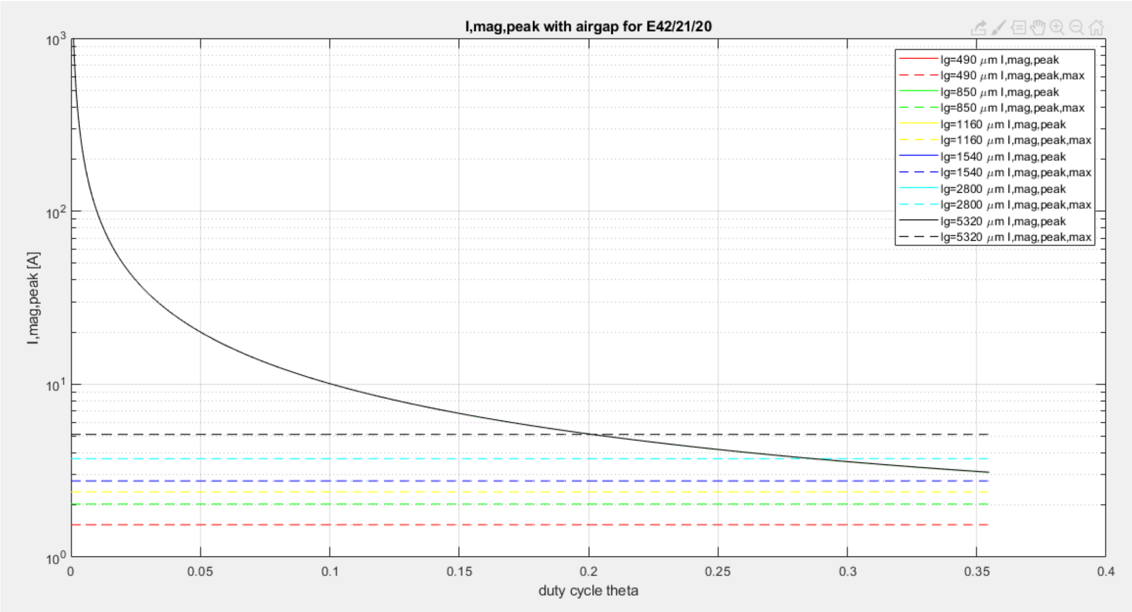


Figure 16: $\imath_{mag,peak}$ with air-gap for E42/21/20

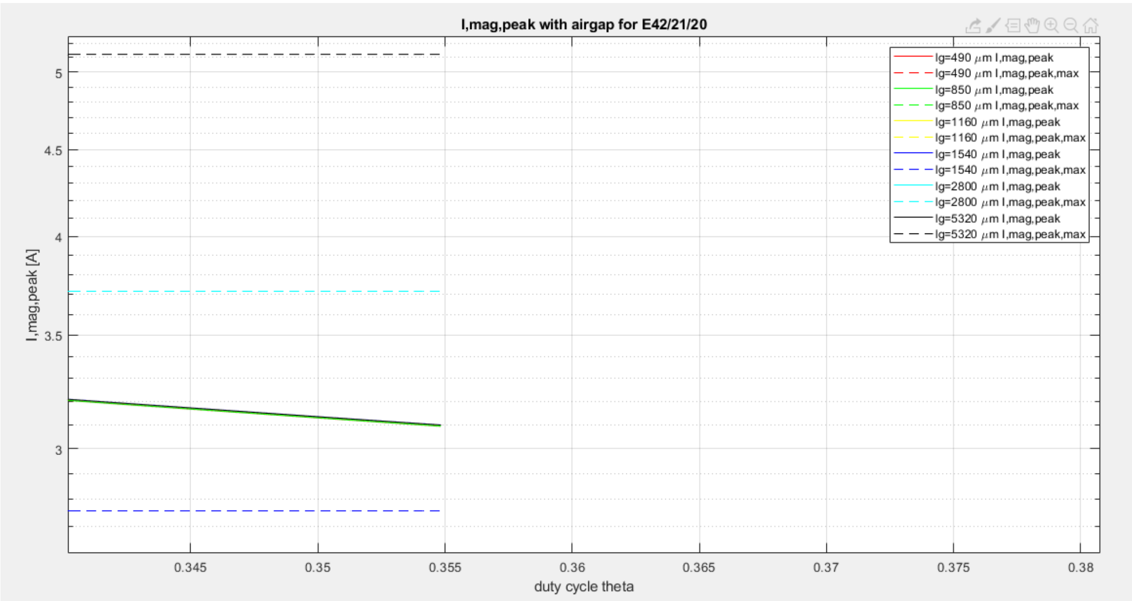


Figure 17: $\imath_{mag,peak}$ with air-gap for E42/21/20 zoom

Now there is two candidates solutions that we keep, for $lg = 2800\mu m$ and for $lg = 5320\mu m$. Finally we check the last one E55/28/21:

	lg	Ae	$n1_{min}$	$i_{mag,peak,max}$
E55/28/21	780 μm	353 mm^2	58	$2.408A$
	1360 μm	353 mm^2	77	$3.162A$
	1840 μm	353 mm^2	89	$3.702A$
	2500 μm	353 mm^2	104	$4.304A$
	4560 μm	353 mm^2	140	$5.832A$
	8740 μm	353 mm^2	193	$8.108A$

Table 6: calculation of $i_{mag,peak,max}$ with air-gap for E42/21/20

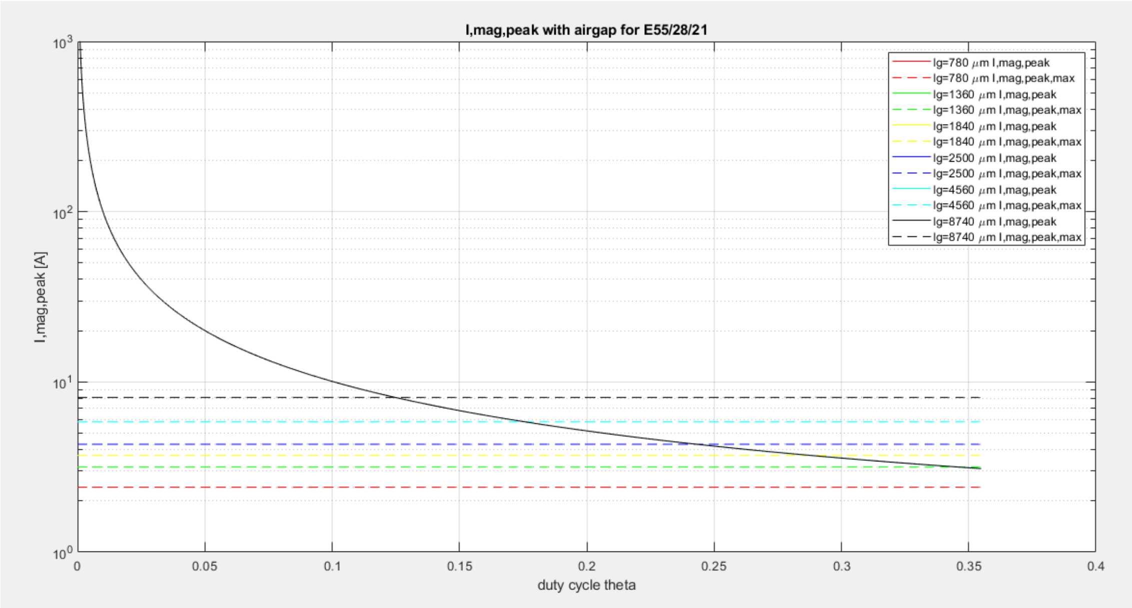


Figure 18: $\imath_{mag,peak}$ with air-gap for E55/28/21

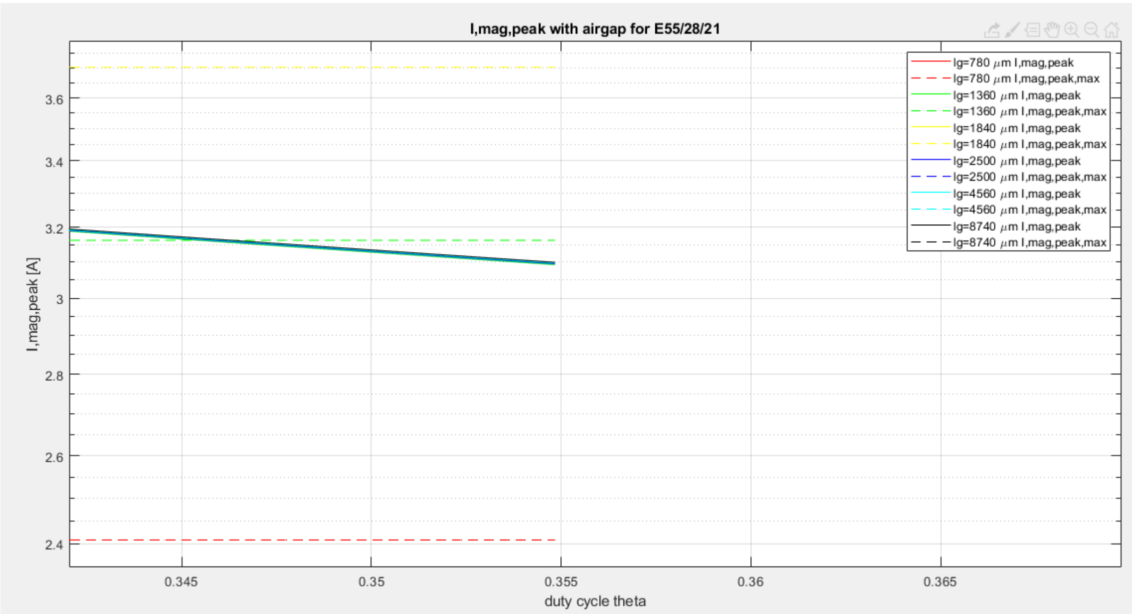


Figure 19: $\imath_{mag,peak}$ with air-gap for E55/28/21 zoom

Here all are candidate except $lg = 780\mu m$, $lg = 1360\mu m$ is limit but as we already have taken a margin for B_{max} we can still keep it as a candidate.
Now to decide among them we will use the constraint number 4. For this we need the value of:

$$i_{1,RMS} = \sqrt{\theta i_{mag,avg}^2 + \Delta i_{mag}^2 \frac{\theta}{12}} \tag{19}$$

$$i_{2,RMS} = \frac{n1}{n2}\sqrt{(1-\theta) i_{mag,avg}^2 + \Delta i_{mag}^2 \frac{(1-\theta)}{12}} \tag{20}$$

Obtained by using the definition of I_{RMS}. Now we see that we need $i_{mag,avg}$ and Δi_{mag} which are given in equation (18) (in case of airgap we take lg and μ_0), by taking $P_{load,max}$ we will obtain $i_{1,RMS,max}$ and $i_{2,RMS,max}$. So we have to choose θ and n1 to get thoose values for every candidate we have kept. θ has to be between $\theta_m in$ (the value for $theta$ at the intersection with the curve $i_m ag_p eak$ and the corresponding $i_m ag_p eak_m ax$ in the previous plots) and $\theta_m ax$, we decide to choose a value close to θ_{max} to make sure we are far away from constraint number 3 (we are then closer to the constraint of the voltage on v_t but we still have the margin of 20V). We can then use equation (1) to obtain $\frac{n1}{n2}$ and by keeping $n1 = n1_{min}$ we can obtain n2, indeed by taking $theta_max$ we get$\frac{n1}{n2} = N_max$ so $n2 = \frac{n1}{N_{max}}$, it has to be a constant so by round it the upper integer to make sure we still have $N \leq Nmax$ we obtain n2 and by utilizing again equation (1) we get the chosen θ). With this chosen n1 and $theta$ we can calculate L_mag using equation (15), $\imath_{mag,avg,max}$ and $\Delta imag$ using equation (18). So by doing all of this for every candidate we obtain:

Transformer	lg	n1	n2	θ choosen	L_{mag}	$i_{mag,avg,max}$	Δi_{mag}	$i_{1,RMS,max}$	$i_{2,RMS,max}$
E42/21/15	3960 μm	183	58	0.3516	1.89 mH	$2.844A$	$558mA$	$1.689A$	$15.548A$
E42/21/20	2800 μm	135	20	0.35065	1.9 mH	2.85 A	0.552 A	1.69 A	15.54 A
	5320 μm	186	28	0.347	1.9 mH	2.88 A	0.547 A	1.7 A	15.49 A
E55/28/21	1360 μm	77	12	0.3392	1.93 mH	2.95 A	0.526 A	1.719 A	15.4 A
	1840 μm	89	13	0.3539	1.9 mH	2.83 A	0.556 A	1.684 A	15.576 A
	2500 μm	104	16	0.3421	1.92 mH	2.92 A	0.535 A	1.712 A	15.433 A
	4560 μm	140	21	0.3478	1.9 mH	2.88 A	0.547 A	1.698	15.502 A
	8740 μm	193	29	0.3474	1.89 mH	2.88 A	0.551 A	1.7 A	15.497 A

Table 7: I,RMS calcultations

Now that we have the RMS currents for each candidate, we can calculate an $W_f = \frac{W_{air}}{S_1+S_2}$ using:

$$S_1 = n1\frac{i_{1,rms}}{J}\text{with J=}5A/mm^2 \tag{21}$$

and

$$S_2 = n2\frac{i_{2,rms}}{J} \tag{22}$$

To be acceptable W_f needs to be lower than 0.5 (S1 and S2 represent area of windings but they cannot take all the area available as the is losses spaces betweens wires). We have (getting W_{air} from the datasheets of each transformer):

Transformer	lg	$S_1 + S_2$ (mm^2)	W_{air} (mm^2)	W_f
E42/21/15	3960 μm	145.8	178 or 180	0.82 and 0.81
E42/21/20	2800 μm	107.8	173 or 255	0.62 and 0.42
	5320 μm	150	173 or 255	0.87 and 0.59
E55/28/21	1360 μm	63.4	250 or 277 or 278	0.25 and 0.23
	1840 μm	70,5	250 or 277 or 278	0.28 and 0.25
	2500 μm	85	250 or 277 or 278	0.34 and 0.31
	4560 μm	112.7	250 or 277 or 278	0.45 and 0.41
	8740 μm	155.5	250 or 277 or 278	0.62 and 0.58

Table 8: last constraint decision

So after eliminating those who have $W_f \geq 0.5$, it seems better to have a value close to 0.5 has a too low W_f would mean a lot of empty space and meaning the transformer would be overdimensioned. So two candidates remains: E42/21/20 with lg=2800 μm and $W_{air} = 255mm^2$ and E55/28/21 with lg=4560μm and $W_{air} = 250mm^2$. We decide to choose the first one who has overall lower dimensions so it would require a lower volume for the circuit.