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LELEC2910 - Antennas and Propagation

Project: Radiation Pattern computing using FFT

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1 Objectives

The main goal of this project is to compute the radiation pattern of a metasurface antenna by using Fast-Fourier Transform algorithm. This antenna is designed to radiate a broadside pencil beam at a frequency $f = 17[GHz]$.
Based on the vectorial tangential electric field at the surface of the antenna, the objective is to compute the surface current distribution first. Then, apply the Fast-Fourier Transform on the current provides the radiation pattern. Finally, the Co-Polar and Cross-Polar radiation pattern can be obtained for a fixed elevation or azimuth angle.

2 Notation

The following equations and results have been obtained using this coordinates systems and these notations.

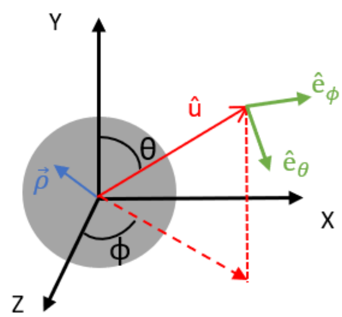

Figure 1: Coordinates system and notations

k	Wave number
$\hat{u}$	Direction of propagation
$\vec{k}$	Wave vector of the radiated field in direction û
$\hat{n}$	Normal to the antenna surface
$\vec{\rho}$	Position vector in the antenna surface
θ	Elevation angle
ϕ	Azimuth angle
λ	Wavelength $(0.017[m])$
$\vec{K}$	Electric current
$\vec{K_m}$	Magnetic current
Δ_x	Sampling period along X-axis $(0.0013[m])$
Δ_y	Sampling period along Y-axis $(0.0013[m])$
η	Free space impedance

Table 1: : Notations

3 Theoritical analysis

The expression of the radiation pattern for a given direction $\hat{u}$ is given by:

$$\vec{F}(\hat{u}) = \frac{-j\eta}{2\lambda} \int\int_S \left[\vec{K}(\vec{\rho}) - \hat{u}\vec{K}(\vec{\rho}) \cdot \hat{u} - \frac{1}{\eta}\hat{u} \times \vec{K_m}(\vec{\rho})\right] e^{jk\hat{u}\cdot\vec{\rho}} dS \tag{1}$$

Applying the equivalence principle the radiation pattern can also be expressed only in terms of the magnetic current $\vec{K_m}$.

$$\vec{F}(\hat{u}) = \frac{-j}{\lambda}\left[\hat{u} \times \hat{n} \times \vec{f_t}\right] \tag{2}$$

$\vec{f_t}$ denotes the spatial fourier transform of the vectorial tangential electric field $\vec{E_t}$.

$$\vec{f_t}(\vec{k}) = \int\int_S \vec{E_t}(\vec{k}) e^{j\vec{k}\cdot\vec{\rho}} dS \tag{3}$$

Equation 2 can be re-expressed in direct relation with the magnetic current.

$$\vec{K_m} = -\hat{n} \times \vec{E_t} \tag{4}$$

$$\vec{F}(\hat{u}) = \frac{j}{\lambda}\left[\hat{u} \times \vec{h_t}\right] \tag{5}$$

$$\vec{h_t}(\vec{k}) = \int\int_S \vec{K_m}(\vec{k}) e^{j\vec{k}\cdot\vec{\rho}} dS \tag{6}$$

Now an analogy with the Fast Fourier Transform algorithm can be obtained. Let first consider the 1-dimensional case and recall the expression of the Discrete Inverse Fourier Transform.

$$h_t(k_x) = \int_X K_m(\rho_x) e^{jku_x\rho_x} d\rho_x \tag{7}$$

$$x[n] = \frac{1}{N} \sum_{p=0}^{N-1} X[p] e^{j\frac{np2\pi}{N}} \tag{8}$$

By the sampling theorem $u_x = \frac{k_x}{k}$ belongs to the interval $\left[\frac{-\lambda}{\Delta_x}, \frac{\lambda}{\Delta_x}\right]$ where k_x can be interpreted as 2π multiplying the spatial frequency.
Based on this, the discrete expression of u_x is given by:

$$u_x(p) = \frac{\lambda}{\Delta_x}\left(\frac{-1}{2} + \frac{p}{N}\right) \tag{9}$$

The spatial position can also be expressed in a discrete way.

$$\rho_x(n) = n\Delta_x \tag{10}$$

By injecting (9) and (10) in (7) and replacing the integration by a summation,

$$h_t(k_x(p)) = \sum_{n=0}^{N-1} K_m(\rho_x(n)) e^{-j\pi n + j\frac{2\pi np}{N}} \tag{11}$$

By analogy with the Inverse Discrete Fourier Transform,

$$h_t(p) = N\, IDFT\left(K_m(n)(-1)^n\right) \tag{12}$$

Finally, by extension to the initial two-dimensional case, the radiation pattern can be obtained by applying the Fast-Fourier Transform algorithm on the magnetic surface current distribution.

$$\vec{h_t}(n,m) = N_x N_y\, IDFT2\left(\vec{K_m}(n,m)(-1)^{n+m}\right) \tag{13}$$

4 Results and discussion

4.1 Magnetic surface current

Here is a representation of the X and Y-components of the magnetic surface current.

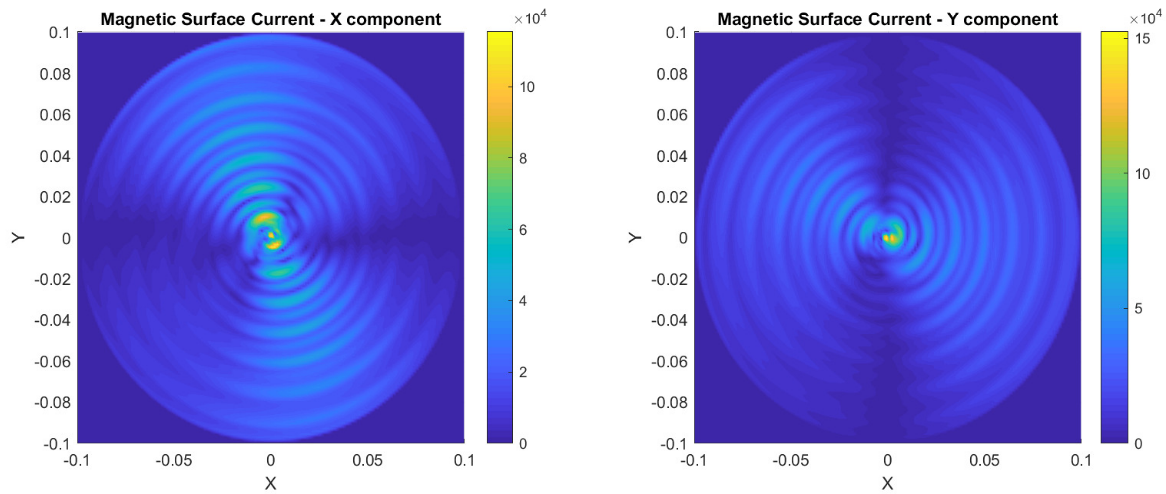

Figure 2: X and Y-components of $\vec{k_m}$

4.2 Radiation pattern

The following figure shows the amplitude of the normalized radiation pattern for a given direction $\hat{u} = (u_x, u_y)$.

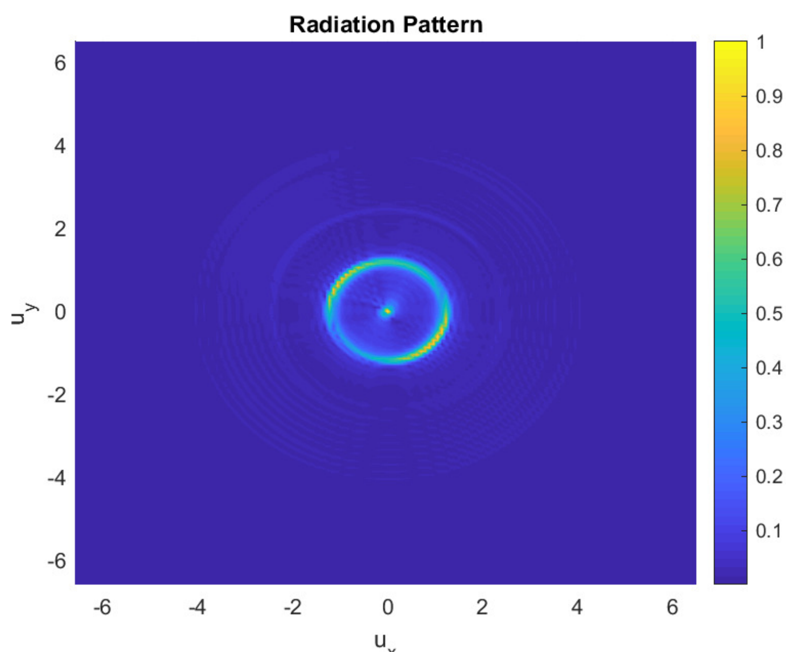

Figure 3: Radiation pattern

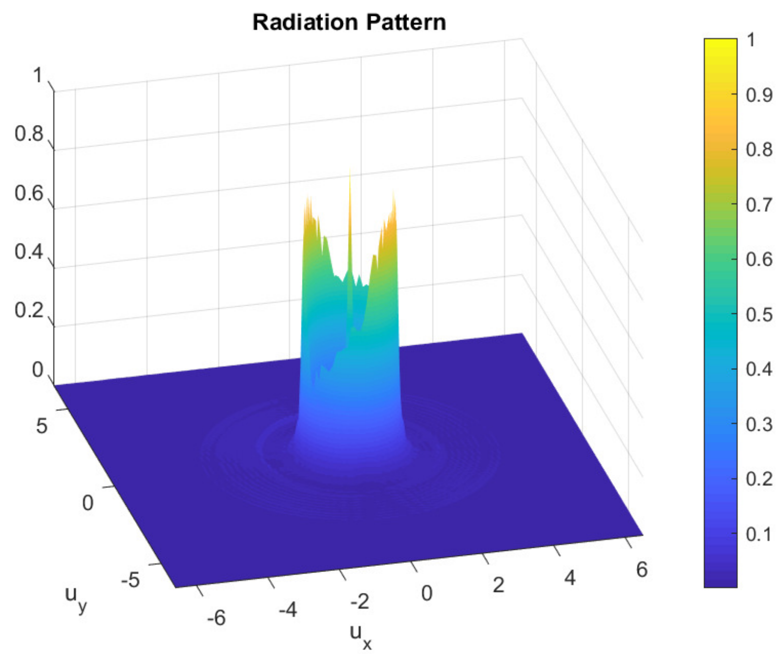

Figure 4: Radiation pattern

Both u_x and u_y takes values greater than one. This comes from the equation (9) and the fact that $\Delta_x < \lambda$. Hence, the radiation pattern is also provided in the invisible domain. In order to express the co-polar and cross-polar coordinates as function of elevation or azimuth angle, it is important to only focus on the visible region. This domain is bounded by this inequality:

$$k_x^2 + k_y^2 \leq 1 \tag{14}$$

Here is the radiation pattern in the visible region.

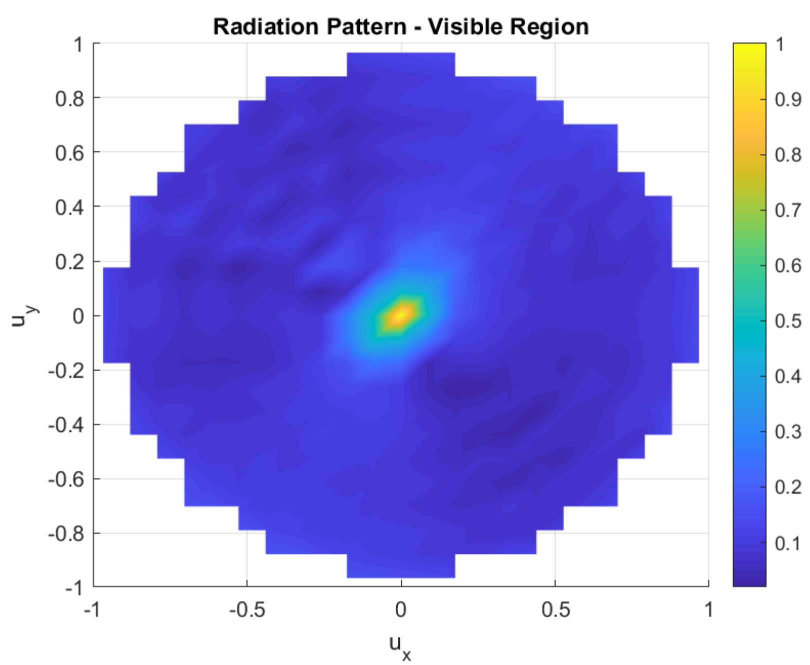

Figure 5: Radiation pattern in visible region

4.3 Co-Polar and Cross-Polar radiation pattern

The Co-Polar radiation pattern is given by:

$$\vec{F_\theta}(\hat{u}) + j\vec{F_\phi}(\hat{u}) \tag{15}$$

and the Cross-Polar:

$$\vec{F_\theta}(\hat{u}) - j\vec{F_\phi}(\hat{u}) \tag{16}$$

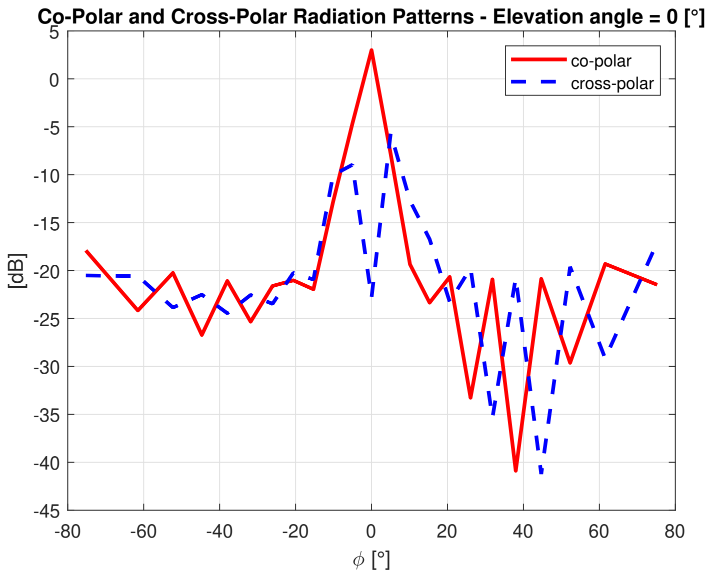

Figure 6: Co-Polar and Cross-Polar radiation pattern according to azimuth angle

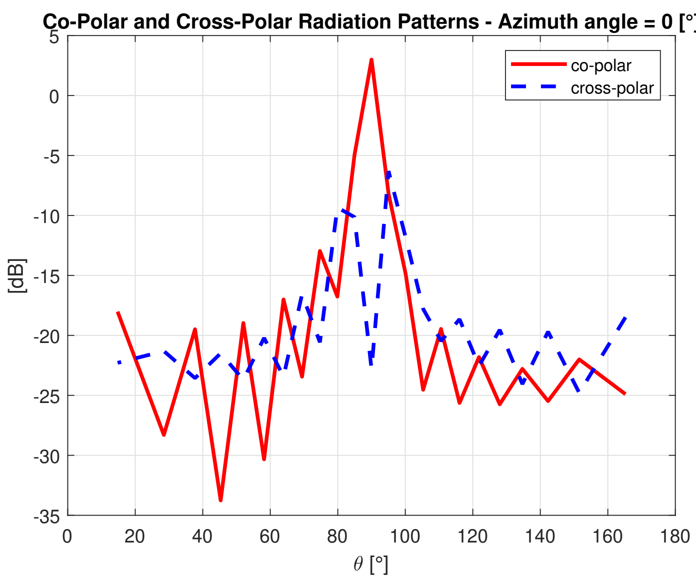

Figure 7: Co-Polar and Cross-Polar radiation pattern according to elevation angle

Since the radiated beam is narrow, there is no interest in representing the co-polar and cross-polar radiation pattern for non-zero fixed elevation or azimuth angle. This is what the antenna is designed for, radiate a broadside pencil beam. The radiation pattern allows to verify this characteristic of the antenna.

Based on these curves in Figures (6) or (7) it is possible to measure how purely the antenna is polarized. This measure is given by the ratio between the maximum radiation intensity of the Co and Cross-Polarizations for $(\theta = 0°, \phi = 0°)$, $25.81[dB]$.

5 Conclusion

By applying the Fast-Fourier transform algorithm on the current distribution, the radiation pattern of a antenna can easily be obtained.