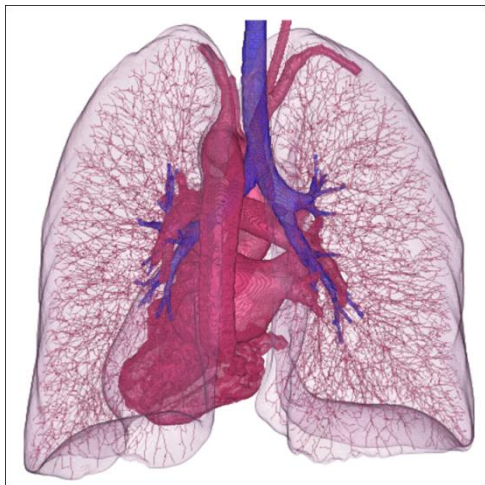


Medical Image Analysis

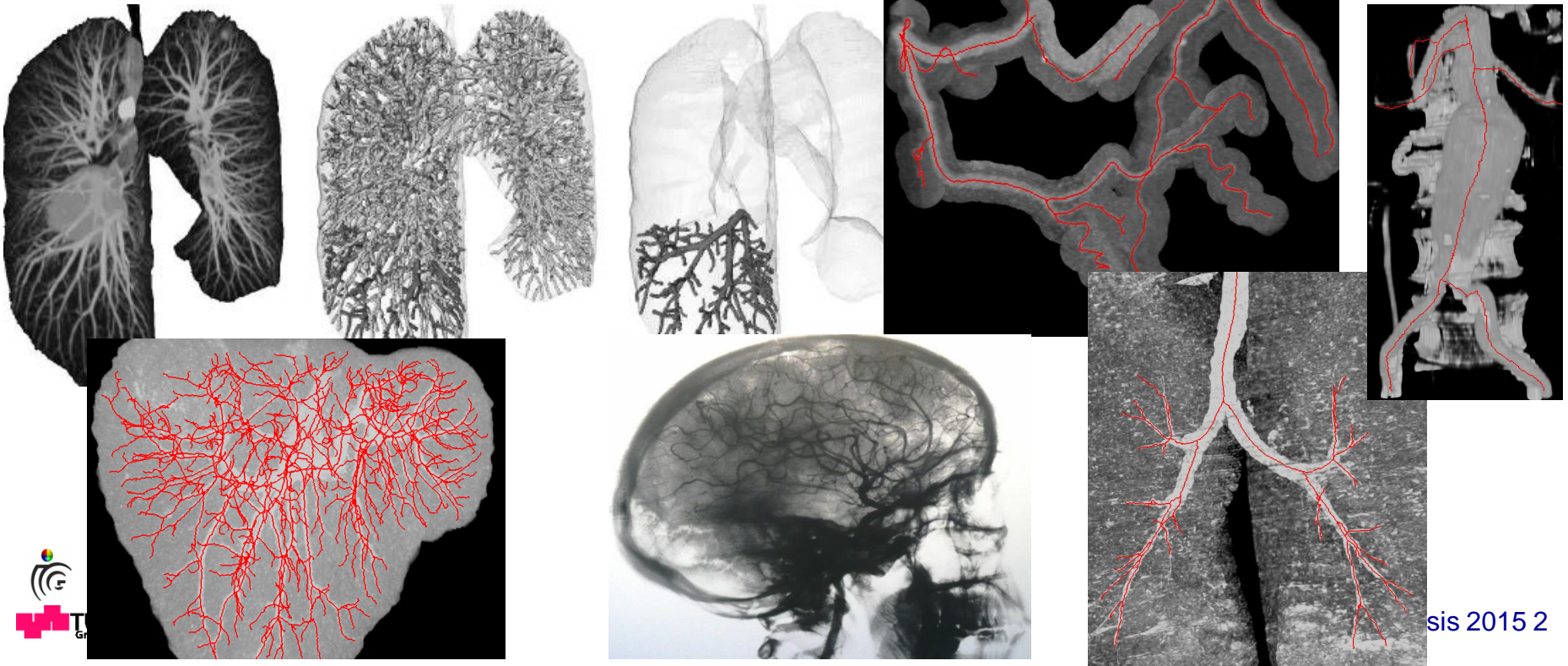
Lecture 11



Vascular Structures

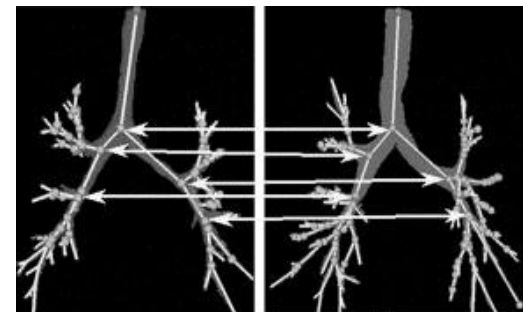
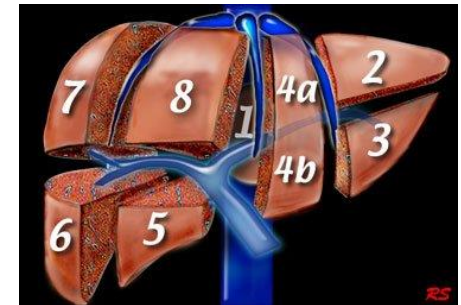
Vascular Structures

- Variety of 3D tubular structures in medical datasets exists



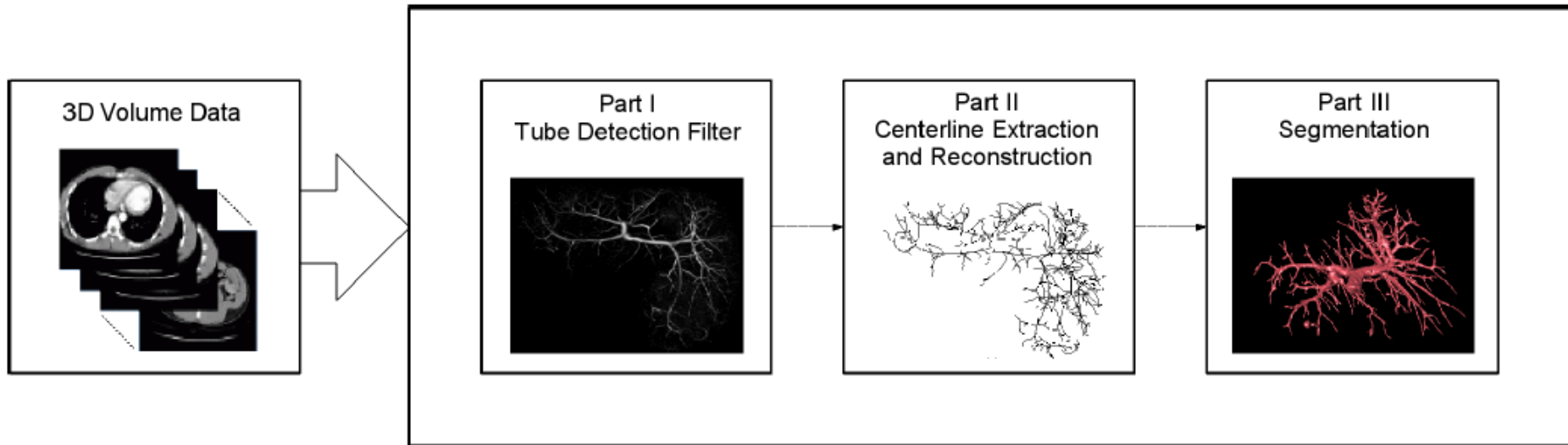
Vascular Structures

- Clinical Motivation
 - Visualization & Detection of Stenoses (calcification) / Aneurisms / Tumors
 - Segmentation of sub-parts
 - Liver segments, lung lobes
 - Registration according to corresponding structures
 - Virtual Broncho-/Colonoscopy



Vascular Structures

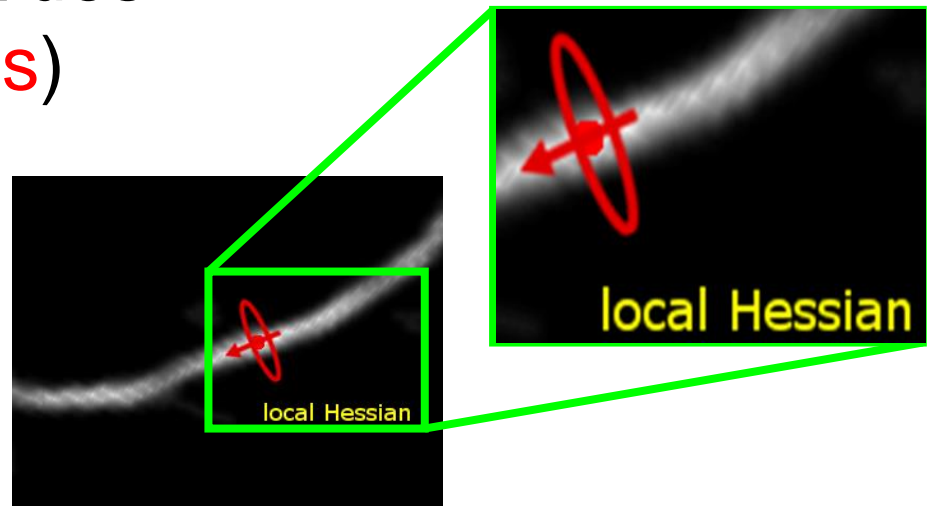
- Overview



*Credits belong to **Christian Bauer***

Vascular Structures

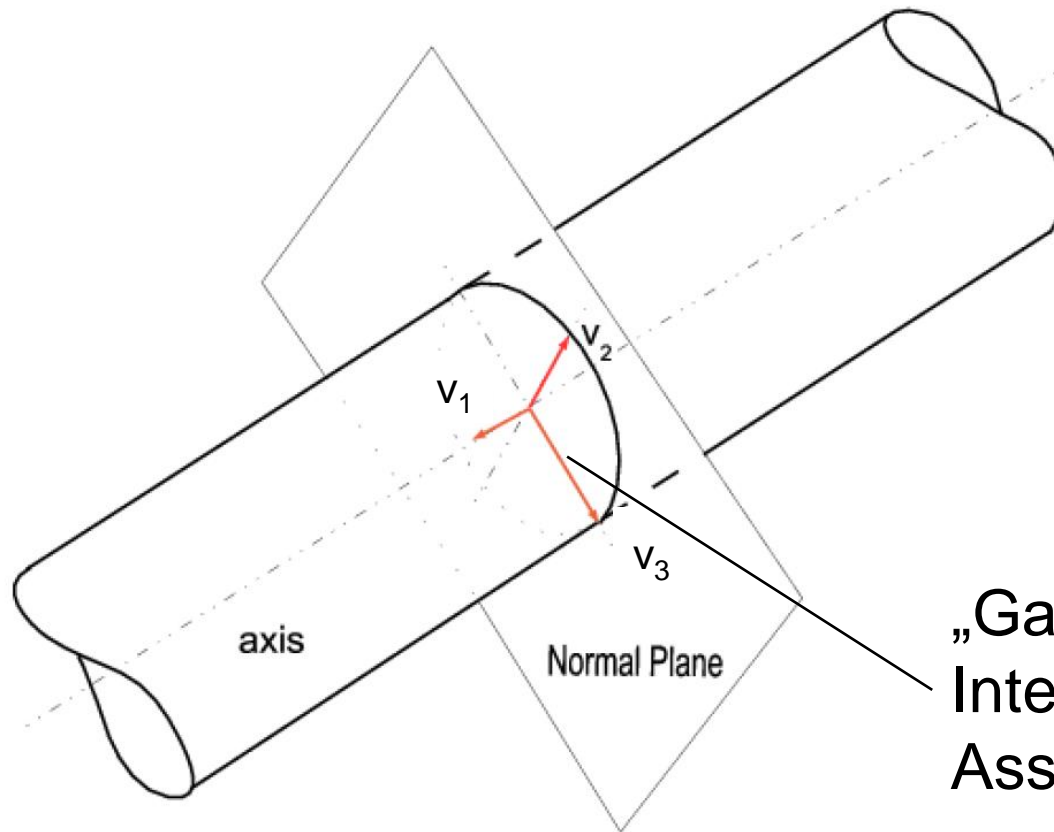
- Mathematically, they have local similarity to a circle in common (**tubularity assumption!**)
- Analyze using point-wise 2nd derivatives of greyvalues (**Hessian eigenvalues**)
- A measure for **curvilinear** structures



Vascular Structures

- Image intensity function $I(x,y,z)$
- Local 2nd order Taylor approximation -> curvature
- Hessian at x,y,z
- Analysis of eigenvalues & eigenvectors of Hessian matrix!

Vascular Structures



„Gaussian
Intensity Profile
Assumption“

Vascular Structures

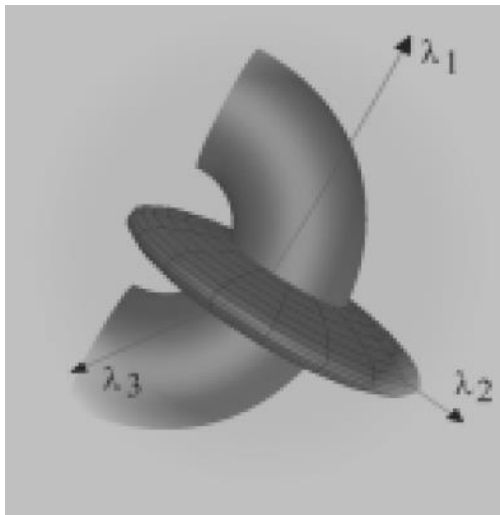
- Algorithm 1:
 - For all pixel (x,y,z) :
 - Compute Hessian matrix at (x,y,z)
 - Compute its Eigenvalues
 - Investigate magnitude of eigenvalues to define a (central) **medialness response** function $R(x,y,z)$

Vascular Structures

- Algorithm 1 according to Frangi et al.

$$R = \begin{cases} 0 & \text{if } \lambda_2 > 0 \text{ or } \lambda_3 > 0 \\ \left(1 - e^{-\frac{\mathcal{R}_A^2}{2\alpha^2}}\right) \left(e^{-\frac{\mathcal{R}_B^2}{2\beta^2}}\right) \left(1 - e^{-\frac{S^2}{2c^2}}\right) & \text{else} \end{cases}$$

with $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$.

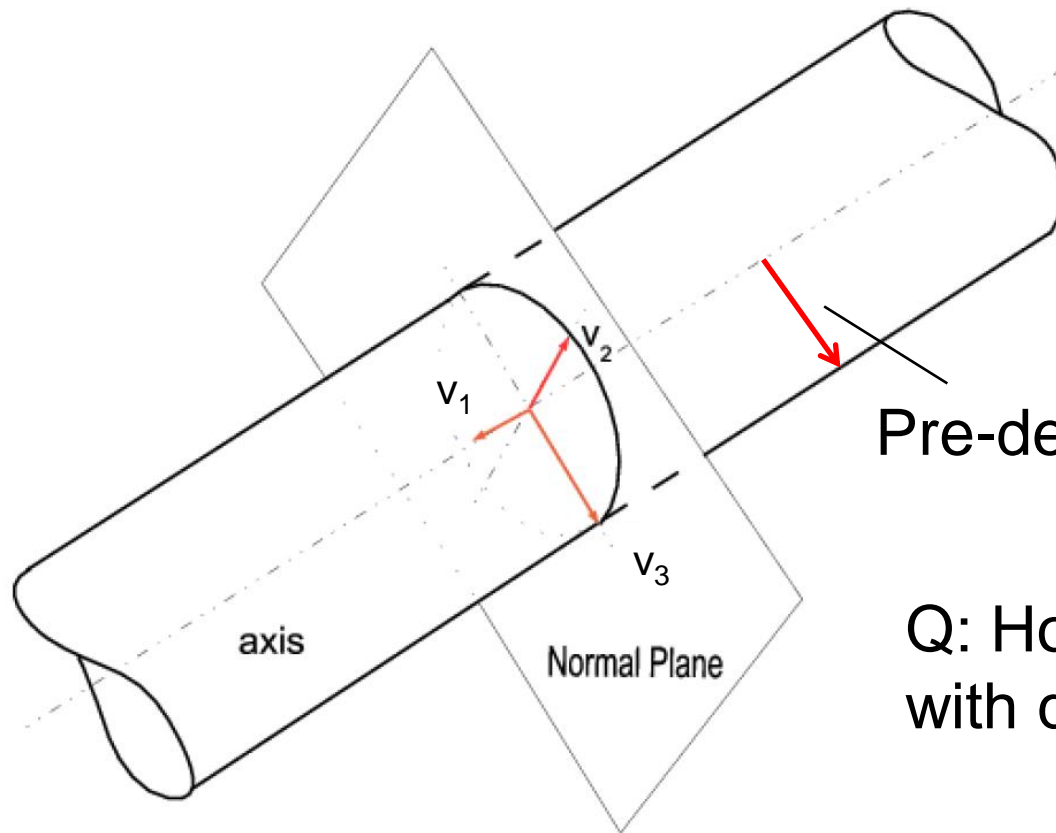


$$\mathcal{R}_B = \frac{\text{Volume}/(4\pi/3)}{(\text{Largest Cross-Section Area}/\pi)^{3/2}} = \frac{|\lambda_1|}{\sqrt{|\lambda_2\lambda_3|}}$$

$$\mathcal{R}_A = \frac{(\text{Largest Cross-Section Area})/\pi}{(\text{Largest Axis Semi-length})^2} = \frac{|\lambda_2|}{|\lambda_3|}$$

$$S = \|\mathcal{H}_\sigma\|_F = \sqrt{\sum_j \lambda_j^2} \quad j = 1, 2, 3$$

Vascular Structures

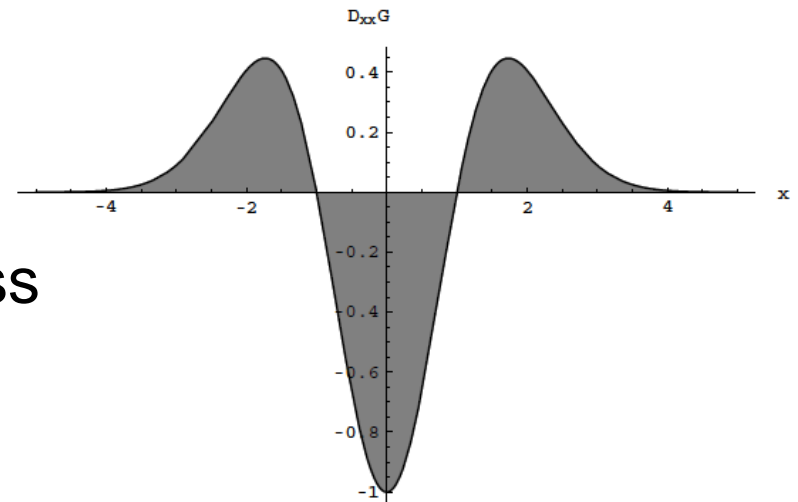


Pre-defined radius r

Q: How do we deal with different r 's?

Vascular Structures

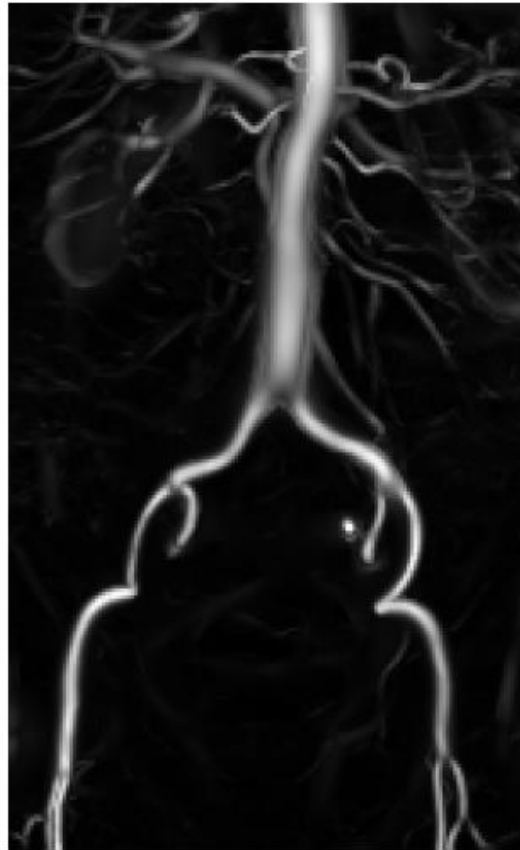
- Multi-Scale Analysis
 - Construct a **scale-space** by convolving the initial image $I(x,y,z)$ with Gaussian derivative kernels of increasing std.dev
 - Perform the medialness filter at each scale
 - Choose the maximum response of the medialness across the scale



Results of Frangi Method



a)



b)

**Maximum Intensity
Projection (MIP)** of
a) initial MRA data set
b) data after vessel
enhancement

Vascular Structures

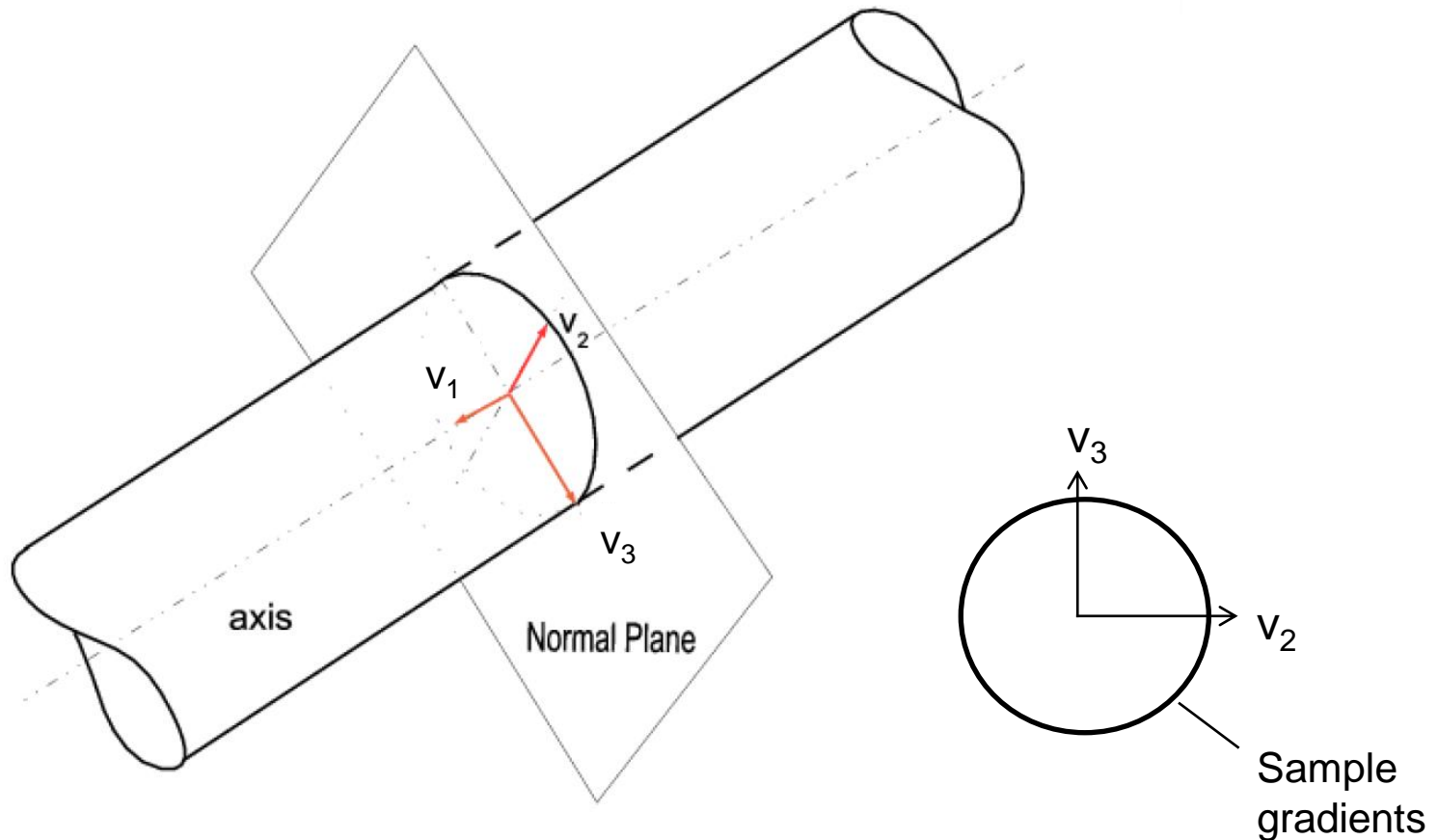
- Disadvantages of Frangi [1] method:
 - Choice of parameters
 - Limited use of directional information (restricted to eigenvalues)
 - Problems when deviation from ideal gaussian intensity profile exists

[1] A. Frangi. Three-Dimensional Model-Based Analysis of Vascular and Cardiac Images. PhD thesis, University Medical Center Utrecht, Netherlands, 2001.

Vascular Structures

- **Offset medialness** functions take eigenvectors of Hessian stronger into account
- Investigate gradients at the border of the tube
- Medialness response now incorporates the **symmetry** of the tubular structure at the border!

Vascular Structures



Vascular Structures

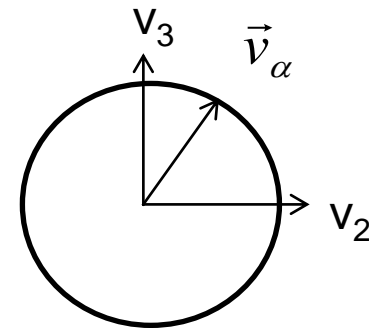
- Algorithm 2:
 - For all pixel (x,y,z)
 - Compute Hessian matrix at (x,y,z)
 - Compute its Eigenvalues & Eigenvectors
 - Investigate eigenvector direction according to smallest eigenvalue -> tube direction
 - Other two eigenvectors form normal plane
 - Sample the (pre-computed) gradient directions at distance r in the normal plane -> compute offset medialness response $R(x,y,z)$

Vascular Structures

- Algorithm 2 according to Krissian et al.[2]

$$R(\vec{x}, \sigma, \theta) = \frac{1}{2\Pi} \int_{\alpha=0}^{2\Pi} -\sigma^\gamma \nabla I^{(\sigma)}(\vec{x} + \theta\sigma \vec{v}_\alpha) \cdot \vec{v}_\alpha d\alpha$$

$$\vec{v}_\alpha = \cos(\alpha) \vec{v}_2 + \sin(\alpha) \vec{v}_3$$



[2] K. Krissian et al. Model-based detection of tubular structures in 3D images.
Computer Vision and Image Understanding, 80(2):130-171, 2000.

Vascular Structures

- Further improving the offset medialness algorithm:
 1. Compute mean gradients along border and weight by circularity measure (dot product of gradient direction \vec{g} and \vec{v})

$$c_i = \begin{cases} -\vec{g}(\vec{x} + r\vec{v}_{\alpha_i}) \cdot \vec{v}_{\alpha_i} & \text{if } -\vec{g}(\vec{x} + r\vec{v}_{\alpha_i}) \cdot \vec{v}_{\alpha_i} > 0 \\ 0 & \text{otherwise} \end{cases}$$

This gives us a weighted mean gradient as **initial medialness** R_i

Vascular Structures

- Further improving the offset medialness algorithm:
 2. Calculate symmetry criterion by looking at the **variance** s^2 of the gradients w.r.t. the initial medialness.

Symmetric structures
have small variance!

$$S(\vec{x}, r) = 1 - \frac{s^2(\vec{x}, r)}{R_i^2(\vec{x}, r)}$$

Then, S measures the homogeneity of the boundary between 0 and 1, and the **symmetry based medialness** R_s

$$R_s(\vec{x}, r) = R_i(\vec{x}, r)S(\vec{x}, r)$$

Vascular Structures

- Further improving the offset medialness algorithm:
 3. Define **an adaptive threshold** to define minimum medialness responses for robustness to noise.

Medialness must be larger than the magnitude of the gradient on the tube centerline R_c .

$$R_c(\vec{x}, r) = \sigma^\gamma |\nabla I^{(\sigma)}|$$

- The **final medialness** is

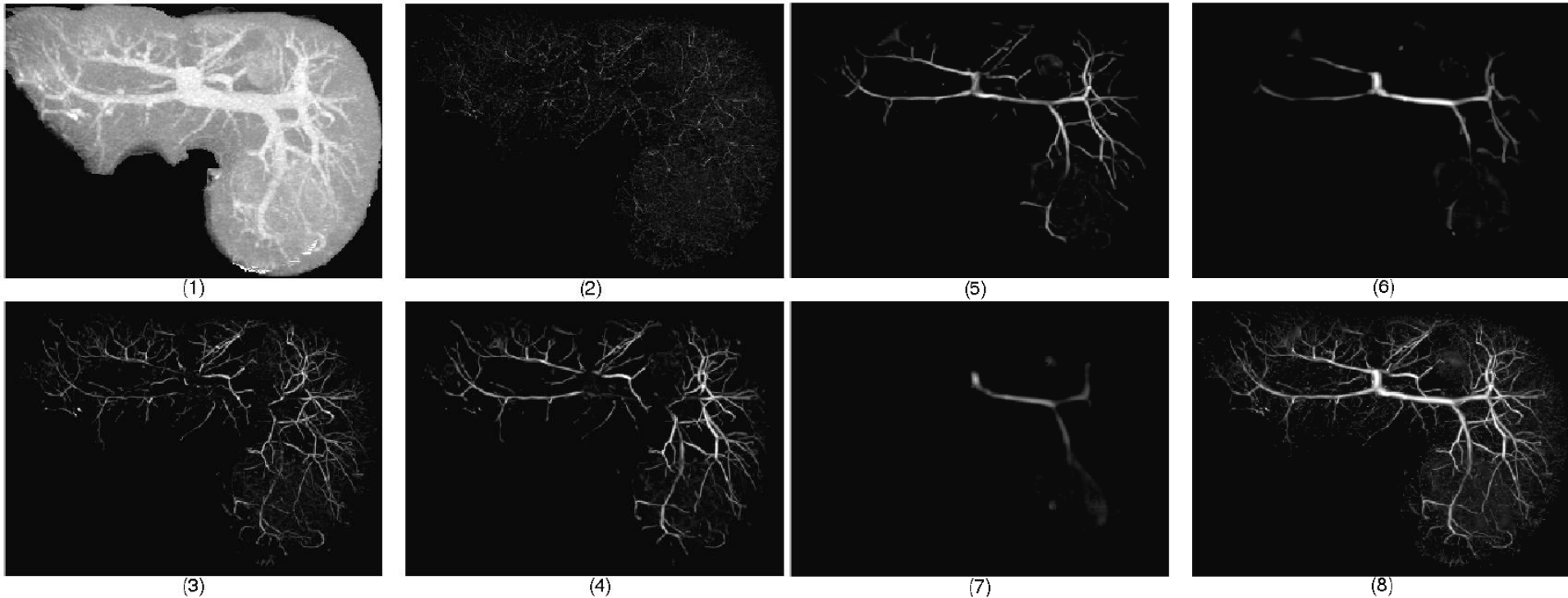
$$R(\vec{x}, r) = \begin{cases} R_s(\vec{x}, r) - R_c(\vec{x}, r) & \text{if } R_s > R_c \\ 0 & \text{otherwise} \end{cases}$$

Vascular Structures

- A very sophisticated vessel enhancement scheme is **Algorithm 3**:
- Repeat
 - For all pixel (x,y,z)
 - Pre-compute gradients & Hessian matrix at (x,y,z)
 - For all pixel (x,y,z)
 - Compute its Hessian Eigenvalues & Eigenvectors
 - Compute initial & symmetry based medialness
 - Using gradient at (x,y,z) : compute final medialness

For all scales

Results of Algorithm 3



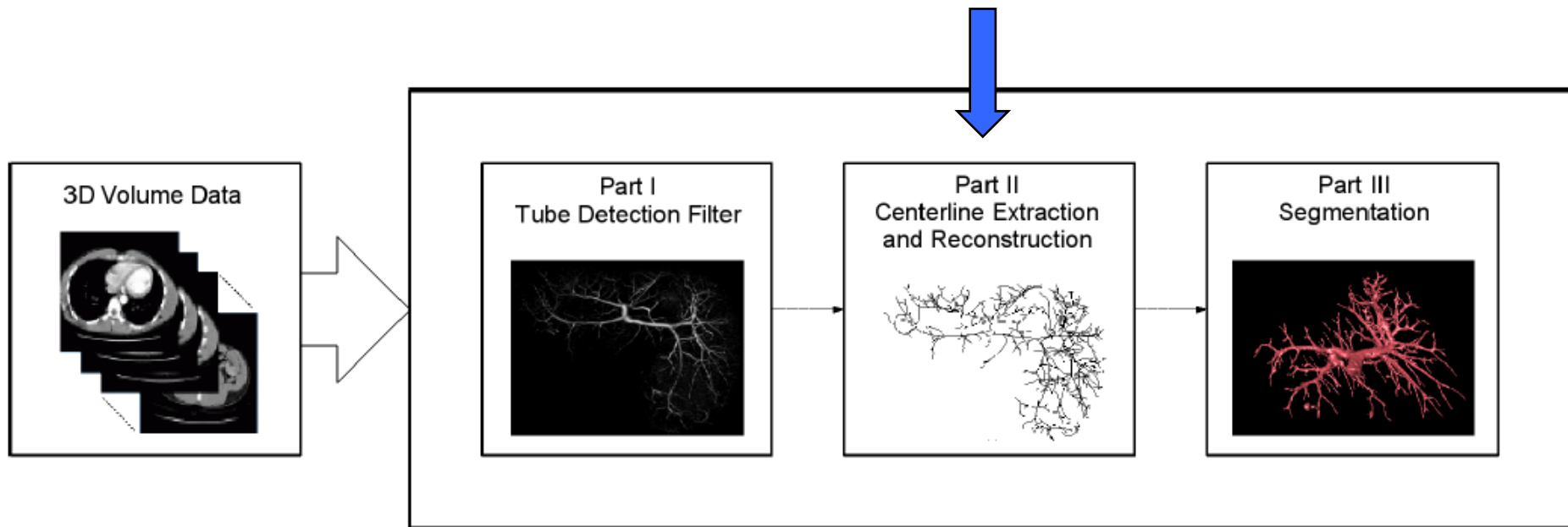
Liver CT data set, responses at different scales,
(8) shows combined multiscale medialness response

Results of Algorithm 3

- LungVessels movie (VLC)

Vascular Structures

- Overview



Vascular Structures

- Centerline Extraction with hysteresis thresholding
 - First, local medialness maxima selection
(=non-max suppression)
 - Second, queue-based
reconnection

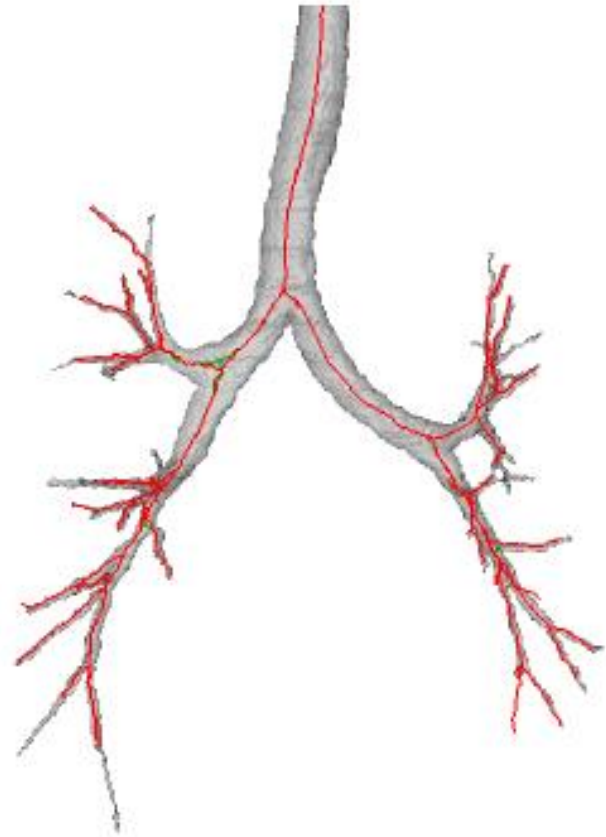
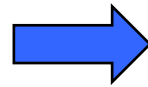
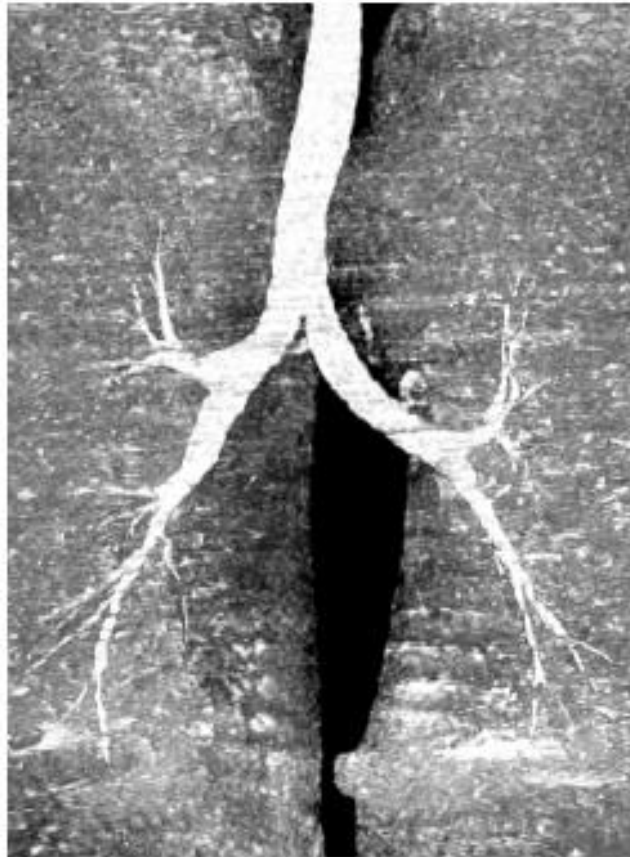
Vascular Structures

- Local medialness maxima
 - For all pixel (x,y,z) we can **look at** the medialness **response of 8 neighboring points** interpolated in the plane orthogonal to the tube direction
 - We keep the medialness response for the local maxima image only if the pixel is a **local maximum** in this plane, and larger than a conservative noise threshold t_{low}

Vascular Structures

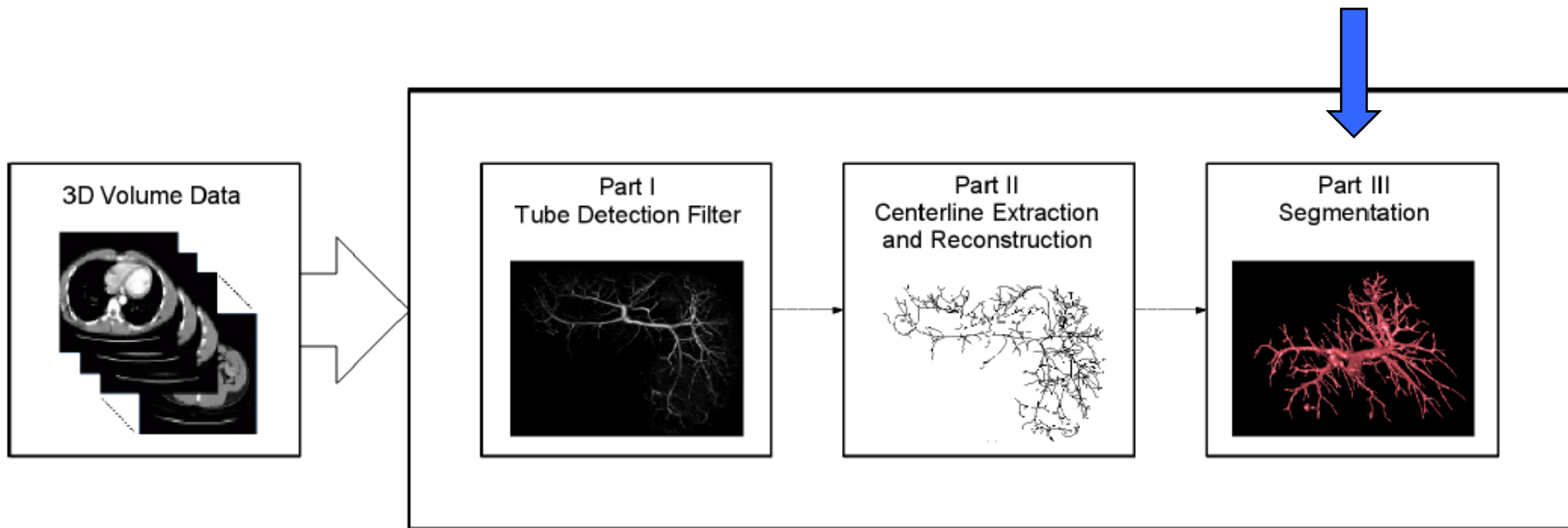
- Queue-based reconnection
 - We put all local maxima into a **queue** if larger than t_{high}
 - While the queue is filled, we **follow** the pos. & neg. tangent directions of the extracted queue entry **as long as the medialness maximum is larger than t_{low}**
 - Pixel are added to centerline as long as this tracing continues
 - We can also incorporate **vessel direction** (eigenvectors!) as a constraint, and try to bridge small gaps (branching points)

Centerline Example



Vascular Structures

- Overview



Use e.g. Geodesic Active Contour based on weighted TV

END

Thank you for your attention,

this concludes MIA 2015!