Medical Image Analysis
Lecture 07

Image Registration

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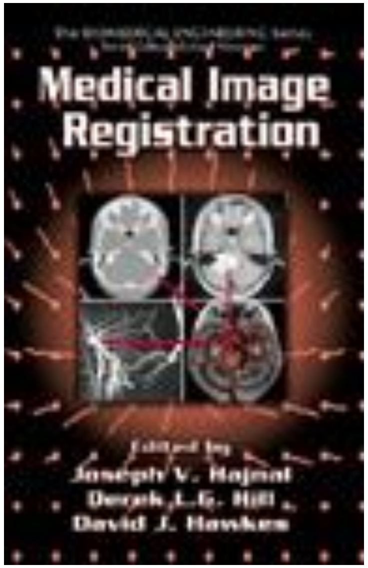
Overview - Image Registration

- What is it?
- Motivation
- Formal Definition & Categorization
- Rigid Registration Methods
  - Feature Based
  - Surface Based
  - Intensity Based

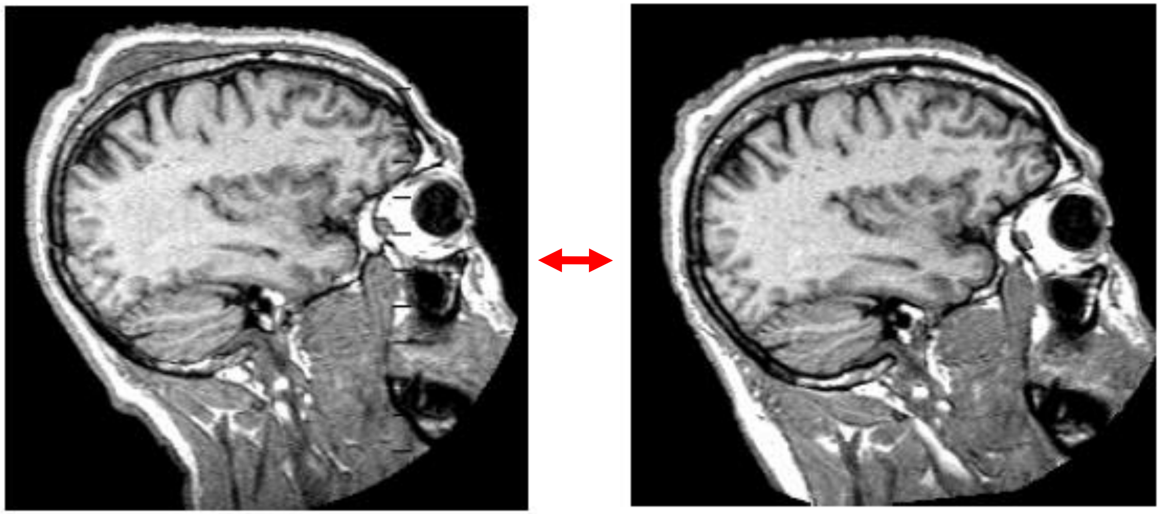
Recommended Literature

- JV Hajnal, DLG Hill, DJ Hawkes, eds., *Medical Image Registration*, CRC, 2001.
  - Chapters on:
    - Registration methodology
    - Applications of rigid body registration
    - Techniques and applications of non-rigid registration

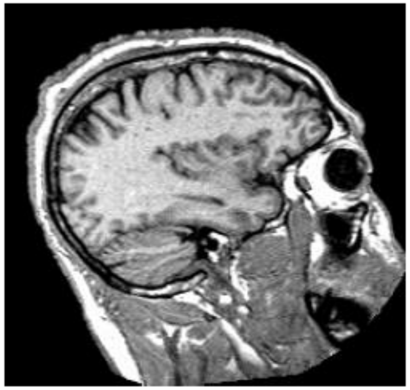


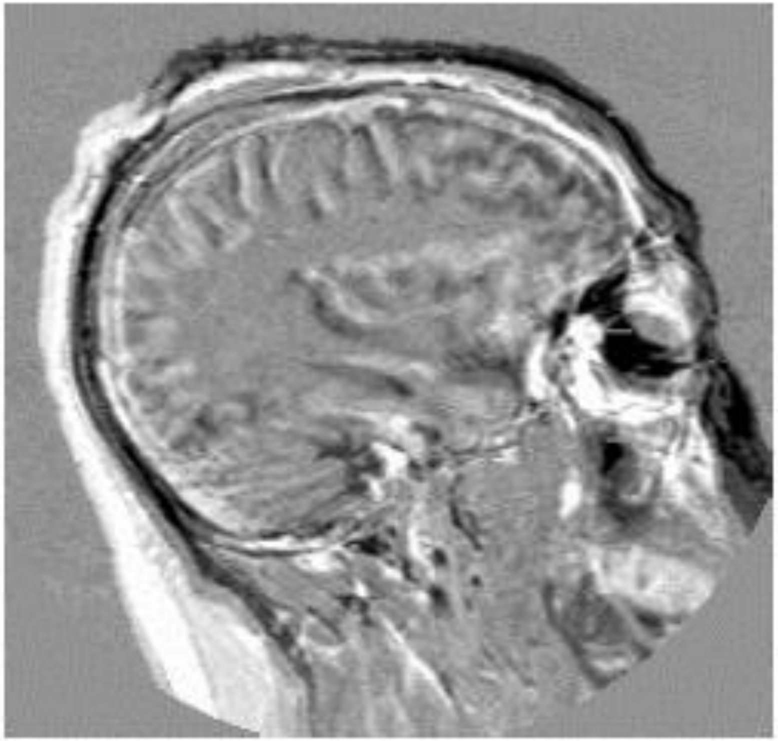
Definition of Image Registration

Registration is the process of determining a one-to one mapping or transformation between the coordinates in one space and those in another, such that points in the two spaces that correspond to the same anatomical point are mapped to each other.



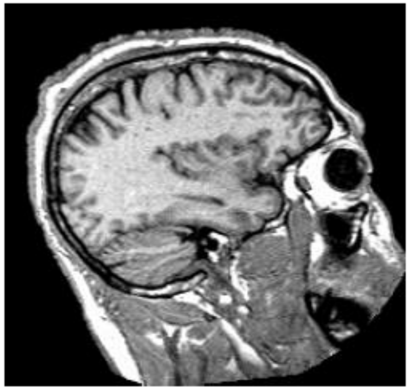
Definition of Image Registration





Difference without registration

Definition of Image Registration

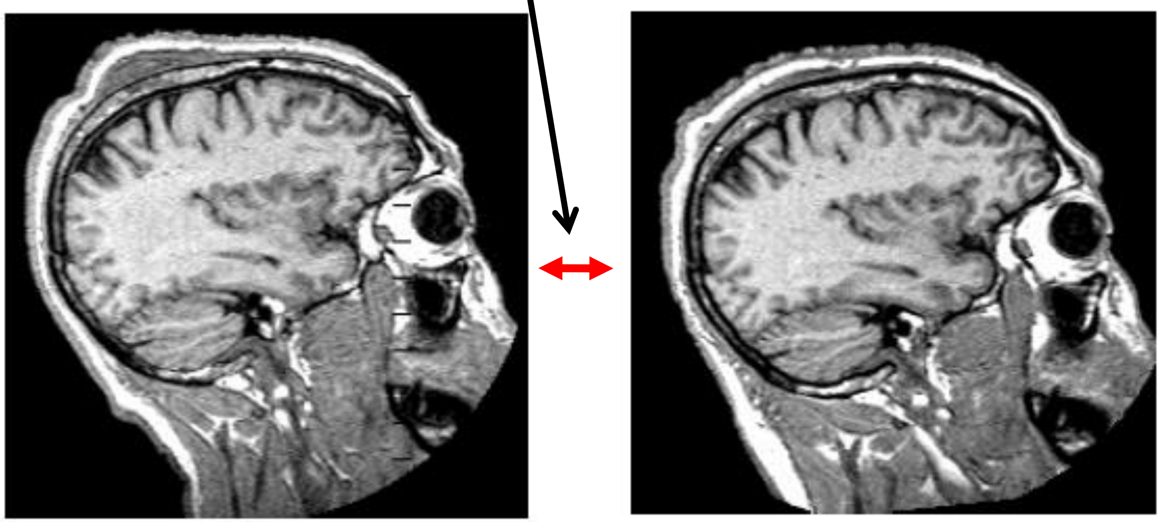


Note: One-to-one correspondence not always possible!

Difference after registration

Definition of Image Registration

Result: a transformation T (can be used to map image A into coordinate system of image B)



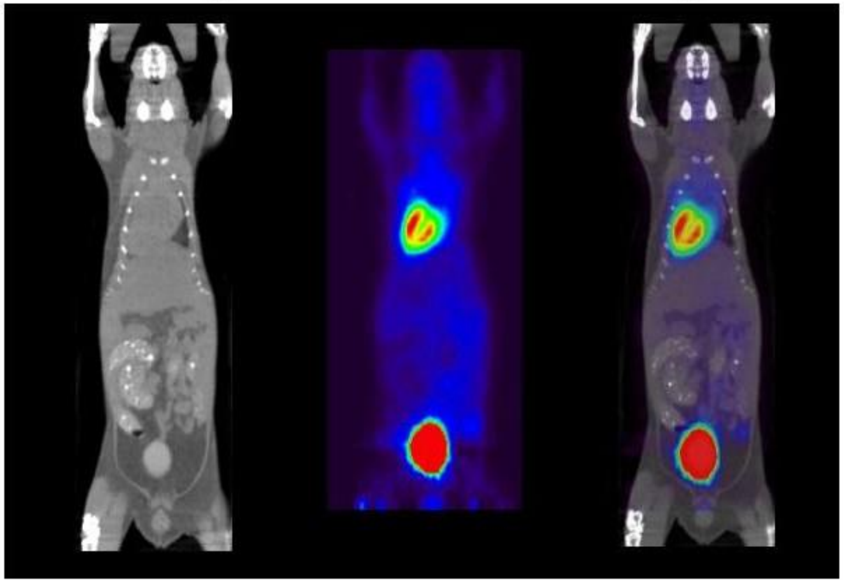
T: e.g. rotation, translation

Overlay of registered A and B -> Image Fusion

Overview Registration Applications

- Multi-modality fusion

Co-registered PET/CT Images

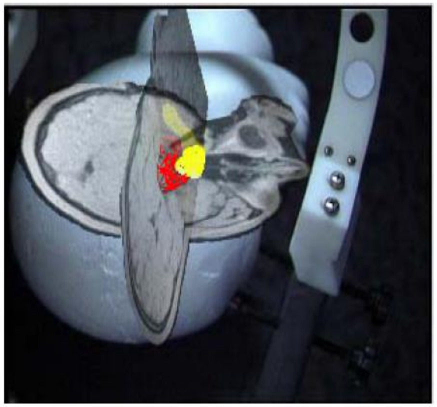


Overview Registration Applications

- Multi-modality fusion
- Monitoring temporal changes

Overview Registration Applications

- Multi-modality fusion
- Monitoring temporal changes
- Image-Guided Surgery



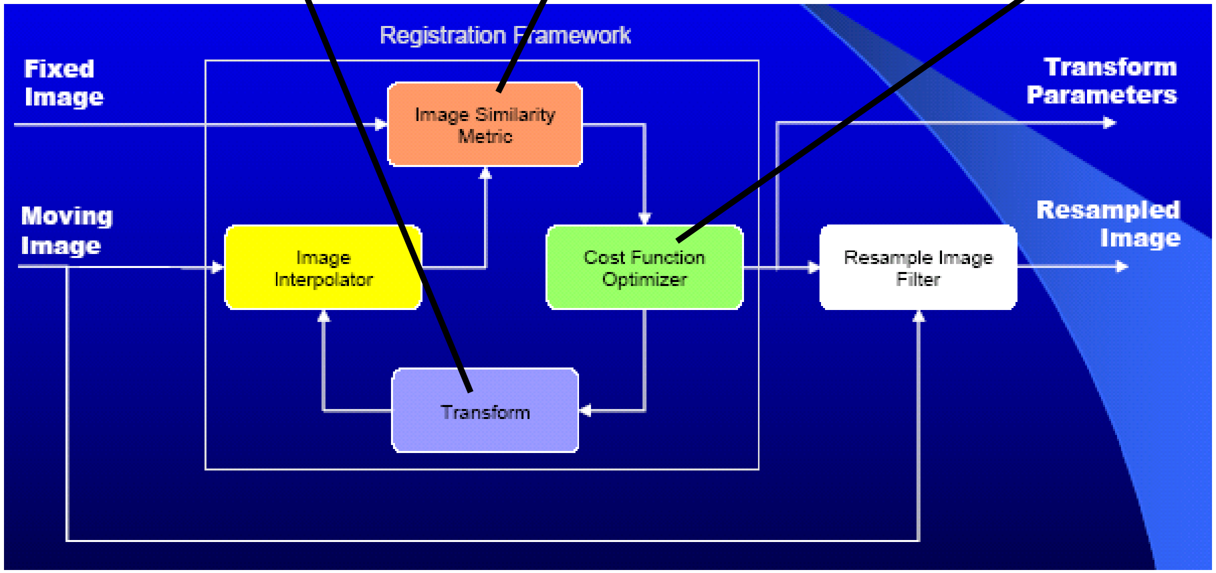
Siemens Corporate Research, Princeton, USA

VarioscopeAR,
Wolfgang Birkfellner, AKH, Wien

Andrew State, UNC

Definition of Image Registration

- Find a mapping h(x) aligning an image („moving“) with a second image („fixed“) such that a defined similarity measure is maximized.



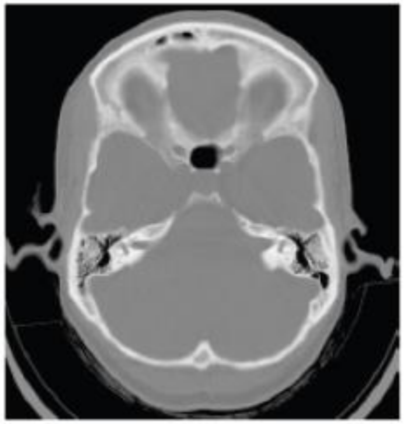
rotation R

Categorization

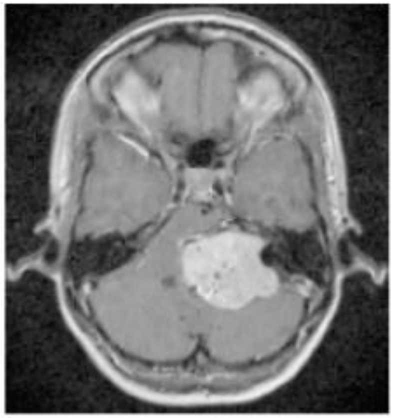
- Types of modalities
  - Single-modal (e.g. MR-MR, CT-CT,…)
    - Dynamic (time series)
  - Multi-modal (e.g. MR-CT, MR-PET, XRay-CT,…)

- Types of image registration
  - Intra-subject (same patient)
    - Image-to-physical space (surgical environment)
  - Inter-subject (different patients)

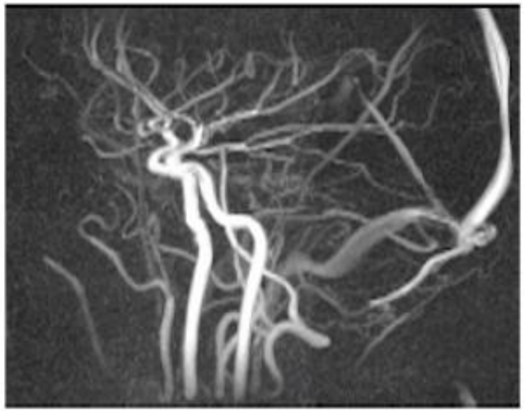
Intra-subject multi-modality registration example



CT

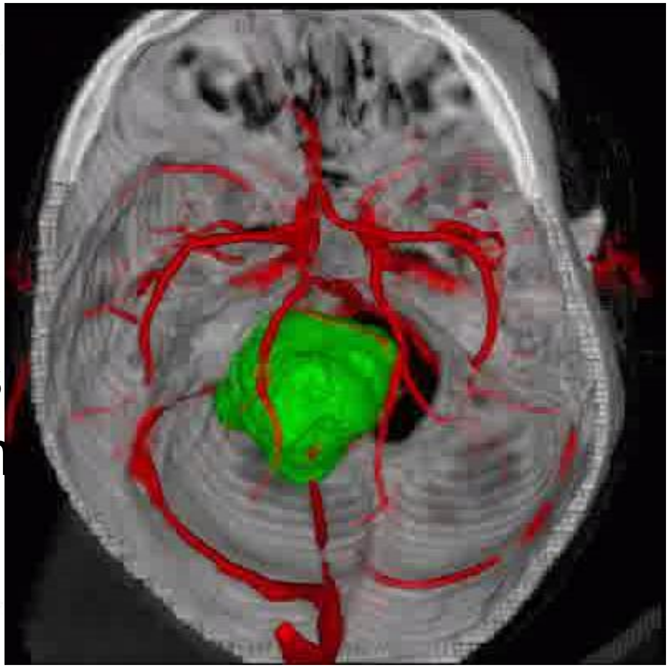


MRI



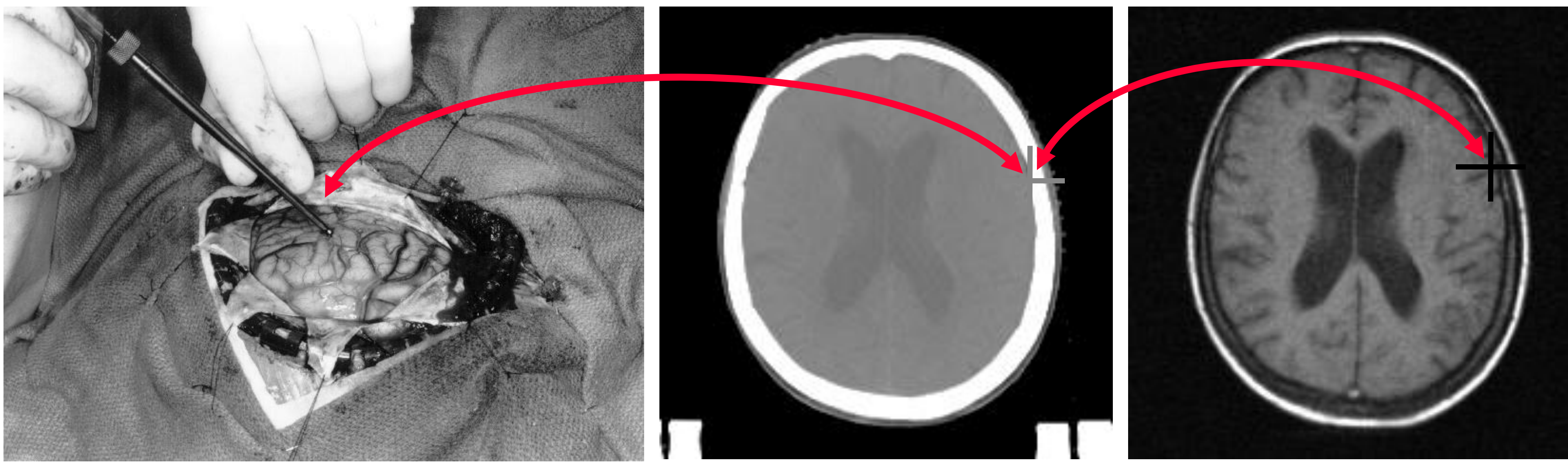
Angiography

→ Fuse 3 Types of Information



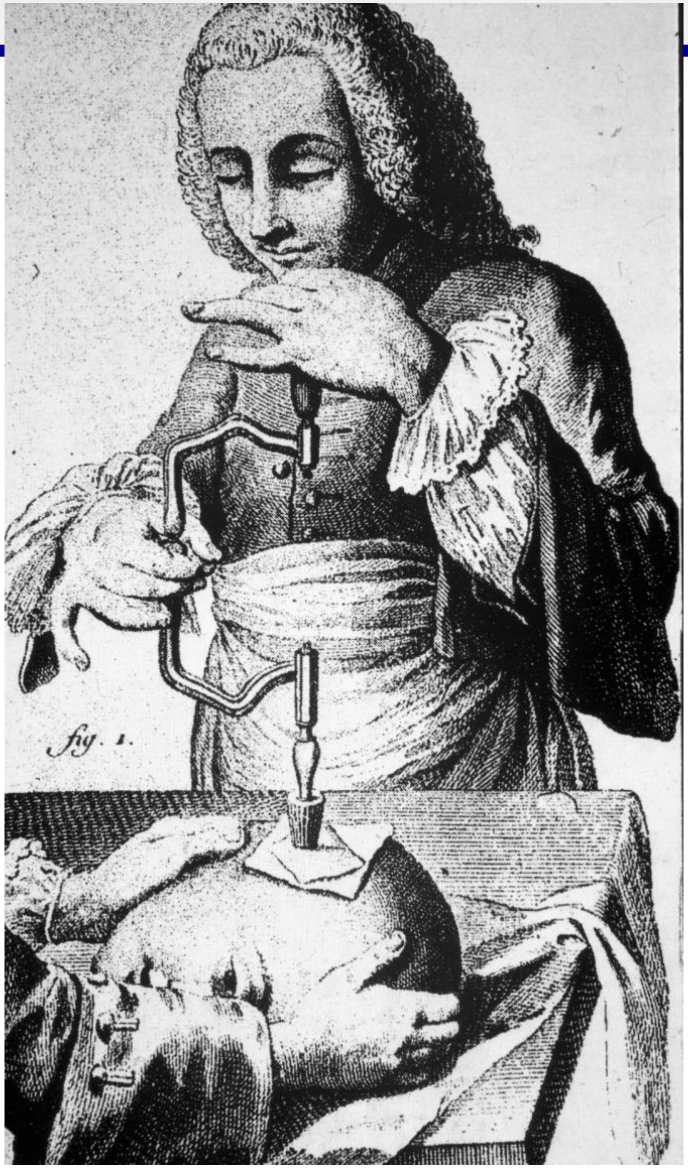
Combined image-to-image and image-to-physical space registration

- Registration of pre-operative CT and MRI to intra-operative scene



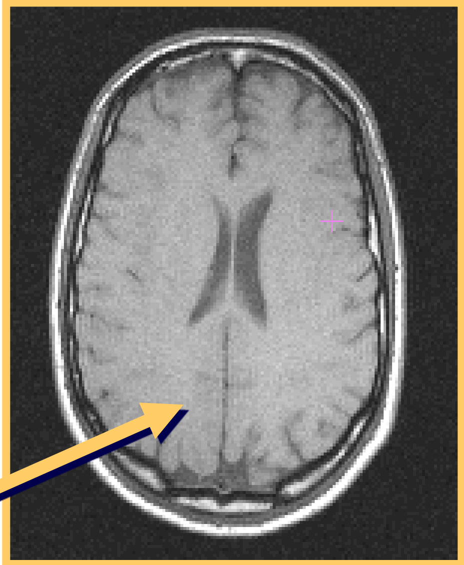
-> Image Guided Surgery

Why image guided surgery?



Accuracy of surgical interventions is important...

Image Guidance is Registration!
Image-to-physical space registration



One view is an image of the patient.

One view is the physical space of the patient viewed by the surgeon

Registration Chain for IGS

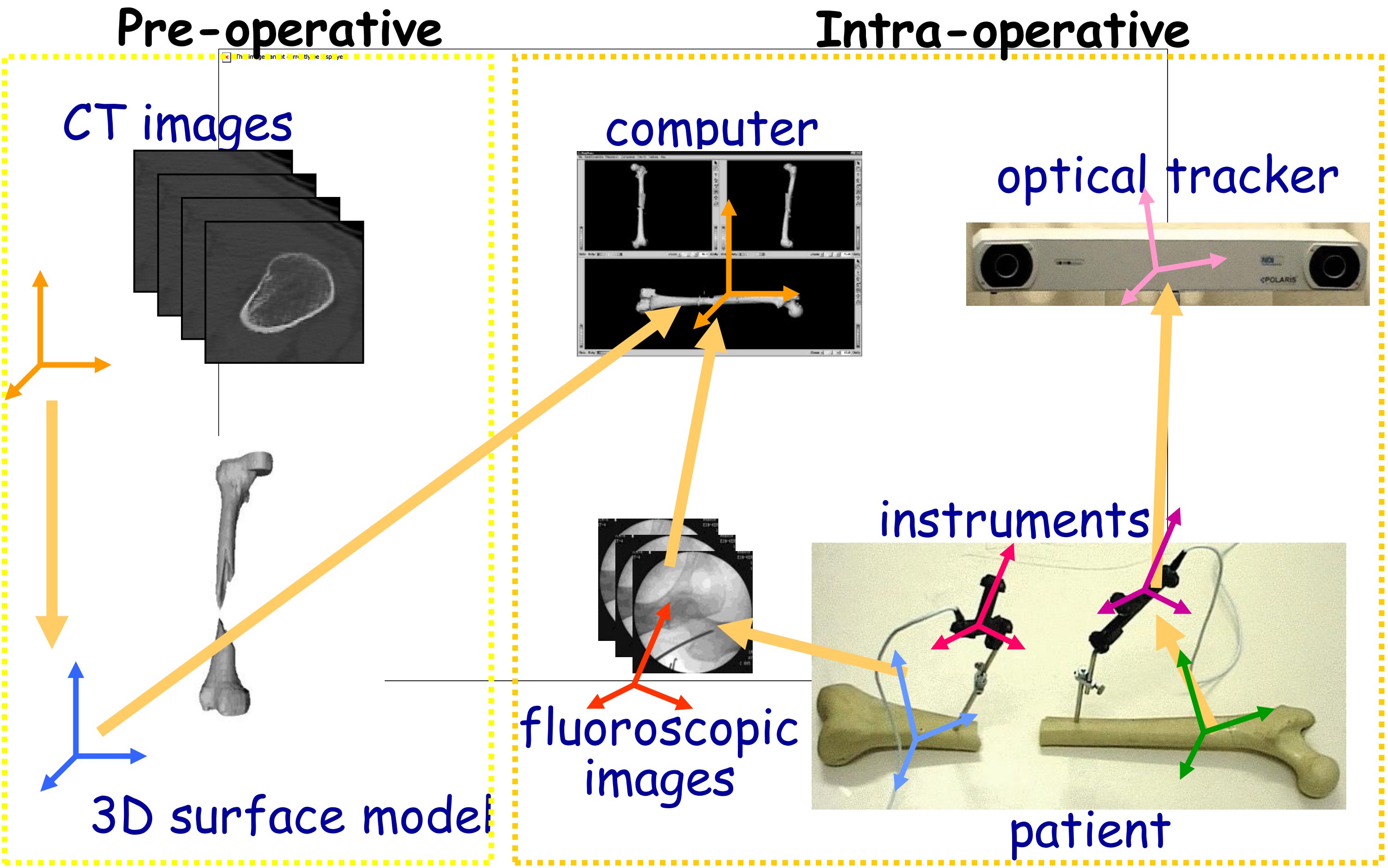
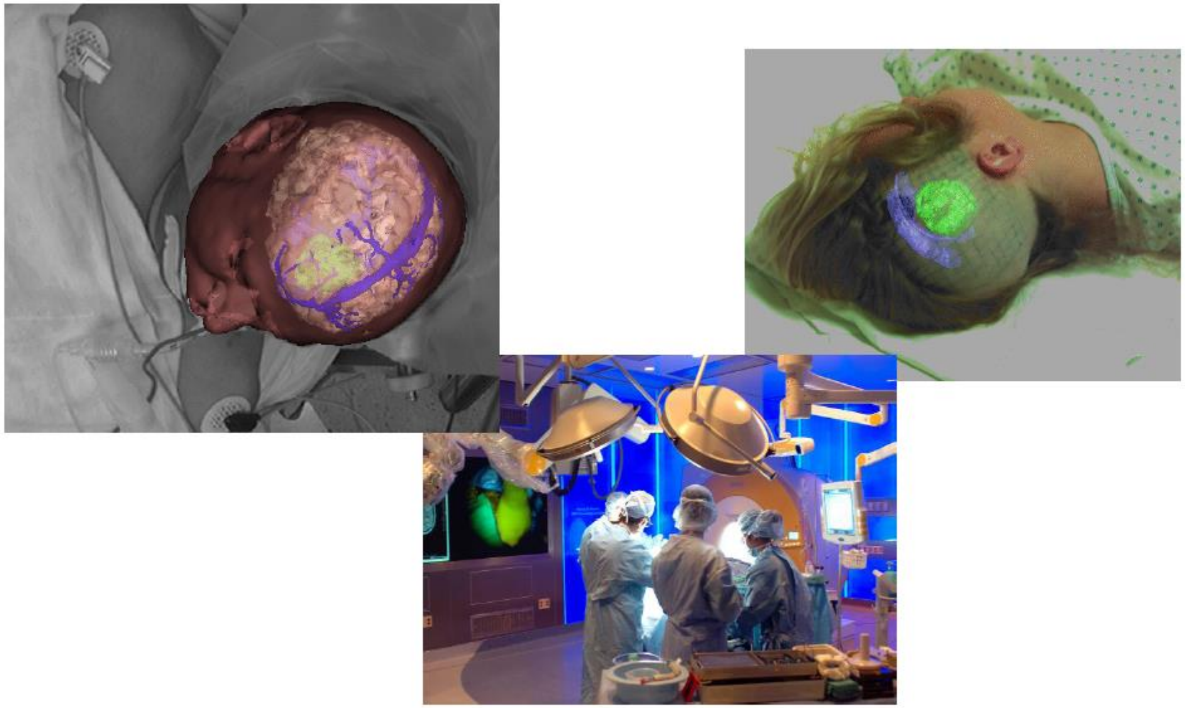


Image Guided Surgery

- Conventional IGS systems:
  - Pre-operative 3D images acquired (e.g., CT, MR)
  - Intra-operative images registered to physical space
  - Surgical probe tracked, registered and displayed on monitor

Image Guided Surgery - AR

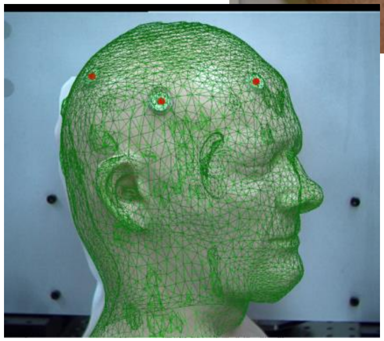
- A disadvantage of conventional IGS systems is that the surgeon has to look away from surgical scene!
- HMD based Augmented Reality (AR) systems allow 3D perception
  - require fast & accurate registration



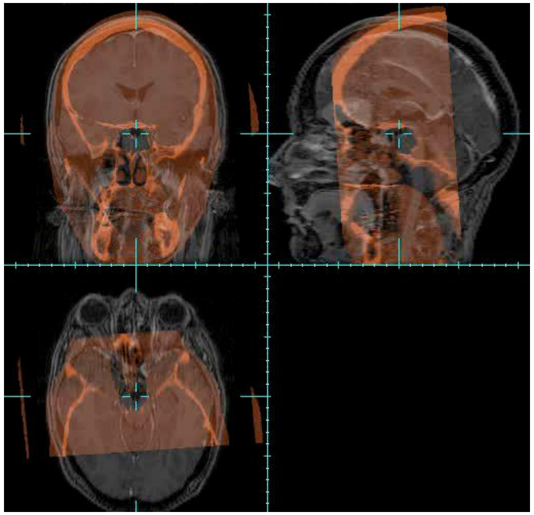
Video: http://www.in.tum.de/forschung/forschungs-highlights/medical-augmented-reality.html

Image Registration Methods

- Feature Based Rigid Registration



- Surface Based Rigid Registration
- Intensity Based Rigid Registration



- Nonlinear (nonrigid) Registration

Increasing complexity

Feature-based rigid registration

Features (Fiducials) are ...

Intrinsic

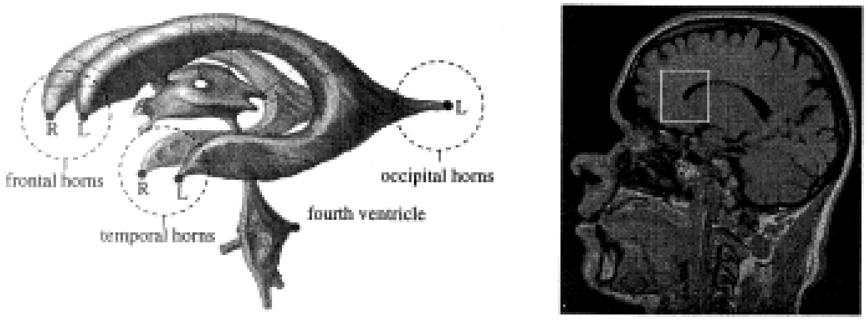
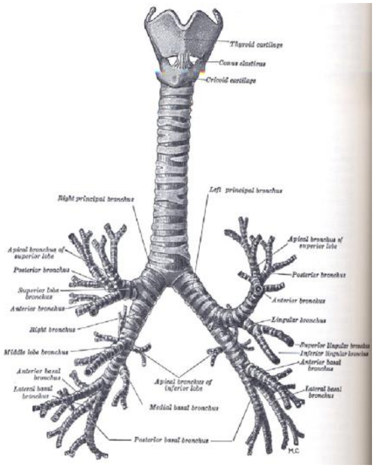


Figure 2.11. Ventricular system of the human brain adapted from [491] with marked landmarks (left), and frontal horn of the ventricular system in a 3D MR image (right).



Anatomical or Geometrical Landmarks

Extrinsic

Skin-Attached or Bone-Implanted Markers

Intrinsic Features for Registration

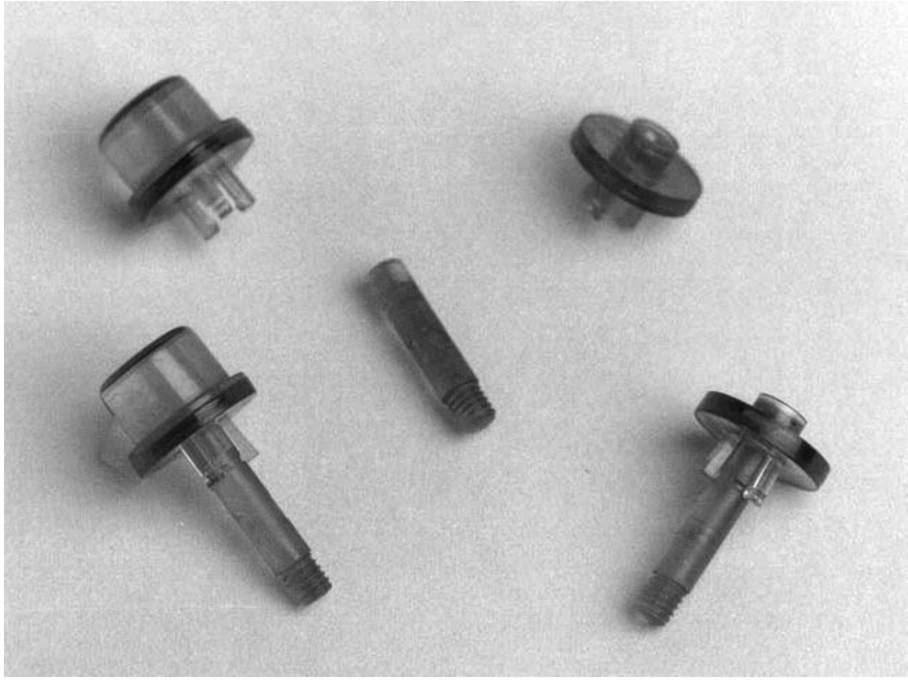
- Anatomic landmarks
  - very distinctive (e.g. branching points of airway tree)
  - uniquely identifiable but often only a low number available or only visible on one of two modalities
- Geometric landmarks
  - curvature extrema of structures
  - often hard to identify
- Manual vs (semi- and fully-) automatic identification
  - Localization error influences registration error!

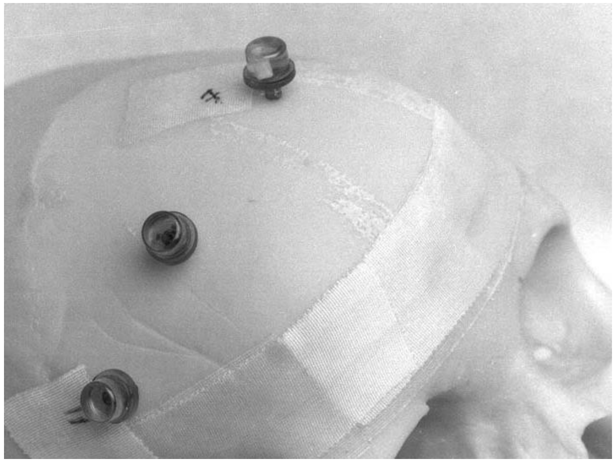
Extrinsic Features - Skin-Affixed Markers

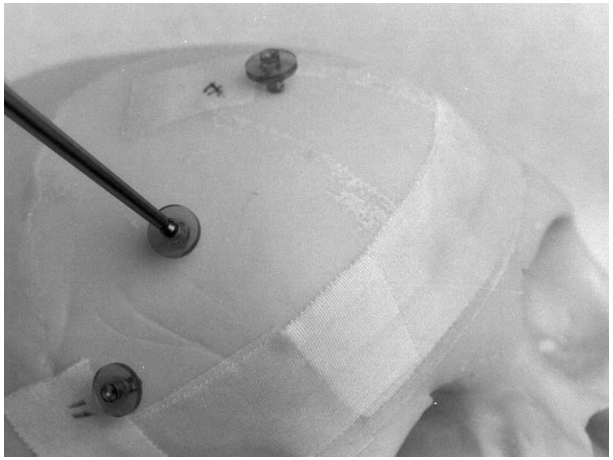
- Advantage: non-invasive
- Disadvantage: can move due to skin mobility

Extrinsic Features - Bone-Implanted Markers

- Advantage: cannot move
- Disadvantage: invasive





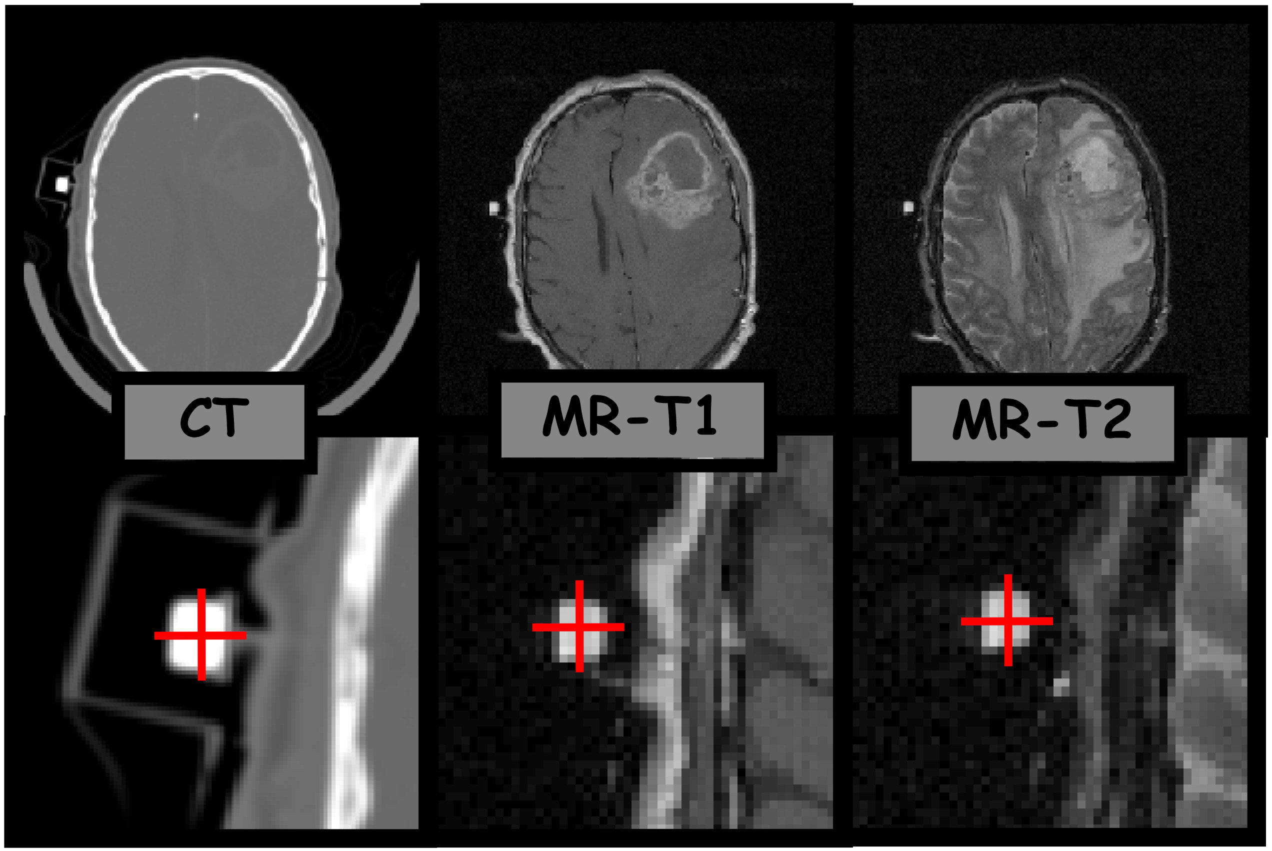


Feature Points for Registration

Advantages of extrinsic markers

- Independent of anatomy
- Automatic algorithms for locating fiducial markers can take advantage of marker’s shape and size in order to accurately and robustly locate the fiducial point

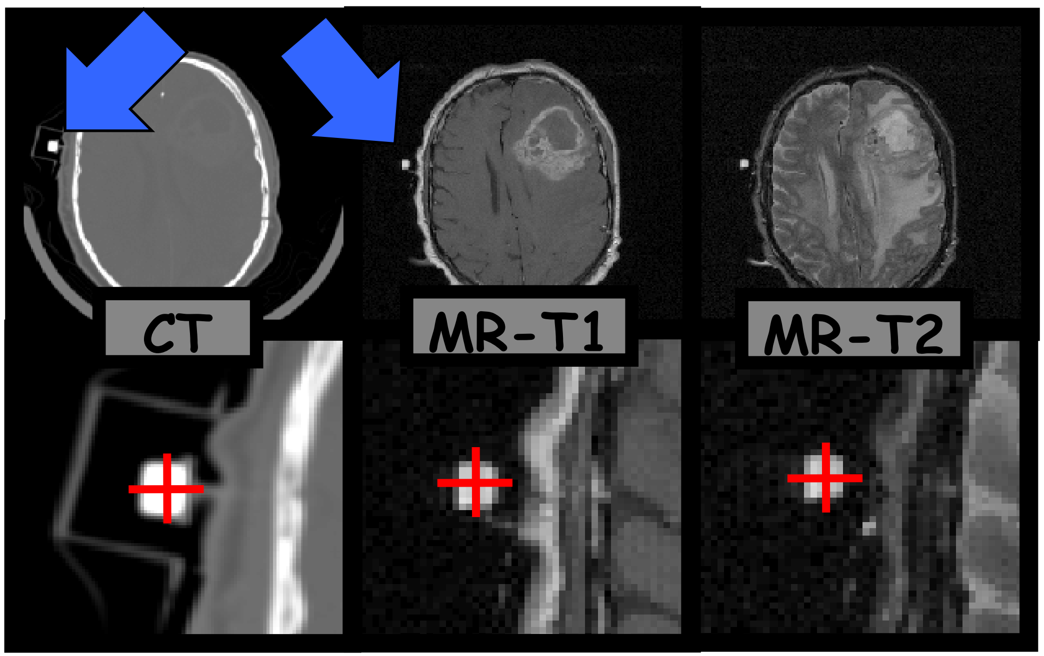
Bone-Implanted Markers



Designed to be easy to identify fully automatically!

What to do with our features?

- Identify corresponding feature points!
  - Feature pairing step



Feature pairs are used to optimize unknown parameters of e.g. a 3D rigid transformation

$$T_{rigid}(x,y,z) = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

6 degrees of freedom!

Point-Based Registration

- Known Point Correspondence
- -> Procrustes Problem
  - Greek Mythology

Point-Based Registration

- The Procrustes Problem:
  - Given two sets of N corresponding points $\mathbf{P} = \{\mathbf{p}_i\}$ and $\mathbf{Q} = \{\mathbf{q}_i\}$
  - Find the rigid-body transformation (rotation matrix $\boldsymbol{R}$ and translation vector $\mathbf{t}$) that minimizes the mean squared distance between the points:

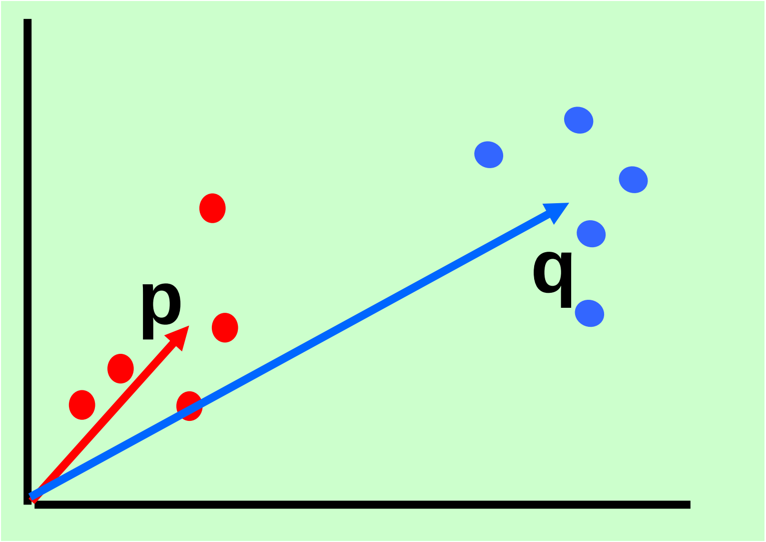
$$\mathrm{E}_{\mathrm{Procr.}} = \frac{1}{N}\sum_{i=1}^{N} |\,\mathbf{Rp}_i + \mathbf{t} - \mathbf{q}_i\,|^2$$

  - E is the *Procrustes registration error*
  - **P** and **Q** represented as matrices

$$\mathbf{P} = \begin{bmatrix} p_{1,x} & p_{1,y} \\ p_{2,x} & p_{2,y} \\ \ldots & \ldots \\ p_{N,x} & p_{N,y} \end{bmatrix}$$

Point-Based Registration

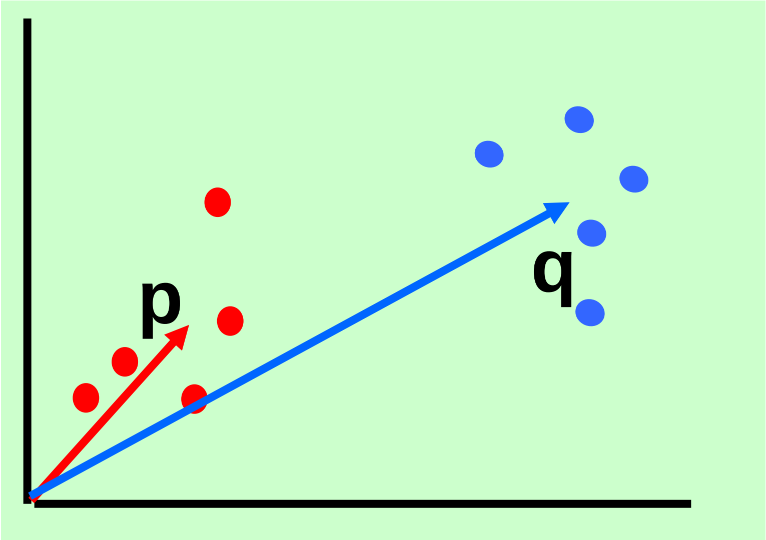
- Procrustes Alignment – 3 steps
  - Center both sets of points
  - Determine rotation
  - Determine translation

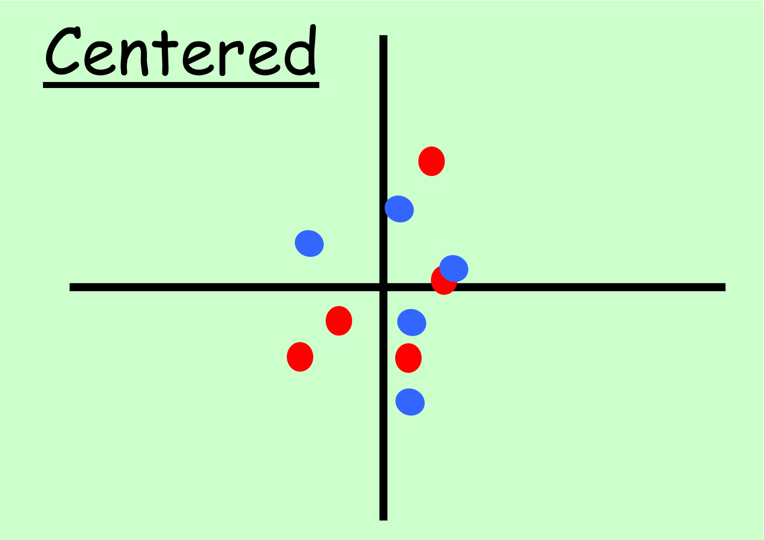


Solution Step 1: Centre points

$$\text{"Centered" points :}$$

$$\tilde{\mathbf{p}}_i = \mathbf{p}_i - \overline{\mathbf{p}}\,; \quad \tilde{\mathbf{q}}_i = \mathbf{q}_i - \overline{\mathbf{q}}$$





Algorithm 1

Solution Step 2: Determine rotation (Orthogonal Procrustes)

$$\boxed{\mathbf{H} \equiv \tilde{\mathbf{P}}^T \tilde{\mathbf{Q}}}$$

$$\mathrm{SVD}: \ \mathbf{H} = \mathbf{U}\mathbf{L}\mathbf{V}^T$$

where

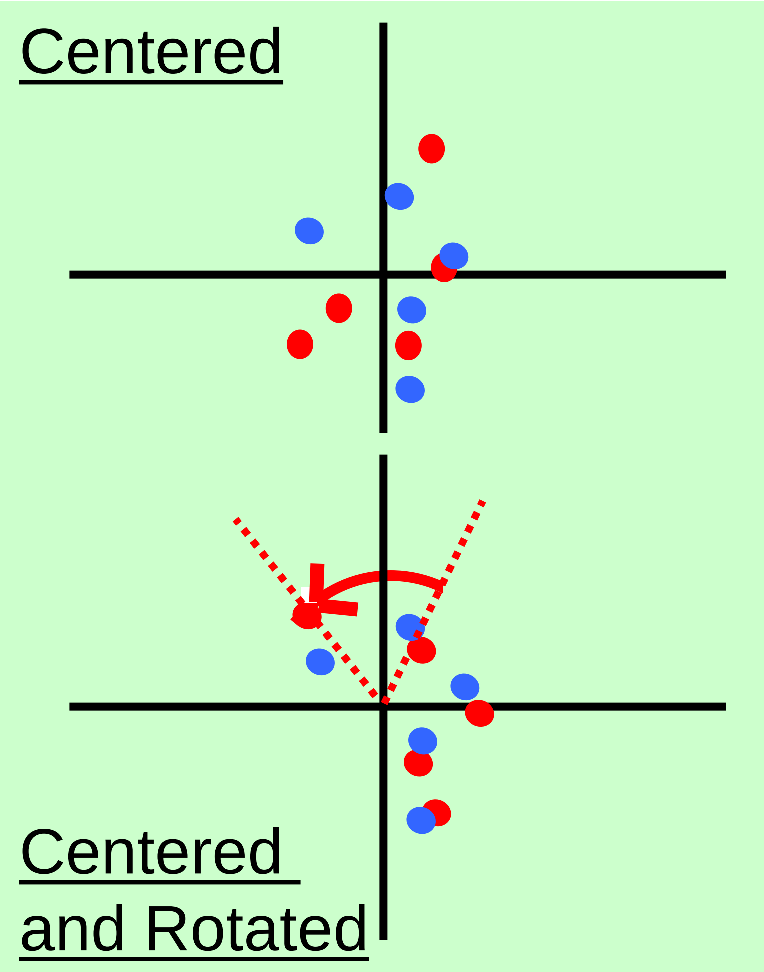
$$\mathbf{U}^T\mathbf{U} = \mathbf{V}^T\mathbf{V} = \mathbf{I}$$

$$\mathbf{L} = \mathrm{diag}(\sigma_1, \sigma_2)$$

$$\sigma_1 \geq \sigma_2 \geq 0$$

$$\boxed{\mathbf{R} = \mathbf{V}\,\mathbf{D}\,\mathbf{U}^T, \ \text{where}}$$

$$\mathbf{D} = \mathrm{diag}\left(1, \ \det(\mathbf{V}\mathbf{U}^{\mathrm{T}})\right)$$



H … 2x2 “prediction” matrix

Algorithm 1

Orthogonal Procrustes Problem

- Is the problem of finding an orthogonal matrix **R** that „rotates“ matrix **Q** into matrix **P**, such that Frobenius norm of **PR-Q** is minimized! [*]

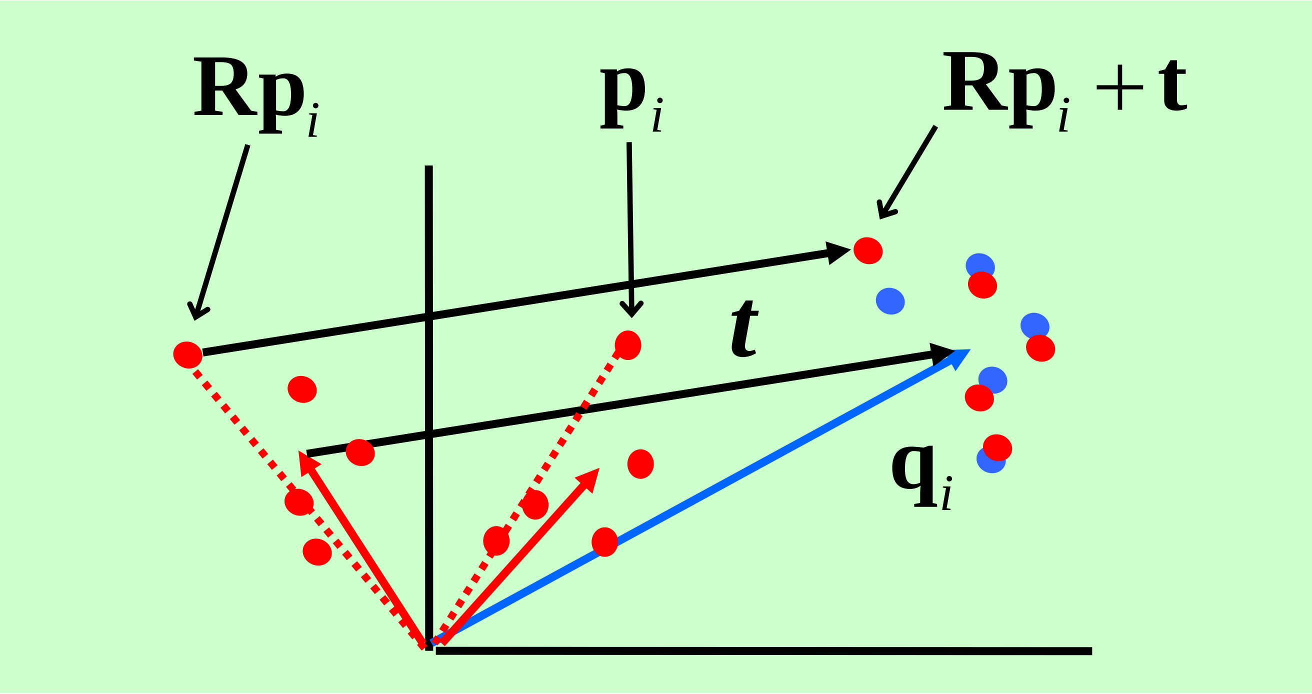
$$\min_{\mathbf{R}} \left\{ \left\| \mathbf{PR} - \mathbf{Q} \right\|_F \right\}$$
$$\text{s.t.}\ \mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}$$

- Equivalent to maximizing tr($\mathbf{R}^T\mathbf{P}^T\mathbf{Q}$), which is achieved by computing SVD of $\mathbf{P}^T\mathbf{Q}$ and taking $\mathbf{R} = \mathbf{V}\mathbf{U}^T$

[*] see e.g. Golub, van Loan: Matrix Computations

Solution Step 3: Determine translation

$$\mathbf{t} = \overline{\mathbf{q}} - \mathbf{R}\overline{\mathbf{p}}$$



Algorithm 1

Point-Based Registration

- Similarity Transformation:
  - Given two sets of *N* corresponding points **P** and **Q**, find the similarity transformation (scale factor s, rotation matrix **R** and translation vector **t**) that minimizes the mean squared distance between the points:

$$E_{\mathrm{Procr.,s}} = \frac{1}{N}\sum_{i=1}^{N} | s\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i |^2$$

Point-Based Similarity Registration

1. Remove mean of point sets
2. Calculate scale of point sets using e.g.

$$s_{\mathbf{p}} = \sqrt{\sum_{i=1}^{N} \left\| \mathbf{p}_i - \bar{\mathbf{p}} \right\|^2},\ s_{\mathbf{q}} = \sqrt{\sum_{i=1}^{N} \left\| \mathbf{q}_i - \bar{\mathbf{q}} \right\|^2}$$

3. Normalize both point sets w.r.t. scale e.g. $s_{\mathbf{p}}$ -> 1, $s_{\mathbf{q}}$ -> 1
4. Calculate Rotation **R** using Algorithm 1
5. Determine scale ratio s to undo scaling
6. Calculate t: $\boxed{\mathbf{t} = \bar{\mathbf{q}} - s\mathbf{R}\bar{\mathbf{p}}}$

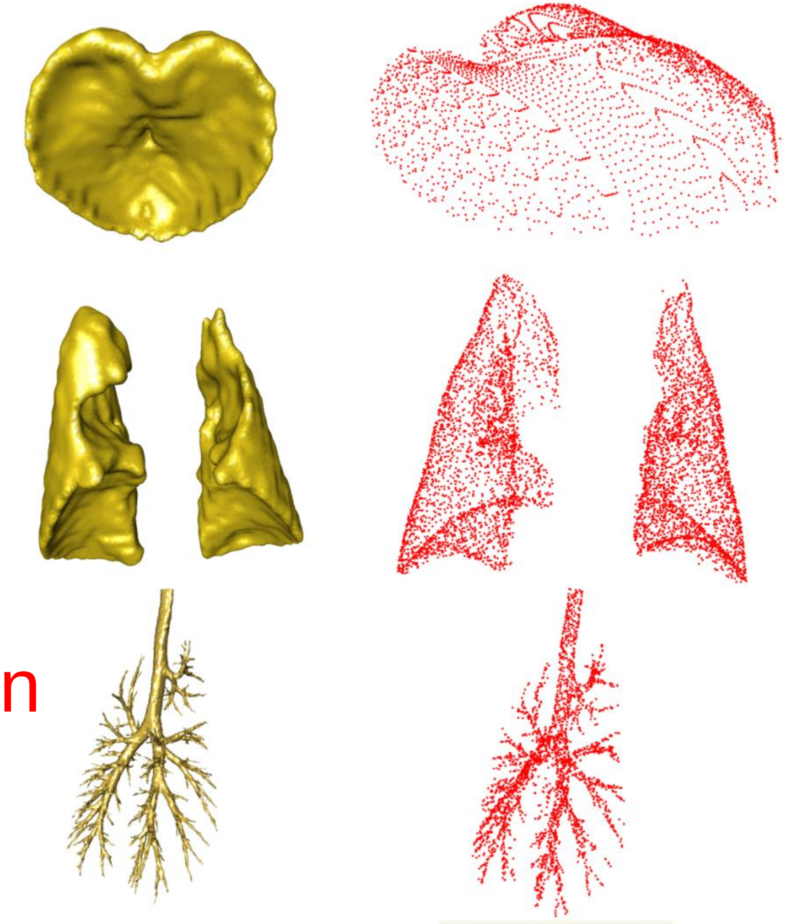
Summary Point-Based Registration

- Point-based registration:
  - Uses anatomical landmarks or extrinsic markers
  - Generally aligns small number of points
  - Requires knowledge of point correspondence

Surface-based rigid registration

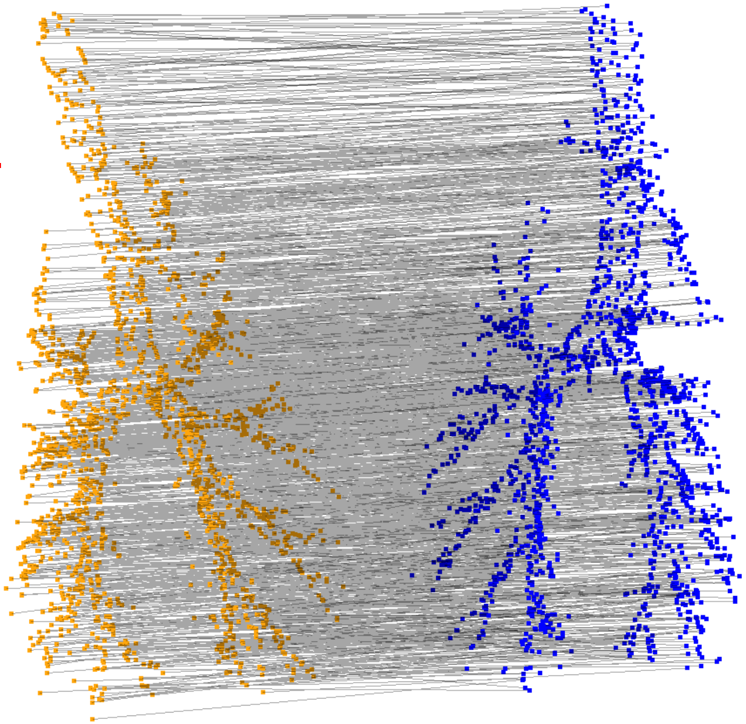
Surface-Based Registration

- Generally aligns large number of points
- The 3D boundary of an object is an intuitive and easily characterized geometrical feature that can be used for registration

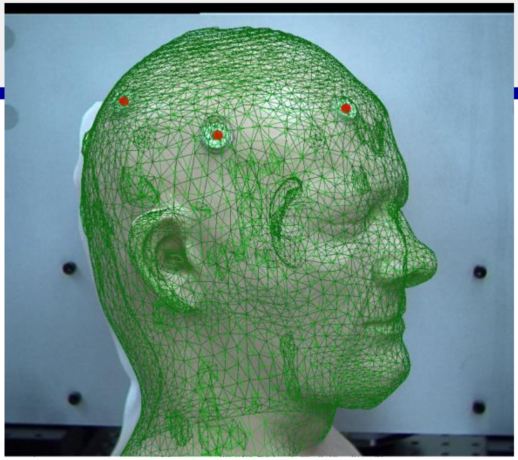


Surface-Based Registration

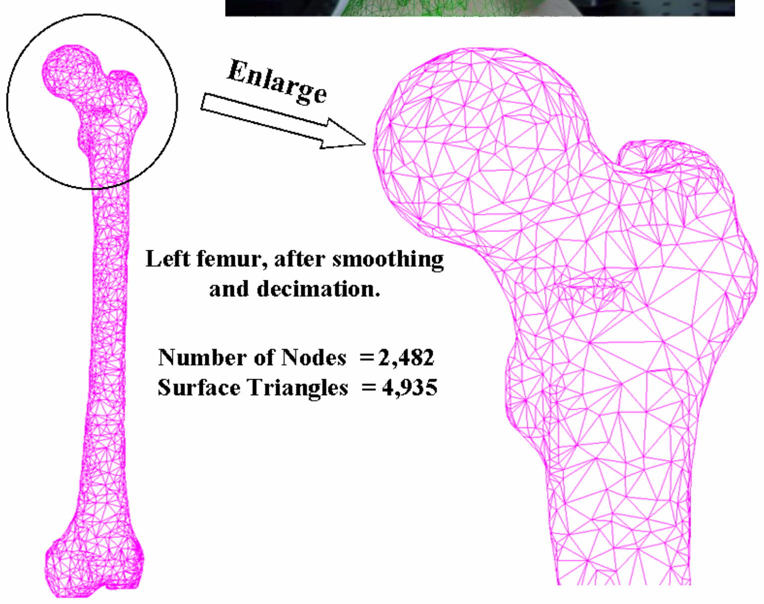
- Segmentation (surface extraction) necessary
  - Easy characterization does not necessarily mean easy segmentation!
- Surface-based methods involve determining corresponding surfaces in different images and/or physical space and finding a transformation that best aligns these surfaces



Typical Surfaces for Rigid Registration



- Skin surface (air-tissue interface)
- Bone surface (tissue-bone interface)
- Representations
  - Point set (collection of points on the surface)
  - Faceted surface, e.g., triangle set approximating surface (Indexed Face Set)
  - Implicit surface (analytically defined in higher-order space, e.g. level sets)



Surface-Based Registration

- Given a set of N_p surface points $\{\mathbf{p}_i\}$ and a surface Q, find the rigid-body transformation T (rotation matrix $\boldsymbol{R}$ and translation vector $\mathbf{t}$) that minimizes the mean squared distance between the points and the surface:

$$d(T) = \frac{1}{N_p} \sum_{i=1}^{N_p} |T(\mathbf{p}_i) - \mathbf{q}_i|^2$$

$$\mathbf{q}_i = C(T(\mathbf{p}_i), \mathrm{Q})$$

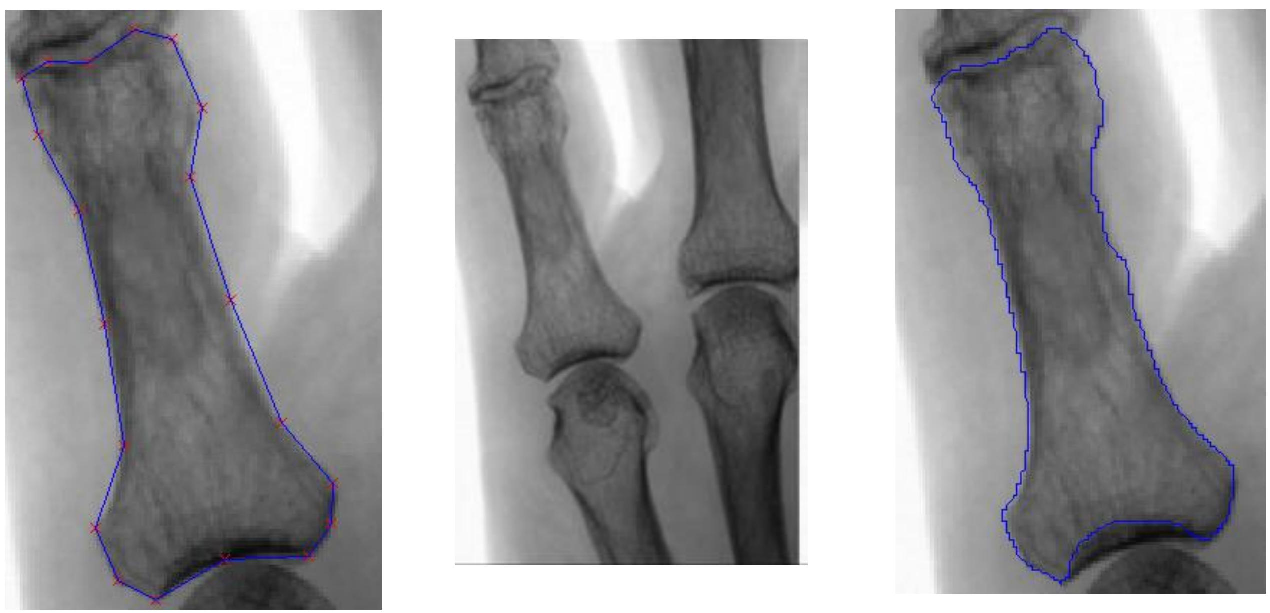
C … correspondence function

Iterative Closest Point (ICP) Registration

- Besl & McKay, PAMI, 1992
- 2 stage iterative algorithm
- Surface P: data shape & Surface Q: model shape

e.g. CAD model,
high-resolution scan

2D Example
(contour)



Iterative Closest Point Algorithm

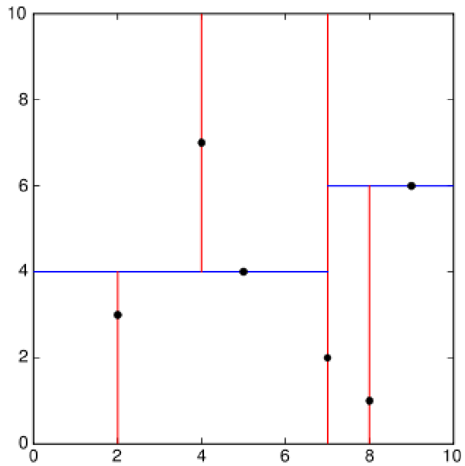
- To register data shape P to model shape *Q:*
  1. decompose *P* into point set { p_i }
  2. Establish initial registration T_0 : $p_i' = T_0 (p_i)$
  3. while not converged do repeat
     1. Compute set of closest points { q_i } on Q, i.e. those x that minimize $$d(\mathbf{p}_i', \mathrm{Q}) = \min_{q \in Q} \left\{ \left\| \mathbf{q} - \mathbf{p}_i' \right\|_2 \right\}$$
     2. Compute optimal registration of corresponding point sets { p_i } and { q_i } , this gives a transformation T_j
     3. Apply T_j to transform point set: $p_i' = T_j (p_i')$

HOW?

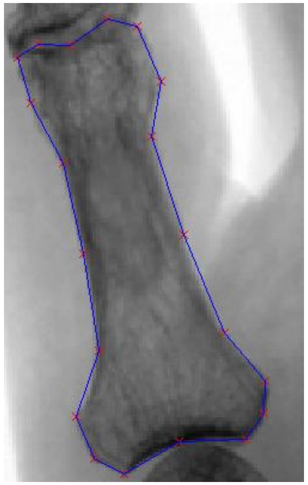
- Matlab illustration

Iterative Closest Point Algorithm

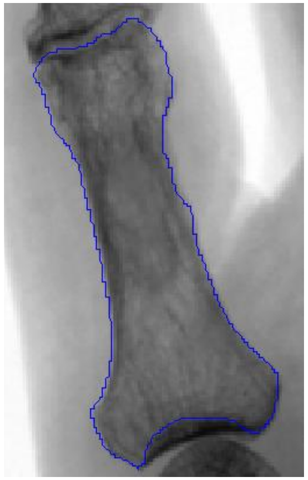
- Computation of closest points is O(N*M)
  - N … number of data shape points
  - M … complexity of model (~ # of model points)
- Large model data -> significant effort for locating closest points! ???
- Speedup by putting model into hierarchical data structures, e.g. quadtrees, octrees, kd-trees -> O(N*log M)



Iterative Closest Point Algorithm



P



Q

- Representation independent
  - Data shape P is converted to point set
  - Model shape Q can be arbitrary surface
  - Only restriction: distance point to surface must be efficiently computable
- Disadvantages:
  - Trapped in local minima -> several random initial guesses
  - Least squares norm not robust to outliers -> L1 norm or other robust norms
    - analytical solution of Procrustes alignment is lost

Summary Surface-Based Registration

- Surface-based registration:
  - Uses extracted surfaces either as point sets, triangle meshes or implicit/parametric surfaces
  - Segmentation is important issue
  - Iterative Closest Point algorithm wide-spread for affine/rigid registration

Homework

- Read paper Klein et al."elastix: A Toolbox for Intensity-Based Medical Image Registration„ (see subversion repository)
- Answer:

  1. Which transformation models are supported by the freely available elastix software?

  2. When optimizing the cost function, the authors propose a strategy to improve the smoothness of the cost function. How does this strategy look like?

  3. Which quality measure is used to evaluate the prostate registration experiment, and what is its meaning?

END

Thank you for your attention,

see you next week!