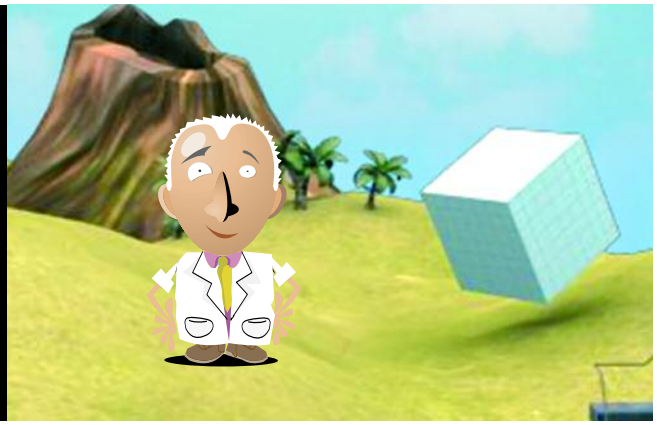
Christmas Lectures 2006

Fascinating fractals

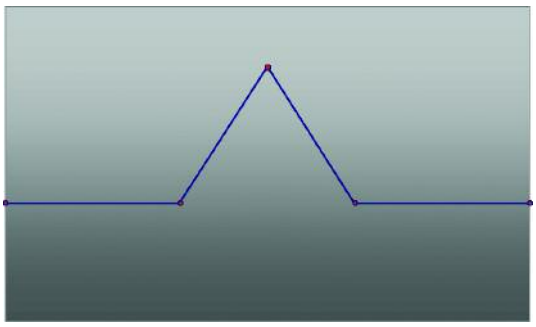


Part one – drawing fractals

Fractals are infinitely complicated shapes, and, as you'll see, it's never possible to finish drawing them. But we can draw closer and closer approximations using a simple method.

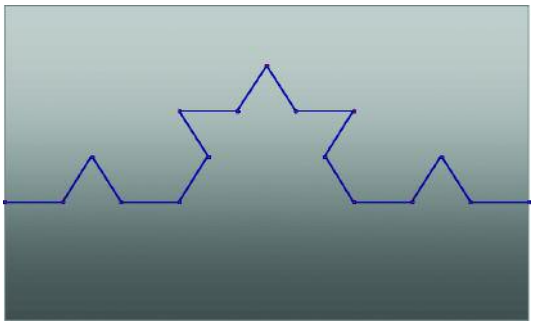
The first step is to draw a shape made up of straight lines and points. You could start with four lines of equal length:

First fractal



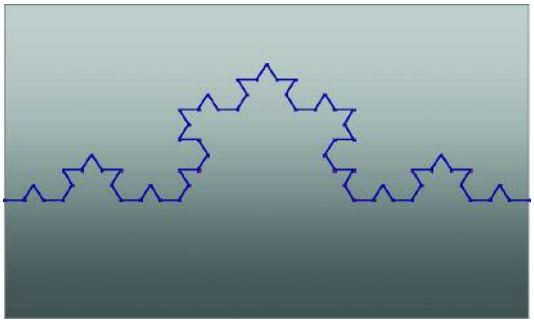
You've drawn your first fractal! Now, replace each of the four straight lines with a scaled-down version of the first fractal to get a second fractal:

Second fractal



In the second fractal there are 16 straight lines. Now, replace each of these with a scaled-down copy of the first fractal to get a third fractal:

Third fractal



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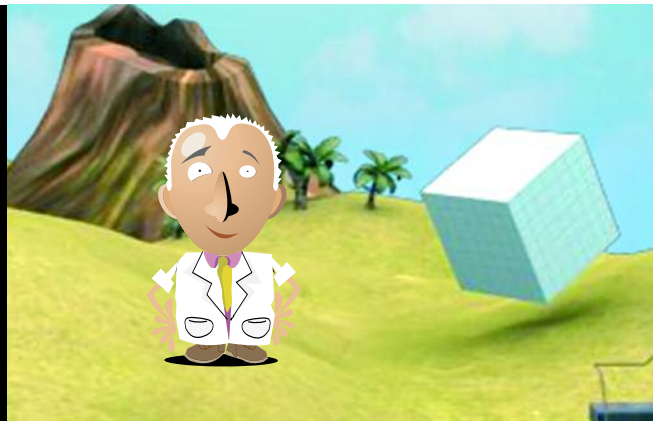
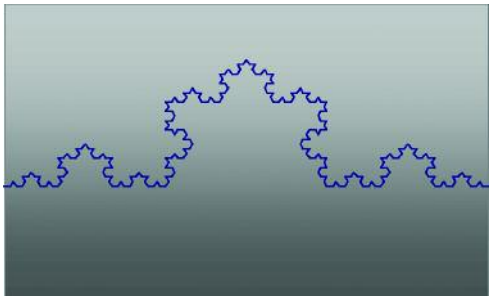
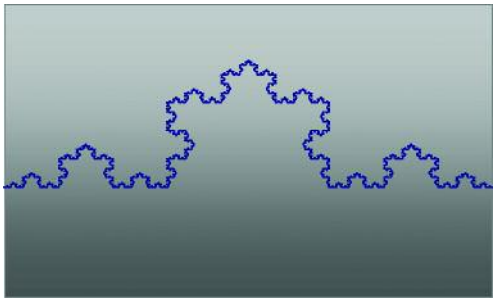


Simple isn't it? Now you've got the hang of it, here are the next two fractals that you would see:

Fourth fractal

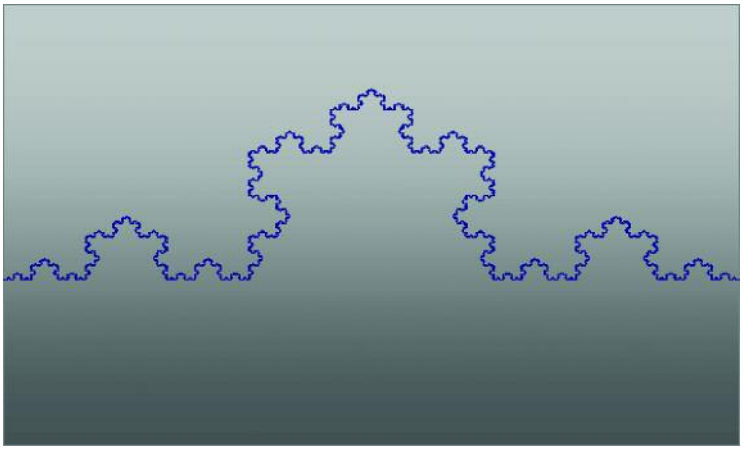


Fifth fractal



Eventually, you have to stop drawing because the changes you make are so small you simply cannot see them. This computer-generated picture contains 4,096 straight lines and shows the sixth fractal.

Sixth fractal

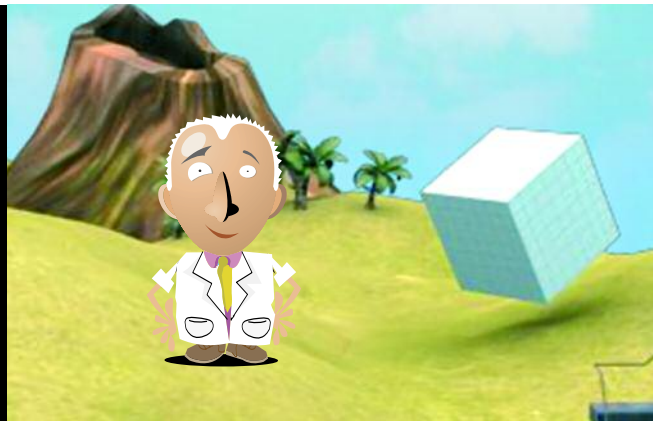


Imagine the length of each line in the first fractal was 1cm, by the sixth fractal the length of each line is just over a hundredth of a millimetre! Does this fractal remind you of anything in nature?

Can you see how there are parts of the picture that look like a small copy of the whole? It's called **self similarity**. It's an important property of fractals and one of the reasons that they are so interesting and so complicated.

We have only been able to draw up to the sixth stage, but we could continue forever. Can you imagine the final picture?

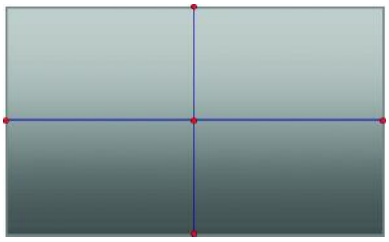
Fascinating fractals



OTHER FRACTAL SHAPES

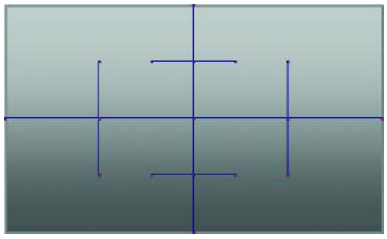
We can apply the same principle to other straight line drawings. For example, a simple cross:

First fractal

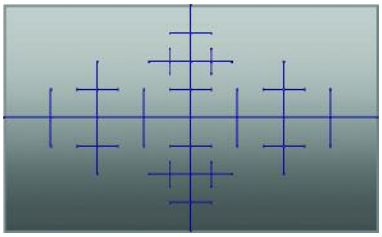


If we follow the rule 'whenever you see a straight line replace it with a copy of the original' we can see how this fractal will develop:

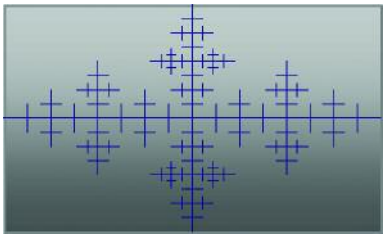
Second fractal



Third fractal

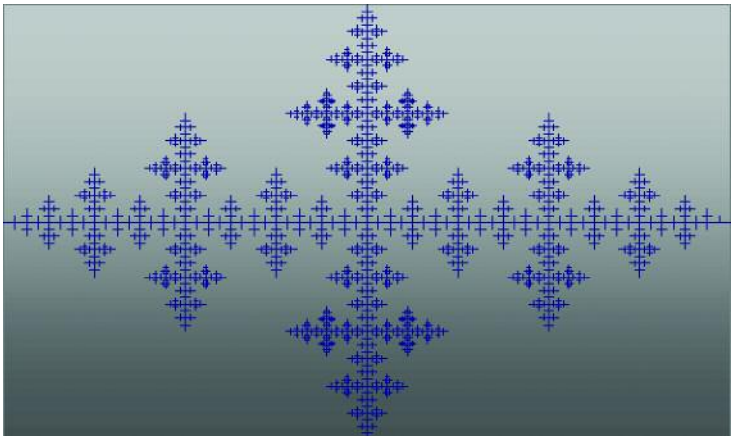


Fourth fractal



By the sixth stage it will look something like this:

Sixth fractal



You may have seen patterns like this embroidered onto tablecloths.

Why not create your own fractal shape? You can use pen and paper.
Web-based interactions like Shodor's Flake Maker
http://www.shodor.org/interactivate/activities/FlakeMaker/ are great because they let you see the stages of construction.