An Introduction to the Network Weight Matrix: Introduction to the Network Weight Matrix

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An Introduction to the Network Weight Matrix

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This study introduces the network weight matrix as a replacement for the spatial weight matrix to measure the spatial dependence between links of a network. This matrix stems from the concepts of betweenness centrality and vulnerability in network science. The elements of the matrix are a function not simply of proximity, but of network topology, network structure, and demand configuration. The network weight matrix has distinctive characteristics, which are capable of reflecting spatial dependence between traffic links: (1) elements are allowed to have negative and positive values capturing the competitive and complementary nature of links, (2) diagonal elements are not fixed to zero, which takes the self-dependence of a link upon itself into consideration, and (3) elements not only reflect the spatial dependence based on the network structure, but they acknowledge the demand configuration as well. We verify the network weight matrix by modeling traffic flows in a 3 × 3 grid test network with 9 nodes and 24 directed links connecting 72 origin-destination (OD) pairs. Models encompassing the network weight matrix outperform both models without spatial components and models with the spatial weight matrix. The network weight matrix represents a more accurate and defensible spatial dependency between traffic links, and offers the potential to augment traffic flow prediction.

Introduction

Network science has become a theoretical framework to understand complex and complicated systems. Particularly noticeable are the words “complex” and “complicated.” A network is complicated when it includes a large number of components, which follow well-defined rules with a limited range of responses to coincidental changes. A network is complex when its components are interconnected, and predicting their overall behavior is not possible from examining the behavior of individual components. In brief, complicated is the opposite of simple and complex is the opposite of independent. Complexity is an integral part of real-world networks and is witnessed in both man-made systems such as the Internet, roads, and railways, and in biological structures such as the brain and genetic networks.

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Roads comprise a complex network displaying self-organized and emergent properties. Transport scientists examine the behavior of its components through the lens of network structure. In particular, work has been aimed at developing methods to identify the interconnection between the components of a road network, and to understand whether and to what extent this interrelationship alleviates the transport network problems. Traffic forecasting was no exception.

Analysts typically tackled traffic forecasting problems using time series approaches (Vlahogianni et al. 2014). Interest in the spatial structure of traffic links opened a new gateway to traffic forecasting problems (Ermagun and Levinson 2016). As a matter of course, transport analysts could benefit from exploiting the spatial structure of traffic links to augment forecasting methods. Capturing the spatial dependence between traffic links has essentially remained untouched since the birth of spatial weight matrices. While now acknowledged in transport science, its roots are found in geography and regional science. A whole host of complex and nuanced approaches have gradually emerged for constructing spatial weight matrices. In spite of attempted remedies, a satisfactory solution has not yet been obtained for the proper creation of spatial weight matrices (Bavaud 1998).

Ermagun and Levinson (2016) systematically reviewed more than 130 studies that used spatial dependence between traffic links to augment traffic forecasting. The review categorizes the methods for capturing spatial dependence into two groups of naïve and modest. The former assumes the adjacent upstream and downstream links affect the study link and ignores the spatial information of the remaining links of the network. The latter uses two criteria to select the influential links: (1) correlation-coefficient assessment and (2) distance adjustment. The correlation-coefficient assessment probes deeply into the data to explore whether and to what extent the information of neighboring links is correlated with the study link. The highly correlated links are then selected as an input of forecasting methods, while an equal weight is assigned to each link. The distance adjustment, which is also known as "l^{th}-order neighbors," assumes the strength of the spatial dependence is reduced by increasing the distance. This is in line with Tobler's "first law of geography" (Tobler 1970). Proposed methods postulate two hypotheses: (1) traffic links are positively correlated and (2) near links are more related than distant links. The current study shows these are restrictive assumptions that misrepresent the flow of traffic links, given the competitive and complementary nature of links (Ermagun et al. 2016).

We introduce concepts, theories, and methods dealing with network weight matrices to bridge the gap between two strands of literature: network science and geospatial analysis. Having this introduction, the remainder of the article is structured as follows. The following section reviews the notion of the spatial weight matrix and its common structures in multifaceted disciplines. We then discuss betweenness centrality and vulnerability properties of the network. We subsequently propose the network weight matrix. We finally verify the network weight matrix and conclude the article by broaching a number of recommendations, which suggests future research.

Spatial weight matrix: A review of existing approaches

In geography and spatial science, spatial weight matrices are foundational building blocks of spatial analysis. A spatial weight matrix for a set of l units is a $l \times l$ matrix, which its components, $w_{i,j}$, regularly satisfy three major rules:

1. $w_{i,j} \geq 0$,
2. $w_{i,i}=0$, and
3. $\sum_{j=1}^{l} w_{i,j}=1$, for all $i=1,2,\ldots,l$ (Bavaud 1998).

Theoretically, $w_{i,j}$ expresses the strength of potential spatial dependence between unit i and unit j, and the greater the $w_{i,j}$ indicates the more dependency. Although the spatial weight matrix has capacity to capture the self-dependence of unit i upon itself, the matrix is typically employed with a zero diagonal matrix. Non-diagonal elements are measured by a number of methods (Getis 2009).

Pertaining to theory, Aldstadt and Getis (2006) argue that three viewpoints are classically inspired spatial analysts to structure a spatial weight matrix: (1) Benefiting from theoretical notion of spatial dependence, (2) benefiting from geometric indicators, and (3) benefiting from descriptive expression of the data. Pertaining to measurement methods, the first two viewpoints embrace two main categories: a distance and a boundary approach, respectively. Each category encompasses a wide spectrum of methods, including use of a contiguity matrix, fixed distance, or inverse distance. The third viewpoint derives its theoretical support from a familiar statistics and geostatistics rule: “Let the data speak for themselves” (Gould 1981). This viewpoint is superior to alternatives that are incapable of detecting the complexities of spatial dependence within the data (Aldstadt and Getis 2006). Use of the empirical variogram function is a well-known instance of this viewpoint. There is prolific literature discussing assorted structural forms of spatial weight matrices. Getis and Aldstadt (2010), for instance, summarize the previous attempts to create a spatial weight matrix. We hence eschew digging into proposed forms of spatial weight matrix in detail, and confine it to list the widely-used spatial weight matrices in Table 1.

Notwithstanding the positive elements and row standardization rules of spatial weight matrix are predominant, the literature has witnessed a few exceptions. In 2013, Bhattacharjee

Table 1. Spatial Weight Matrix Configurations

Weight method	Formulation	Definition
Boundary approach		
Contiguity	$w_{i,j}=\begin{cases} 1 : l_{ij} > 0 \\ 0 : l_{ij}=0 \end{cases}$	l_{ij}: length of shared boundary
Shared boundary	$w_{i,j}=\frac{l_{ij}}{\sum_{i\neq j} l_{ij}}$	l_{ij}: length of shared boundary
Distance approach		
Radial distance	$w_{i,j}=\begin{cases} 1 : 0 \leq d_{ij} \leq d \\ 0 : d_{ij} > d \end{cases}$	d_{ij}: distances between spatial units, d: distance threshold
Power distance	$w_{i,j}=d_{ij}^{\alpha}$	α: any positive value
Exponential distance	$w_{i,j}=\exp(-\alpha d_{ij})$	α: any positive value
Double-power distance	$w_{i,j}=\begin{cases} \left[1-\left(\frac{d_{ij}}{d}\right)^k\right] : 0 \leq d_{ij} \leq d \\ 0 : d_{ij} > d \end{cases}$	d_{ij}: distances between spatial units, d: distance threshold

and Jensen-Butler (2013) estimated a spatial weight matrix under structural constraint. They assumed the matrix is unknown and potentially asymmetric with both positive and negative off-diagonal elements. Likewise, in 2015, Chung and Hewings (2015) introduced a spatial weight matrix encompassing both negative and positive elements to examine the competitive and complementary relationship between regional economies. The matrix also has nonzero diagonal elements to capture the self-dependent nature of regions.

Knowledge of the appropriate spatial weight matrix is still rudimentary. The literature on creation of the spatial weight matrix leaves room to grow in extracting spatial dependence from data. While spatial analysts acknowledge the value of theoretical conceptualization toward examining the spatial dependence of phenomena that manifests itself in a spatial weight matrix, empirically convenient approaches are superseding theory (Getis and Aldstadt 2010). Consequently, creating a spatial weight matrix that is theoretically defensible has become troublesome in geography and spatial science. This research aims to fill the lacuna by introducing a novel network weight matrix. Constructing the network weight matrix requires understanding network topology and infrastructure. Accordingly, we represent the notion of link centrality and link vulnerability in the following sections as fundamental properties of a network.

Network topology: Centrality

In this section, we provide a brief historical overview of the centrality concept and measurements. Let us begin with a simple definition of a network (also called a graph in the mathematics literature). A network encompasses a set of edges and vertices (also known as links and nodes, respectively). Mathematically, a network is represented by an adjacency matrix, which in the simplest case is a square, binary, and symmetric matrix with the order of number of vertices. The element of this matrix is one, if there is an edge between corresponding vertices; otherwise, it is zero (Harary 1962). An important variant in transport applications is the directed network, in which links have a particular path direction between two nodes.

Large networks coalesce with numerous routes that use a single edge between two vertices. Flows on edges are not arbitrarily distributed over a network, but rather exhibit dependency. This leads to questions, including:

- What are the most central links of a network?
- What links have the most influence over others?
- Which link connections are most crucial to the functioning of a network?
- Will removing a link break down the network into smaller networks? If so, which links are those?

These questions are of primary interest since they concern crucial subjects, including network resilience and reliability (Sudakov and Vu 2008). To answer these questions, the concept of centrality was introduced in the 19th century (Bavelas 1948). In the discipline of network science, the definition of centrality originates from two distinct structural perspectives. The first perspective assigns the role of centrality based on closeness to every other link in a network. This perception stems from the idea that the closest link has more access to other links (Bavelas 1950). The second perspective is motivated by the idea that links are central to the degree they stand between other links on the path of flow (Freeman 1977). Accordingly, the medial character of such links facilitates the flow between origins and destinations.

Over the past decade, a number of centrality measures have emerged to capture the importance of the links (Borgatti 2005). The two most widely used measurements are degree centrality and betweenness centrality. Concretely, the following terms define and formulate these measurements.

(1) Degree centrality, which is also known as connectivity (a term which is also defined with other meanings in the literature, so we will not use it here), measures the number of links that connect to a specific link (Freeman 1978). In a network with l links, this is formally defined by equation (1). In this equation, $r(i,j)=1$ if link i and j are connected; otherwise, $r(i,j)=0$.

$$D(i)=\sum_{j=1}^{l} r(i,j) \tag{1}$$

(2) Betweenness centrality is formulated in light of the hypothesis that flow is passed from an origin to a destination only along a network path linking them. This concept was first proposed by Freeman (1977) in social network science. Pursuant to this hypothesis, the betweenness of link i in a network with n nodes is measured as equation (2). In this equation, P_{od} stands for the number of shortest paths between o and d, and P_{oid} represents the number of shortest paths between o and d that pass through link i.

$$B(i)=\sum_{o<d}^{n}\sum^{n}\frac{P_{oid}}{P_{od}} \tag{2}$$

The degree centrality is incapable of quantifying the network weight of a link for two main reasons. First, it is probable a link with high degree centrality is poorly connected to all other links in a network. Second, a link with few direct connections is not necessarily less important, since it may play a “cut-link” role in a network, and elimination of this breaks down the network into two disconnected pieces. This role is captured by betweenness centrality. As a result, we use the measure of betweenness centrality to quantify the network weight of links.

Network infrastructure: Vulnerability

Vulnerability indicates whether and to what extent degradation of a link affects the network topology and performance, irrespective of the probability of degradation. Network performance is typically assessed by network characteristics, including accessibility, mobility, connectivity, circuity, and centrality. Taylor and D’Este (2007) broadly discussed the concept of vulnerability, and defined vulnerability by using the notion of accessibility in the following terms:

- “*A network node is vulnerable if loss (or substantial degradation) of a small number of links significantly diminishes the accessibility of the node, as measured by a standard index of accessibility.*”
- “*A network link is critical if loss (or substantial degradation) of the link significantly diminishes the accessibility of the network or of particular nodes, as measured by a standard index of accessibility.*”

We define the criticality of the link using the notion of betweenness in the following:

> "*The criticality of the link is determined by the change in the betweenness of other links upon the elimination of the link*."

Consider a network $G(N, L)$, where N and L stand for a set of n nodes and a set of l links. Let us define $B(i, G(N, L))$ as betweenness centrality of link i in network $G(N, L)$, and $B(i, G(N, L - \{j\}))$ as betweenness centrality of link i following the elimination of link j from network $G(N, L)$. At the link level, the change in betweenness centrality of link i, stemming from the elimination of link j, indicates the spatial influence of link j on link i. This shift reveals the degree of spatial dependence between link j and link i. Accordingly, we introduce $\Delta B(ij)$ term denoting the change in betweenness centrality of link i, when network link j fails. In more formal terms, the $\Delta B(ij)$ term is formulated in equation (3).

$$\Delta B(ij) = B(i, G(N, L)) - B(i, G(N, L - \{j\})) \tag{3}$$

At the network level, we formulate the criticality index of the link j in a network per equation (4). The criticality index of link j indicates the change in betweenness centrality of network following the removal of link j from the network.

$$I_c(j) = \sum_{i=1}^{l} B(i, G(N, L))) - \sum_{i=1}^{l-1} B(i, G(N, L - \{j\}))) \tag{4}$$

Having the concepts of link centrality and link vulnerability, we discuss the creation of the network weight matrix in the next section. Specifically, we benefit from equation (3) to construct the network weight matrix.

Network weight matrix

Consider two links i and j in the network $G(N, L)$. We define the competitive and complementary links in a traffic network with the following terms:

Definition 1. Link i is complementary to link j, when an increase in the cost of link i decreases the flow of link j.

Definition 2. Link i is competitive with link j, when an increase in the cost of link i increases the flow of link j.

The traditional spatial weight matrices used in spatial analysis cannot capture the competitive nature of links, as all the components are positive. They are also simply structured in terms of either geometric or proximity indicators. These matrices suffer from a lack of robust traffic theory, and thereby cannot reflect the actual spatial dependence between traffic links.

We remedy this deficiency by introducing the network weight matrix, which allows the data to speak for themselves. The network weight matrix is derived from an algorithm, which is built from the concept of vulnerability in network science. The $\Delta B(ij)$ term stems from equation (3) as the corresponding component of network weight matrix. Accordingly, the components of the network weight matrix are allowed to take both negative and positive values. In

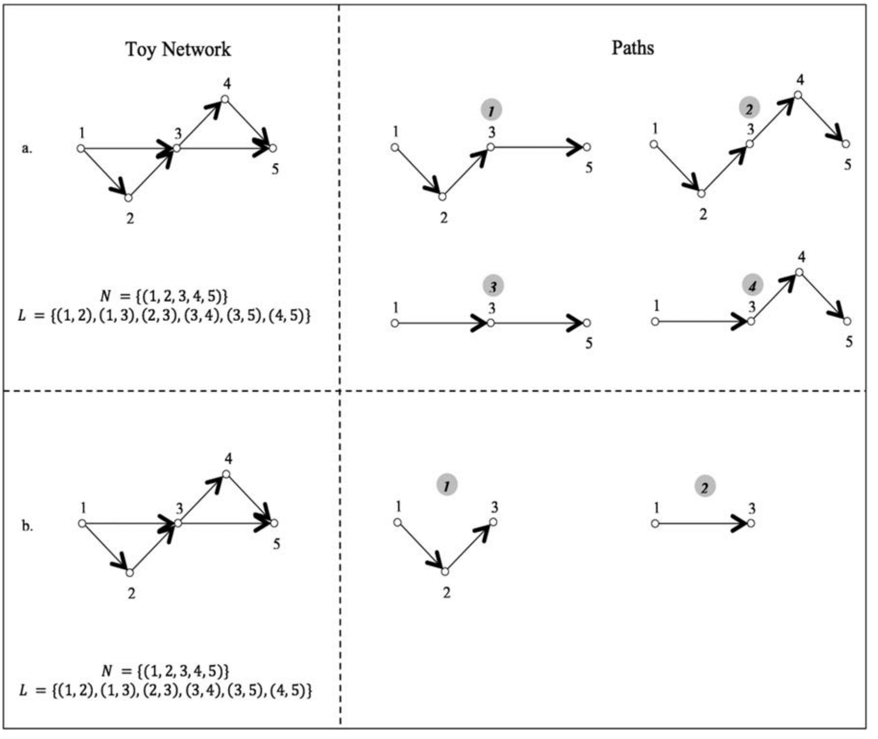

Figure 1. A toy network and its link path graph for Example 1.

addition, the weights in the matrix are a function not simply of proximity, but of network topology and demand configuration. To structure the network weight matrix, we postulate three major hypotheses:

Hypothesis 1. The spatial dependence of competitive links is negative, and more critical competitive links have a greater absolute spatial dependence weight in a network.

Hypothesis 2. The spatial dependence of complementary links is positive, and more critical complementary links have a greater spatial dependence weight in a network.

Hypothesis 3. Near links are not necessarily more related than distant links.

Let us consider two distinct toy networks $G(N, L)$, as classroom instances for understanding the creation process of the network weight matrix. For simplicity, we test different demand configurations to show the impact of demand configuration on network weight of links and test our hypotheses.

Example 1. The first instance encompasses five nodes and six links with equal travel costs. We consider two distinct demand configurations in this example. The first includes four routes, which pass the flow by directional links from node 1 to node 5. The second comprises two routes, which pass the flow by directional links from node 1 to node 3. Fig. 1 depicts the toy network and its distinct paths for these configurations.

Table 2. Betweenness Centrality Calculation for Example 1

Network scenario	1–5 OD Pair						1–3 OD Pair					
	(1,2)	*(1,3)*	*(2,3)*	*(3,4)*	*(3,5)*	*(4,5)*	*(1,2)*	*(1,3)*	*(2,3)*	*(3,4)*	*(3,5)*	*(4,5)*
All links	0	1	0	0	1	0	0	1	0	0	0	0
Without (1,2)	0	1	0	0	1	0	0	1	0	0	0	0
Without (1,3)	1	0	1	0	1	0	1	0	1	0	0	0
Without (2,3)	0	1	0	0	1	0	0	1	0	0	0	0
Without (3,4)	0	1	0	0	1	0	0	1	0	0	0	0
Without (3,5)	0	1	0	1	0	1	0	1	0	0	0	0
Without (4,5)	0	1	0	0	1	0	0	1	0	0	0	0

1-5 OD Pair

$$W_{ij} = \begin{array}{c} \text{Links} \\ (1,2) \\ (1,3) \\ (2,3) \\ (3,4) \\ (3,5) \\ (4,5) \end{array} \begin{array}{cccccc} (1,2) & (1,3) & (2,3) & (3,4) & (3,5) & (4,5) \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} & \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}$$

1-3 OD Pair

$$W_{ij} = \begin{array}{c} \text{Links} \\ (1,2) \\ (1,3) \\ (2,3) \\ (3,4) \\ (3,5) \\ (4,5) \end{array} \begin{array}{cccccc} (1,2) & (1,3) & (2,3) & (3,4) & (3,5) & (4,5) \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} & \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}$$

Figure 2. Network weight matrices for Example 1.

The network weight matrix for each demand configurations is structured in three steps:

Step 1. The betweenness centrality of all links is calculated when all links are in the network.

Step 2. Each link is removed from the network (only one at a time) and betweenness centrality is recomputed for the network with the missing link. The results are shown in Table 2.

Step 3. Equation (3) is used to compute each element of the network weight matrix. The network weight matrices for two distinct demand configurations are depicted in Fig. 2.

For illustration, looking at the very first row of the network weight matrix for 1–5 OD Pair in Fig. 2, it indicates there is no spatial dependence between link (1,2) and other links, when flow passes from origin 1 to destination 5. It is empirically true as removing link (1,2) does not change the path of flow on the network. Likewise, looking at the second row of this matrix, it shows the negative spatial dependence between link (1,3) and both link (1,2) and link (2,3). This reveals the competitive nature of links in line with our first hypothesis. It is empirically true as removing link (1,3) shifts the path of flow from link (1,3) to link (1,2), and thereby link (2,3) in the network. As expected, the network weight matrix is an asymmetric matrix since the toy network is directional. It is then not surprising that link (1,3) affects link (1,2), but not vice versa. The nature of this is illustrated by the centrality role of link (1,3) in passing flow between the given OD pair.

As shown in Fig. 2, the network weight matrices are dissimilar for two demand configurations. It emphasizes the essential role of demand configuration in spatial dependence between traffic links.

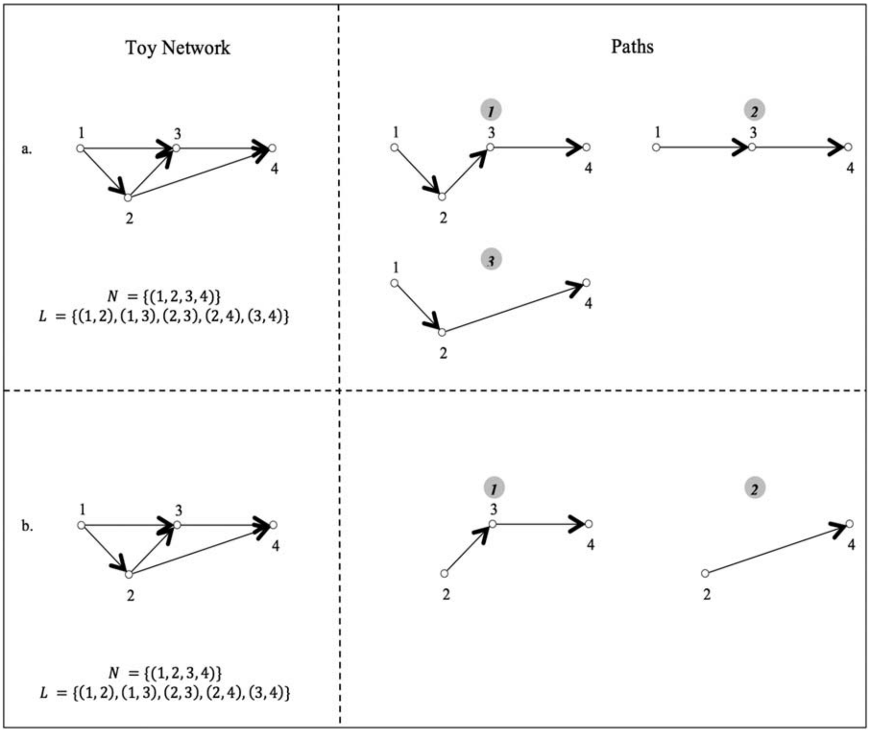

Figure 3. A toy network and its link path graph for Example 2.

Table 3. Betweenness Centrality Calculation for Example 2

Network scenario	1–4 OD Pair					2–4 OD Pair				
	(1,2)	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*	*(1,2)*	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*
All links	0	1	0	0	1	0	0	0	1	0
Without (1,2)	0	1	0	0	1	0	0	0	1	0
Without (1,3)	1	0	0	1	0	0	0	1	1	0
Without (2,3)	0	1	0	0	1	0	0	0	1	0
Without (2,4)	0	1	0	0	1	0	0	1	0	1
Without (3,4)	1	0	0	1	0	0	0	0	1	0

Example 2. The second instance encompasses four nodes and five links. The travel costs of links (1,2), (2,3), (1,3), and (3,4) are equal, while the travel costs of link (2,4) is $\sqrt{3}$ times of other links. Similar to Example 1, we test two distinct demand configurations in this example. The first includes three routes, which pass the flow by directional links from node 1 to node 4. The second comprises two routes, which pass the flow by directional links from node 2 to node 4. Fig. 3 represents the toy network and its distinct paths for these configurations.

We construct the network weight matrix for each demand configuration as two steps illustrated in Example 1. We show the results of step 1 and step 2 in Table 3 and Fig. 4, respectively. The network weight matrix for 1–4 OD pair in Fig. 4 reveals the competitive and

1-4 OD Pair

$W_{ij}=$

Links	(1,2)	(1,3)	(2,3)	(2,4)	(3,4)
(1,2)	0	0	0	0	0
(1,3)	-1	1	0	-1	1
(2,3)	0	0	0	0	0
(2,4)	0	0	0	0	0
(3,4)	-1	1	0	-1	1

2-4 OD Pair

$W_{ij}=$

Links	(1,2)	(1,3)	(2,3)	(2,4)	(3,4)
(1,2)	0	0	0	0	0
(1,3)	0	0	0	0	0
(2,3)	0	0	0	0	0
(2,4)	0	0	-1	1	-1
(3,4)	0	0	0	0	0

Figure 4. Network weight matrices for Example 2.

complementary nature of traffic links. Looking at the second row of this matrix, we find there is a negative spatial dependence between link (1,3) and both link (1,2) and link (2,4). It means removing link (1,3) shifts the original flow from path 2 to path 3, which is shorter than path 1. According to Hypothesis 2, the spatial dependence between link (1,3) and link (3,4) is positive, which demonstrates the complimentary nature of these links. It is illustrated by the fact that no flow passes through link (3,4) when link (1,3) is removed from the network.

As assumed, the network weight matrix comprises both negative and positive elements, which supports the dual competitive and complementary nature of traffic links. In addition, the more centrality of a link indicates the greater the network weight. These findings are in light of our first and second hypotheses. We also infer near links are not necessarily more related than distant links. For instance, link (1,3) in Fig. 1 is the same distance from links (1,2), (2,3), (3,4), and (3,5). In the case of 1–5 OD pair, while there is a spatial dependency between link (1,3) and both links (1,2) and (2,3), the dependency with links (3,4) and (4,5) is zero. Likewise, link (3,5) and link (3,4) are connected. The dependency between these links is −1 when demand flows from node 1 to node 5, and it is zero when demand flows from node 1 to node 3. These examples not only show how the topology of a network affects the spatial dependence between links, but how the configuration of demand, which shapes the flow on the network, manipulates this dependency.

The traffic network is a complex system, and this complexity forms a convoluted spatial dependence structure between traffic links. The complementary and competitive nature of links is not simply captured by looking only at the structure of the network. For instance, one may infer link (1,3) and link (3,5) in Example 1 are complementary since they are a series in the network. Likewise, the same is inferred for link (1,3) and link (3,4) in Example 2. While the former is not the case, the latter is correct. The demand configuration causes this complexity and deference. Unquestionably, the traditional spatial weight matrix is incapable of reflecting such a complicated spatial dependence structure, as it simply supposes the spatial dependence is a function of structural proximity. The following section verifies and validate the network weight matrix in a more complex simulation-based problem.

Network weight matrix verification: Simulation-based problem

To assess the capability of the network weight matrix over the spatial weight matrix, we consider a 3 × 3 complete grid test network with 9 nodes and 24 directed links connecting 72 OD pairs. We represent the schematic of the test network in Fig. 5.

Pertaining to the traffic network theory, travel demand in a network may be fixed or allowed to vary as a function of travel costs. Likewise, the travel costs for each traffic link may

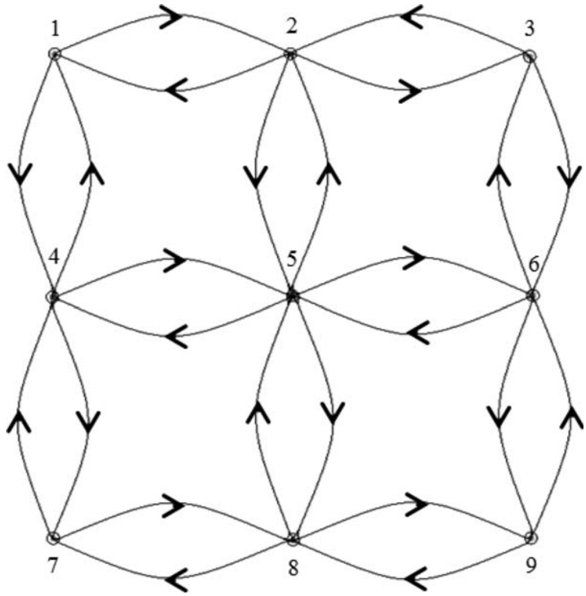

Figure 5. Simulation-based test problem.

be fixed or an increasing function of link flows (Boyce and Janson 1980). To solve the test network problem, we postulate a fundamental assumption: The traffic flow on each link is significantly lower than link capacity, and thereby no congestion exists in the network. Consequently, we relax both variable travel demand and variable link travel costs. The traffic assignment problem is then solved by the all-or-nothing assignment condition, since both the user equilibrium and the system-optimal assignment conditions collapse to the all-or-nothing assignment in the case of fixed link travel costs.

For simulation, we assume 100 units of demand for each OD pair, and randomly assign the cost of travel to each link. For the travel cost assignment, we generate a random decimal value between 1 and 1.5 using $a+(b-a)\times Rand()$ formula in Microsoft Excel. In this formula, a stands for the lower bound and b stands for the upper bound of random decimal values, while the $Rand()$ function generates a random decimal value between 0 and 1. The lower and upper bounds selected for the travel cost of each link are large enough to differ between traffic links, and small enough to prevent the removal of a link and its corresponding routes because of large cost differentials. The trips between origins and destinations are implicitly assigned to the minimum travel cost path. To conduct the network and flow analysis, we developed a script in the AIMSUN traffic simulation software package. To assure the stability of the analysis, we repeat the link cost assignment process and the flow analysis 100 times with different random seeds for the link costs. To make the simulation analysis replicable, we provided the travel cost assigned to each link for all 100 iterations in Appendix A1.

We construct the network weight matrix and spatial weight matrix of the test network. We use equation (3) to create the network weight matrix. To calculate the elements of the network weight matrix, we developed code in Matlab to use Dijkstra's algorithm (Dijkstra 1959) to find the shortest travel cost between each OD pair. To configure the spatial weight matrix, we use the first-order adjacency form that has been used in the literature of traffic flow analysis (Kamarianakis and Prastacos 2003; Cheng et al. 2012). In an undirected network, this is a

Network Weights Matrix

Links	(1,2)	(2,1)	(3,2)	(2,3)	(1,4)	(4,1)	(2,5)	(5,2)	(6,3)	(3,6)	(4,5)	(5,4)	(5,6)	(6,5)	(7,4)	(4,7)	(8,5)	(5,8)	(6,9)	(9,6)	(7,8)	(8,7)	(8,9)	(9,8)
(1,2)	6	0	0	1	-6	0	4	-1	-1	0	-5	0	-1	0	0	-1	0	1	0	0	-1	0	0	0
(2,1)	0	2	0	0	0	-2	-2	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0
(3,2)	0	1	8	0	0	-1	4	-3	2	-6	0	-1	0	-7	0	0	-1	0	0	1	0	0	0	-1
(2,3)	0	0	0	2	0	0	-2	0	-2	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0
(1,4)	-2	0	0	0	2	0	-2	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0
(4,1)	0	-6	-2	0	0	6	0	-4	-2	0	-1	4	0	1	1	0	0	0	0	-1	-1	0	0	1
(2,5)	4	-2	4	-4	-6	0	14	0	0	-8	-3	2	2	-6	0	-1	0	1	0	0	-1	0	0	0
(5,2)	-2	0	-2	0	0	-2	0	4	-2	0	1	0	-2	0	-1	0	1	0	0	0	1	0	0	0
(6,3)	0	0	2	-6	0	0	0	-8	8	0	0	0	4	-2	0	0	-2	0	0	2	0	0	0	-2
(3,6)	0	0	-2	0	0	0	-2	0	0	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0
(4,5)	-4	0	0	-1	0	-4	-2	1	1	0	5	0	2	0	0	-1	0	0	1	0	-1	0	-1	0
(5,4)	0	-7	-3	0	-3	4	2	-2	-3	0	0	14	0	2	-3	4	3	-5	0	-1	0	-7	0	1
(5,6)	0	0	0	-8	0	0	2	-6	4	-4	1	0	14	0	0	-1	2	-3	4	-2	-1	0	-6	0
(6,5)	0	-1	-3	0	0	1	-2	0	-3	0	0	2	0	5	0	1	0	1	-2	0	0	-1	0	-2
(7,4)	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	2	0	-2	0	0	0	-2	0	0	0
(4,7)	-1	0	0	0	1	0	-1	0	0	0	-2	4	0	0	0	7	0	-7	0	0	1	-6	0	0
(8,5)	0	0	-1	0	0	0	0	1	-1	0	-5	2	2	-3	-7	0	13	0	0	-6	4	-3	-3	3
(5,8)	1	0	1	0	-1	0	2	0	0	-1	0	-2	0	1	0	-3	0	5	-2	0	-3	0	0	-2
(6,9)	0	0	-1	0	0	0	-1	0	0	1	1	0	4	-1	0	-1	0	-5	6	0	-1	0	-6	0
(9,6)	0	0	1	0	0	0	0	-1	1	0	0	0	-2	0	0	0	-3	0	0	3	0	0	0	-3
(7,8)	0	0	0	0	0	0	0	0	0	0	-7	0	-1	0	-6	1	4	-2	-1	0	7	0	1	0
(8,7)	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-2	-2	0	0	0	0	2	0	0
(8,9)	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-2	0	-2	0	0	0	2	0
(9,8)	0	0	0	0	0	0	0	0	0	0	0	-1	0	-5	0	-1	3	-1	0	-5	0	1	0	5

Spatial Weights Matrix

Links	(1,2)	(2,1)	(3,2)	(2,3)	(1,4)	(4,1)	(2,5)	(5,2)	(6,3)	(3,6)	(4,5)	(5,4)	(5,6)	(6,5)	(7,4)	(4,7)	(8,5)	(5,8)	(6,9)	(9,6)	(7,8)	(8,7)	(8,9)	(9,8)
(1,2)	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
(2,1)	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
(3,2)	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
(2,3)	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
(1,4)	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
(4,1)	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
(2,5)	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0
(5,2)	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
(6,3)	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
(3,6)	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
(4,5)	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
(5,4)	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
(5,6)	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
(6,5)	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
(7,4)	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
(4,7)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
(8,5)	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
(5,8)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
(6,9)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
(9,6)	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
(7,8)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
(8,7)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
(8,9)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
(9,8)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0

Figure 6. Network and spatial weight matrices of the test network.

binary matrix, in which the non-zero elements signify two links directly connected by a node. In a directed network, the traffic flows in one direction, and a link influences its downstream links. Hence, the spatial weight matrix is asymmetric as it allows the impact in one direction (Cheng et al. 2012). We show both matrices in Fig. 6 for comparison. In all 100 example repetitions, the spatial weight matrix is a fixed matrix as it is formed based on the network structure. The network weight matrix, however, changes as it is a function of not only the network structure, but the link cost and thus the flow pattern. In the following subsection, we compare the network weight matrix and spatial weight matrix in spatial analysis. We also validate the capability of models comprising the network weight matrix to predict the traffic flow.

Calibration and model specification

The traffic flow on a link is not only related to the cost of the link, but the cost of other links in the network. This relation is captured by the network weight matrix. We assume reflecting the cost of other traffic links has the potential to improve the traffic flow prediction. In this section, we develop three distinct models to test this assumption.

Model 1. This model simply considers a linear relationship between traffic flow on each link and its corresponding travel cost.

Model 2. This model includes not only a linear relationship between traffic flow on each link and its corresponding travel cost, but the cost of other traffic links. To capture this spatial relationship, we test the first order adjacency spatial weight matrix as shown in Fig. 6.

Model 3. This model includes not only a linear relationship between traffic flow on each link and its corresponding travel cost, but the cost of other traffic links. To capture this spatial relationship, we test the network weight matrix as shown in Fig. 6.

We develop a bivariate linear regression (BLR) for Model 1, and a spatial cross-regressive for Model 2 and Model 3. The spatial cross-regressive model, also called spatial lag of X (SLX), takes the spatial lags of explanatory variables into consideration. This model allows us to examine the effects of travel cost spillovers. We use ordinary least squares (OLS) estimation, as it is the best linear unbiased estimator (BLUE) of the coefficients for both types of models. Comparing these models enables us to assess and verify the power of the proposed network weight matrix in traffic analysis. A point worthy of attention is that we do not aim to delve into an analysis of traffic flow and travel cost, but we intend to test whether and to what extent the network weight matrix augments spatial analysis of a traffic network. Hereafter, we assume a simple linear relationship between traffic flow and travel cost.

We represent the results of the models over 100 iterations in Table 4. In Model 1, the travel cost of a link is negatively correlated with the flow on that link, which is in line with our hypothesis. This variable is significant at the 90% confidence interval in all iterations. The goodness-of-fit of the model fluctuates between 0.089 and 0.583 with the average value of 0.290.

In Model 2, the travel cost of a link has a negative correlation with flow on that link. The coefficient of the spatial term varies from negative to positive values, and it is statistically insignificant in 83% of the cases. This indicates there is no steady spatial dependency between flow of a link and the cost of other links in our test network. In addition, comparing the goodness-of-fit of Model 1 and Model 2 does not show any model improvement. On average, adding the spatial term in the model augments the adjusted R^2 of the model by only 1%.

In Model 3, the coefficient of the travel cost has a negative sign and is almost always statistically significant at the 99% confidence interval. The goodness-of-fit of the model ranges from 0.435 to 0.950 with the average value of 0.713. The results disclose that using the weight matrix augments the adjusted R^2 of the model, on average, by about 40%.

To give the reader a better sense of comparing the general goodness-of-fit of the models in all iterations, we graph the value of the adjusted R^2 of the models in Fig. 7. The adjusted R^2 of Model 1 and Model 2 is almost equal in each iteration, which indicates the deficiency of spatial weight matrices in capturing spatial dependence between traffic links. Model 3 significantly

Table 4. Linear regression and spatial cross-regressive models for 100 iterations

Models	Model 1			Model 2			Model 3		
	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.
Intercept	1495.74	3222.17	2177.22	1462.79	3351.64	2073.791	2177.22	−1177.67	−101.88
t-test	2.82	7.04	4.50	1.99	7.25	4.09	4.50	−0.77	−7.90
Coefficient	−2126.79	−717.94	−1272.20	−2194.71	−703.99	−1278.24	−1272.20	−321.46	−119.27
t-test	−1.80	−5.76	−3.26	−1.71	−6.48	−3.24	−4.33	−21.03	−8.16
Coefficient	–	–	–	−94.07	282.95	49.56	–	–	–
t-test	–	–	–	0.09	3.79	0.64	–	–	–
Adj. R^2	0.089	0.583	0.290	0.062	0.669	0.304	0.435	0.950	0.713

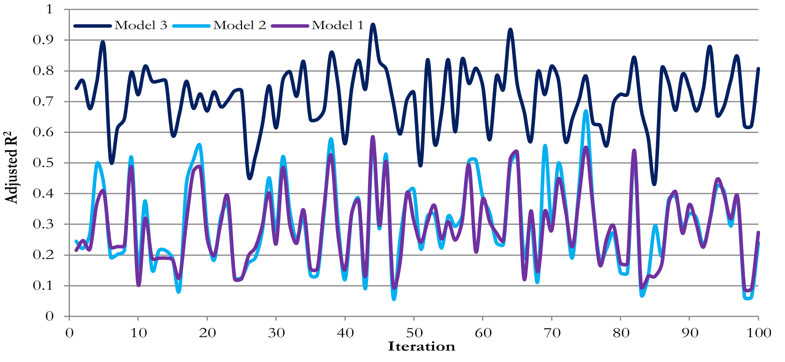

Figure 7. Adjusted R^2 of models in 100 iterations.

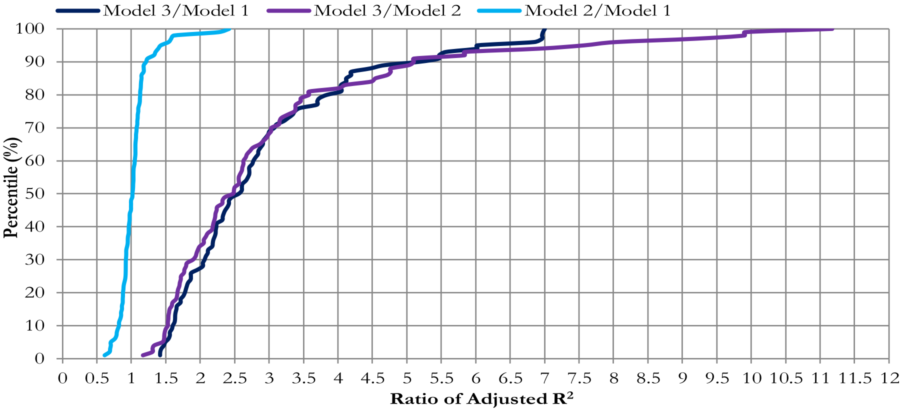

Figure 8. Cumulative density function for the ratio of Adjusted R^2 of models in 100 iterations.

performs better than the other models in all 100 iterations. Drilling down further, we calculate the ratio of the adjusted R^2 of the models to observe the magnitude of difference between the goodness-of-fit of the models. We depict the cumulative density function of these ratios in Fig. 8. Each line specifies the ratio of the adjusted R^2 of two selected models.

Looking at the light blue line, it is found that Model 2 outperforms Model 1 in 50% of cases as the ratio of the adjusted R^2 is greater than 1 in Fig. 8. This superiority is less than 1.5 times in 90% of situations. Model 1 also outperforms Model 2 in 50% of cases as the ratio of the adjusted R^2 is less than 1 in Fig. 8. Only in one of the iterations this superiority is more than 1.5 times. Looking at the purple line, it is found that Model 3 outperforms Model 2 in all 100 iterations. The adjusted R^2 of Model 3 is as great as 11 according to the right tail of the purple line. The excellence of Model 3 is more than 2 times in 65% of situations. The navy blue line shows the distribution of the ratio of the adjusted R^2 for Model 3 and Model 1. This distribution illustrates that Model 3 surpasses Model 1 in all 100 iterations. This supremacy is greater than 2 times in 80% of cases and greater than 4 times in 20% of cases. This leads to the conclusion that the network weight matrix provides a more accurate method to represent spatial dependency between traffic links, and has the potential to improve traffic flow prediction. It also emphasizes that the restrictive assumptions of the traditional spatial weight matrices overlook the realistic spatial dependence between traffic links, and result in failure of those types of traffic forecasting models in a network comprising both competitive and complementary links.

Concluding remarks

The spatial dependence between traffic links is potentially important but largely unexplored in traffic network analysis. To examine the existent spatial dependence in a traffic network, we propose a novel network weight matrix. This matrix stems from the concept of betweenness centrality and vulnerability in network science. We structure the network weight matrix to take into consideration not only network topology and infrastructure, but demand configuration. This study introduces a new way of viewing spatial traffic flow dependence in the context of network econometrics. The network weight matrix introduced in the current research has distinctive characteristic, which are capable of reflecting actual spatial dependence between traffic links:

- The elements are allowed to have negative and positive values, which capture competitive and complementary nature of links.
- The diagonal elements are not fixed to zero, which takes the self-dependence of link i upon itself into consideration.
- The elements not only reflect the spatial dependence based on the network topology and the costs of links, but they acknowledge the demand configuration as well.

The network weight matrices are theoretically defensible as they are based on the transport network theory. As the elements of the network weight matrix more closely reflect the dependence structure of the links, the weight matrix becomes more accurate and defensible. Not only do they capture the competitive and complementary nature of links, they embed additional network dynamics such as cost of links and demand configuration. This study begins a new line of research in examining actual spatial dependence between links by introducing a network weight matrix.

The following topics are recommended for further exploration:

- How does the network weight matrix illustrate the spatial correlation of traffic components in a real-world network?
- How does the network weight matrix augment the prediction power of the traditional traffic forecasting models?
- How does the performance of the network weight matrix change in a congested traffic network?

Although we tested the network weight matrix in a traffic network problem, it has potential for implementation in other disciplines of spatial and network science. We believe the network weight matrix has further applications in models of physical flow, but also social networks for which links may be either competitive or complementary with each other.

Table A1. Travel Cost Assigned to Each Link for all 100 Iterations of the Simulation Problem

	(1,2)	(2,1)	(3,2)	(2,3)	(1,4)	(4,1)	(2,5)	(5,2)	(6,3)	(3,6)	(4,5)	(5,4)	(5,6)	(6,5)	(7,4)	(4,7)	(8,5)	(5,8)	(6,9)	(9,6)	(7,8)	(8,7)	(8,9)	(9,8)
1	1.47	1.39	1.46	1.01	1.13	1.33	1.06	1.21	1.12	1.34	1.21	1.05	1.36	1.22	1.19	1.11	1.42	1.02	1.03	1	1.48	1.13	1.38	1.08
2	1.11	1.47	1.24	1.38	1.15	1.39	1.31	1.11	1.11	1.41	1.39	1.03	1.48	1.02	1.24	1.14	1.12	1.38	1.47	1.43	1.19	1.33	1.44	1.3
3	1.33	1.42	1.33	1.4	1.05	1.13	1.36	1.49	1.11	1.38	1.41	1.03	1.34	1.17	1.16	1.47	1.1	1.47	1.17	1.45	1.44	1.22	1.47	1.37
4	1.23	1.42	1.45	1.22	1.4	1.19	1.07	1.19	1.48	1.01	1.14	1.18	1.12	1.39	1.5	1.33	1.19	1.21	1.01	1.1	1.41	1.13	1.21	1.49
5	1.22	1.01	1.38	1.31	1.32	1.36	1.43	1.08	1.35	1.22	1.35	1.22	1.31	1.13	1.37	1.25	1.43	1.22	1.38	1.37	1.12	1.19	1.39	1.31
6	1.41	1.36	1.22	1.25	1.47	1.14	1.3	1.43	1.09	1.03	1.34	1.49	1.14	1.47	1.14	1.23	1.26	1.18	1.26	1.42	1.26	1.45	1.08	1.08
7	1.1	1.16	1.47	1.12	1.37	1.45	1.31	1.39	1.16	1.35	1.33	1.28	1.37	1.01	1.37	1.44	1.47	1.25	1.09	1.47	1.48	1.48	1.05	1.12
8	1.07	1.46	1.2	1.14	1.48	1.42	1.04	1.34	1.49	1.19	1.05	1.13	1.13	1.38	1.16	1.25	1.21	1.47	1.49	1.21	1.15	1.06	1.31	1.27
9	1.38	1.03	1.08	1.27	1.11	1.26	1.01	1.31	1.09	1.49	1.44	1.28	1.15	1.25	1.46	1.48	1.28	1.03	1.44	1.4	1.38	1.36	1.19	1.32
10	1.35	1.18	1.38	1.18	1.01	1.19	1.05	1.15	1.13	1.12	1.34	1.16	1.1	1.49	1.4	1.05	1.05	1	1.15	1.15	1.37	1.07	1.47	1.26
11	1.26	1.19	1.31	1.16	1.17	1.36	1.37	1.15	1.2	1.19	1.38	1.32	1.3	1.16	1.36	1.23	1.49	1.2	1.4	1.42	1.26	1.13	1.07	1.43
12	1.02	1.2	1.45	1.25	1.09	1.1	1.05	1.09	1.08	1.14	1.19	1.16	1.37	1.11	1.28	1.32	1.06	1.39	1.35	1	1.14	1.29	1.04	1.25
13	1.01	1.31	1.33	1.16	1.25	1.06	1.17	1.39	1.09	1.35	1.15	1.47	1.34	1.15	1.1	1.23	1.03	1.43	1.17	1.09	1.4	1.02	1.29	1.17
14	1.06	1.44	1.31	1.31	1.42	1.23	1.44	1.35	1.05	1.31	1.42	1.3	1.36	1.32	1.41	1.02	1.36	1.1	1.45	1.45	1.08	1.04	1.38	1.48
15	1.42	1.08	1.29	1.23	1.07	1.05	1.42	1.18	1.15	1.3	1.02	1.34	1.45	1.03	1.16	1.01	1.45	1.05	1.02	1.44	1.14	1.26	1.18	1.11
16	1.04	1.3	1.41	1.34	1.09	1.47	1.41	1.2	1.03	1.45	1.33	1.14	1.19	1.47	1.08	1.05	1.12	1.26	1.32	1.07	1.49	1.32	1.12	1.35
17	1.04	1.37	1.17	1.42	1.07	1.14	1.45	1.36	1	1.06	1.41	1.44	1.23	1.16	1.12	1.31	1.09	1.44	1.3	1.13	1.26	1.19	1.4	1.13
18	1.41	1.49	1.34	1.02	1.36	1.03	1.09	1.03	1.35	1.35	1.13	1.09	1.47	1.47	1.42	1.33	1.22	1.26	1.26	1.1	1.35	1.21	1.5	1.35
19	1.32	1.34	1.05	1.42	1.09	1.04	1.44	1.45	1.4	1.26	1.45	1.36	1.41	1.28	1.04	1.2	1.11	1.31	1.42	1.27	1.27	1.12	1.17	1.32
20	1.28	1.33	1.35	1.23	1.31	1.33	1.02	1.28	1.07	1.27	1.09	1.38	1.15	1.28	1.18	1.25	1.06	1.19	1.38	1.36	1.06	1.43	1.33	1.18
21	1.24	1.32	1.41	1.16	1.02	1.2	1.35	1.38	1.45	1.3	1.44	1.2	1.37	1.36	1.1	1.43	1.24	1.16	1.03	1.21	1.15	1.47	1.31	1.26
22	1.45	1.32	1.17	1.24	1.42	1.12	1.14	1.15	1.1	1.09	1.45	1.11	1.15	1.07	1.45	1.12	1.19	1.32	1.28	1.19	1.3	1.48	1.49	1.08
23	1.17	1.1	1.33	1.2	1.48	1.37	1.44	1.39	1.35	1.19	1.46	1.2	1.17	1.35	1.34	1.07	1.49	1.34	1.38	1.22	1.45	1.42	1.07	1.31
24	1.22	1.19	1.21	1.24	1.2	1.13	1.13	1.45	1.2	1.02	1.07	1.42	1.42	1.03	1.46	1.12	1.23	1.45	1.31	1.32	1.11	1.44	1.15	1.01
25	1.23	1.13	1.25	1.17	1.08	1.3	1.33	1.49	1.47	1.41	1.48	1.41	1.19	1.27	1.38	1.5	1.04	1.25	1.15	1.01	1.02	1.08	1.11	1.1
26	1.41	1.33	1.01	1.04	1.42	1.11	1.28	1.4	1.31	1.18	1.15	1.4	1.21	1.43	1.43	1.15	1.21	1.29	1.44	1.26	1.19	1.3	1.49	1.41
27	1.22	1.05	1.02	1.41	1.44	1.48	1.4	1.28	1.33	1.14	1.05	1.3	1.35	1.49	1.12	1.42	1.12	1.29	1.18	1.45	1.11	1.22	1.03	1.16

Table A1 *Continued*

	(1,2)	(2,1)	(3,2)	(2,3)	(1,4)	(4,1)	(2,5)	(5,2)	(6,3)	(3,6)	(4,5)	(5,4)	(5,6)	(6,5)	(7,4)	(4,7)	(8,5)	(5,8)	(6,9)	(9,6)	(7,8)	(8,7)	(8,9)	(9,8)
28	1	1.45	1.27	1.05	1.18	1.49	1.22	1.01	1.49	1.02	1.19	1.07	1.43	1.44	1.07	1	1.46	1.09	1.08	1.06	1.01	1.27	1.34	1.13
29	1.15	1.06	1.25	1.16	1.33	1.45	1.08	1.19	1.12	1.43	1.12	1.45	1.11	1.09	1.15	1.31	1.3	1.2	1.05	1.3	1.11	1.12	1.5	1.35
30	1.18	1.5	1.35	1.21	1.5	1.43	1.03	1.33	1.22	1.09	1.23	1.22	1.1	1.12	1.22	1.04	1.06	1.43	1.48	1.12	1.02	1.08	1.07	1.28
31	1.48	1.19	1.38	1.12	1.01	1.32	1.38	1.07	1.26	1.3	1.47	1.44	1.36	1.3	1.38	1.06	1.37	1.39	1.36	1.13	1.3	1.24	1.11	1.3
32	1.31	1.2	1.45	1.17	1.04	1.06	1.35	1.47	1.16	1.15	1.04	1.07	1.15	1.47	1.49	1.31	1.07	1.37	1.43	1.21	1.38	1.08	1.47	1.43
33	1.35	1.14	1.42	1.49	1.04	1.04	1.38	1.33	1.35	1.1	1.27	1.09	1.01	1.44	1.09	1.28	1.44	1.17	1.45	1.02	1.31	1.12	1.03	1.42
34	1.19	1.24	1.06	1.4	1.48	1.49	1.17	1.43	1.34	1.33	1.43	1.36	1.04	1.16	1.09	1.11	1.25	1.48	1.1	1.13	1.07	1.36	1.5	1.28
35	1.05	1.05	1.09	1.06	1.11	1.2	1.07	1.13	1.39	1.22	1.05	1.24	1.23	1.01	1.34	1.09	1.21	1.27	1.48	1.15	1.29	1.08	1.23	1.05
36	1.03	1.22	1.29	1.44	1.15	1.42	1.47	1.19	1.45	1.46	1.35	1.09	1.08	1.27	1.33	1.38	1.16	1.46	1.15	1.06	1.1	1.43	1.13	1.12
37	1.09	1.22	1.01	1.06	1.04	1.35	1.1	1.15	1.11	1.18	1.21	1.33	1.27	1.09	1.09	1.11	1.31	1.06	1.02	1.37	1.37	1.07	1.43	1.28
38	1.48	1.4	1.08	1.17	1.18	1.3	1.13	1.12	1.27	1.36	1.15	1.05	1.02	1.14	1.5	1.13	1	1.38	1.41	1.22	1.48	1.37	1.31	1.27
39	1.42	1.24	1.19	1.18	1.26	1.11	1.23	1.08	1.29	1.47	1.24	1.43	1.25	1.09	1.13	1.09	1.36	1.05	1.06	1.08	1.28	1.36	1.41	1.47
40	1.41	1.04	1.41	1.33	1.03	1.35	1.01	1.24	1.18	1.02	1.3	1.26	1.41	1.11	1.3	1.24	1.31	1.34	1.02	1.15	1.46	1.09	1	1.39
41	1.3	1.01	1.31	1.09	1.39	1.29	1.29	1.4	1.27	1.44	1.38	1.44	1.13	1.44	1.38	1.49	1.19	1.36	1.03	1.4	1.13	1.02	1.39	1.46
42	1.02	1.1	1.42	1.17	1.16	1.32	1.09	1.18	1.24	1.02	1.44	1.08	1.37	1.17	1.24	1.17	1.39	1.08	1.23	1.24	1.01	1.28	1.13	1.41
43	1.01	1.35	1.01	1.17	1.04	1.28	1.11	1.1	1	1.13	1.38	1.09	1.12	1.34	1.12	1.26	1.38	1.14	1.24	1.19	1.25	1.47	1.04	1.04
44	1.2	1.49	1.21	1.39	1.37	1.37	1.01	1.27	1.06	1.38	1.24	1.14	1.07	1.14	1.35	1.29	1.01	1.45	1.47	1.36	1.28	1.32	1.47	1.48
45	1.15	1.4	1.04	1.46	1.06	1.48	1.14	1.25	1.13	1.24	1.32	1.03	1.1	1.09	1.31	1.49	1.33	1.11	1.49	1.1	1.02	1.17	1.24	1.34
46	1.26	1.43	1.04	1.13	1.3	1.25	1.5	1.42	1.25	1.02	1.31	1.45	1.41	1.29	1.31	1.5	1.25	1.25	1.46	1.26	1.02	1.27	1.37	1.47
47	1.38	1.16	1.18	1.04	1.27	1.12	1.02	1.1	1.14	1.11	1.27	1.46	1.18	1.05	1.01	1.19	1.2	1.14	1.49	1.14	1.1	1.33	1.19	1.06
48	1.39	1.06	1.34	1.35	1.3	1.06	1.22	1.03	1.26	1.3	1.31	1.11	1.48	1.26	1.4	1.16	1.09	1.34	1.39	1.29	1.07	1.03	1.34	1.4
49	1.28	1.02	1.45	1.37	1.23	1.4	1.31	1.11	1.32	1.49	1.17	1.31	1.11	1.11	1.21	1.13	1.25	1.4	1.49	1.11	1.2	1.04	1.12	1.01
50	1.35	1.32	1.43	1.19	1.32	1.37	1.3	1.14	1.41	1.47	1.17	1.01	1.21	1.46	1.31	1.17	1	1.09	1.43	1.06	1.38	1.25	1.23	1.49
51	1.16	1.22	1.15	1.35	1.13	1.46	1.43	1.15	1.15	1.32	1.38	1.3	1.19	1.3	1.27	1.03	1.27	1.01	1.4	1.18	1.29	1.03	1.34	1.43
52	1.36	1.15	1.49	1.07	1.28	1.32	1.44	1.16	1.43	1.05	1.15	1.19	1.43	1.05	1.13	1.02	1.09	1.25	1.43	1.26	1.14	1.37	1.16	1.15
53	1.23	1.2	1.41	1.38	1.38	1.4	1.33	1.25	1.15	1.25	1.32	1.33	1.46	1.42	1.46	1.03	1.24	1.3	1.44	1.12	1.22	1.15	1	1.24
54	1.18	1.43	1.08	1.1	1.26	1.42	1.36	1.16	1.49	1.23	1.22	1.36	1.45	1.4	1.35	1.06	1.17	1.28	1.22	1.29	1.45	1.23	1.11	1.11
55	1.25	1.06	1.35	1.17	1.08	1.25	1.29	1.06	1.05	1.32	1.37	1.13	1.02	1.41	1.2	1.11	1.03	1.25	1.3	1.26	1.36	1.31	1.2	1.13
56	1.33	1.07	1.05	1.29	1.03	1.05	1.26	1.02	1.04	1.23	1.22	1.11	1.31	1.5	1.37	1.25	1.23	1.11	1.27	1.49	1.41	1.33	1.46	1.32
57	1.09	1.28	1.39	1.33	1.45	1.42	1.16	1.48	1.38	1.26	1.45	1.39	1.37	1.39	1.43	1.26	1.1	1.44	1.45	1.24	1.01	1.02	1.37	1.15
58	1.34	1	1.4	1.27	1.14	1.2	1.49	1.29	1.38	1.13	1.07	1.17	1.01	1.08	1.04	1.12	1	1.41	1.2	1.16	1.27	1.39	1.3	1.27

Table A1 *Continued*

	(1,2)	(2,1)	(3,2)	(2,3)	(1,4)	(4,1)	(2,5)	(5,2)	(6,3)	(3,6)	(4,5)	(5,4)	(5,6)	(6,5)	(7,4)	(4,7)	(8,5)	(5,8)	(6,9)	(9,6)	(7,8)	(8,7)	(8,9)	(9,8)
59	1.37	1.42	1.49	1.25	1.38	1.22	1.44	1.17	1.16	1.46	1.2	1.25	1.13	1.5	1.41	1.43	1.1	1.08	1.14	1.36	1.29	1.3	1.11	1.5
60	1.45	1.38	1.12	1.17	1.43	1.34	1.02	1.05	1.18	1.33	1.11	1.11	1.4	1.38	1.34	1.37	1.28	1.31	1.09	1.29	1.46	1.01	1.43	1.39
61	1.13	1.43	1.36	1.3	1.21	1.04	1.27	1.3	1.3	1.4	1.2	1.1	1.38	1.02	1.22	1.36	1.02	1.3	1.4	1.24	1.47	1.16	1.45	1.08
62	1.26	1.11	1.08	1.22	1.03	1.12	1.29	1.38	1.17	1.21	1.36	1.01	1.47	1.11	1.3	1.44	1.08	1.3	1.11	1.35	1.22	1.43	1.23	1.02
63	1.23	1.26	1.5	1.4	1.01	1.08	1.19	1.17	1.34	1.08	1.2	1.46	1.13	1.14	1.26	1.34	1.02	1.34	1.46	1.16	1.18	1.12	1.11	1.33
64	1.34	1.07	1.29	1.06	1.3	1.17	1.2	1.06	1.4	1.08	1.26	1.45	1.47	1.41	1.43	1.19	1.03	1.18	1.26	1.07	1.16	1.12	1.14	1
65	1.06	1.15	1.34	1.43	1.03	1.1	1.17	1.23	1.44	1.18	1.28	1.48	1.14	1.13	1.03	1	1.32	1.04	1.43	1.29	1.41	1.21	1.18	1.24
66	1.46	1.3	1.31	1.19	1.45	1.32	1.27	1.47	1.49	1.41	1.43	1.17	1.17	1.34	1.27	1.45	1.5	1.18	1.16	1.36	1.02	1.18	1.3	1.48
67	1.17	1	1.45	1.15	1.49	1.38	1.31	1.44	1.26	1.02	1.09	1.26	1.48	1.27	1.39	1.17	1.44	1.31	1.38	1.44	1.4	1.3	1.08	1.22
68	1.34	1.47	1.03	1.4	1.44	1.03	1.21	1.21	1.36	1.06	1.32	1.47	1.08	1.47	1.06	1.06	1.07	1.08	1.12	1.43	1.43	1.26	1.37	1.36
69	1.34	1.05	1.46	1.03	1	1.13	1.4	1.41	1.08	1.18	1.22	1.05	1.13	1.05	1.28	1.29	1.42	1.01	1.22	1.3	1.27	1.31	1.39	1.43
70	1.04	1.39	1.42	1.1	1.24	1.36	1.22	1.4	1.38	1.48	1.3	1.34	1.31	1.05	1.37	1.32	1.16	1.07	1.47	1.07	1.09	1.12	1.47	1.26
71	1.46	1.21	1.05	1.42	1.07	1.25	1.35	1.13	1.37	1.14	1.39	1.43	1.5	1.28	1.09	1.33	1.41	1.44	1.19	1.03	1.21	1.21	1.14	1.19
72	1.3	1.33	1.42	1.03	1.44	1.36	1.32	1.47	1.37	1.42	1.32	1.29	1.45	1.49	1.16	1.15	1.19	1.34	1.49	1.25	1.21	1.31	1.12	1.08
73	1.05	1.44	1.12	1.12	1.02	1.06	1.47	1.17	1.25	1.36	1.11	1.5	1.27	1.03	1.32	1.41	1.43	1.4	1.22	1.23	1.24	1.14	1.3	1.25
74	1.38	1.05	1.2	1.49	1.38	1.18	1.05	1.43	1.06	1.44	1.08	1.18	1.01	1.3	1.49	1.1	1.46	1.42	1.45	1.34	1.22	1.13	1.35	1.42
75	1.24	1.28	1.01	1.08	1.25	1.27	1.01	1.1	1.24	1.28	1.37	1.2	1.19	1.42	1.48	1.12	1.07	1.05	1.22	1.39	1.19	1.27	1.23	1.26
76	1.05	1.39	1.25	1.03	1.48	1.48	1.46	1.39	1.29	1.05	1.3	1.07	1.3	1.49	1.32	1.27	1.35	1.09	1.01	1.17	1.08	1.11	1.08	1.18
77	1.37	1.36	1.37	1.5	1.46	1.21	1.38	1.41	1.31	1.06	1.07	1.19	1.04	1.34	1.36	1.04	1.45	1.06	1.04	1.38	1.19	1.26	1.2	1.16
78	1.15	1.44	1.09	1.16	1.33	1.13	1.47	1.23	1.09	1.18	1.01	1.39	1.43	1.26	1.17	1.08	1.21	1.16	1.48	1.35	1.09	1.4	1.09	1.33
79	1.02	1.18	1.4	1.12	1.3	1.49	1.2	1.25	1.34	1.21	1.39	1.47	1.28	1.32	1.39	1.16	1.23	1.27	1.33	1.37	1	1.31	1.16	1.37
80	1.11	1.08	1.11	1.17	1.27	1.15	1.06	1.4	1.44	1.16	1.11	1.3	1.28	1.1	1.27	1.08	1.06	1.15	1.45	1.21	1.4	1.18	1.08	1.04
81	1.15	1.26	1.25	1.45	1.21	1.05	1.34	1.43	1.03	1.31	1.18	1.25	1.27	1.25	1.26	1	1.44	1.07	1.27	1.31	1.41	1.23	1.45	1.41
82	1.41	1.36	1.1	1.26	1.08	1.41	1.36	1.24	1.21	1.45	1.1	1.12	1	1.44	1.29	1.27	1.09	1.49	1.38	1.09	1.25	1.44	1.39	1.27
83	1.02	1.49	1.18	1.21	1.39	1.16	1.36	1.33	1.15	1.17	1.01	1.4	1.25	1.4	1.26	1.24	1.37	1.22	1.09	1.41	1.09	1.44	1.34	1.49
84	1.32	1.05	1.43	1.45	1.25	1.37	1.16	1.47	1.27	1.23	1.01	1.12	1.5	1.46	1.11	1.23	1.22	1.05	1	1.18	1.42	1.37	1.18	1.44
85	1.5	1.14	1.41	1.03	1.46	1.13	1.28	1.48	1.11	1.37	1.42	1.4	1.41	1.27	1.3	1.38	1.03	1.07	1.14	1.33	1.25	1.2	1.13	1.22
86	1.42	1.45	1.38	1.04	1.11	1.41	1.3	1.25	1.34	1.18	1.4	1.29	1.06	1.24	1.33	1.17	1.2	1.32	1.14	1.45	1.37	1.41	1.18	1.33
87	1.23	1.01	1.07	1.34	1.45	1.21	1.03	1.29	1.35	1.4	1.31	1.32	1.21	1.23	1.08	1.06	1.14	1.39	1.41	1.46	1.25	1.17	1.27	1.42
88	1	1.29	1.1	1.16	1.24	1.5	1.18	1.07	1.31	1.18	1.09	1.18	1.29	1.24	1.38	1.21	1.39	1.48	1.11	1.03	1.13	1.46	1.32	1.32
89	1.19	1.14	1.17	1.18	1.22	1.1	1.19	1.27	1.37	1.34	1.29	1.11	1.1	1.49	1.2	1.33	1.05	1.33	1.08	1.45	1.14	1.33	1.16	1.48

Table A1 *Continued*

	(1,2)	(2,1)	(3,2)	(2,3)	(1,4)	(4,1)	(2,5)	(5,2)	(6,3)	(3,6)	(4,5)	(5,4)	(5,6)	(6,5)	(7,4)	(4,7)	(8,5)	(5,8)	(6,9)	(9,6)	(7,8)	(8,7)	(8,9)	(9,8)
90	1.43	1.07	1.18	1.49	1.38	1.41	1.06	1.11	1.29	1.18	1.25	1.09	1.46	1.41	1.03	1.02	1.16	1.42	1.35	1.04	1.18	1.08	1.18	1.23
91	1.21	1.45	1.12	1.05	1.17	1.13	1	1.29	1.07	1.24	1.02	1.18	1.02	1.33	1.22	1.3	1.04	1.17	1.1	1	1.13	1.33	1.17	1.18
92	1.21	1.2	1.44	1.04	1.46	1.01	1.48	1.11	1.01	1.2	1.21	1.2	1.2	1.22	1.07	1.24	1.19	1.03	1.43	1.31	1.43	1.33	1.49	1.38
93	1.13	1.21	1.32	1.43	1.19	1.29	1.13	1.09	1.23	1.18	1.36	1.15	1.06	1.26	1.13	1.41	1.21	1.1	1.4	1.14	1.24	1.23	1.44	1.29
94	1.2	1.29	1.07	1.12	1.31	1.22	1.24	1.09	1.25	1.04	1.01	1.17	1.46	1.22	1.22	1.07	1.25	1.27	1.42	1.16	1.13	1.28	1.1	1.42
95	1.49	1.15	1.44	1.35	1.12	1.44	1.28	1	1.18	1.35	1.42	1.25	1.15	1.13	1.07	1.31	1.35	1.32	1.23	1.36	1.08	1.17	1.34	1.4
96	1.22	1.05	1.07	1.11	1.49	1.3	1.1	1.4	1.23	1.42	1.17	1.4	1.04	1.1	1.02	1.35	1.25	1.27	1.14	1.46	1.45	1.46	1.42	1.29
97	1.03	1.38	1.21	1.01	1.19	1.44	1.16	1.13	1.19	1.4	1.11	1.15	1.46	1.46	1.4	1.42	1.02	1.38	1.01	1.44	1.21	1.29	1.04	1.16
98	1.4	1.05	1.45	1.11	1.43	1.17	1.48	1.27	1.3	1.38	1.2	1.05	1.29	1.45	1.27	1.16	1.19	1.46	1.31	1.13	1.14	1.04	1.21	1.26
99	1.08	1.44	1.31	1.32	1.13	1.23	1.29	1.46	1.14	1.1	1.22	1.15	1.12	1.25	1.12	1.14	1.46	1.33	1.24	1.03	1.09	1.12	1.04	1.13
100	1.36	1.11	1.32	1.22	1.13	1.37	1.1	1.1	1.39	1.26	1.14	1.34	1.42	1.27	1.45	1.29	1.35	1.5	1.1	1.48	1.03	1.18	1.38	1.41
Mean	1.24	1.24	1.26	1.22	1.24	1.24	1.26	1.25	1.24	1.26	1.22	1.24	1.24	1.24	1.22	1.26	1.25	1.24	1.27	1.25	1.23	1.23	1.25	1.27
St.dev.	0.14	0.15	0.14	0.13	0.13	0.14	0.15	0.14	0.15	0.14	0.14	0.14	0.13	0.13	0.14	0.13	0.14	0.14	0.15	0.14	0.14	0.13	0.14	0.13

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