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Lista 4

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Sumário

1 Matrix Operations

1.1 Exercício 1

Tarefa: Calcule todos os autovalores da matriz simétrica e tridiagonal A, definida por:

$$A_{mm} = -2, \quad A_{m,m-1} = A_{m-1,m} = 1$$

a qual está relacionada à discretização da derivada segunda em uma dimensão:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

Use as seguintes relações de recorrência para o polinômio característico $P_n(\lambda) = \det(A - \lambda I)$:

$$
\begin{aligned}
P_1(\lambda) &= A_{11} - \lambda, \\
P_2(\lambda) &= (A_{22} - \lambda)P_1(\lambda) - A_{21}A_{12}, \\
P_3(\lambda) &= (A_{33} - \lambda)P_2(\lambda) - A_{32}A_{23}P_1(\lambda), \\
&\vdots \\
P_m(\lambda) &= (A_{mm} - \lambda)P_{m-1}(\lambda) - A_{m,m-1}A_{m-1,m}P_{m-2}(\lambda), \\
&\vdots \\
P_n(\lambda) &= (A_{nn} - \lambda)P_{n-1}(\lambda) - A_{n,n-1}A_{n-1,n}P_{n-2}(\lambda).
\end{aligned}
$$

Use o método de Newton-Raphson para calcular numericamente as raízes de $P_n(\lambda)$, ou seja, os autovalores da matriz A.

Sugestão: escreva uma relação de recorrência também para as derivadas primeira e segunda, em relação à variável λ, das funções $P_m(\lambda)$. Considere o caso $n \times n$, onde n é uma entrada (input) do código.

Compare os resultados numéricos com o resultado exato:

$$\lambda_m = -4\sin^2\left(\frac{m\pi}{2(n+1)}\right), \quad m = 1, 2, \ldots, n.$$

Sugestão: use as desigualdades

$$\max_{j=1,\ldots,n}\left(A_{jj} + \sum_{k \neq j}|A_{jk}|\right) \geq \lambda_m \geq \min_{j=1,\ldots,n}\left(A_{jj} - \sum_{k \neq j}|A_{jk}|\right)$$

para organizar a busca inicial das raízes de $P_n(\lambda)$.

Derivando a relação de recorrência com relação a lambda, obtemos:

$$P'_m(\lambda) = (A_{mm} - \lambda)P'_{m-1}(\lambda) - A_{m,m-1}A_{m-1,m}P'_{m-2}(\lambda) - P_{m-1}(\lambda)$$

Para determinar o chute inicial do Newton-Raphson, temos:

$$A_{jj} = -2,$$
$$\sum_{k \neq j} |A_{jk}| = |A_{j,j-1}| + |A_{j,j+1}| = 1 + 1 = 2, \quad \text{para } j = 2, \ldots, n-1,$$
$$\sum_{k \neq j} |A_{jk}| = 1, \quad \text{para } j = 1 \text{ ou } j = n,$$

E, utilizando a desigualde dada, temos:

$$-4 \geq \lambda_{\text{máx}} \geq 0$$

Logo, é razoável começar com o chute inicial entre 0 e -4.

O código foi compilado com o comando:

```
gfortran L4-5255417-ex-1.f90 -Wall -Wextra -pedantic -ffree-form -o L4-
```

Resultados:

Testou-se os seguintes casos:

Figura 1: Dimensão da Matriz = 2

Com o resultado:

```
1 eigenvalue:    -1.000000000000  real eigenvalue:    -1.000000000000
    difference: 0.000000000000 +/-  1.00E-10
2 eigenvalue:    -3.000000000000  real eigenvalue:    -3.000000000000
    difference: 0.000000000000 +/-  1.00E-10
```


Figura 2: Dimensão da Matriz = 10

Com o resultado:

```
 1  eigenvalue:     -0.081014052771  real  eigenvalue:      -0.081014052771
       difference: 0.000000000000 +/-   1.00E-10
 2  eigenvalue:     -0.317492934338  real  eigenvalue:      -0.317492934338
       difference: 0.000000000000 +/-   1.00E-10
 3  eigenvalue:     -0.690278532109  real  eigenvalue:      -0.690278532109
       difference: 0.000000000000 +/-   1.00E-10
 4  eigenvalue:     -1.169169973996  real  eigenvalue:      -1.169169973996
       difference: 0.000000000000 +/-   1.00E-10
 5  eigenvalue:     -1.715370323453  real  eigenvalue:      -1.715370323453
       difference: 0.000000000000 +/-   1.00E-10
 6  eigenvalue:     -2.284629676547  real  eigenvalue:      -2.284629676547
       difference: 0.000000000000 +/-   1.00E-10
 7  eigenvalue:     -2.830830026004  real  eigenvalue:      -2.830830026004
       difference: 0.000000000000 +/-   1.00E-10
 8  eigenvalue:     -3.309721467891  real  eigenvalue:      -3.309721467891
       difference: 0.000000000000 +/-   1.00E-10
 9  eigenvalue:     -3.682507065662  real  eigenvalue:      -3.682507065662
       difference: 0.000000000000 +/-   1.00E-10
10  eigenvalue:     -3.918985947229  real  eigenvalue:      -3.918985947229
       difference: 0.000000000000 +/-   1.00E-10
```



```
> $ ./L4-5255417-ex-1.exe
 Insert matrix dimension:
100
 Insert tolerance
1.0e-10
```

Figura 3: Dimensão da Matriz = 100

Com o resultado:

```
 1  eigenvalue:     -0.000967435416  real  eigenvalue:      -0.000967435416
       difference: 0.000000000000 +/-   1.00E-10
 2  eigenvalue:     -0.003868805733  real  eigenvalue:      -0.003868805733
       difference: 0.000000000000 +/-   1.00E-10
 3  eigenvalue:     -0.008701304062  real  eigenvalue:      -0.008701304062
       difference: 0.000000000000 +/-   1.00E-10
 4  eigenvalue:     -0.015460255273  real  eigenvalue:      -0.015460255273
       difference: 0.000000000000 +/-   1.00E-10
 5  eigenvalue:     -0.024139120518  real  eigenvalue:      -0.024139120518
       difference: 0.000000000000 +/-   1.00E-10
 6  eigenvalue:     -0.034729503555  real  eigenvalue:      -0.034729503555
       difference: 0.000000000000 +/-   1.00E-10
 7  eigenvalue:     -0.047221158873  real  eigenvalue:      -0.047221158873
       difference: 0.000000000000 +/-   1.00E-10
 8  eigenvalue:     -0.061602001601  real  eigenvalue:      -0.061602001601
       difference: 0.000000000000 +/-   1.00E-10
 9  eigenvalue:     -0.077858119203  real  eigenvalue:      -0.077858119203
       difference: 0.000000000000 +/-   1.00E-10
10  eigenvalue:     -0.095973784935  real  eigenvalue:      -0.095973784935
       difference: 0.000000000000 +/-   1.00E-10
11  eigenvalue:     -0.115931473060  real  eigenvalue:      -0.115931473060
       difference: 0.000000000000 +/-   1.00E-10
12  eigenvalue:     -0.137711875805  real  eigenvalue:      -0.137711875805
       difference: 0.000000000000 +/-   1.00E-10
```

```
13 eigenvalue:    -0.161293922036 real eigenvalue:    -0.161293922036
      difference: 0.000000000000 +/-   1.00E-10
14 eigenvalue:    -0.186654797646 real eigenvalue:    -0.186654797646
      difference: 0.000000000000 +/-   1.00E-10
15 eigenvalue:    -0.213769967626 real eigenvalue:    -0.213769967626
      difference: 0.000000000000 +/-   1.00E-10
16 eigenvalue:    -0.242613199801 real eigenvalue:    -0.242613199801
      difference: 0.000000000000 +/-   1.00E-10
17 eigenvalue:    -0.273156590207 real eigenvalue:    -0.273156590207
      difference: 0.000000000000 +/-   1.00E-10
18 eigenvalue:    -0.305370590084 real eigenvalue:    -0.305370590084
      difference: 0.000000000000 +/-   1.00E-10
19 eigenvalue:    -0.339224034470 real eigenvalue:    -0.339224034470
      difference: 0.000000000000 +/-   1.00E-10
20 eigenvalue:    -0.374684172343 real eigenvalue:    -0.374684172343
      difference: 0.000000000000 +/-   1.00E-10
21 eigenvalue:    -0.411716698310 real eigenvalue:    -0.411716698310
      difference: 0.000000000000 +/-   1.00E-10
22 eigenvalue:    -0.450285785794 real eigenvalue:    -0.450285785794
      difference: 0.000000000000 +/-   1.00E-10
23 eigenvalue:    -0.490354121693 real eigenvalue:    -0.490354121693
      difference: 0.000000000000 +/-   1.00E-10
24 eigenvalue:    -0.531882942481 real eigenvalue:    -0.531882942481
      difference: 0.000000000000 +/-   1.00E-10
25 eigenvalue:    -0.574832071705 real eigenvalue:    -0.574832071705
      difference: 0.000000000000 +/-   1.00E-10
26 eigenvalue:    -0.619159958857 real eigenvalue:    -0.619159958857
      difference: 0.000000000000 +/-   1.00E-10
27 eigenvalue:    -0.664823719568 real eigenvalue:    -0.664823719568
      difference: 0.000000000000 +/-   1.00E-10
28 eigenvalue:    -0.711779177099 real eigenvalue:    -0.711779177099
      difference: 0.000000000000 +/-   1.00E-10
29 eigenvalue:    -0.759980905078 real eigenvalue:    -0.759980905078
      difference: 0.000000000000 +/-   1.00E-10
30 eigenvalue:    -0.809382271447 real eigenvalue:    -0.809382271447
      difference: 0.000000000000 +/-   1.00E-10
31 eigenvalue:    -0.859935483572 real eigenvalue:    -0.859935483572
      difference: 0.000000000000 +/-   1.00E-10
32 eigenvalue:    -0.911591634488 real eigenvalue:    -0.911591634488
      difference: 0.000000000000 +/-   1.00E-10
33 eigenvalue:    -0.964300750203 real eigenvalue:    -0.964300750203
      difference: 0.000000000000 +/-   1.00E-10
34 eigenvalue:    -1.018011838053 real eigenvalue:    -1.018011838053
      difference: 0.000000000000 +/-   1.00E-10
35 eigenvalue:    -1.072672936029 real eigenvalue:    -1.072672936029
      difference: 0.000000000000 +/-   1.00E-10
36 eigenvalue:    -1.128231163049 real eigenvalue:    -1.128231163049
      difference: 0.000000000000 +/-   1.00E-10
37 eigenvalue:    -1.184632770116 real eigenvalue:    -1.184632770117
      difference: 0.000000000000 +/-   1.00E-10
38 eigenvalue:    -1.241823192319 real eigenvalue:    -1.241823192319
      difference: 0.000000000000 +/-   1.00E-10
39 eigenvalue:    -1.299747101617 real eigenvalue:    -1.299747101617
      difference: 0.000000000000 +/-   1.00E-10
40 eigenvalue:    -1.358348460369 real eigenvalue:    -1.358348460369
      difference: 0.000000000000 +/-   1.00E-10
41 eigenvalue:    -1.417570575545 real eigenvalue:    -1.417570575545
      difference: 0.000000000000 +/-   1.00E-10
```

```
42 eigenvalue:    -1.477356153574 real eigenvalue:    -1.477356153574
   difference: 0.000000000000 +/-  1.00E-10
43 eigenvalue:    -1.537647355770 real eigenvalue:    -1.537647355770
   difference: 0.000000000000 +/-  1.00E-10
44 eigenvalue:    -1.598385854289 real eigenvalue:    -1.598385854289
   difference: 0.000000000000 +/-  1.00E-10
45 eigenvalue:    -1.659512888553 real eigenvalue:    -1.659512888555
   difference: 0.000000000002 +/-  1.00E-10
46 eigenvalue:    -1.720969322112 real eigenvalue:    -1.720969322112
   difference: 0.000000000000 +/-  1.00E-10
47 eigenvalue:    -1.782695699829 real eigenvalue:    -1.782695699829
   difference: 0.000000000000 +/-  1.00E-10
48 eigenvalue:    -1.844632305422 real eigenvalue:    -1.844632305422
   difference: 0.000000000000 +/-  1.00E-10
49 eigenvalue:    -1.906719219225 real eigenvalue:    -1.906719219225
   difference: 0.000000000000 +/-  1.00E-10
50 eigenvalue:    -1.968896376160 real eigenvalue:    -1.968896376159
   difference: 0.000000000001 +/-  1.00E-10
51 eigenvalue:    -2.031103623840 real eigenvalue:    -2.031103623841
   difference: 0.000000000001 +/-  1.00E-10
52 eigenvalue:    -2.093280780775 real eigenvalue:    -2.093280780775
   difference: 0.000000000000 +/-  1.00E-10
53 eigenvalue:    -2.155367694578 real eigenvalue:    -2.155367694578
   difference: 0.000000000000 +/-  1.00E-10
54 eigenvalue:    -2.217304300171 real eigenvalue:    -2.217304300171
   difference: 0.000000000000 +/-  1.00E-10
55 eigenvalue:    -2.279030677888 real eigenvalue:    -2.279030677888
   difference: 0.000000000000 +/-  1.00E-10
56 eigenvalue:    -2.340487111447 real eigenvalue:    -2.340487111445
   difference: 0.000000000002 +/-  1.00E-10
57 eigenvalue:    -2.401614145711 real eigenvalue:    -2.401614145711
   difference: 0.000000000000 +/-  1.00E-10
58 eigenvalue:    -2.462352644230 real eigenvalue:    -2.462352644230
   difference: 0.000000000000 +/-  1.00E-10
59 eigenvalue:    -2.522643846426 real eigenvalue:    -2.522643846426
   difference: 0.000000000000 +/-  1.00E-10
60 eigenvalue:    -2.582429424455 real eigenvalue:    -2.582429424455
   difference: 0.000000000000 +/-  1.00E-10
61 eigenvalue:    -2.641651539631 real eigenvalue:    -2.641651539631
   difference: 0.000000000000 +/-  1.00E-10
62 eigenvalue:    -2.700252898383 real eigenvalue:    -2.700252898383
   difference: 0.000000000000 +/-  1.00E-10
63 eigenvalue:    -2.758176807681 real eigenvalue:    -2.758176807681
   difference: 0.000000000000 +/-  1.00E-10
64 eigenvalue:    -2.815367229884 real eigenvalue:    -2.815367229883
   difference: 0.000000000000 +/-  1.00E-10
65 eigenvalue:    -2.871768836951 real eigenvalue:    -2.871768836951
   difference: 0.000000000000 +/-  1.00E-10
66 eigenvalue:    -2.927327063971 real eigenvalue:    -2.927327063971
   difference: 0.000000000000 +/-  1.00E-10
67 eigenvalue:    -2.981988161947 real eigenvalue:    -2.981988161947
   difference: 0.000000000000 +/-  1.00E-10
68 eigenvalue:    -3.035699249797 real eigenvalue:    -3.035699249797
   difference: 0.000000000000 +/-  1.00E-10
69 eigenvalue:    -3.088408365512 real eigenvalue:    -3.088408365512
   difference: 0.000000000000 +/-  1.00E-10
70 eigenvalue:    -3.140064516428 real eigenvalue:    -3.140064516428
   difference: 0.000000000000 +/-  1.00E-10
```

```
71  eigenvalue:    -3.190617728553  real eigenvalue:    -3.190617728553
       difference: 0.000000000000 +/-  1.00E-10
72  eigenvalue:    -3.240019094922  real eigenvalue:    -3.240019094922
       difference: 0.000000000000 +/-  1.00E-10
73  eigenvalue:    -3.288220822901  real eigenvalue:    -3.288220822901
       difference: 0.000000000000 +/-  1.00E-10
74  eigenvalue:    -3.335176280432  real eigenvalue:    -3.335176280432
       difference: 0.000000000000 +/-  1.00E-10
75  eigenvalue:    -3.380840041143  real eigenvalue:    -3.380840041143
       difference: 0.000000000000 +/-  1.00E-10
76  eigenvalue:    -3.425167928295  real eigenvalue:    -3.425167928295
       difference: 0.000000000000 +/-  1.00E-10
77  eigenvalue:    -3.468117057519  real eigenvalue:    -3.468117057519
       difference: 0.000000000000 +/-  1.00E-10
78  eigenvalue:    -3.509645878307  real eigenvalue:    -3.509645878307
       difference: 0.000000000000 +/-  1.00E-10
79  eigenvalue:    -3.549714214206  real eigenvalue:    -3.549714214206
       difference: 0.000000000000 +/-  1.00E-10
80  eigenvalue:    -3.588283301690  real eigenvalue:    -3.588283301690
       difference: 0.000000000000 +/-  1.00E-10
81  eigenvalue:    -3.625315827657  real eigenvalue:    -3.625315827656
       difference: 0.000000000000 +/-  1.00E-10
82  eigenvalue:    -3.660775965530  real eigenvalue:    -3.660775965530
       difference: 0.000000000000 +/-  1.00E-10
83  eigenvalue:    -3.694629409916  real eigenvalue:    -3.694629409916
       difference: 0.000000000000 +/-  1.00E-10
84  eigenvalue:    -3.726843409793  real eigenvalue:    -3.726843409793
       difference: 0.000000000000 +/-  1.00E-10
85  eigenvalue:    -3.757386800199  real eigenvalue:    -3.757386800199
       difference: 0.000000000000 +/-  1.00E-10
86  eigenvalue:    -3.786230032374  real eigenvalue:    -3.786230032374
       difference: 0.000000000000 +/-  1.00E-10
87  eigenvalue:    -3.813345202354  real eigenvalue:    -3.813345202354
       difference: 0.000000000000 +/-  1.00E-10
88  eigenvalue:    -3.838706077964  real eigenvalue:    -3.838706077964
       difference: 0.000000000000 +/-  1.00E-10
89  eigenvalue:    -3.862288124195  real eigenvalue:    -3.862288124195
       difference: 0.000000000000 +/-  1.00E-10
90  eigenvalue:    -3.884068526940  real eigenvalue:    -3.884068526940
       difference: 0.000000000000 +/-  1.00E-10
91  eigenvalue:    -3.904026215065  real eigenvalue:    -3.904026215065
       difference: 0.000000000000 +/-  1.00E-10
92  eigenvalue:    -3.922141880797  real eigenvalue:    -3.922141880797
       difference: 0.000000000000 +/-  1.00E-10
93  eigenvalue:    -3.938397998399  real eigenvalue:    -3.938397998399
       difference: 0.000000000000 +/-  1.00E-10
94  eigenvalue:    -3.952778841127  real eigenvalue:    -3.952778841127
       difference: 0.000000000000 +/-  1.00E-10
95  eigenvalue:    -3.965270496445  real eigenvalue:    -3.965270496445
       difference: 0.000000000000 +/-  1.00E-10
96  eigenvalue:    -3.975860879482  real eigenvalue:    -3.975860879482
       difference: 0.000000000000 +/-  1.00E-10
97  eigenvalue:    -3.984539744727  real eigenvalue:    -3.984539744727
       difference: 0.000000000000 +/-  1.00E-10
98  eigenvalue:    -3.991298695938  real eigenvalue:    -3.991298695938
       difference: 0.000000000000 +/-  1.00E-10
99  eigenvalue:    -3.996131194267  real eigenvalue:    -3.996131194267
       difference: 0.000000000000 +/-  1.00E-10
```

```
100  eigenvalue:      -3.999032564584 real eigenvalue:      -3.999032564584
     difference: 0.000000000000 +/-   1.00E-10
```


Figura 4: Dimensão da Matriz = 200

Com o resultado:

```
1  eigenvalue:      -0.000244286119 real eigenvalue:      -0.000244286119
     difference: 0.000000000000 +/-   1.00E-10
2  eigenvalue:      -0.000977084799 real eigenvalue:      -0.000977084799
     difference: 0.000000000000 +/-   1.00E-10
3  eigenvalue:      -0.002198217029 real eigenvalue:      -0.002198217029
     difference: 0.000000000000 +/-   1.00E-10
4  eigenvalue:      -0.003907384502 real eigenvalue:      -0.003907384502
     difference: 0.000000000000 +/-   1.00E-10
5  eigenvalue:      -0.006104169692 real eigenvalue:      -0.006104169692
     difference: 0.000000000000 +/-   1.00E-10
6  eigenvalue:      -0.008788035956 real eigenvalue:      -0.008788035956
     difference: 0.000000000000 +/-   1.00E-10
7  eigenvalue:      -0.011958327662 real eigenvalue:      -0.011958327662
     difference: 0.000000000000 +/-   1.00E-10
8  eigenvalue:      -0.015614270353 real eigenvalue:      -0.015614270353
     difference: 0.000000000000 +/-   1.00E-10
9  eigenvalue:      -0.019754970931 real eigenvalue:      -0.019754970931
     difference: 0.000000000000 +/-   1.00E-10
10 eigenvalue:      -0.024379417881 real eigenvalue:      -0.024379417881
     difference: 0.000000000000 +/-   1.00E-10
11 eigenvalue:      -0.029486481515 real eigenvalue:      -0.029486481515
     difference: 0.000000000000 +/-   1.00E-10
12 eigenvalue:      -0.035074914249 real eigenvalue:      -0.035074914249
     difference: 0.000000000000 +/-   1.00E-10
13 eigenvalue:      -0.041143350905 real eigenvalue:      -0.041143350905
     difference: 0.000000000000 +/-   1.00E-10
14 eigenvalue:      -0.047690309049 real eigenvalue:      -0.047690309049
     difference: 0.000000000000 +/-   1.00E-10
15 eigenvalue:      -0.054714189350 real eigenvalue:      -0.054714189350
     difference: 0.000000000000 +/-   1.00E-10
16 eigenvalue:      -0.062213275972 real eigenvalue:      -0.062213275972
     difference: 0.000000000000 +/-   1.00E-10
17 eigenvalue:      -0.070185736991 real eigenvalue:      -0.070185736991
     difference: 0.000000000000 +/-   1.00E-10
18 eigenvalue:      -0.078629624846 real eigenvalue:      -0.078629624846
     difference: 0.000000000000 +/-   1.00E-10
19 eigenvalue:      -0.087542876813 real eigenvalue:      -0.087542876813
     difference: 0.000000000000 +/-   1.00E-10
20 eigenvalue:      -0.096923315508 real eigenvalue:      -0.096923315508
     difference: 0.000000000000 +/-   1.00E-10
21 eigenvalue:      -0.106768649419 real eigenvalue:      -0.106768649419
     difference: 0.000000000000 +/-   1.00E-10
```

```
22 eigenvalue:    -0.117076473469 real eigenvalue:    -0.117076473469
   difference: 0.000000000000 +/-  1.00E-10
23 eigenvalue:    -0.127844269599 real eigenvalue:    -0.127844269599
   difference: 0.000000000000 +/-  1.00E-10
24 eigenvalue:    -0.139069407386 real eigenvalue:    -0.139069407386
   difference: 0.000000000000 +/-  1.00E-10
25 eigenvalue:    -0.150749144684 real eigenvalue:    -0.150749144684
   difference: 0.000000000000 +/-  1.00E-10
26 eigenvalue:    -0.162880628297 real eigenvalue:    -0.162880628297
   difference: 0.000000000000 +/-  1.00E-10
27 eigenvalue:    -0.175460894670 real eigenvalue:    -0.175460894670
   difference: 0.000000000000 +/-  1.00E-10
28 eigenvalue:    -0.188486870620 real eigenvalue:    -0.188486870620
   difference: 0.000000000000 +/-  1.00E-10
29 eigenvalue:    -0.201955374081 real eigenvalue:    -0.201955374081
   difference: 0.000000000000 +/-  1.00E-10
30 eigenvalue:    -0.215863114885 real eigenvalue:    -0.215863114885
   difference: 0.000000000000 +/-  1.00E-10
31 eigenvalue:    -0.230206695564 real eigenvalue:    -0.230206695564
   difference: 0.000000000000 +/-  1.00E-10
32 eigenvalue:    -0.244982612180 real eigenvalue:    -0.244982612180
   difference: 0.000000000000 +/-  1.00E-10
33 eigenvalue:    -0.260187255182 real eigenvalue:    -0.260187255182
   difference: 0.000000000000 +/-  1.00E-10
34 eigenvalue:    -0.275816910287 real eigenvalue:    -0.275816910287
   difference: 0.000000000000 +/-  1.00E-10
35 eigenvalue:    -0.291867759387 real eigenvalue:    -0.291867759387
   difference: 0.000000000000 +/-  1.00E-10
36 eigenvalue:    -0.308335881482 real eigenvalue:    -0.308335881482
   difference: 0.000000000000 +/-  1.00E-10
37 eigenvalue:    -0.325217253638 real eigenvalue:    -0.325217253638
   difference: 0.000000000000 +/-  1.00E-10
38 eigenvalue:    -0.342507751972 real eigenvalue:    -0.342507751972
   difference: 0.000000000000 +/-  1.00E-10
39 eigenvalue:    -0.360203152653 real eigenvalue:    -0.360203152653
   difference: 0.000000000000 +/-  1.00E-10
40 eigenvalue:    -0.378299132942 real eigenvalue:    -0.378299132942
   difference: 0.000000000000 +/-  1.00E-10
41 eigenvalue:    -0.396791272241 real eigenvalue:    -0.396791272241
   difference: 0.000000000000 +/-  1.00E-10
42 eigenvalue:    -0.415675053178 real eigenvalue:    -0.415675053178
   difference: 0.000000000000 +/-  1.00E-10
43 eigenvalue:    -0.434945862706 real eigenvalue:    -0.434945862706
   difference: 0.000000000000 +/-  1.00E-10
44 eigenvalue:    -0.454598993236 real eigenvalue:    -0.454598993236
   difference: 0.000000000000 +/-  1.00E-10
45 eigenvalue:    -0.474629643779 real eigenvalue:    -0.474629643779
   difference: 0.000000000000 +/-  1.00E-10
46 eigenvalue:    -0.495032921126 real eigenvalue:    -0.495032921126
   difference: 0.000000000000 +/-  1.00E-10
47 eigenvalue:    -0.515803841040 real eigenvalue:    -0.515803841040
   difference: 0.000000000000 +/-  1.00E-10
48 eigenvalue:    -0.536937329473 real eigenvalue:    -0.536937329473
   difference: 0.000000000000 +/-  1.00E-10
49 eigenvalue:    -0.558428223807 real eigenvalue:    -0.558428223807
   difference: 0.000000000000 +/-  1.00E-10
50 eigenvalue:    -0.580271274114 real eigenvalue:    -0.580271274114
   difference: 0.000000000000 +/-  1.00E-10
```

```
51 eigenvalue:    -0.602461144442 real eigenvalue:    -0.602461144442
   difference: 0.000000000000 +/-  1.00E-10
52 eigenvalue:    -0.624992414112 real eigenvalue:    -0.624992414113
   difference: 0.000000000000 +/-  1.00E-10
53 eigenvalue:    -0.647859579050 real eigenvalue:    -0.647859579050
   difference: 0.000000000000 +/-  1.00E-10
54 eigenvalue:    -0.671057053122 real eigenvalue:    -0.671057053122
   difference: 0.000000000000 +/-  1.00E-10
55 eigenvalue:    -0.694579169509 real eigenvalue:    -0.694579169509
   difference: 0.000000000000 +/-  1.00E-10
56 eigenvalue:    -0.718420182084 real eigenvalue:    -0.718420182084
   difference: 0.000000000000 +/-  1.00E-10
57 eigenvalue:    -0.742574266818 real eigenvalue:    -0.742574266818
   difference: 0.000000000000 +/-  1.00E-10
58 eigenvalue:    -0.767035523204 real eigenvalue:    -0.767035523204
   difference: 0.000000000000 +/-  1.00E-10
59 eigenvalue:    -0.791797975697 real eigenvalue:    -0.791797975697
   difference: 0.000000000000 +/-  1.00E-10
60 eigenvalue:    -0.816855575173 real eigenvalue:    -0.816855575173
   difference: 0.000000000000 +/-  1.00E-10
61 eigenvalue:    -0.842202200408 real eigenvalue:    -0.842202200408
   difference: 0.000000000000 +/-  1.00E-10
62 eigenvalue:    -0.867831659574 real eigenvalue:    -0.867831659574
   difference: 0.000000000000 +/-  1.00E-10
63 eigenvalue:    -0.893737691750 real eigenvalue:    -0.893737691750
   difference: 0.000000000000 +/-  1.00E-10
64 eigenvalue:    -0.919913968450 real eigenvalue:    -0.919913968451
   difference: 0.000000000000 +/-  1.00E-10
65 eigenvalue:    -0.946354095176 real eigenvalue:    -0.946354095176
   difference: 0.000000000000 +/-  1.00E-10
66 eigenvalue:    -0.973051612970 real eigenvalue:    -0.973051612970
   difference: 0.000000000000 +/-  1.00E-10
67 eigenvalue:    -1.000000000000 real eigenvalue:    -1.000000000000
   difference: 0.000000000000 +/-  1.00E-10
68 eigenvalue:    -1.027192673148 real eigenvalue:    -1.027192673148
   difference: 0.000000000000 +/-  1.00E-10
69 eigenvalue:    -1.054622989623 real eigenvalue:    -1.054622989623
   difference: 0.000000000000 +/-  1.00E-10
70 eigenvalue:    -1.082284248578 real eigenvalue:    -1.082284248578
   difference: 0.000000000000 +/-  1.00E-10
71 eigenvalue:    -1.110169692752 real eigenvalue:    -1.110169692752
   difference: 0.000000000000 +/-  1.00E-10
72 eigenvalue:    -1.138272510118 real eigenvalue:    -1.138272510118
   difference: 0.000000000000 +/-  1.00E-10
73 eigenvalue:    -1.166585835548 real eigenvalue:    -1.166585835548
   difference: 0.000000000000 +/-  1.00E-10
74 eigenvalue:    -1.195102752489 real eigenvalue:    -1.195102752489
   difference: 0.000000000000 +/-  1.00E-10
75 eigenvalue:    -1.223816294655 real eigenvalue:    -1.223816294656
   difference: 0.000000000000 +/-  1.00E-10
76 eigenvalue:    -1.252719447726 real eigenvalue:    -1.252719447726
   difference: 0.000000000000 +/-  1.00E-10
77 eigenvalue:    -1.281805151063 real eigenvalue:    -1.281805151063
   difference: 0.000000000000 +/-  1.00E-10
78 eigenvalue:    -1.311066299431 real eigenvalue:    -1.311066299431
   difference: 0.000000000000 +/-  1.00E-10
79 eigenvalue:    -1.340495744739 real eigenvalue:    -1.340495744740
   difference: 0.000000000000 +/-  1.00E-10
```

```
 80  eigenvalue:    -1.370086297783 real eigenvalue:    -1.370086297783
     difference: 0.000000000000 +/-  1.00E-10
 81  eigenvalue:    -1.399830729999 real eigenvalue:    -1.399830729999
     difference: 0.000000000000 +/-  1.00E-10
 82  eigenvalue:    -1.429721775237 real eigenvalue:    -1.429721775237
     difference: 0.000000000000 +/-  1.00E-10
 83  eigenvalue:    -1.459752131530 real eigenvalue:    -1.459752131530
     difference: 0.000000000000 +/-  1.00E-10
 84  eigenvalue:    -1.489914462877 real eigenvalue:    -1.489914462877
     difference: 0.000000000000 +/-  1.00E-10
 85  eigenvalue:    -1.520201401040 real eigenvalue:    -1.520201401040
     difference: 0.000000000000 +/-  1.00E-10
 86  eigenvalue:    -1.550605547340 real eigenvalue:    -1.550605547340
     difference: 0.000000000000 +/-  1.00E-10
 87  eigenvalue:    -1.581119474467 real eigenvalue:    -1.581119474468
     difference: 0.000000000000 +/-  1.00E-10
 88  eigenvalue:    -1.611735728292 real eigenvalue:    -1.611735728293
     difference: 0.000000000001 +/-  1.00E-10
 89  eigenvalue:    -1.642446829690 real eigenvalue:    -1.642446829690
     difference: 0.000000000000 +/-  1.00E-10
 90  eigenvalue:    -1.673245276363 real eigenvalue:    -1.673245276363
     difference: 0.000000000000 +/-  1.00E-10
 91  eigenvalue:    -1.704123544679 real eigenvalue:    -1.704123544679
     difference: 0.000000000000 +/-  1.00E-10
 92  eigenvalue:    -1.735074091507 real eigenvalue:    -1.735074091507
     difference: 0.000000000000 +/-  1.00E-10
 93  eigenvalue:    -1.766089356056 real eigenvalue:    -1.766089356056
     difference: 0.000000000000 +/-  1.00E-10
 94  eigenvalue:    -1.797161761728 real eigenvalue:    -1.797161761728
     difference: 0.000000000001 +/-  1.00E-10
 95  eigenvalue:    -1.828283717967 real eigenvalue:    -1.828283717967
     difference: 0.000000000000 +/-  1.00E-10
 96  eigenvalue:    -1.859447622109 real eigenvalue:    -1.859447622109
     difference: 0.000000000000 +/-  1.00E-10
 97  eigenvalue:    -1.890645861246 real eigenvalue:    -1.890645861247
     difference: 0.000000000001 +/-  1.00E-10
 98  eigenvalue:    -1.921870814082 real eigenvalue:    -1.921870814082
     difference: 0.000000000000 +/-  1.00E-10
 99  eigenvalue:    -1.953114852793 real eigenvalue:    -1.953114852793
     difference: 0.000000000001 +/-  1.00E-10
100  eigenvalue:    -1.984370344895 real eigenvalue:    -1.984370344895
     difference: 0.000000000000 +/-  1.00E-10
101  eigenvalue:    -2.015629655105 real eigenvalue:    -2.015629655105
     difference: 0.000000000000 +/-  1.00E-10
102  eigenvalue:    -2.046885147207 real eigenvalue:    -2.046885147207
     difference: 0.000000000001 +/-  1.00E-10
103  eigenvalue:    -2.078129185918 real eigenvalue:    -2.078129185918
     difference: 0.000000000000 +/-  1.00E-10
104  eigenvalue:    -2.109354138754 real eigenvalue:    -2.109354138753
     difference: 0.000000000001 +/-  1.00E-10
105  eigenvalue:    -2.140552377891 real eigenvalue:    -2.140552377891
     difference: 0.000000000000 +/-  1.00E-10
106  eigenvalue:    -2.171716282033 real eigenvalue:    -2.171716282033
     difference: 0.000000000000 +/-  1.00E-10
107  eigenvalue:    -2.202838238272 real eigenvalue:    -2.202838238272
     difference: 0.000000000001 +/-  1.00E-10
108  eigenvalue:    -2.233910643944 real eigenvalue:    -2.233910643944
     difference: 0.000000000000 +/-  1.00E-10
```

```
109  eigenvalue:    -2.264925908493 real eigenvalue:    -2.264925908493
      difference: 0.000000000000 +/-   1.00E-10
110  eigenvalue:    -2.295876455321 real eigenvalue:    -2.295876455321
      difference: 0.000000000000 +/-   1.00E-10
111  eigenvalue:    -2.326754723637 real eigenvalue:    -2.326754723637
      difference: 0.000000000000 +/-   1.00E-10
112  eigenvalue:    -2.357553170310 real eigenvalue:    -2.357553170310
      difference: 0.000000000000 +/-   1.00E-10
113  eigenvalue:    -2.388264271708 real eigenvalue:    -2.388264271707
      difference: 0.000000000001 +/-   1.00E-10
114  eigenvalue:    -2.418880525533 real eigenvalue:    -2.418880525532
      difference: 0.000000000000 +/-   1.00E-10
115  eigenvalue:    -2.449394452660 real eigenvalue:    -2.449394452660
      difference: 0.000000000000 +/-   1.00E-10
116  eigenvalue:    -2.479798598960 real eigenvalue:    -2.479798598960
      difference: 0.000000000000 +/-   1.00E-10
117  eigenvalue:    -2.510085537123 real eigenvalue:    -2.510085537123
      difference: 0.000000000000 +/-   1.00E-10
118  eigenvalue:    -2.540247868470 real eigenvalue:    -2.540247868470
      difference: 0.000000000000 +/-   1.00E-10
119  eigenvalue:    -2.570278224763 real eigenvalue:    -2.570278224763
      difference: 0.000000000000 +/-   1.00E-10
120  eigenvalue:    -2.600169270001 real eigenvalue:    -2.600169270001
      difference: 0.000000000000 +/-   1.00E-10
121  eigenvalue:    -2.629913702217 real eigenvalue:    -2.629913702217
      difference: 0.000000000000 +/-   1.00E-10
122  eigenvalue:    -2.659504255261 real eigenvalue:    -2.659504255260
      difference: 0.000000000000 +/-   1.00E-10
123  eigenvalue:    -2.688933700569 real eigenvalue:    -2.688933700569
      difference: 0.000000000000 +/-   1.00E-10
124  eigenvalue:    -2.718194848937 real eigenvalue:    -2.718194848937
      difference: 0.000000000000 +/-   1.00E-10
125  eigenvalue:    -2.747280552274 real eigenvalue:    -2.747280552274
      difference: 0.000000000000 +/-   1.00E-10
126  eigenvalue:    -2.776183705345 real eigenvalue:    -2.776183705344
      difference: 0.000000000000 +/-   1.00E-10
127  eigenvalue:    -2.804897247511 real eigenvalue:    -2.804897247511
      difference: 0.000000000000 +/-   1.00E-10
128  eigenvalue:    -2.833414164452 real eigenvalue:    -2.833414164452
      difference: 0.000000000000 +/-   1.00E-10
129  eigenvalue:    -2.861727489882 real eigenvalue:    -2.861727489882
      difference: 0.000000000000 +/-   1.00E-10
130  eigenvalue:    -2.889830307248 real eigenvalue:    -2.889830307248
      difference: 0.000000000000 +/-   1.00E-10
131  eigenvalue:    -2.917715751422 real eigenvalue:    -2.917715751422
      difference: 0.000000000000 +/-   1.00E-10
132  eigenvalue:    -2.945377010377 real eigenvalue:    -2.945377010377
      difference: 0.000000000000 +/-   1.00E-10
133  eigenvalue:    -2.972807326852 real eigenvalue:    -2.972807326852
      difference: 0.000000000000 +/-   1.00E-10
134  eigenvalue:    -3.000000000000 real eigenvalue:    -3.000000000000
      difference: 0.000000000000 +/-   1.00E-10
135  eigenvalue:    -3.026948387030 real eigenvalue:    -3.026948387030
      difference: 0.000000000000 +/-   1.00E-10
136  eigenvalue:    -3.053645904824 real eigenvalue:    -3.053645904824
      difference: 0.000000000000 +/-   1.00E-10
137  eigenvalue:    -3.080086031550 real eigenvalue:    -3.080086031549
      difference: 0.000000000000 +/-   1.00E-10
```

```
138  eigenvalue:   -3.106262308250  real eigenvalue:    -3.106262308250
     difference: 0.000000000000 +/-  1.00E-10
139  eigenvalue:   -3.132168340426  real eigenvalue:    -3.132168340426
     difference: 0.000000000000 +/-  1.00E-10
140  eigenvalue:   -3.157797799592  real eigenvalue:    -3.157797799592
     difference: 0.000000000000 +/-  1.00E-10
141  eigenvalue:   -3.183144424827  real eigenvalue:    -3.183144424827
     difference: 0.000000000000 +/-  1.00E-10
142  eigenvalue:   -3.208202024303  real eigenvalue:    -3.208202024303
     difference: 0.000000000000 +/-  1.00E-10
143  eigenvalue:   -3.232964476796  real eigenvalue:    -3.232964476796
     difference: 0.000000000000 +/-  1.00E-10
144  eigenvalue:   -3.257425733182  real eigenvalue:    -3.257425733182
     difference: 0.000000000000 +/-  1.00E-10
145  eigenvalue:   -3.281579817916  real eigenvalue:    -3.281579817916
     difference: 0.000000000000 +/-  1.00E-10
146  eigenvalue:   -3.305420830491  real eigenvalue:    -3.305420830491
     difference: 0.000000000000 +/-  1.00E-10
147  eigenvalue:   -3.328942946878  real eigenvalue:    -3.328942946878
     difference: 0.000000000000 +/-  1.00E-10
148  eigenvalue:   -3.352140420950  real eigenvalue:    -3.352140420950
     difference: 0.000000000000 +/-  1.00E-10
149  eigenvalue:   -3.375007585888  real eigenvalue:    -3.375007585887
     difference: 0.000000000000 +/-  1.00E-10
150  eigenvalue:   -3.397538855558  real eigenvalue:    -3.397538855558
     difference: 0.000000000000 +/-  1.00E-10
151  eigenvalue:   -3.419728725886  real eigenvalue:    -3.419728725886
     difference: 0.000000000000 +/-  1.00E-10
152  eigenvalue:   -3.441571776193  real eigenvalue:    -3.441571776193
     difference: 0.000000000000 +/-  1.00E-10
153  eigenvalue:   -3.463062670527  real eigenvalue:    -3.463062670527
     difference: 0.000000000000 +/-  1.00E-10
154  eigenvalue:   -3.484196158960  real eigenvalue:    -3.484196158960
     difference: 0.000000000000 +/-  1.00E-10
155  eigenvalue:   -3.504967078874  real eigenvalue:    -3.504967078874
     difference: 0.000000000000 +/-  1.00E-10
156  eigenvalue:   -3.525370356221  real eigenvalue:    -3.525370356221
     difference: 0.000000000000 +/-  1.00E-10
157  eigenvalue:   -3.545401006764  real eigenvalue:    -3.545401006764
     difference: 0.000000000000 +/-  1.00E-10
158  eigenvalue:   -3.565054137294  real eigenvalue:    -3.565054137294
     difference: 0.000000000000 +/-  1.00E-10
159  eigenvalue:   -3.584324946822  real eigenvalue:    -3.584324946822
     difference: 0.000000000000 +/-  1.00E-10
160  eigenvalue:   -3.603208727759  real eigenvalue:    -3.603208727759
     difference: 0.000000000000 +/-  1.00E-10
161  eigenvalue:   -3.621700867058  real eigenvalue:    -3.621700867058
     difference: 0.000000000000 +/-  1.00E-10
162  eigenvalue:   -3.639796847347  real eigenvalue:    -3.639796847347
     difference: 0.000000000000 +/-  1.00E-10
163  eigenvalue:   -3.657492248028  real eigenvalue:    -3.657492248028
     difference: 0.000000000000 +/-  1.00E-10
164  eigenvalue:   -3.674782746362  real eigenvalue:    -3.674782746362
     difference: 0.000000000000 +/-  1.00E-10
165  eigenvalue:   -3.691664118518  real eigenvalue:    -3.691664118518
     difference: 0.000000000000 +/-  1.00E-10
166  eigenvalue:   -3.708132240613  real eigenvalue:    -3.708132240613
     difference: 0.000000000000 +/-  1.00E-10
```

```
167 eigenvalue:    -3.724183089713 real eigenvalue:    -3.724183089713
    difference: 0.000000000000 +/-  1.00E-10
168 eigenvalue:    -3.739812744818 real eigenvalue:    -3.739812744818
    difference: 0.000000000000 +/-  1.00E-10
169 eigenvalue:    -3.755017387820 real eigenvalue:    -3.755017387820
    difference: 0.000000000000 +/-  1.00E-10
170 eigenvalue:    -3.769793304436 real eigenvalue:    -3.769793304436
    difference: 0.000000000000 +/-  1.00E-10
171 eigenvalue:    -3.784136885115 real eigenvalue:    -3.784136885115
    difference: 0.000000000000 +/-  1.00E-10
172 eigenvalue:    -3.798044625919 real eigenvalue:    -3.798044625919
    difference: 0.000000000000 +/-  1.00E-10
173 eigenvalue:    -3.811513129380 real eigenvalue:    -3.811513129380
    difference: 0.000000000000 +/-  1.00E-10
174 eigenvalue:    -3.824539105330 real eigenvalue:    -3.824539105330
    difference: 0.000000000000 +/-  1.00E-10
175 eigenvalue:    -3.837119371703 real eigenvalue:    -3.837119371703
    difference: 0.000000000000 +/-  1.00E-10
176 eigenvalue:    -3.849250855316 real eigenvalue:    -3.849250855316
    difference: 0.000000000000 +/-  1.00E-10
177 eigenvalue:    -3.860930592614 real eigenvalue:    -3.860930592614
    difference: 0.000000000000 +/-  1.00E-10
178 eigenvalue:    -3.872155730401 real eigenvalue:    -3.872155730401
    difference: 0.000000000000 +/-  1.00E-10
179 eigenvalue:    -3.882923526531 real eigenvalue:    -3.882923526531
    difference: 0.000000000000 +/-  1.00E-10
180 eigenvalue:    -3.893231350581 real eigenvalue:    -3.893231350581
    difference: 0.000000000000 +/-  1.00E-10
181 eigenvalue:    -3.903076684492 real eigenvalue:    -3.903076684492
    difference: 0.000000000000 +/-  1.00E-10
182 eigenvalue:    -3.912457123187 real eigenvalue:    -3.912457123187
    difference: 0.000000000000 +/-  1.00E-10
183 eigenvalue:    -3.921370375154 real eigenvalue:    -3.921370375154
    difference: 0.000000000000 +/-  1.00E-10
184 eigenvalue:    -3.929814263009 real eigenvalue:    -3.929814263009
    difference: 0.000000000000 +/-  1.00E-10
185 eigenvalue:    -3.937786724028 real eigenvalue:    -3.937786724028
    difference: 0.000000000000 +/-  1.00E-10
186 eigenvalue:    -3.945285810650 real eigenvalue:    -3.945285810650
    difference: 0.000000000000 +/-  1.00E-10
187 eigenvalue:    -3.952309690951 real eigenvalue:    -3.952309690951
    difference: 0.000000000000 +/-  1.00E-10
188 eigenvalue:    -3.958856649095 real eigenvalue:    -3.958856649095
    difference: 0.000000000000 +/-  1.00E-10
189 eigenvalue:    -3.964925085751 real eigenvalue:    -3.964925085751
    difference: 0.000000000000 +/-  1.00E-10
190 eigenvalue:    -3.970513518485 real eigenvalue:    -3.970513518485
    difference: 0.000000000000 +/-  1.00E-10
191 eigenvalue:    -3.975620582119 real eigenvalue:    -3.975620582119
    difference: 0.000000000000 +/-  1.00E-10
192 eigenvalue:    -3.980245029069 real eigenvalue:    -3.980245029069
    difference: 0.000000000000 +/-  1.00E-10
193 eigenvalue:    -3.984385729647 real eigenvalue:    -3.984385729647
    difference: 0.000000000000 +/-  1.00E-10
194 eigenvalue:    -3.988041672338 real eigenvalue:    -3.988041672338
    difference: 0.000000000000 +/-  1.00E-10
195 eigenvalue:    -3.991211964044 real eigenvalue:    -3.991211964044
    difference: 0.000000000000 +/-  1.00E-10
```

```
196  eigenvalue:   -3.993895830308 real eigenvalue:   -3.993895830308
       difference: 0.000000000000 +/-  1.00E-10
197  eigenvalue:   -3.996092615498 real eigenvalue:   -3.996092615498
       difference: 0.000000000000 +/-  1.00E-10
198  eigenvalue:   -3.997801782971 real eigenvalue:   -3.997801782971
       difference: 0.000000000000 +/-  1.00E-10
199  eigenvalue:   -3.999022915201 real eigenvalue:   -3.999022915201
       difference: 0.000000000000 +/-  1.00E-10
200  eigenvalue:   -3.999755713881 real eigenvalue:   -3.999755713881
       difference: 0.000000000000 +/-  1.00E-10
```

Percebe-se que a diferença se manteve dentro da tolerância em todos os casos mostrando a eficiência do método empregado.

1.2 Exercício 2

Tarefa: Considere a mesma matriz A do exercício anterior e o vetor $\vec{d}$ com elementos $d_i = \delta_{i,j}$, onde j é escolhido aleatoriamente entre os valores $1, 2, \ldots, n$. Deseja-se resolver numericamente o sistema de equações lineares $A\vec{x} = \vec{d}$, utilizando o **algoritmo de Thomas**, específico para matrizes tridiagonais.

O algoritmo consiste em duas fases: uma substituição direta modificada para calcular coeficientes intermediários d'_m e c'_m, e uma substituição retroativa para obter a solução final $\vec{x}$. Abaixo estão as fórmulas utilizadas:

$$d'_1 = \frac{d_1}{b_1}, \qquad\qquad c'_1 = \frac{c_1}{b_1},$$
$$d'_m = \frac{d_m - a_m d'_{m-1}}{b_m - a_m c'_{m-1}}, \qquad c'_m = \frac{c_m}{b_m - a_m c'_{m-1}}, \quad \text{para } m = 2, 3, \ldots, n-1,$$
$$d'_n = \frac{d_n - a_n d'_{n-1}}{b_n - a_n c'_{n-1}}.$$

A solução $\vec{x}$ é então obtida pela substituição retroativa:

$$x_n = d'_n,$$
$$x_m = d'_m - c'_m x_{m+1}, \quad \text{para } m = n-1, n-2, \ldots, 1.$$

Considere o caso de uma matriz tridiagonal de dimensão $n \times n$, com n sendo um input do código. Após resolver o sistema, verifique a solução calculando o vetor resíduo:

$$\vec{r} = A\vec{x} - \vec{d}.$$

15

Esse vetor deve ser próximo de zero (em norma) se a solução numérica estiver correta.

O código foi compilado com o comando:

```
gfortran L4-5255417-ex-2.f90 -Wall -Wextra -pedantic -ffree-form -o L4-
```

Resultados:

Testou-se os seguintes casos:


```
pedro@Pedro-Lenovo ~/Documentos/GitHub/quanticacomp/lista4
> $ ./L4-5255417-ex-2.exe
 Insert matrix dimension:
8
The solution x of Ax = d (with j = 1) and the residue vector are:
   1      -0.888889        0.000000
   2      -0.777778        0.000000
   3      -0.666667        0.000000
   4      -0.555556        0.000000
   5      -0.444444        0.000000
   6      -0.333333        0.000000
   7      -0.222222        0.000000
   8      -0.111111        0.000000

pedro@Pedro-Lenovo ~/Documentos/GitHub/quanticacomp/lista4
> $ ./L4-5255417-ex-2.exe
 Insert matrix dimension:
8
The solution x of Ax = d (with j = 5) and the residue vector are:
   1      -0.444444        0.000000
   2      -0.888889        0.000000
   3      -1.333333        0.000000
   4      -1.777778        0.000000
   5      -2.222222        0.000000
   6      -1.666667        0.000000
   7      -1.111111        0.000000
   8      -0.555556        0.000000
```

Figura 5: Testes para matriz de dimensão 8

Figura 6: Testes para matriz de dimensão 10

(a)

(b)

Figura 7: Teste para matriz de dimensão 20

Como todos os vetores resíduo obtidos foram nulos, sabemos que o código funciona de acordo com o esperado - obtendo o autovetor x corretamente.