

The revision of the manuscript has significantly improved its overall quality. The theoretical background on hyperbolic non-conservative problems has been appropriately addressed. In particular, proving non-negativity of the phase density and the phasic volume fraction strengthens the quality of the method, since a rigorous proof is lacking in many publications on methods for two-phase flow. Therefore, it cannot be considered a simple extension of the original work by Guermond and Popov. Moreover, the readability is improved by summarizing the main findings in lemmata. Finally, all my remarks have been appropriately addressed. Therefore I strongly recommend publication the revised version. In the final version, the authors are recommended to account for the following minor changes:

- p. 2, l. 43 and p. 8, l. 25:
Replace "Del Maso-Le Floch-Mariat" by "Dal Maso-LeFloch-Murat"
- p. 8, l. 30: replace "Le Flcoh" by "LeFloch"
- Figures 7 and 8: The authors might consider to rescale the pressure and density figures, e.g., $\log(p)$ or $\log(p + P_\infty)$ and $\rho_l/\rho_{l,init}$, $\rho_g/\rho_{g,init}$. Then spurious oscillations will become visible that are suppressed otherwise due to the scaling.
- References [18], [19], [20] and [39]: replace "Floch, G.L." by "LeFloch, P."
- Reference [32]: the list of authors seems to be not correct
- References [39]: replace "Maso, D." by "Dal Maso, G."
- Reference [36]: Replace "Le Mtayer" by "Le Métayer"