Answers to Reviewer #1

Ms. Ref. No.: JOMP-D-15-00204
Title: "Viscous Regularization for the Non-equilibrium Seven-Equation Two-Phase Flow Model',
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Viscous regularization of hyperbolic conservation laws is a meaningful approach to stabilize the numerical discretization of such problems. In the present manuscript the authors derive a viscous regularization for a two-phase model following previous work by Guermond and Popov for a single-phase uid. The basic idea is to design regularization terms such that the entropy residuals (corresponding to the physical entropy and Harten's generalized entropies) are positive. For scalar problems in conservative form it can be proven that this ensures uniqueness of the weak solution. Therefore, positivity of the entropy residuals is considered a useful property also for systems. The subject can be considered very interesting and relevant. The presentation is clear and the analysis is sound. However, some issues are not properly addressed and the presentation of the numerical results needs to be improved before I can recommend this paper for publication.

1. The hyperbolic system (1) corresponding to the seven-equation two-phase ow model is not in conservative form because of the gradients $\nabla\alpha_k$ in (1a), (1c) and (1d). Therefore classical results from the theory of conservation laws cannot be directly applied to the present system. Based on the DLM theory [2] the classical definitions of a weak solution and the entropy solution, respectively, can be extended for quasi- conservative problems introducing a path, see [3, 4, 2]. Existence of a solution to a problem in non-conservative form has been investigated in [5]. In [6] existence of a solution to the Riemann problem has been verified. In particular, shock waves are influenced by the path since it enters the generalized Rankine-Hugoniot jump conditions, see [4]. Note that rarefaction waves and contact discontinuities are not affected by the choice of the path. It would be interesting for the reader to know what notion of weak solution and entropy solution is applied here.

2. In the abstract, see page 1, line 20, the authors claim that regularization ensures uniqueness of the weak solution. No evidence is given in the manuscript, i.e., neither a reference nor a proof, that justifies this statement. For scalar problems in conservative form it is well-known that uniqueness is ensured if an entropy inequality holds for all entropy-entropy flux pairs. However, the problem at hand is not in conservative form. See also the first remark.

3. In (3) the authors give a particular choice for the interfacial pressure and velocity. It should be mentioned that other choices are considered in the

literature for which the second law of thermodynamics can be verified, see e.g. [7].

4. On page 5, l. 34-37, it should be mentioned that hyperbolicity also requires a full set of linearly independent (left/right) eigenvectors. For the considered system this only holds true if the non-resonance condition is satisfied, see [1].

5. In view of my first remark it would be interesting to know whether the field associated to the eigenvalue λ_{3+dim} in (6) is linearly degenerated. In this case the jump in α corresponds to a contact discontinuity.

6. For the numerical computations in Section 5.2 the stiffened gas law (32) has to be modified to account for the heat of formation q_k.

7. For both computations presented in Section 5 the authors should provide more information on the discretization, e.g., the type of scheme (FVM/DG, explicit/implicit) and its main ingredients. In particular, the discretization of the non-conservative terms in (1) is of interest. Furthermore, the spatial/temporal dicretization and the CFL number should be mentioned.

8. For both test cases it would be interesting to investigate grid convergence because the chosen regularization terms are grid-dependent and vanish with increasing resolution, i.e., stabilization by viscous regularization is successively reduced. In particular, the grid convergence study is recommended for the shock tube problem because the numerical results show a severe smearing for both the contact wave and the shock wave.

9. Since the authors only give the initial data for velocity, pressure and temperature, the authors should also provide a caloric equation of state to compute the density, i.e., T_k.

10. The authors verified positivity of the entropy residual. Therefore it would be interesting to see whether the numerical scheme maintains this property. Therefore I would be interested to see plots of the numerical entropy residual to check whether it remains positive.

11. Besides the two computations presented here I would like to recommend further numerical tests exhibiting stronger variations in the volume fractions and the density, e.g., a shock tube problem for a water-gas configuration with $(\rho, uP, \alpha)_{water} = (1000kg.m^3, 0m/s, 10^5Pa, 0 : 99)$ and $(\rho, uP, \alpha)_{gas} = (0.2kg.m^3, 0m/s, 2000Pa, 0 : 99)$, i.e., the initial states are almost "pure" states of either water or gas, with material parameters as given in [8].

12. In the conclusion, see page 23, line 41/42 the authors claim that the regularization ensures uniqueness of the numerical solution. Since the authors

do not give a uniqueness proof this statement seems to be too strong and should be rephrased. See also my first and second remark.