

## Answers to Reviewer #2

Ms. Ref. No.: JOMP-D-15-00204

Title: “Viscous Regularization for the Non-equilibrium Seven-Equation Two-Phase Flow Model”,

*Springer Journal of Scientific Computing*

In blue are reviewer’s comments, in black are our answers, in red are Jean’s comments, and in green are Marco’s comments.

After the recent work by Guermond and Popov where a general class of viscous regularizations of compressible Euler equations is investigated, the present work proposes an extension in the case of the non-equilibrium seven-equation two-phase flow model. To address such an issue, the authors introduce a general viscosity within the adopted model. Then, they derive the entropy evolution law, now perturbed by the additional viscosity. By adopting a relevant definition of the additional viscosity terms, the authors claim that a minimum entropy principle is satisfied. This study is completed by a Chapman-Enskog extension to get the associated five-equation model in the limit of infinite relaxation coefficients. The analysis is achieved by considering the incompressible regime governed by low Mach number. The authors claim that the adopted viscosity regularization does not modify the required incompressible regime. Finally, numerical illustrations are displayed in order to attempt to illustrate the relevance of the viscosity regularizations.

My opinion about this work is not good at all since this paper looks like a poor extension of the work by Guermond and Popov.

We disagree with your assessment. add all the things we learn thanks to reviewer #1; extending to 2-phase flow is by far not a small extension because of everything reviewer # 1

1. The main point of this work concerns the derivation of the minimum entropy principle. Here, the establishment of this property is not clear at all. I think that the proof is incomplete. For instance, Guermond and Popov need (and prove) the positiveness of the density. I think that the positiveness of partial density is here needed but no proof is given. Moreover, I am convinced that  $\alpha_k \in [0, 1]$  is also necessary and must be proved. I urge the author to read carefully the paper by Guermond and Popov and reconsider the establishment of their results.

we can cite Guermond for the phasic regularized continuity equation and show that  $\alpha\rho$  is non-negative. Then we can work on the alpha equations (both of them) to show that  $\alpha_k \in [0, 1]$ . From this, we conclude that  $\rho_k$  is non-negative. Showing that  $\alpha\rho$  is non-negative is trivial based on Guermond’s work. Showing, however, that  $\rho$  is non-negative seems trickier.

2. Several times, the author speaks about uniqueness of the numerical solution. I don't understand the meaning of these words. Moreover, this paper does not contain numerical derivations. Page 7, the authors refer to Leveque (pages 27-28 in Numerical Methods for Conservation Laws), but these two pages in the Leveque book coincide with the introduction of weak solutions and entropy inequalities. Nothing about uniqueness of the numerical solution.

Using Leveque for everything is a mistake. We will correct this using piece of the answer we gave to reviewer #1. Once again, 'entropy solution' and not 'uniqueness of the weak solution'.

3. The numerical schemes, used to get the numerical illustrations, are not specified. However, the derivation of a numerical scheme to approximate the weak solution of the model under consideration is a very difficult task.

We will add this

4. The presentation of the entropy residual is absolutely not relevant. In section 3.2, I understand the opportunity to omit the underlined terms. However, the equation (15) turns out to be wrong. The authors have to introduce a specific notation to designate the entropy residual.

This is where the reviewer really shows he doesn't get it. We can still add a specific notation for the entropy residual as he requested it.

In addition, I think that these results are not suitable to be published in Journal of Scientific Computing. As a consequence, I do not recommend the publication of this work.

Pure BS