

The liquid phase obeys the following mass, momentum and energy balance equations, supplemented by a non-conservative volume-fraction equation:

$$\frac{\partial (\alpha \rho)_{liq} A}{\partial t} + \vec{\nabla} \cdot (\alpha \rho \mathbf{u} A)_{liq} = 0 \quad (1a)$$

$$\begin{aligned} & \frac{\partial (\alpha \rho \mathbf{u})_{liq} A}{\partial t} + \vec{\nabla} \cdot [\alpha_{liq} A (\rho \mathbf{u} \otimes \mathbf{u} + P \mathbb{I})_{liq}] \\ &= P_{int} A \vec{\nabla} \alpha_{liq} + P_{liq} \alpha_{liq} \vec{\nabla} A + A \lambda_u (\mathbf{u}_{vap} - \mathbf{u}_{liq}) \end{aligned} \quad (1b)$$

$$\begin{aligned} & \frac{\partial (\alpha \rho E)_{liq} A}{\partial t} + \vec{\nabla} \cdot [\alpha_{liq} \mathbf{u}_{liq} A (\rho E + P)_{liq}] = P_{int} \mathbf{u}_{int} A \vec{\nabla} \alpha_{liq} \\ & - \bar{P}_{int} A \mu_P (P_{liq} - P_{vap}) + \bar{\mathbf{u}}_{int} A \lambda_u (\mathbf{u}_{vap} - \mathbf{u}_{liq}) \end{aligned}$$

$$\frac{\partial \alpha_{liq} A}{\partial t} + A \mathbf{u}_{int} \cdot \vec{\nabla} \alpha_{liq} = A \mu_P (P_{liq} - P_{vap}) \quad (1c)$$

On the same model, the equations for the vapor phase are:

$$\frac{\partial (\alpha \rho A)_{vap}}{\partial t} + \vec{\nabla} \cdot (\alpha \rho \mathbf{u})_{vap} A = 0 \quad (2a)$$

$$\begin{aligned} & \frac{\partial (\alpha \rho \mathbf{u})_{vap} A}{\partial t} + \vec{\nabla} \cdot [\alpha_{vap} A (\rho \mathbf{u} \otimes \mathbf{u} + P \mathbb{I})_{vap}] \\ &= P_{int} A \vec{\nabla} \alpha_{vap} + P_{vap} \alpha_{vap} \vec{\nabla} A + A \lambda_u (\mathbf{u}_{liq} - \mathbf{u}_{vap}) \end{aligned} \quad (2b)$$

$$\begin{aligned} & \frac{\partial (\alpha \rho E)_{vap} A}{\partial t} + \vec{\nabla} \cdot [\alpha_{vap} \mathbf{u}_{vap} A (\rho E + P)_{vap}] = P_{int} \mathbf{u}_{int} A \vec{\nabla} \alpha_{vap} \\ & - \bar{P}_{int} A \mu_P (P_{vap} - P_{liq}) + \bar{\mathbf{u}}_{int} A \lambda_u (\mathbf{u}_{liq} - \mathbf{u}_{vap}) \end{aligned}$$

$$\frac{\partial \alpha_{vap} A}{\partial t} + A \mathbf{u}_{int} \cdot \vec{\nabla} \alpha_{vap} = A \mu_P (P_{vap} - P_{liq}) \quad (2c)$$

Interfacial variables:

$$P_{int} = \bar{P}_{int} + \frac{Z_{liq} Z_{vap}}{Z_{liq} + Z_{vap}} \frac{\vec{\nabla} \alpha_{liq}}{\|\vec{\nabla} \alpha_{liq}\|} \cdot (\mathbf{u}_{vap} - \mathbf{u}_{liq}) \quad (3a)$$

$$\bar{P}_{int} = \frac{Z_{vap} P_{liq} + Z_{liq} P_{vap}}{Z_{liq} + Z_{vap}} \quad (3b)$$

$$\mathbf{u}_{int} = \bar{\mathbf{u}}_{int} + \frac{\vec{\nabla} \alpha_{liq}}{\|\vec{\nabla} \alpha_{liq}\|} \frac{P_{vap} - P_{liq}}{Z_{liq} + Z_{vap}} \quad (3c)$$

$$\bar{\mathbf{u}}_{int} = \frac{Z_{liq} \mathbf{u}_{liq} + Z_{vap} \mathbf{u}_{vap}}{Z_{liq} + Z_{vap}}. \quad (3d)$$

The pressure,  $\mu_P$ , and velocity,  $\lambda_u$ , relaxation coefficients are proportional to each other and function of the interfacial area  $A_{int}$ :

$$\lambda_u = \frac{1}{2} \mu_P Z_{liq} Z_{vap} \quad (4)$$

$$\mu_P = \frac{A_{int}}{Z_{liq} + Z_{vap}} \quad (5)$$

The specific interfacial area (i.e., the interfacial surface area per unit volume of two-phase mixture),  $A_{int}$ , must be specified from some type of flow regime map or function under the form of a correlation:

$$A_{int} = A_{int}^{max} \left[ 6.75 (1 - \alpha_{liq})^2 \alpha_{liq} \right], \quad (6)$$

where  $A_{int}^{max} = 5100 \text{ m}^2/\text{m}^3$ . With such definition, the interfacial area is zero in the limits  $\alpha_{liq} = 0$  and  $\alpha_{liq} = 1$ .

**Initial conditions**

Uniform pressure for both phases:  $P_{vap} = P_{liq} = 10^6 \text{ Pa}$ .

The two phases are at rest:  $u_{vap} = u_{liq} = 0 \text{ m} \cdot \text{s}^{-1}$

Uniform temperature for both phases:  $T_{vap} = T_{liq} = 300 \text{ K}$

Volume fraction linearly varies from 0.9 to 0.4.

**Boundary conditions**

Static pressure:  $\alpha = 0.9$ ,  $P = 10^6$  and  $T = 300$ .

Solid wall.