Application of the entropy viscosity method to the 1-D 7-equation Model for Two-Phase Flows

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INTRODUCTION

In this paper, we extend the entropy viscosity method, proposed by Guermond et al. [1, 2] to the well-posed 1-D 7-equation two-phase model [3, 4]. This model is obtained by integrating the single-phase flow balance equations weighed by a characteristic function for each phase. The resulting system of equations contains non-conservative terms that describe the interaction between phases but also an equation for the volume fraction. It is also known to be unconditionally hyperbolic which is highly desirable when working with approximate Riemann solvers and can treat a wide range of applications. Its particularity comes from the pressure and velocity relaxation terms in the volume fraction, momentum and energy equations that can bring the two phases in equilibrium when using large values of the relaxation parameters. In other words, the 7-equation model can degenerate into the 6 [5] and the 5-equation models [6]. The 7-equation model is currently solved on *discontinuous schemes* with approximate Riemann solvers derived from the well-established approach for single-phase flows, while using an upwind-type flux for the non-conservative terms [7, 8, 9, 10, 11].

The entropy viscosity technique is a viscous regularization technique that satisfies the entropy minimum principle; adequate dissipation terms (viscous fluxes) are added to the governing laws while ensuring the entropy minimum principle still holds. Viscosity coefficients modulates the magnitude of the added dissipation such that it is large in shock regions and vanishingly small elsewhere. The entropy viscosity coefficients are taken proportional to the entropy production while, at the same time, being bounded from above by a first-order viscosity coefficient that reduces the spatial discretization to be similar to a first-order Godunov scheme (the latter being known to be overly dissipative but monotone [12]). Hence, entropy production in shocks will result in large viscosity coefficients and thus will avoid spurious oscillations.

The entropy method is independent of the type of spatial discretization (finite volume, continuous or discontinuous finite elements, ...) and thus can be applied ubiquitously. The results presented in this summary were obtained with RELAP-7 [13], the next-generation reactor system code build upon the MOOSE multiphysics framework [14] that uses a *Continuous Galerkin Finite Element Method*.

In this summary, the 1-D 7-equation model is recalled along with the dissipative terms used in the entropy method. Definition of the viscosity coefficients is also given and numerical results are presented for a 1-D two-phase flow shock tube.

THEORY

We recall the 1-D 7-equation model for a phase k in interaction with a phase j and also added the viscous regularization based on the entropy-viscosity method (equation of the phase j are obtained by simply substituting k by j):

$$\frac{\partial \alpha_k A}{\partial t} + u_{int} A \frac{\partial \alpha_k}{\partial x} = A\mu_P(p_k - p_j) + \boxed{\frac{\partial}{\partial x}(A l_k)} \tag{1a}$$

$$\frac{\partial (\alpha\rho)_k A}{\partial t} + \frac{\partial (\alpha\rho u)_k A}{\partial x} = \boxed{\frac{\partial}{\partial x}(A f_k)} \tag{1b}$$

$$\begin{aligned}&\frac{\partial (\alpha\rho u)_k A}{\partial t} + \frac{\partial \alpha_k A \left(\rho u^2 + p\right)_k}{\partial x} = p_{int} A \frac{\partial \alpha_k}{\partial x} \\ &+ p_k \alpha_k \frac{\partial A}{\partial x} + A\lambda_u(u_j - u_k) \\ &+ \boxed{\frac{\partial}{\partial x}\left[A\left(g_k + u_k f_k\right)\right]}\end{aligned} \tag{1c}$$

$$\begin{aligned}&\frac{\partial (\alpha\rho E)_k A}{\partial t} + \frac{\partial \alpha_k u_k A (\rho E + p)_k}{\partial x} = p_{int} u_{int} A \frac{\partial \alpha_k}{\partial x} \\ &- \bar{p}_{int} A \mu_P (p_k - p_j) + \bar{u}_{int} A \lambda_u (u_j - u_k) \\ &+ \boxed{\frac{\partial}{\partial x}\left[A\left(h_k + u_k g_k - \frac{u_k^2}{2} f_k + \rho_k e_k l_k\right)\right]}\end{aligned} \tag{1d}$$

where the dissipative terms, boxed in Eq .1, have the following definition:

$$l_k = \beta_k \partial_x \alpha_k \tag{2a}$$

$$f_k = \alpha_k \kappa_k \partial_x \rho_k + \rho_k l_k \tag{2b}$$

$$g_k = \alpha_k \mu_k \rho_k \partial_x u_k \tag{2c}$$

$$h_k = \alpha_k \kappa_k \partial_x (\rho_k e_k) \tag{2d}$$

The notation is standard: α_k, ρ_k, $(\rho\vec{u})_k$ and $(\rho E)_k$ are the volume fraction, the density, the momentum and the total energy for phase k, respectively, and will be referred to as the conservative variables. $\vec{u}_k$ is the fluid velocity for phase k and its specific internal energy is denoted by $e_k = E_k - \frac{u_k^2}{2}$. The area A is a given and can be spatial-dependent. An equation of state is used to compute the pressure P_k. Definitions of the interfacial variables denoted by the subscript *int* can be found in [4]. As mentioned in the introduction, each viscosity coefficient is function of an upper bound refer to as the first-order viscosity and denoted by the subscript

max, and a high-order viscosity coefficient denoted by the subscript e as follows:

$$\mu_k^K(\vec{r}_q,t) = \min(\mu_{max,k}^K(\vec{r}_q,t), \mu_{e,k}^K(\vec{r}_q,t)),$$
$$\kappa_k^K(\vec{r}_q,t) = \min(\kappa_{max,k}^K(\vec{r}_q,t), \kappa_{e,k}^K(\vec{r}_q,t))$$
$$\text{and } \beta_k^K(\vec{r}_q,t) = \min(\beta_{max,k}^K(\vec{r}_q,t), \beta_{e,k}^K(\vec{r}_q,t))$$

where K is a given element of the mesh, and $\vec{r}_q$ is a quadrature point location within cell K. The first-order viscosity coefficients, $\mu_{max,k}^K$ and $\kappa_{max,k}^K$, are only present in the continuity, momentum and energy equations of each phase, and thus are defined proportional to the local maximum eigenvalue $\|\vec{u}_k\| + c_k$.

$$\kappa_{max,k}^K(\vec{r}_q,t) = \mu_{max,k}^K(\vec{r}_q,t) = \frac{h^k}{2}(\|\vec{u}_k(\vec{r}_q,t)\| + c_k(\vec{r}_q,t))$$
$$\beta_{max,k}^K(\vec{r}_q,t) = \frac{h^K}{2}\|\vec{u}_{int}(\vec{r}_q,t)\|,$$

where h^k is the grid size. On the other hand, the first-order viscosity coefficient $\beta_{max,k}^K$ is defined proportional to the eigenvalue $\vec{u}_{int}$ since intimately related to the void fraction equation. The high-order viscosity coefficients, $\beta_{e,k}^K$, $\kappa_{e,k}^K$ and $\mu_{e,k}^K$ are distinct positive viscosity coefficients and are based on the local entropy production in phase k. The definition of the coefficients $\mu_{e,k}^K$ and $\kappa_{e,k}^K$ is identical to the one used for the multi-D Euler equations [15] and yield well-scaled dissipative terms in the low-Mach regime [16, 17, 18]:

$$\kappa_{e,k}^K(\vec{r}_q,t) = h_K^2 \frac{\max\left(|\widetilde{R}_k^K(\vec{r}_q,t)|, J_P^K\right)}{\rho_k c_k^2} \tag{3a}$$

$$\mu_{e,k}^K(\vec{r}_q,t) = Pr_k\, \mu_e^K(\vec{r}_q,t) \tag{3b}$$

where c_k is the phase speed of sound and the weighting factor is the local Mach number, $M_k = \|\vec{u}_k\|/c_k$. Pr_k is the Prandtl number; see [1] for additional details. The entropy residual is denoted by $\widetilde{R}$ and its definition is recalled in Eq. (4).

$$\widetilde{R}(\vec{r}_q,t) := \left(\frac{\mathrm{D}P_k}{\mathrm{D}t} - c_k^2\frac{\mathrm{D}\rho_k}{\mathrm{D}t}\right) \propto \partial_t s_k + \vec{u}\cdot\vec{\nabla}s_k = \frac{\mathrm{D}s_k}{\mathrm{D}t}, \tag{4}$$

where s_k is the phase entropy, function of the density ρ_k and the internal energy e_k. Proof of Eq. (4) can be found in [15]. Lastly, the quantity J_P denotes the inter element jumps of the gradient of the pressure and the density (see [15] for details).

The approach to define the viscosity coefficient $\beta_{e,k}^K$ is similar to the logic followed for hyperbolic scalar equations [1, 2]: an entropy equation can be derived from the volume fraction equation (Eq. (1a)) and used in the definition of the coefficient $\beta_{e,k}^K$. Following the work by Guermond et al. [1, 2], one obtains:

$$\beta_{e,k}^K(\vec{r}_q,t) = h_K^2 \frac{\max\left(|R_{\alpha,k}^K(\vec{r}_q,t)|, J_\alpha^K\right)}{\alpha_k}, \tag{5a}$$

where the entropy residual associated to the volume fraction equation, Eq. (1a), is

$$R_{\alpha,k}^K(\vec{r}_q,t) = \frac{1}{2}\left(\frac{\partial\alpha_k^2}{\partial t} + u_{int}\frac{\partial\alpha_k^2}{\partial x}\right). \tag{5b}$$

where $s_{\alpha,k} = \alpha_k^2/2$ can be interpreted as the entropy function associated to the volume fraction equation of phase k. Once again, J_α^K denotes the inter element jump of the gradient of the volume fraction.

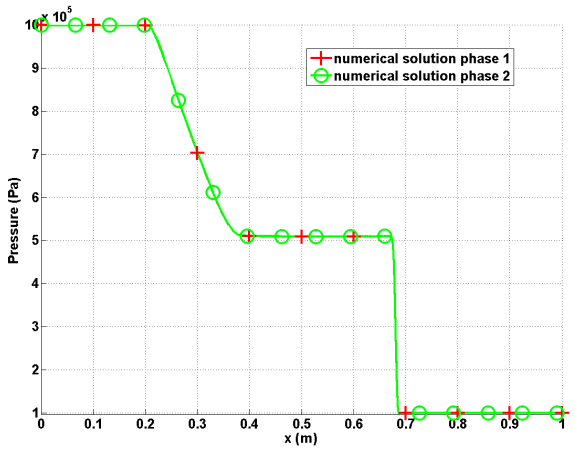
RESULTS AND ANALYSIS

The 1-D 7-equation model with viscous stabilization is discretized with *continuous* finite elements in space and BDF2 in time using the multiphysics framework RELAP-7. The resulting nonlinear system of equations at each time step is solved using a Jacobian-free Newton Krylov technique. We present one sample result for a 1-D two-phase flow shock tube of length $L = 1\ m$ and area $A = 1\ m^2$ filled with two gas phases in equilibrium (same pressure and velocity) described by the Ideal Gas equation of state with $\gamma_1 = 3$ and $\gamma_2 = 1.4$ with the objective of testing our numerical stabilization method. The diaphragm separates the pipe in two chambers with a high pressure ($P_{left} = 1\ MPa$) on the left side and a low pressure ($P_{right} = 0.1\ MPa$) in the right side. Both fluids are initially at rest. The volume fraction is set to 0.5 which means each side of the chamber contains a mixture of two fluids with different equation of state parameters. The pressure and velocity relaxation coefficients are computed using the expression provided in Eq. (6).

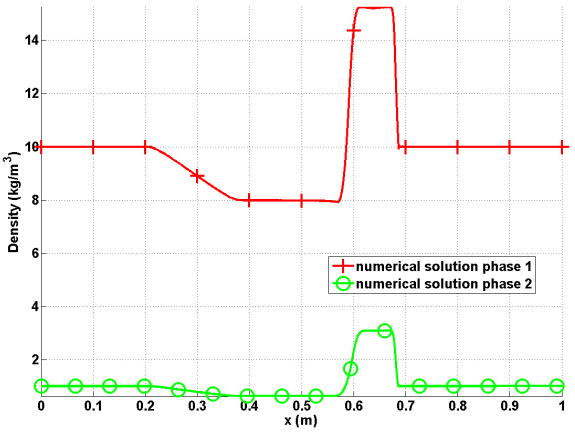
$$\lambda_u = \frac{1}{2}\mu_P Z_1 Z_2 \text{ and } \mu_P = \frac{A_{int}}{Z_1 + Z_2} \tag{6}$$

where the interfacial area is set to a large value, $A_{int} = 10^4 m^{-1}$, so that the two phases achieve pressure and velocity equilibrium at all time. The geometry is discretized with an uniform mesh of 500 cells and run with a CFL of one until $t_{final} = 305\ \mu s$. The numerical results are presented from Fig. 1a to 1g. An exact solution for this shock tube test is not available but numerical results obtained with a discontinuous scheme on a moving mesh can be found in [19] and use for eyeball comparison.

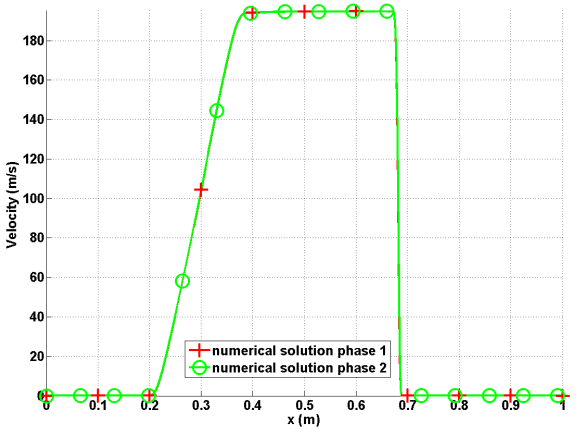
As expected, the two fluids have the same pressure and velocity profiles as shown in Fig. 1a and Fig. 1c, respectively. The shock is well resolved and does not display any instability. The density of phase 1 and 2 have different values but experience the same variations (shock, contact and rarefaction waves), as shown in Fig. 1b. The volume fraction varies because of the pressure relaxation term (Eq. (1a)) and displays a shock wave around $x = 0.7\ m$ as shown in Fig. 1d. Consequently, the viscosity coefficient β_k is peaked in the shock region and also displays a second peak of lower amplitude because of the jump. Overall, the numerical solution is efficiently stabilized by the entropy viscosity method and the discontinuity are well resolved. The viscosity coefficients are peaked in the shock region and thus, behave as expected.



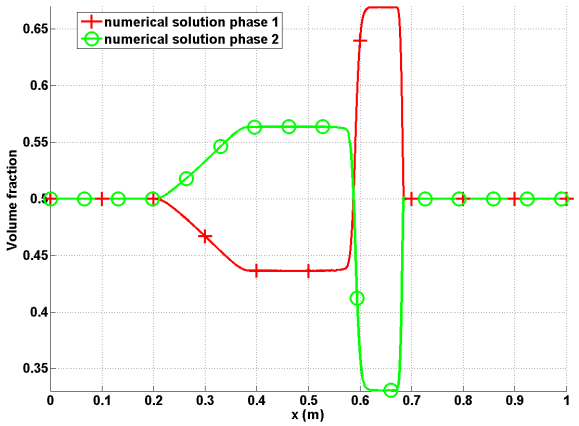
(a) Pressure



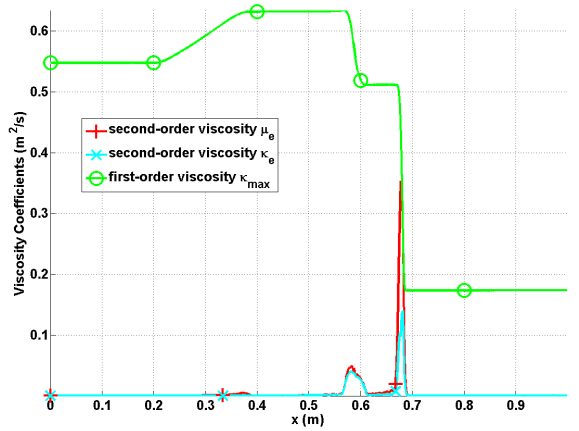
(b) Density



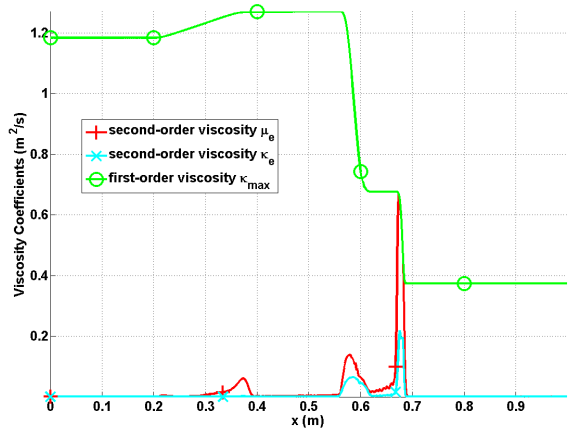
(c) Velocity



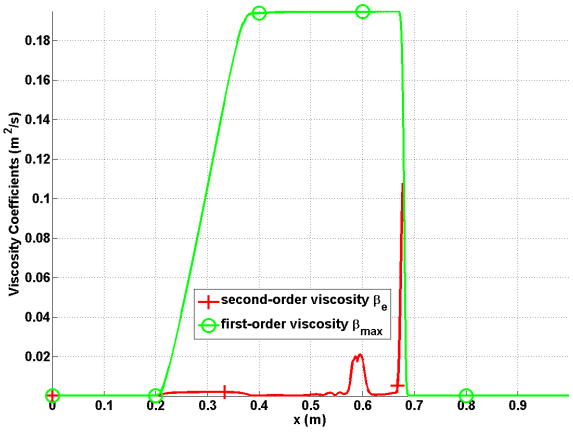
(d) Volume fraction



(e) Viscosity coefficients for phase 1



(f) Viscosity coefficients for phase 2



(g) Viscosity coefficients for volume fraction

CONCLUSIONS

We have presented an extension of the entropy viscosity method to the 1-D 7-equation model and applied it to a 1-D two-phase flow shock tube with large relaxation coefficients. The numerical results show that the stabilization method is capable of stabilizing the schemes and that the viscosity coefficients are well-scaled. This work will further contribute to the assessment of the stabilization technique for reactor flows in RELAP-7.

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