Extension of the Entropy Viscosity Method to the Seven-Equation two-phase flow Model

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Outline:

Introduction / Background

Interest for Two-Phase Flow Models (T-PFM)

- Engineering applications: oil/gas, combustion, nuclear ...
- Accurately predict flow behavior → improve safety margins

Two-Phase Flow Models

- Example of models: HEM, 5-equ model of Kapila, 6-equ and 7-equ models
- Use a well-posed compressible model → real eigenvalues
- Allow to develop/use numerical methods

Discretization and numerical methods

- T-PFM are usually solved on discontinuous schemes (FV, DGFEM)
- Numerical methods: approximate Riemann solver → Godunov-type solvers
- All-speed fluid flow solver → low-Mach, transonic and supersonic flows
- Achieve spatial and temporal high-order accuracy.

The Seven-Equation two-phase flow Model (SEM)

- Each phase obeys Euler equations + void fraction equation + exchange terms
- Has 7 real eigenvalues: acoustic, contact and interfacial waves
- Degenerates to Euler equations when one phase disappears

All-speed fluid flow solver

Goal

To use compressible fluid equations for all Mach numbers
To solve them using *continuous* FEM using MOOSE → RELAP-7

All-speed fluid flow solver

Low-Mach: huge disparity in speeds (pressure waves move much faster)

- Severely CFL-constrained if using explicit time-stepping
- Best to use implicit time stepping
- Nonlinear system of equations
- Fits the JFNK formalism in MOOSE where all physic components are tightly coupled

Regularization technique for discretization of fluid flow

We will employ novel artificial viscosity schemes based on the entropy production residual (Guermond et al., *Entropy viscosity method for nonlinear conservation laws*, J. of Comput. Phys. (2011).

The entropy viscosity method is discretization-independent and was significantly tested in the low-Mach, transonic and supersonic regimes for the single-phase Euler equations (including using continuous FEM).

The Seven-Equation two-phase flow Model (1/2)

We consider two phases j, k. Phase k obeys the following system of equations:

Void fraction equation:

$$\partial_t \alpha_k + \vec{u}_{int} \cdot \vec{\nabla} \alpha_k = \mu_{rel} \left(P_k - P_j \right)$$

Continuity equation:

$$\partial_t \left(\alpha_k \rho_k \right) + \vec{\nabla} \cdot \left(\alpha_k \rho_k \vec{u}_k \right) = 0$$

Momentum equation:

$$\partial_t \left(\alpha_k \rho_k \vec{u}_k \right) + \vec{\nabla} \cdot \left[\alpha_k \left(\rho_k \vec{u}_k \otimes \vec{u}_k \right) \right] + \vec{\nabla} (\alpha_k P_k) = P_{int} \vec{\nabla} \alpha_k + \lambda_{rel} \left(\vec{u}_j - \vec{u}_k \right)$$

Energy equation:

$$\partial_t \left(\alpha_k \rho_k E_k \right) + \vec{\nabla} \cdot \left[\alpha_k \vec{u}_j \left(\rho_k E_k + P_k \right) \right] = P_{int} \vec{u}_{int} \cdot \vec{\nabla} \alpha_k - \mu_{rel} \bar{P}_{int} \left(P_k - P_j \right) + \lambda_{rel} \bar{\vec{u}}_{int} \cdot \left(\vec{u}_j - \vec{u}_k \right)$$

The mass, momentum and energy exchange terms between the phases k and j are missing in the above equations.

The Seven-Equation two-phase flow Model (2/2)

Interfacial (P_{int} and $\vec{u}_{int}$) and relaxation parameters (λ_{rel} and μ_{rel})

$$\begin{cases} P_{int} = \bar{P}_{int} - \frac{Z_k Z_j}{Z_k + Z_j} \frac{\vec{\nabla}\alpha_k}{|\vec{\nabla}\alpha_k|} \cdot (\vec{u}_k - \vec{u}_j) \\ \bar{P}_{int} = \frac{Z_k P_j + Z_j P_k}{Z_k + Z_j} \\ \vec{u}_{int} = \bar{\vec{u}}_{int} - \frac{\vec{\nabla}\alpha_k}{|\vec{\nabla}\alpha_k|} \frac{P_k - P_j}{Z_k + Z_j} \\ \bar{\vec{u}}_{int} = \frac{Z_k \vec{u}_k + Z_j \vec{u}_j}{Z_k + Z_j} \end{cases} \text{and} \begin{cases} \mu_{rel} = \frac{A_{int}}{Z_k + Z_j} \\ \lambda_{rel} = \frac{\mu_{rel}}{2} Z_k Z_j \\ A_{int} = 6.25\ A_{int}^{max}\ \alpha_k (1 - \alpha_k)^2 \end{cases}$$

Phasic entropy residual

$$(s_e)_k^{-1} \alpha_k \rho_k \frac{Ds_k}{Dt} = \mu_{rel} \frac{Z_k}{Z_k + Z_j} (P_j - P_k)^2 + \lambda_{rel} \frac{Z_j}{Z_k + Z_j} (\vec{u}_j - \vec{u}_k)^2$$

$$\frac{Z_k}{(Z_k + Z_j)^2} \left[Z_j (\vec{u}_j - \vec{u}_k) + \frac{\vec{\nabla}\alpha_k}{||\vec{\nabla}\alpha_k||} (P_k - P_j) \right]^2 \geq 0$$

R. Berry, R. Saurel, O. LeMetayer, *The discrete equation method (DEM) for fully compressible, two-phase flows in ducts of spatially varying cross-section*, Nuclear Engineering and Design, 240 (2010) 3797-3818.

Quick overview of the entropy-based artificial viscosity formalism

General scalar conservation law: $\partial_t u + \vec{\nabla} \cdot \vec{f}(u) = 0$.

1. Determine an entropy pair $(s(u), \vec{\Psi}(u))$ for the PDE under consideration
2. Compute the entropy residual $R_e := \partial_t s(u_h) + \vec{\nabla} \cdot \Psi(u_h)$, in each cell K, at each quadrature point x_q
3. Compute the speed associated with this residual

$$v_e := h \frac{|R_e(x_q)|_K}{|s - \bar{s}|_\infty} \tag{2}$$

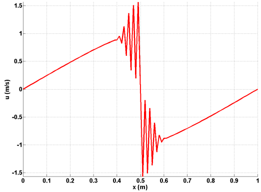
4. Define the dynamic viscosity (m^2/s) as

$$\mu := h \min\left(\frac{1}{2}|\vec{f}'(u)|,\ v_e\right) \tag{3}$$

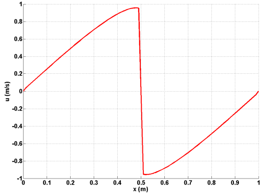
5. Plug in the standard Galerkin weak form as a viscous regularization

$$\int_V \left(\partial_t u_h + \vec{\nabla} \cdot \vec{f}(u_h)\right) b \, dx + \sum_K \int_K \mu_K \vec{\nabla} u_h \vec{\nabla} b \, dx = 0 \quad \forall b \tag{4}$$

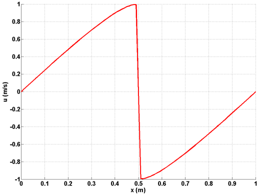
Example: Burgers equation



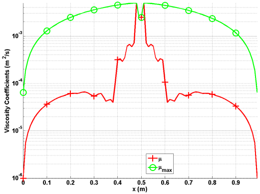
(a) Without stabilization.



(b) With first-order viscosity.



(c) With the EVM.



(d) Viscosity coefficient profiles.

A viscous regularization for the Seven-Equation two-phase flow Model

Void fraction equation:

$$\partial_t \alpha_k + \vec{u}_{int} \cdot \vec{\nabla} \alpha_k = \mu_{rel} \left(P_k - P_j \right) + \vec{\nabla} \cdot \left(\beta_k \vec{\nabla} \alpha_k \right)$$

Continuity equation:

$$\partial_t \left(\alpha_k \rho_k \right) + \vec{\nabla} \cdot \left(\alpha_k \rho_k \vec{u}_k \right) = \vec{\nabla} \cdot \vec{f} = \vec{\nabla} \cdot \left[\alpha_k \kappa_k \vec{\nabla} \rho_k + \rho_k \beta_k \vec{\nabla} \alpha_k \right]$$

Momentum equation:

$$\partial_t \left(\alpha_k \rho_k \vec{u}_k \right) + \vec{\nabla} \cdot \left[\alpha_k \left(\rho_k \vec{u}_k \otimes \vec{u}_k \right) \right] + \vec{\nabla} \left(\alpha_k P_k \right) =$$

$$P_{int} \vec{\nabla} \alpha_k + \lambda_{rel} \left(\vec{u}_j - \vec{u}_k \right) + \vec{\nabla} \cdot \left[\alpha_k \rho_k \mu_k \vec{\nabla}^s \vec{u} + \vec{u} \otimes \vec{f} \right]$$

Energy equation:

$$\partial_t \left(\alpha_k \rho_k E_k \right) + \vec{\nabla} \cdot \left[\alpha_k \vec{u}_j \left(\rho_k E_k + P_k \right) \right] =$$

$$P_{int} \vec{u}_{int} \cdot \vec{\nabla} \alpha_k - \mu_{rel} \bar{P}_{int} \left(P_k - P_j \right) + \lambda_{rel} \bar{\vec{u}}_{int} \cdot \left(\vec{u}_j - \vec{u}_k \right) +$$

$$\vec{\nabla} \cdot \left[\alpha_k \kappa_k \vec{\nabla} (\rho e)_k - \frac{||\vec{u}_k||^2}{2} \vec{f} + \alpha_k \mu_k \vec{u}_k : \vec{\nabla}^s \vec{u}_k \right]$$

M. Delchini, J. Ragusa and R. Berry, *A Viscous Regularization for the Seven-Equation Two-Phase Flow Model*, in preparation.

An all-Mach flow definition of the viscosity coefficients

$\mu_k(\vec{r},t) = \min\left(\mu_{k,\max}(\vec{r},t), \mu_{k,e}(\vec{r},t)\right)$, and $\kappa_k(\vec{r},t) = \min\left(\mu_{k,\max}(\vec{r},t), \kappa_{k,e}(\vec{r},t)\right)$

- $\kappa_{k,\max}(\vec{r},t) = \mu_{k,\max}(\vec{r},t) = \frac{h}{2}\left(||\vec{u}_k|| + c_k\right)$.
- $\kappa_{k,e}(\vec{r},t) = \frac{h^2 \max(\widetilde{R}_k, J_k)}{\rho_k c_k^2}$ and $\mu_{k,e}(\vec{r},t) = \frac{h^2 \max(\widetilde{R}_k, J_k)}{\mathrm{norm}^{\mu}_{k,P}}$
- $\widetilde{R}_k = \frac{DP_k}{Dt} - c_k^2 \frac{D\rho_k}{Dt}$ and $J_k = ||\vec{u}_k|| \max\left([[\ \vec{\nabla} P_k \cdot \vec{n}\ ]],\ [[c_k^2 \vec{\nabla}\rho_k \cdot \vec{n}\ ]]\right)$
- $\mathrm{norm}^{\mu}_{k,P} = \mathbb{G}(M_k)\rho_k ||\vec{u}_k||^2 + (1 - \mathbb{G}(M_k))\rho_k c_k^2$
- $\lim_{M_k \to 0} \mathbb{G}(M_k) = 0$ and $\lim_{M_k \to +\infty} \mathbb{G}(M_k) = 1$

$\beta_k(\vec{r},t) = \min\left(\beta_{k,\max}(\vec{r},t), \beta_{k,e}(\vec{r},t)\right)$

- $\beta_{max} = \frac{h}{2} ||\vec{u}_{int}||$ and $\beta_{k,e} = h^2 \frac{\max(R_{k,\alpha}, J_{k,\alpha})}{||\eta_k - \bar{\eta}_k||_\infty}$
- $R_{k,\alpha} = \partial_t \eta_k + \vec{u}_{int} \cdot \vec{\nabla}\eta_k$ with $\eta_k = \frac{\alpha_k^2}{2}$ and $J_{k,\alpha} = ||\vec{u}_{int}||\ [[\ \vec{\nabla}\alpha_k \cdot \vec{n}\ ]]$

M. Delchini, J. Ragusa and R. Berry, *1-D Numerical Solution of the Seven-Equation Two-Phase Flow Model Using an Entropy-Based Artificial Dissipative Method*, in preparation.

Spatial and temporal discretizations

- BDF2
- Linear polynomial test functions → second-order accuracy
- Ideal gas equation of states: $\gamma_1 = 3$ and $\gamma_2 = 1.4$
- 1-D pipe

Two 1-D shock tests for two limit cases (heavy fluid 1 and light fluid 2):

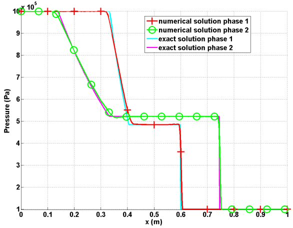
First test: two independent fluids

- Both fluids initially at rest, ($P_{left} = 1$ MPa, $P_{right} = 0.1$ MPa) and ($\rho_1 = 10$, $\rho_2 = 1\ kg/m^3$)
- Zero relaxation coefficients: $\mu_{rel} = \lambda_{rel} = 0$
- Exact solutions are available

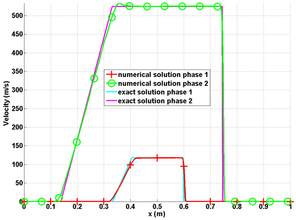
Second test: infinite relaxation parameters (Kapila)

- Same initial conditions
- Infinite relaxation coefficients: $\mu_{rel} = \lambda_{rel} \to \infty$

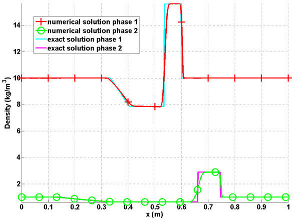
First test: two independent fluids



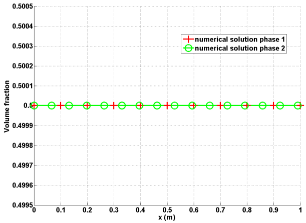
(e) Pressure at $t = 305\ \mu s$



(f) Velocity at $t = 305\ \mu s$

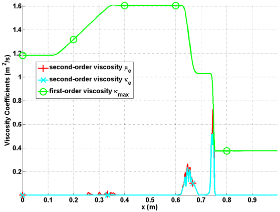


(g) Density at $t = 305\ \mu s$

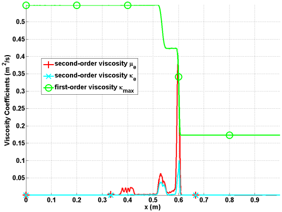


(h) Volume fraction at $t = 305\ \mu s$

First test: two independent fluids



(i) Viscosity coefficients phase 1



(j) Viscosity coefficients phase 2

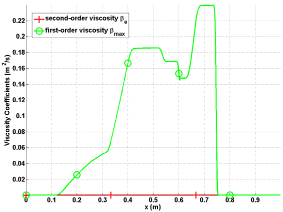
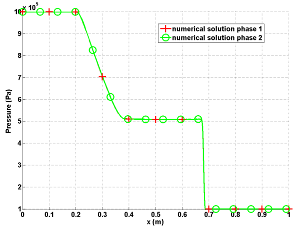
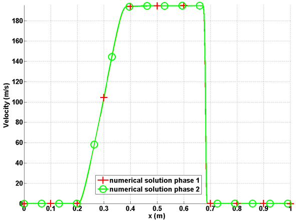


Figure: Viscosity coefficients volume fraction

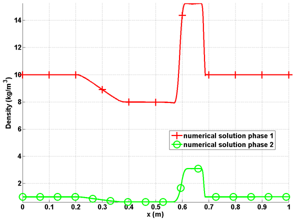
Second test: with infinite relaxation coefficients



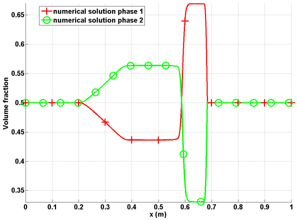
(a) Pressure at $t = 305\ \mu s$



(b) Velocity at $t = 305\ \mu s$

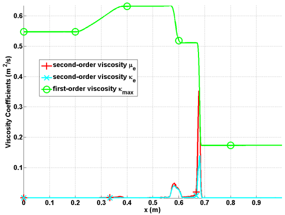


(c) Density at $t = 305\ \mu s$

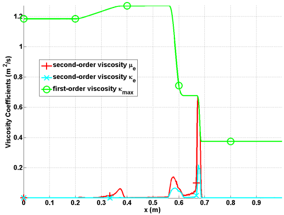


(d) Volume fraction at $t = 305\ \mu s$

Second test: with infinite relaxation coefficients



(e) Viscosity coefficients phase 1



(f) Viscosity coefficients phase 2

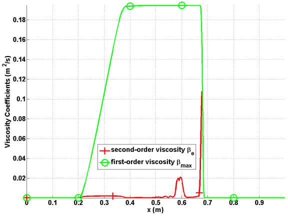


Figure: Viscosity coefficients volume fraction

Conclusions and future work

Conclusions

- Derived a viscous regularization for the SEM that is consistent with the entropy minimum principle
- All-Mach flow definition of the viscosity coefficients
- Presented numerical results using a *continuous* FEM spatial discretization and an implicit (BDF2) temporal integration
- Method is implemented in RELAP-7, a MOOSE-based application of the INL

Future work

- Further 1-D tests: hydrostatic tests, stronger shocks
- Multi-D simulations → requires a preconditioner
- Implement the EVM using discontinuous schemes for comparison against approximate Riemann solver

QUESTIONS/COMMENTS ?