

# MDS by Majorizing Gauss-Newton

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TBD

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**Note:** This is a working manuscript which will be expanded/updated frequently. All suggestions for improvement are welcome. All Rmd, tex, html, pdf, R, and C files are in the public domain. Attribution will be appreciated, but is not required. The files can be found at <https://github.com/deleeuw>

## 1 Loss

rStress with  $r > 0$  is defined as

$$\sigma_r(x) := \sum_k w_k (\delta_k^r - d_k^r(x))^2$$

## 2 First-order Taylor approximation to powered distances

$$d_k^r(x) \approx d_k^r(y) + r d_k^{r-1}(y)(d_k(x) - d_k(y))$$

Note that if  $r = 1$  both sides are equal to  $d_k(x)$ . If  $r = 2$

$$d_k^2(x) \approx d_k^2(y) + 2d_k(y)(d_k(x) - d_k(y)) = 2d_k(x)d_k(y) - d_k^2(y).$$

## 3 Gauss-Newton approximation to rStress

$$\sigma_r(x) \approx \sum_k w_k (\delta_k^r - d_k^r(y) - r d_k^{r-1}(y)(d_k(x) - d_k(y)))^2 = \quad (1)$$

$$\sum_k w_k \{r d_k^{r-1}(y)\}^2 \left( \frac{\delta_k^r - d_k^r(y)}{r d_k^{r-1}(y)} - (d_k(x) - d_k(y)) \right)^2 \quad (2)$$

Let

$$\tilde{w}_k(y) := r^2 w_k d_k^{2(r-1)}(y)$$

and

$$\tilde{\delta}_k(y) := \frac{\delta_k^r - d_k^r(y)}{r d_k^{r-1}(y)} + d_k(y) = \frac{\delta_k^r + (r-1)d_k^r(y)}{r d_k^{r-1}(y)}$$

then

$$\sigma_r(x; y) \approx \sum_k \tilde{w}_k(y) (\tilde{\delta}_k(y) - d_k(x))^2$$

which can be minimized by majorization. Also if  $x = y$  then  $\sigma_r(x; x) = \sigma_r(x)$ .

For  $r \geq 1$  we have  $\delta_k(y) \geq 0$ .

For  $r = 1$  we have  $\tilde{w}_k(y) = w_k$  and  $\tilde{\delta}_k(y) = \delta_k$ . This is regular smacof.

For  $r = 2$  we have  $\tilde{w}_k(y) = 4w_k d_k^2(y)$  and  $\tilde{\delta}_k(y) = \frac{\delta_k^2 + d_k^2(y)}{2d_k(y)}$ .

## 4 Algorithm

$$\sigma_r(x^+) \approx \sigma_r(x^+; x) \leq \sigma_r(x; x) = \sigma_r(x)$$