Smacof at 50: A Manual
Part x: Linearly Constrained Smacof

Jan de Leeuw - University of California Los Angeles

Started May 08, 2024, Version of July 19, 2024

Abstract

TBD

Contents

Note: This is a working manuscript which will be expanded/updated frequently. All suggestions for improvement are welcome. All Rmd, tex, html, pdf, R, and C files are in the public domain. Attribution will be appreciated, but is not required. All files can be found at https://github.com/deleeuw in the repositories smacofCode, smacofManual, and smacofExamples.

1 Introduction

$$X = YA = \sum_{j=1}^{m} Y_j A_j$$

with Y_j known $n \times k_j$ "design" matrices and A an unknown $k_j \times p$ matrix of coefficients.

Various constraints on A.

1. No constraints
2. A_j are diagonal
3. A_j are scalar
4. A_j are block diagonal

Special choices of Y_j

1. Previous iterations of X
2. Indicator matrices

3. Design matrices (example the rectangles)

In general the constraint phase must solve minimization of

$$\omega(A) := \text{tr } (\overline{X} - \sum_{j=1}^{m} Y_j A_j)' V (\overline{X} - \sum_{j=1}^{m} Y_j A_j)$$

Minimize

$$-2 \sum_{j=1}^{m} \text{tr } A_j' Y_j' V \overline{X} + \sum_{j=1}^{m} \sum_{l=1}^{m} \text{tr } A_j' Y_j' V Y_l \, A_l$$

diagonal (not very interesting)

$$-2 \sum_{j=1}^{m} a_j' \text{diag}\{Y_j' V \overline{X}\} + \sum_{j=1}^{m} \sum_{l=1}^{m} a_j' \text{diag}\{Y_j' V Y_l \, \} a_l$$

scalar

$$-2 \sum_{j=1}^{m} \alpha_j \text{tr } Y_j' V \overline{X} + \sum_{j=1}^{m} \sum_{l=1}^{m} \alpha_j \alpha_l \text{tr } Y_j' V Y_l$$

rank one $A_j = z_j t_j'$ (not very interesting)

$$-2 \sum_{j=1}^{m} z_j' Y_j' V \overline{X} t_j + \sum_{j=1}^{m} \sum_{l=1}^{m} t_l' t_j \; z_j' Y_j' V Y_l \, z_l$$

block diagonal

$$A = \bigoplus_{j=1}^{m} A_j$$

$$X = (Y_1 A_1 \cdots Y_m A_m)$$

$$\omega(A) = \sum_{j=1}^{m} \text{tr } (\overline{X}_j - Y_j A_j)' V (\overline{X}_j - Y_j A_j)$$

2 References