simple

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Majorization

$$\sigma(X)=\sum w_k(\delta_k-d_k(X))^2$$

Now $d_k(X)=d_k(\tilde X)+(d_k(X)-d_k(\tilde X))$. Thus

$$\sigma(X)=\sum w_k((\delta_k-d_k(\tilde X))-(d_k(X)-d_k(\tilde X)))^2\leq\sigma(\tilde X)-2w_\star\sum\frac{w_k}{w_\star}(\delta_k-d_k(\tilde X))(d_k(X)-d_k(\tilde X))+w_\star\sum(d_k(X)-$$

$$\hat d_k(\tilde X)=\frac{w_k}{w_\star}\delta_k+(1-\frac{w_k}{w_\star})d_k(\tilde X)$$

So decrease

$$\sum(d_k(X)-\hat d_k(\tilde X))^2$$

Note this can also be applied if some weights are zero.

$$\overline{B}(\tilde X)=\sum\frac{\frac{w_k}{w_\star}\delta_k+(1-\frac{w_k}{w_\star})d_k(\tilde X)}{d_k(\tilde X)}=w_\star^{-1}\{B(\tilde X)+(w_\star J-V)\}$$

Alt:

$$\eta^2(X)=\text{tr }X'VX=\sum w_{ij}d_{ij}^2(X)\leq w_\star\sum d_{ij}^2(X)=nw_\star\text{ tr }X'X$$

$$\eta^2(\tilde X+(X-\tilde X))\leq\eta^2(\tilde X)+2\text{ tr }\tilde X'V(X-\tilde X)+nw_\star\text{ tr }(X-\tilde X)'(X-\tilde X)$$

$$\mathcal{D}\omega=V\tilde X+nw_\star(X-\tilde X)$$

$$B(\tilde X)\tilde X=V\tilde X+nw_\star(X-\tilde X)$$

$$X=\tilde X+\frac{1}{nw_\star}\{B(\tilde X)\tilde X-V\tilde X\}=\tilde X+\frac{1}{nw_\star}V\{\Phi(\tilde X)-\tilde X\}$$