# Smacof at 50: A Manual
# Part 5: Unfolding in Smacof

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**Abstract**

TBD

# Contents



**Note:** This is a working paper which will be expanded/updated frequently. All suggestions for improvement are welcome.

# 1 Elegant

$$\sigma(C) = \sigma(\tilde{C} + (C - \tilde{C})) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\delta_{ij}^2 - \text{tr } A_{ij}\tilde{C}) - \text{tr } A_{ij}(C - \tilde{C}))^2$$

$$\sigma(C) = \sigma(\tilde{C}) - 2\sum_{i=1}^{n}\sum_{j=1}^{m}(\delta_{ij}^2 - \text{tr } A_{ij}\tilde{C})\text{tr } A_{ij}(C - \tilde{C}) + \sum_{i=1}^{n}\sum_{j=1}^{m}\{\text{tr } A_{ij}(C - \tilde{C})\}^2$$

From (**deleeuw_groenen_pietersz?**)

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\{\text{tr } A_{ij}(C - \tilde{C})\}^2 \leq (n + m + 2)\text{tr } (C - \tilde{C})^2$$

Define

$$B(\tilde{C}) := \frac{1}{n + m + 2}\sum_{i=1}^{n}\sum_{j=1}^{m}(\delta_{ij}^2 - \text{tr } A_{ij}\tilde{C})A_{ij}$$

So we minimize

$$-2 \text{ tr } B(\tilde{C})C + \text{tr } C^2 - 2\text{tr } C\tilde{C} = \text{tr } (C - \{\tilde{C} + B(\tilde{C})\})^2$$

# References