Smacof at 50: A Manual
Part 2: Non-metric Smacof

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Abstract

TBD

Contents

1 Introduction **3**

2 Paired Comparisons **4**

3 Triads **5**

4 Rank Orders **5**

Note: This is a working manuscript which will be expanded/updated frequently. All suggestions for improvement are welcome. All Rmd, tex, html, pdf, R, and C files are in the public domain. Attribution will be appreciated, but is not required. The files can be found at https://github.com/deleeuw/smacofCode.

1 Introduction

pick and rank

$$\sigma_R(X) = \sum_{r=1}^{s} \min_{\delta \in \mathcal{K}_r} (\delta - d(X))' W_r (\delta - d(X))$$

with $\mathcal{K}_r$ pointed polyhedral convex cones, defined by a partial order $\leq_r$ and with W_r diagonal and positive semi-definite.

$$\sigma_R(X) = \sum_{r=1}^{s} (\hat{d}_r - d(X))' W_r (\hat{d}_r - d(X))$$

Let

$$W_\star := \sum_{r=1}^{s} W_r$$

and

$$\hat{d}_\star = \sum_{r=1}^{s} W_r \hat{d}_r$$

$$\sum w_{\star ij} d_{ij}^2(X) = \text{tr } X'VX$$

$$\sum \hat{d}_{\star ij} d_{ij}(X) = \text{tr } X'B(X)X$$

2 Paired Comparisons

Positive Orthant / Absolute Value / Pairwise

$$\min_{\hat{D}} r_{ijkl}(X, \hat{D}) = \begin{cases} 0 & \text{if correct,} \\ \frac{1}{2} w_{ijkl}(d_{ij}(X) - d_{kl}(X))^2 & \text{if wrong.} \end{cases}$$

$$w_{ijkl}\{(\hat{d}_{ij} - d_{ij}(X))^2 + (\hat{d}_{kl} - d_{kl}(X))^2\}$$

$$\sigma(X, \hat{D}) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} w_{ijkl}\{(\hat{d}_{ij} - d_{ij}(X))^2 + (\hat{d}_{kl} - d_{kl}(X))^2\}.$$

$$\text{sign}(\hat{d}_{ij} - \hat{d}_{kl}) = s_{ijkl}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

3 Triads

4 Rank Orders