Smacof at 50: A Manual
Part 5: Unfolding in Smacof

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Abstract

TBD

Contents

Note: This is a working manuscript which will be expanded/updated frequently. All suggestions for improvement are welcome. All Rmd, tex, html, pdf, R, and C files are in the public domain. Attribution will be appreciated, but is not required. The files can be found at https://github.com/deleeuw in the repositories smacofCode, smacofManual, and smacofExamples.

0.1 Metric Unfolding

Metric unfolding is a special case of metric MDS, as implemented for example in smacofAC. Thus we minimize stress defined as

$$\sigma(X)=\sum_{i=1}^N\sum_{j=1}^N w_{ij}(\delta_{ij}-d_{ij}(X))^2. \tag{1}$$

For unfolding

$$W=\begin{array}{c c} & \begin{array}{cc} n & m \end{array} \\ \begin{array}{c} n \\ m \end{array} & \begin{bmatrix} 0 & U \\ U' & 0 \end{bmatrix} \end{array}$$

In other words there is a set of n row-objects and a set of m column-objects. The withiin-set dissimilarities are missing, or ignored, and only the between-set dissimilarities are fitted. If we partition X as

$$X=\begin{array}{c c} & p \\ \begin{array}{c} n \\ m \end{array} & \begin{bmatrix} Y \\ Z \end{bmatrix} \end{array}$$

then we can also write

$$\sigma(Y,Z)=\sum_{i=1}^n\sum_{j=1}^m u_{ij}(\delta_{ij}-d(y_i,z_j))^2. \tag{2}$$

By defining

$$X=\begin{array}{c c} & p \\ \begin{array}{c} n \\ m \end{array} & \begin{bmatrix} Y \\ Z \end{bmatrix} \end{array}$$

and

$$W=\begin{array}{c c} & \begin{array}{cc} n & m \end{array} \\ \begin{array}{c} n \\ m \end{array} & \begin{bmatrix} 0 & U \\ U' & 0 \end{bmatrix} \end{array}$$

we can write

The multidimensional unfolding model for preference judgments is often attributed to (**coombs?**), (**kruskal_carroll?**), (**roskam?**)

1 Initial Configuration

$$\sigma(C) = \sigma(\tilde{C} + (C - \tilde{C})) = \sum_{i=1}^{n}\sum_{j=1}^{m}((\delta_{ij}^2 - \text{tr } A_{ij}\tilde{C}) - \text{tr } A_{ij}(C - \tilde{C}))^2$$

$$\sigma(C) = \sigma(\tilde{C}) - 2\sum_{i=1}^{n}\sum_{j=1}^{m}(\delta_{ij}^2 - \text{tr } A_{ij}\tilde{C})\text{tr } A_{ij}(C - \tilde{C}) + \sum_{i=1}^{n}\sum_{j=1}^{m}\{\text{tr } A_{ij}(C - \tilde{C})\}^2$$

From De Leeuw, Groenen, and Pietersz (2006)

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\{\text{tr } A_{ij}(C - \tilde{C})\}^2 \leq (n + m + 2)\text{tr } (C - \tilde{C})^2$$

Define

$$B(\tilde{C}) := \frac{1}{n + m + 2}\sum_{i=1}^{n}\sum_{j=1}^{m}(\delta_{ij}^2 - \text{tr } A_{ij}\tilde{C})A_{ij}$$

So we minimize

$$-2 \text{ tr } B(\tilde{C})C + \text{tr } C^2 - 2\text{tr } C\tilde{C} = \text{tr } (C - \{\tilde{C} + B(\tilde{C})\})^2$$

References

De Leeuw, J., P. J. F. Groenen, and R. Pietersz. 2006. "Optimizing Functions of Squared Distances." UCLA Department of Statistics. https://jansweb.netlify.app/publication/deleeuw-groenen-pietersz-u-06/deleeuw-groenen-pietersz-u-06.pdf.