

Suppose we have a real (not machine) number

$$x = 0.d_1d_2\dots d_kd_{k+1}\dots \times 10^n.$$

If we're working with  $k$ -digit decimal arithmetic, then chopping  $x$  would produce the machine number

$$\text{chop}(x) = 0.d_1d_2\dots d_k \times 10^n,$$

i.e., to chop for  $k$ -digit arithmetic you drop every piece of information past the  $k$ th digit.

Rounding for  $k$ -digit arithmetic depends on the  $(k + 1)$ st digit. If  $d_{k+1} \geq 5$ , then we round up (which will change  $d_k$  but may or may not change any other digits). If  $d_{k+1} < 5$ , then we round down (which is equivalent to chopping for  $k$ -digit arithmetic).

The formula

$$fl(x) = \text{chop}(x + 5 \times 10^{n-(k+1)})$$

summarizes succinctly the rules for rounding. If  $d_{k+1} < 5$ , then adding 5 to  $d_{k+1}$  and then chopping is the same as rounding down. If  $d_{k+1} \geq 5$ , then adding 5 to  $d_{k+1}$  will add 1 to  $d_k$  (because you have to carry the 1 when you add), which is the same as rounding up.

Here are some examples.

e.g.) Suppose  $x = 193.2534$  are we use 5-digit arithmetic. Since

$$x = 0.1932534 \times 10^3,$$

we have

$$\text{chop}(x) = 0.19325 \times 10^3,$$

and according to the formula for rounding

$$\begin{aligned} fl(x) &= \text{chop}(0.1932534 \times 10^3 + 5 \times 10^{3-(5+1)}) \\ &= \text{chop}(0.1932534 \times 10^3 + 0.000005 \times 10^3) \\ &= \text{chop}(0.1932584 \times 10^3) \\ &= 0.19325 \times 10^3 \\ &= 193.25 \end{aligned}$$

e.g.) Suppose  $x = 92.346098$  are we use 7-digit arithmetic. Since

$$x = 0.92346098 \times 10^2,$$

we have

$$\text{chop}(x) = 0.9234609 \times 10^2,$$

and according to the formula for rounding

$$\begin{aligned} fl(x) &= \text{chop}(0.92346098 \times 10^2 + 5 \times 10^{2-(7+1)}) \\ &= \text{chop}(0.92346098 \times 10^2 + 0.00000005 \times 10^2) \\ &= \text{chop}(0.92346103 \times 10^2) \\ &= 0.923461 \times 10^2 \\ &= 92.3461. \end{aligned}$$