Recall Newton's form of the interpolating polynomial

$$\begin{cases} P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) \\ \qquad + \dots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1}) \\ a_k = f[x_0, x_1, \dots, x_k] \\ f[x_i, x_{i+1}, \dots, x_{i+k}] = \dfrac{f[x_{i+1},\dots,x_{i+k}] - f[x_i,\dots,x_{i+k-1}]}{x_{i+k}-x_i} \end{cases}$$

The polynomial $P_n(x)$ interpolates $(x_0, f(x_0)), (x_1, f(x_1)), \dots, (x_n, f(x_n))$.

$a_0 = f(x_0)$

$a_1 = f[x_0, x_1] = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$

e.g.) Consider the data $(1, 0.77)$, $(1.3, 0.22)$, $(1.6, 0.86)$, $(1.9, 0.28)$, $(2.2, 0.11)$.

0th divided diff.

$f[x_0,x_1] = \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \dfrac{0.22-0.77}{1.3-1.0}$

$f[x_1,x_2] = \dfrac{f(x_2)-f(x_1)}{x_2-x_1} = \dfrac{0.86-0.22}{1.6-1.3}$

i	x_i	$f(x_i)$	$f[x_{i-1},x_i]$	$f[x_{i-2},x_{i-1},x_i]$
0	1.0	0.77	–	–
1	1.3	0.22	$\boxed{-1.83}$	–
2	1.6	0.86	$\boxed{2.13}$	6.61
3	1.9	0.28	-1.93	$\boxed{-6.78}$
4	2.2	0.11	$\boxed{-0.57}$	2.28

$\leftarrow f[x_0,x_1,x_2] = \dfrac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \dfrac{2.13+1.83}{1.6-1.0}$

$f[x_{i-3},x_{i-2},x_{i-1},x_i]$	$f[x_{i-4},x_{i-3},x_{i-2},x_{i-1},x_i]$
–	–
–	–
–	–
-14.48	–
10.1	20.8

The polynomial of degree 4 interpolates all the data

$$P_4(x) = 0.77 + -1.83\,(x-1) + 6.61\,(x-1.3)(x-1)$$

$$a_0 + a_1\,(x-x_0) + a_2\,(x-x_1)(x-x_0)$$

$$+ -14.48\,(x-1)(x-1.3)(x-1.6)$$

$$a_3\,(x-x_0)(x-x_1)(x-x_2)$$

$$+ 20.8\,(x-1)(x-1.3)(x-1.9)(x-1.6)$$

$$\boxed{a_4}\,(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$a_4 = f[x_0, x_1, x_2, x_3, x_4]$$

```
input x_0, x_1, ..., x_n, f(x_0) = F_{0,0}, f(x_1) = F_{1,0}, ..., f(x_n) = F_{n,0}
for i = 1, 2, ..., n
    for j = 1, 2, ..., i
        F_{i,j} = (F_{i,j-1} - F_{i-1,j-1}) / (x_i - x_{i-j})
    end
end
```

Generates a table of divided differences for data.

$$\begin{pmatrix} x_0 & F_{0,0} & & & \\ x_1 & F_{1,0} & F_{1,1} & & \\ x_2 & F_{2,0} & F_{2,1} & F_{2,2} & \\ x_3 & F_{3,0} & F_{3,1} & \vdots & \ddots \\ \vdots & \vdots & \vdots & & \end{pmatrix}$$