

Ojo, ocuparme de estos typos: Typos en diapositivas:

6 - entagled en lugar de mixed 8 - PEC en lugar de PCE 10 - mas en lugar de map 11 - actully 13 - con en lugar de where 14 - implica en lugar de implies 15 - opertation - asosciated 18 - título en español "algunas propiedades"

Otras notas: En la diapositiva 15 mencionaste que una figura está volteada. En la diapositiva 12, el itemize de las propiedades del conjunto podría ser más claro (el primer punto no lo caché hasta que se explicó).

# Pauli component erasing operations

Carlos Pineda<sup>1</sup>

José Alfredo de León<sup>1,2</sup> Alejandro Fonseca<sup>3</sup> François Leyvraz<sup>1</sup> David Davalos<sup>4</sup>

<sup>1</sup>Universidad Nacional Autónoma de México

<sup>2</sup>Universidad de San Carlos de Guatemala

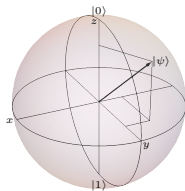
<sup>3</sup>Universidade Federal de Pernambuco (Brazil)

<sup>4</sup>Slovak Academy of Sciences

arXiv:2205.05808

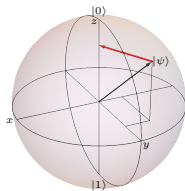
## General goal

Quantum channels can project some components. But not all projections are allowed.



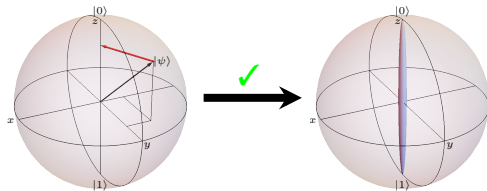
## General goal

Quantum channels can project some components. But not all projections are allowed.



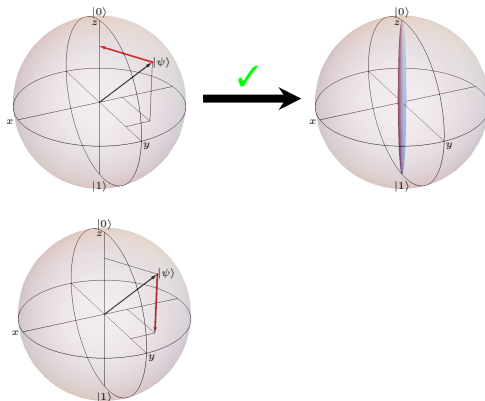
## General goal

Quantum channels can project some components. But not all projections are allowed.



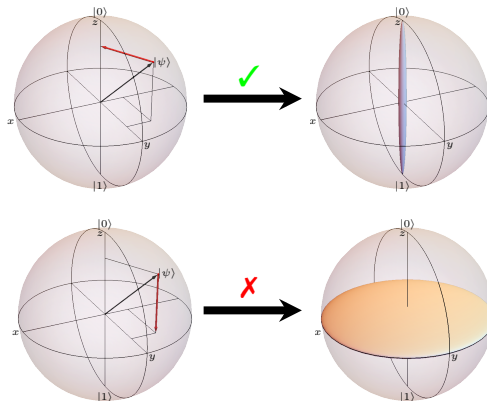
## General goal

Quantum channels can project some components. But not all projections are allowed.



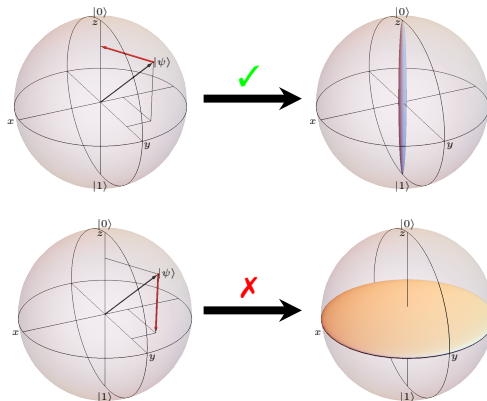
## General goal

Quantum channels can project some components. But not all projections are allowed.



## General goal

Quantum channels can project some components. But not all projections are allowed.



We want to develop intuition about what's going on and generalize.



# Outline

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

## Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

# Quantum channels

A quantum channel ( $\Lambda$ ) generalizes conveniently unitary evolution, in a similar way as a density matrix generalizes a ket.

Quantum channels can describe:

- ▶ Unitary evolution  $\Lambda(\rho) = U\rho U^\dagger$
- ▶ Reduced dynamics (decoherence)  $\Lambda(\rho) = \text{tr}_{\text{env}} [U\rho \otimes \rho_{\text{env}} U^\dagger]$
- ▶ Measurements

## Quantum channels

A quantum channel ( $\Lambda$ ) generalizes conveniently unitary evolution, in a similar way as a density matrix generalizes a ket.

Quantum channels can describe:

- ▶ Unitary evolution  $\Lambda(\rho) = U\rho U^\dagger$
- ▶ Reduced dynamics (decoherence)  $\Lambda(\rho) = \text{tr}_{\text{env}} [U\rho \otimes \rho_{\text{env}} U^\dagger]$
- ▶ Measurements

Quantum channels are linear operators acting on the spaces of operators, i.e. superoperators.

## Quantum channels, some properties

Quantum channels map states into states  $\mathcal{E}(\rho) = \rho'$

- ▶ They are linear  $\mathcal{E}(\alpha\varsigma + \sigma) = \alpha\mathcal{E}(\alpha\varsigma) + \mathcal{E}(\sigma)$ ,
- ▶ preserve the trace  $\text{tr}\mathcal{E}(\sigma) = \text{tr}\sigma$ ,
- ▶ preserve hermiticity  $\mathcal{E}(\sigma) = \mathcal{E}(\sigma)^\dagger$  if  $\sigma = \sigma^\dagger$ , and
- ▶ preserve positivity,  $\mathcal{E}(\sigma) \geq 0$  if  $\sigma \geq 0$ .

## Quantum channels, some properties

Quantum channels map states into states  $\mathcal{E}(\rho) = \rho'$

- ▶ They are linear  $\mathcal{E}(\alpha\varsigma + \sigma) = \alpha\mathcal{E}(\alpha\varsigma) + \mathcal{E}(\sigma)$ ,
- ▶ preserve the trace  $\text{tr}\mathcal{E}(\sigma) = \text{tr}\sigma$ ,
- ▶ preserve hermiticity  $\mathcal{E}(\sigma) = \mathcal{E}(\sigma)^\dagger$  if  $\sigma = \sigma^\dagger$ , and
- ▶ preserve positivity,  $\mathcal{E}(\sigma) \geq 0$  if  $\sigma \geq 0$ .

They send extended states to extended states.

- ▶ Complete positivity,  $(\mathcal{E} \otimes \mathbb{1}_k)(\sigma) \geq 0$  if  $\sigma \geq 0$ .

There are *maps* that send states to states, but not extended states to extended states, for example the transpose.

## Quantum channels, complete positivity

- Complete positivity means that for all  $k$ , and all  $\rho \geq 0$ ,

$$(\mathcal{E}_n \otimes \mathbb{1}_k)(\rho) \geq 0.$$

## Quantum channels, complete positivity

- ▶ Complete positivity means that for all  $k$ , and all  $\rho \geq 0$ ,

$$(\mathcal{E}_n \otimes \mathbb{1}_k)(\rho) \geq 0.$$

- ▶ It is enough to study the maximally mixed state for  $k = n$  levels:

$$(\mathcal{E}_n \otimes \mathbb{1}_n)(|\text{Bell}_n\rangle\langle\text{Bell}_n|) = \rho_{\mathcal{E}} \geq 0.$$

This induces a correspondence between states and quantum channels called Choi–Jamiołkowski isomorphism.



# Quantum channels, complete positivity

- ▶ Complete positivity means that for all  $k$ , and all  $\rho \geq 0$ ,

$$(\mathcal{E}_n \otimes \mathbb{1}_k)(\rho) \geq 0.$$

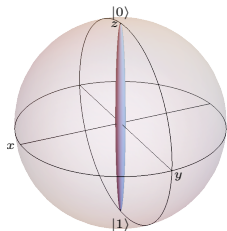
- ▶ It is enough to study the maximally mixed state for  $k = n$  levels:

$$(\mathcal{E}_n \otimes \mathbb{1}_n)(|\text{Bell}_n\rangle\langle\text{Bell}_n|) = \rho_{\mathcal{E}} \geq 0.$$

This induces a correspondence between states and quantum channels called Choi–Jamiołkowski isomorphism.

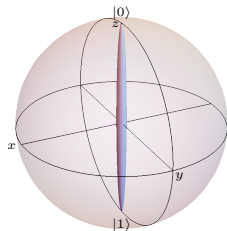
- ▶ We must check, diagonalizing  $\rho_{\mathcal{E}}$ , (which is proportional to the Choi matrix) if all its eigenvalues are semipositive.

# Quantum channels



$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_z \sigma_z$$

# Quantum channels



$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_z \sigma_z$$

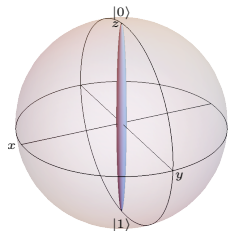
$$\mathcal{E} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\rho_{\mathcal{E}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\lambda_{\rho_{\mathcal{E}}} = (\frac{1}{2}, \frac{1}{2}, 0, 0)$$

GOOD!

# Quantum channels



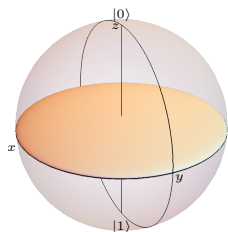
$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_z \sigma_z$$

$$\mathcal{E} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\rho_{\mathcal{E}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

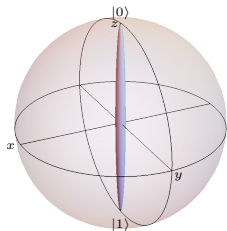
$$\lambda_{\rho_{\mathcal{E}}} = \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$$

GOOD!



$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_x \sigma_x + r_y \sigma_y$$

# Quantum channels



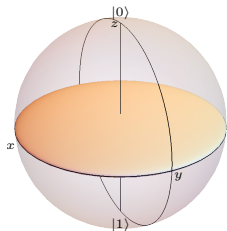
$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_z \sigma_z$$

$$\mathcal{E} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\rho_{\mathcal{E}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\lambda_{\rho_{\mathcal{E}}} = (\frac{1}{2}, \frac{1}{2}, 0, 0)$$

GOOD!



$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_x \sigma_x + r_y \sigma_y$$

$$\mathcal{E} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\rho_{\mathcal{E}} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

$$\lambda_{\rho_{\mathcal{E}}} = (\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$$

BAD!

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

## Single qubit PCE

A single qubit state can be expressed as

$$\rho = \frac{\mathbb{1} + \vec{r} \cdot \vec{\sigma}}{2} = \frac{\mathbb{1} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z}{2},$$

## Single qubit PCE

A single qubit state can be expressed as

$$\rho = \frac{\mathbb{1} + \vec{r} \cdot \vec{\sigma}}{2} = \frac{\mathbb{1} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z}{2},$$

PCE maps project over some of these components:

$$\mathcal{E}(\rho) = \frac{\mathbb{1} + \vec{r}' \cdot \vec{\sigma}}{2}, \quad r'_i = \tau_i r_i, \quad \tau_i \in \{0, 1\}.$$



## Single qubit PCE

A single qubit state can be expressed as

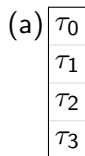
$$\rho = \frac{\mathbb{1} + \vec{r} \cdot \vec{\sigma}}{2} = \frac{\mathbb{1} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z}{2},$$

PCE maps project over some of these components:

$$\mathcal{E}(\rho) = \frac{\mathbb{1} + \vec{r}' \cdot \vec{\sigma}}{2}, \quad r'_i = \tau_i r_i, \quad \tau_i \in \{0, 1\}.$$

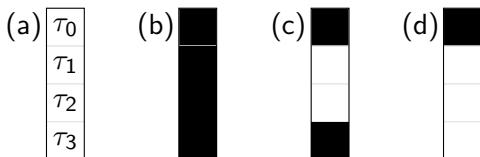
- ▶ They map density matrices to density matrices
- ▶ The previous maps are examples of PCE maps
- ▶ They are a subset of Pauli maps
- ▶ They are not always completely positive

# Single qubit PCE and its diagrams



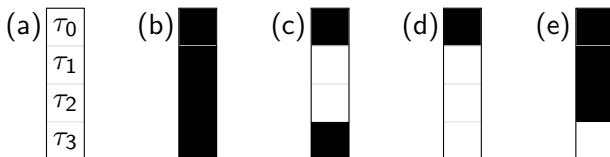
- ▶ Each component is represented by a square.
- ▶ It is either black (if its preserved) or white (if its erased)

# Single qubit PCE and its diagrams



- ▶ Each component is represented by a square.
- ▶ It is either black (if its preserved) or white (if its erased)
- ▶ (b) is the identity channel, (c) is dephasing and (d) depolarizing.

# Single qubit PCE and its diagrams



- ▶ Each component is represented by a square.
- ▶ It is either black (if its preserved) or white (if its erased)
- ▶ (b) is the identity channel, (c) is dephasing and (d) depolarizing.
- ▶ (e) is the map that projects to the  $xy$  plane

## $N$ qubit PCEs

- An  $N$  qubit state can be written as

$$\rho = \frac{1}{2^N} \sum_{\vec{\alpha}} r_{\vec{\alpha}} \sigma_{\vec{\alpha}},$$

with  $\vec{\alpha} = (\alpha_1, \dots, \alpha_N)$ , and  $\sigma_{\vec{\alpha}} := \sigma_{\alpha_1} \otimes \dots \otimes \sigma_{\alpha_N}$ .

## $N$ qubit PCEs

- An  $N$  qubit state can be written as

$$\rho = \frac{1}{2^N} \sum_{\vec{\alpha}} r_{\vec{\alpha}} \sigma_{\vec{\alpha}},$$

with  $\vec{\alpha} = (\alpha_1, \dots, \alpha_N)$ , and  $\sigma_{\vec{\alpha}} := \sigma_{\alpha_1} \otimes \dots \otimes \sigma_{\alpha_N}$ .

- A PCE map is such that

$$r_{\vec{\alpha}} \mapsto \tau_{\vec{\alpha}} r_{\vec{\alpha}}, \quad \tau_{\vec{\alpha}} = 0, 1.$$

This are the objects we are interested in.

# $N$ qubit PCEs

- ▶ An  $N$  qubit state can be written as

$$\rho = \frac{1}{2^N} \sum_{\vec{\alpha}} r_{\vec{\alpha}} \sigma_{\vec{\alpha}},$$

with  $\vec{\alpha} = (\alpha_1, \dots, \alpha_N)$ , and  $\sigma_{\vec{\alpha}} := \sigma_{\alpha_1} \otimes \dots \otimes \sigma_{\alpha_N}$ .

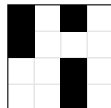
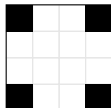
- ▶ A PCE map is such that

$$r_{\vec{\alpha}} \mapsto \tau_{\vec{\alpha}} r_{\vec{\alpha}}, \quad \tau_{\vec{\alpha}} = 0, 1.$$

This are the objects we are interested in.

- ▶ For 2 qubits, we have equivalent diagrams:

$\tau_{(0,0)}$	$\tau_{(0,1)}$	$\tau_{(0,2)}$	$\tau_{(0,3)}$
$\tau_{(1,0)}$	$\tau_{(1,1)}$	$\tau_{(1,2)}$	$\tau_{(1,3)}$
$\tau_{(2,0)}$	$\tau_{(2,1)}$	$\tau_{(2,2)}$	$\tau_{(2,3)}$
$\tau_{(3,0)}$	$\tau_{(3,1)}$	$\tau_{(3,2)}$	$\tau_{(3,3)}$



Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

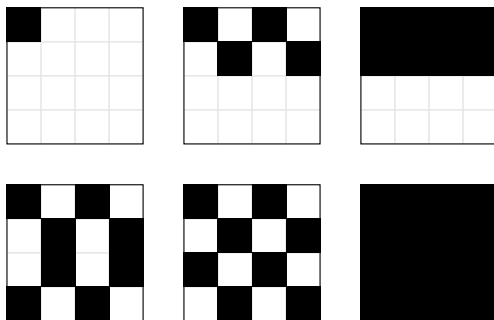


## Statement of the problem

Which PCE maps are actually quantum channels?

This will hopefully help us understand better the geometry of many-body quantum channels

## Examples and rules



- ▶ Only PCEs that preserve  $2^k$  components.
- ▶ Not all PCEs that preserve  $2^k$  components are channels.
- ▶ The same number of channels that preserve  $2^k$  and  $2^{2N-k}$  components.
- ▶ There is a generating set that preserves half of the components.

## Choi matrix diagonalization

- The channel can be expressed as  $\hat{\mathcal{E}} = \frac{1}{2^N} \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} |\sigma_{\vec{\alpha}}\rangle\rangle\langle\langle\sigma_{\vec{\alpha}}|$ .

## Choi matrix diagonalization

- ▶ The channel can be expressed as  $\hat{\mathcal{E}} = \frac{1}{2^N} \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} |\sigma_{\vec{\alpha}}\rangle\rangle\langle\langle\sigma_{\vec{\alpha}}|$ .
- ▶ And the corresponding state reads  $\rho_{\mathcal{E}} \propto \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} \bigotimes_{j=1}^N \sigma_{\alpha_j} \otimes \sigma_{\alpha_j}^*$ .

# Choi matrix diagonalization

- ▶ The channel can be expressed as  $\hat{\mathcal{E}} = \frac{1}{2^N} \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} |\sigma_{\vec{\alpha}}\rangle\langle\sigma_{\vec{\alpha}}|$ .
- ▶ And the corresponding state reads  $\rho_{\mathcal{E}} \propto \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} \bigotimes_{j=1}^N \sigma_{\alpha_j} \otimes \sigma_{\alpha_j}^*$ .
- ▶ The tensor matrices commute  $[\sigma_{\vec{\alpha}} \otimes \sigma_{\vec{\alpha}}^*, \sigma_{\vec{\alpha}'} \otimes \sigma_{\vec{\alpha}'}^*] = 0$ .

# Choi matrix diagonalization

- ▶ The channel can be expressed as  $\hat{\mathcal{E}} = \frac{1}{2^N} \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} |\sigma_{\vec{\alpha}}\rangle\langle\sigma_{\vec{\alpha}}|$ .
- ▶ And the corresponding state reads  $\rho_{\mathcal{E}} \propto \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} \bigotimes_{j=1}^N \sigma_{\alpha_j} \otimes \sigma_{\alpha_j}^*$ .
- ▶ The tensor matrices commute  $[\sigma_{\vec{\alpha}} \otimes \sigma_{\vec{\alpha}}^*, \sigma_{\vec{\alpha}'} \otimes \sigma_{\vec{\alpha}'}^*] = 0$ .
- ▶ And the eigenvectors fulfill  $(\sigma_{\vec{\alpha}} \otimes \sigma_{\vec{\alpha}}^*) \mathbf{v}_{\vec{\beta}} = \frac{1}{2^N} A_{\vec{\alpha}\vec{\beta}} \mathbf{v}_{\vec{\beta}}$  con

$$A = a^{\otimes N}, \quad a = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

# Choi matrix diagonalization

- ▶ The channel can be expressed as  $\hat{\mathcal{E}} = \frac{1}{2^N} \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} |\sigma_{\vec{\alpha}}\rangle\langle\sigma_{\vec{\alpha}}|$ .
- ▶ And the corresponding state reads  $\rho_{\mathcal{E}} \propto \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} \bigotimes_{j=1}^N \sigma_{\alpha_j} \otimes \sigma_{\alpha_j}^*$ .
- ▶ The tensor matrices commute  $[\sigma_{\vec{\alpha}} \otimes \sigma_{\vec{\alpha}}^*, \sigma_{\vec{\alpha}'} \otimes \sigma_{\vec{\alpha}'}^*] = 0$ .
- ▶ And the eigenvectors fulfill  $(\sigma_{\vec{\alpha}} \otimes \sigma_{\vec{\alpha}}^*) \mathbf{v}_{\vec{\beta}} = \frac{1}{2^N} A_{\vec{\alpha}\vec{\beta}} \mathbf{v}_{\vec{\beta}}$  con

$$A = a^{\otimes N}, \quad a = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

- ▶ This implies that the eigenvectors also fulfill  $\lambda_{\vec{\alpha}} = \frac{1}{2^N} A_{\vec{\alpha}\vec{\beta}} \tau_{\vec{\beta}}$ .
- ▶ So we just diagonalize the Choi matrix ( $\propto \rho_{\mathcal{E}}$ ) for *all* Pauli channels

## An important relation among indices of a channel

Based on the following

- ▶  $A$  is almost idempotent:  $A^{-1} = \frac{1}{4^N} A$
- ▶ If  $\lambda_{\vec{\alpha}} \geq 0$  (as should be for channels), then  $\sum_{\vec{\alpha} \in \Omega} \lambda_{\vec{\alpha}} = 0$  implies  $\lambda_{\vec{\alpha}} = 0, \forall \vec{\alpha} \in \Omega$ ,



## An important relation among indices of a channel

Based on the following

- ▶  $A$  is almost idempotent:  $A^{-1} = \frac{1}{4^N} A$
- ▶ If  $\lambda_{\vec{\alpha}} \geq 0$  (as should be for channels), then  $\sum_{\vec{\alpha} \in \Omega} \lambda_{\vec{\alpha}} = 0$  implies  $\lambda_{\vec{\alpha}} = 0, \forall \vec{\alpha} \in \Omega$ ,

we can say that  $\tau_{\vec{\alpha}} = 1$  implies  $\tau_{\vec{\gamma}} = \tau_{\vec{\gamma}'}$  if

$$A_{\vec{\beta}\vec{\gamma}} = A_{\vec{\beta}\vec{\gamma}'}, \quad \forall \vec{\beta} : A_{\vec{\alpha}\vec{\beta}} = 1.$$

This is a relation between the indices that are preserved in a channel

## An important relation among indices of a channel

Based on the following

- ▶  $A$  is almost idempotent:  $A^{-1} = \frac{1}{4^N} A$
- ▶ If  $\lambda_{\vec{\alpha}} \geq 0$  (as should be for channels), then  $\sum_{\vec{\alpha} \in \Omega} \lambda_{\vec{\alpha}} = 0$  implies  $\lambda_{\vec{\alpha}} = 0, \forall \vec{\alpha} \in \Omega$ ,

we can say that  $\tau_{\vec{\alpha}} = 1$  implies  $\tau_{\vec{\gamma}} = \tau_{\vec{\gamma}'}$  if

$$A_{\vec{\beta}\vec{\gamma}} = A_{\vec{\beta}\vec{\gamma}'}, \quad \forall \vec{\beta} : A_{\vec{\alpha}\vec{\beta}} = 1.$$

This is a relation between the indices that are preserved in a channel

This can be reformulated saying that if  $\tau_{\vec{\alpha}} = \tau_{\vec{\beta}} = 1$ , then  $\tau_{\vec{\alpha} \oplus \vec{\beta}} = 1$  with the sum of the components defined by:

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

## A *beautiful* vector space

- ▶ We have an addition operation ( $\oplus$ ) and, if we consider the field  $\{0, 1\}$ , multiplication by a scalar.
- ▶ That is, we have a vector space (but no interior product).

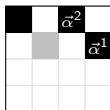
## A *beautiful* vector space

- ▶ We have an addition operation ( $\oplus$ ) and, if we consider the field  $\{0, 1\}$ , multiplication by a scalar.
- ▶ That is, we have a vector space (but no interior product).
- ▶ Given that the components of a channels are closed under addition, to each channel we have associated a vector subspace, and viceversa.

## A *beautiful* vector space

- ▶ We have an addition operation ( $\oplus$ ) and, if we consider the field  $\{0, 1\}$ , multiplication by a scalar.
- ▶ That is, we have a vector space (but no interior product).
- ▶ Given that the components of a channel are closed under addition, to each channel we have associated a vector subspace, and viceversa.

Examples:



Two of its elements are

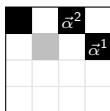
$$\vec{\alpha}^{(1)} = (0, 2), \vec{\alpha}^{(2)} = (1, 3)$$

but  $\vec{\alpha}^{(1)} \oplus \vec{\alpha}^{(2)} = (1, 1)$  which is not preserved. Thus, this diagram does not correspond to a channel.

## A beautiful vector space

- ▶ We have an addition operation ( $\oplus$ ) and, if we consider the field  $\{0, 1\}$ , multiplication by a scalar.
- ▶ That is, we have a vector space (but no interior product).
- ▶ Given that the components of a channel are closed under addition, to each channel we have associated a vector subspace, and viceversa.

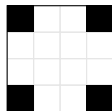
Examples:



Two of its elements are

$$\vec{\alpha}^{(1)} = (0, 2), \vec{\alpha}^{(2)} = (1, 3)$$

but  $\vec{\alpha}^{(1)} \oplus \vec{\alpha}^{(2)} = (1, 1)$  which is not preserved. Thus, this diagram does not correspond to a channel.



Its elements are

$$\vec{\alpha}^{(0)} = (0, 0), \quad \vec{\alpha}^{(1)} = (0, 3)$$

$$\vec{\alpha}^{(2)} = (3, 0), \quad \vec{\alpha}^{(3)} = (3, 3)$$

which are closed under  $\oplus$ . Thus this diagram represents a channel.

## Some consequences

What we have observed, are direct consequences of the subspace structure of the channels.

- ▶ The number of elements in a vector space with scalar field of  $q$  elements is  $q^k$ . That is, our channels preserve *exactly*  $2^k$  components.
- ▶ We can count the number of subspaces of a dimension  $k$ , and it yields  $S_{n,k} = \prod_{m=0}^{k-1} \frac{2^{2n-m}-1}{2^{k-m}-1}$ .
- ▶ So we verify the symmetry  $S_{N,K} = S_{N,2N-K}$ .
- ▶ There are maximal proper subspaces (next slide).

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence



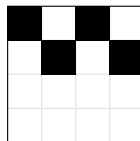
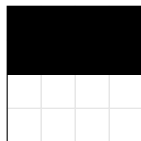
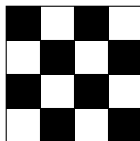
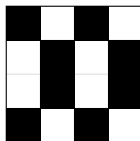
# The generators

- ▶ Any vector subspace is the intersection of maximal proper subspaces.
- ▶ Any channel can be generated as the composition of channels that preserve half of the components.

# The generators

- ▶ Any vector subspace is the intersection of maximal proper subspaces.
- ▶ Any channel can be generated as the composition of channels that preserve half of the components.

Example:

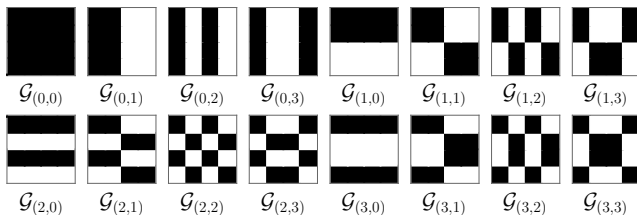


Any pair of the first channels generates the last one.

# Algunas propiedades de los generadores

Some consequences:

- ▶ The generators have right/left and up/down symmetry. More generally reflection or anti-reflection for any number of particles.
- ▶ Any generator can be classified by the action on each qubit.



# Decoherence

We can construct physical processes that generate the channels, by noticing that the Kraus operators corresponding to generators are

$$\mathcal{G}_{\vec{\alpha}} \longrightarrow \left\{ \frac{\mathbb{1}}{\sqrt{2}}, \frac{\sigma_{\vec{\alpha}}}{\sqrt{2}} \right\}.$$

This can be interpreted easily and we can also generate its Lindblad equation.

Other PCEs can be created and interpreted with the aid of the generators that produce them.

# Conclusions and outlook

- ▶ We introduced Pauli component erasing maps.
- ▶ We managed to diagonalize the Choi matrix of any Pauli channel.
- ▶ We associate vector subspaces to PCE channels, and exploit this relation.
- ▶ We find generators, Kraus operators, Lindblad equations etc for PCEs, which allows to associate them to physical processes.

## Conclusions and outlook

- ▶ We introduced Pauli component erasing maps.
- ▶ We managed to diagonalize the Choi matrix of any Pauli channel.
- ▶ We associate vector subspaces to PCE channels, and exploit this relation.
- ▶ We find generators, Kraus operators, Lindblad equations etc for PCEs, which allows to associate them to physical processes.

### Outlook

- ▶ qudits
- ▶ Allow  $\tau = 0, 1, -1$
- ▶ Study the geometry of the set of channels