

Pauli component erasing operations

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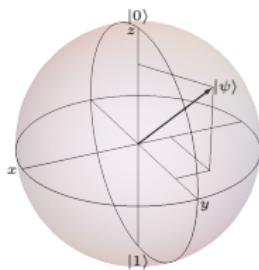
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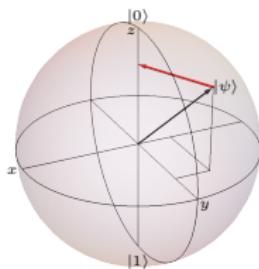
General goal

Quantum channels can project some components. But not all projections are allowed.



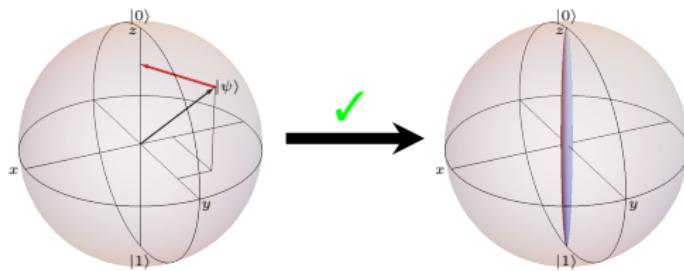
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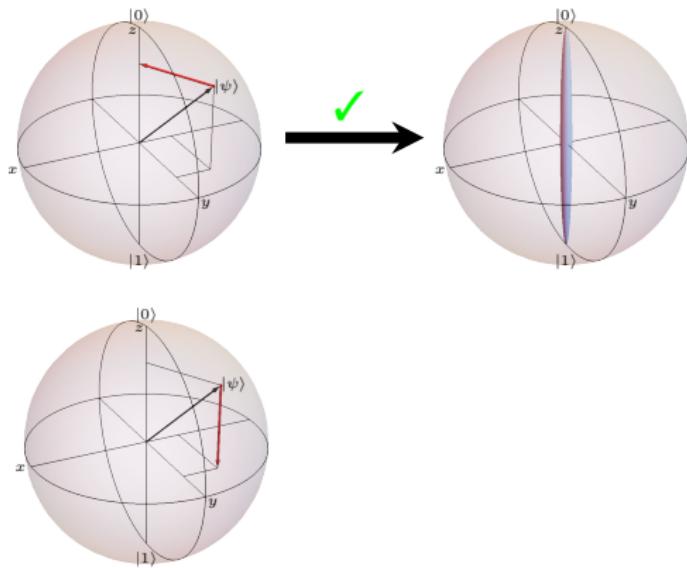
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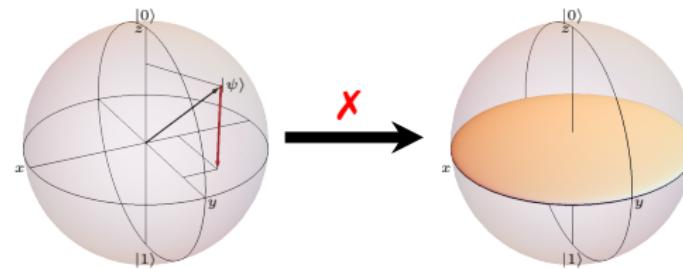
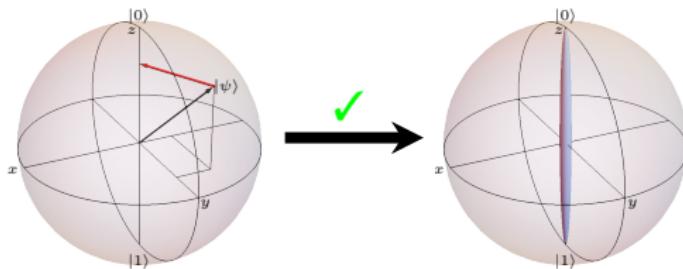
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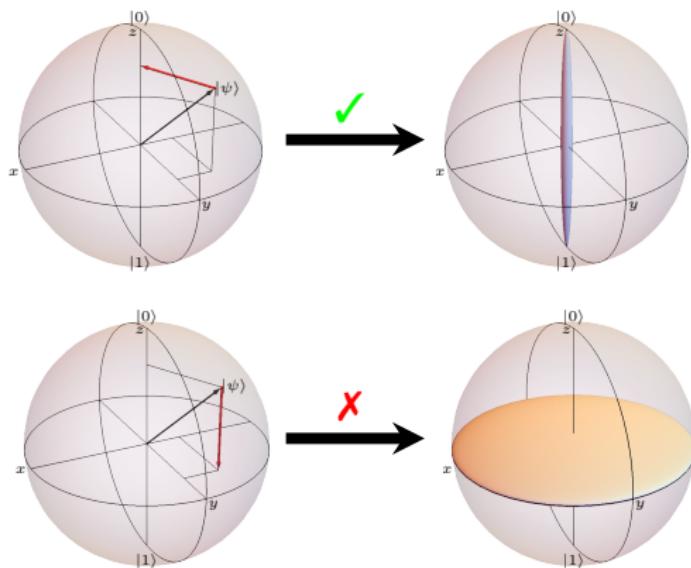
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We want to develop intuition about what's going on and generalize.

Outline

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

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Quantum channels

A quantum channel (Λ) generalizes conveniently unitary evolution, in a similar way as a density matrix generalizes a ket.

Quantum channels can describe:

- ▶ Unitary evolution $\Lambda(\rho) = U\rho U^\dagger$
- ▶ Reduced dynamics (decoherence) $\Lambda(\rho) = \text{tr}_{\text{env}} [U\rho \otimes \rho_{\text{env}} U^\dagger]$
- ▶ Measurements

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- ▶ Measurements

Quantum channels are linear operators acting on the spaces of operators, i.e. superoperators.

Quantum channels, some properties

Quantum channels map states into states $\mathcal{E}(\rho) = \rho'$

- ▶ They are linear $\mathcal{E}(\alpha\varsigma + \sigma) = \alpha\mathcal{E}(\varsigma) + \mathcal{E}(\sigma)$,
- ▶ preserve the trace $\text{tr}\mathcal{E}(\sigma) = \text{tr}\sigma$,
- ▶ preserve hermiticity $\mathcal{E}(\sigma) = \mathcal{E}(\sigma)^\dagger$ if $\sigma = \sigma^\dagger$, and
- ▶ preserve positivity, $\mathcal{E}(\sigma) \geq 0$ if $\sigma \geq 0$.

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They send extended states to extended states.

- ▶ Complete positivity, $(\mathcal{E} \otimes \mathbb{1}_k)(\sigma) \geq 0$ if $\sigma \geq 0$.

There are *maps* that send states to states, but not extended states to extended states, for example the transpose.

Quantum channels, complete positivity

- ▶ Complete positivity means that for all k , and all $\rho \geq 0$,

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$$(\mathcal{E}_n \otimes \mathbb{1}_n)(|\text{Bell}_n\rangle\langle\text{Bell}_n|) = \rho_{\mathcal{E}} \geq 0.$$

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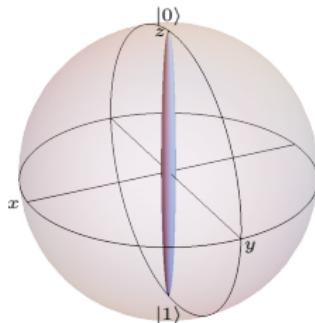
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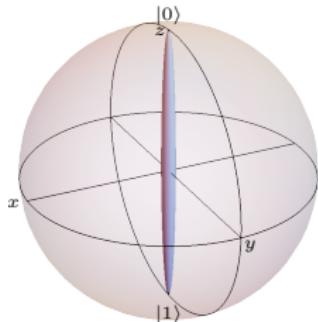
- ▶ We must check, diagonalizing $\rho_{\mathcal{E}}$, (which is proportional to the Choi matrix) if all its eigenvalues are semipositive.

Quantum channels



$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_z \sigma_z$$

Quantum channels

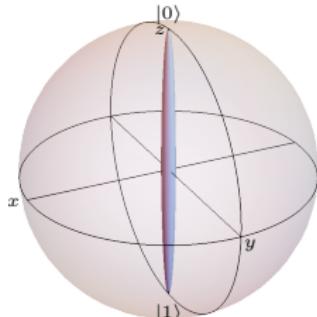


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$$\mathcal{E} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \rho_{\mathcal{E}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$\lambda_{\rho_{\mathcal{E}}} = (\frac{1}{2}, \frac{1}{2}, 0, 0)$
GOOD!

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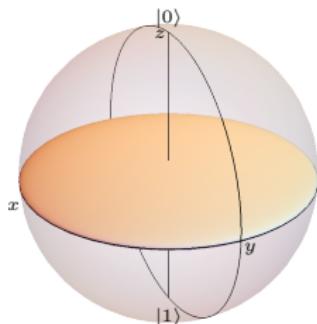


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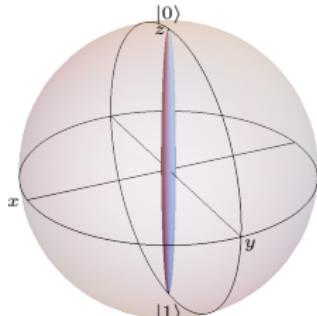
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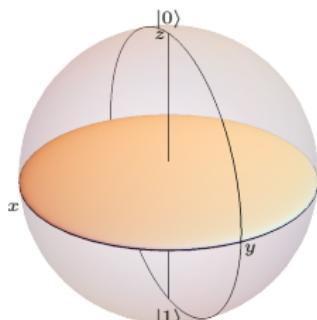


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$$\lambda_{\rho_{\mathcal{E}}} = (\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$$

BAD!

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

Single qubit PCE

A single qubit state can be expressed as

$$\rho = \frac{\mathbb{1} + \vec{r} \cdot \vec{\sigma}}{2} = \frac{\mathbb{1} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z}{2},$$

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PCE maps project over some of these components:

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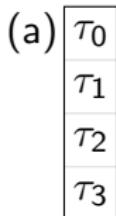
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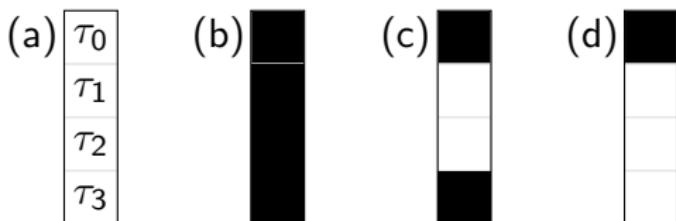
- ▶ They map density matrices to density matrices
- ▶ The previous maps are examples of PCE maps
- ▶ They are a subset of Pauli maps
- ▶ They are not always completely positive

Single qubit PCE and its diagrams



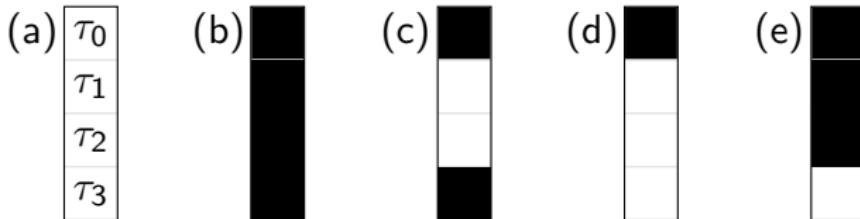
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Single qubit PCE and its diagrams



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- ▶ It is either black (if its preserved) or white (if its erased)
- ▶ (b) is the identity channel, (c) is dephasing and (d) depolarizing.
- ▶ (e) is the map that projects to the xy plane

N qubit PCEs

- ▶ An N qubit state can be written as

$$\rho = \frac{1}{2^N} \sum_{\vec{\alpha}} r_{\vec{\alpha}} \sigma_{\vec{\alpha}},$$

with $\vec{\alpha} = (\alpha_1, \dots, \alpha_N)$, and $\sigma_{\vec{\alpha}} := \sigma_{\alpha_1} \otimes \dots \otimes \sigma_{\alpha_N}$.

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This are the objects we are interested in.

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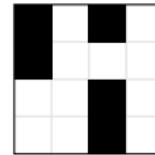
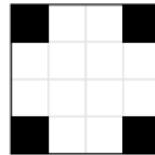
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- ▶ For 2 qubits, we have equivalent diagrams:

$\tau_{(0,0)}$	$\tau_{(0,1)}$	$\tau_{(0,2)}$	$\tau_{(0,3)}$
$\tau_{(1,0)}$	$\tau_{(1,1)}$	$\tau_{(1,2)}$	$\tau_{(1,3)}$
$\tau_{(2,0)}$	$\tau_{(2,1)}$	$\tau_{(2,2)}$	$\tau_{(2,3)}$
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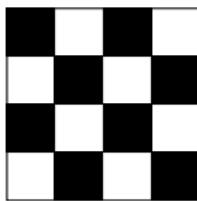
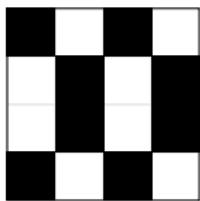
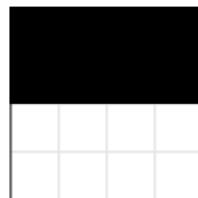
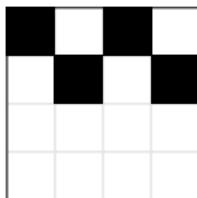
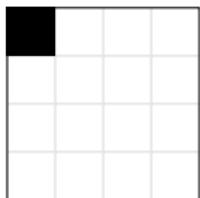
Generators and decoherence

Planteamiento del problema

¿Cuales mapas PCE son canales cuánticos válidos?

Esto nos ayudará a entender mejor la geometría de canales cuánticos de muchos cuerpos.

Ejemplos y reglas



- ▶ Solo se admiten PCEs que preservan 2^k componentes.
- ▶ No todos los que preservan 2^k componentes son canales.
- ▶ Hay el mismo número de canales que preservan 2^k y 2^{2N-k} componentes.
- ▶ Hay un conjunto generador que conserva la mitad de componentes.

Diagonalizacion de la matriz de Choi

- El canal tiene la forma $\hat{\mathcal{E}} = \frac{1}{2^N} \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} |\sigma_{\vec{\alpha}}\rangle\rangle\langle\langle\sigma_{\vec{\alpha}}|$.

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- ▶ Los eigenvectores comunes cumplen $(\sigma_{\vec{\alpha}} \otimes \sigma_{\vec{\alpha}}^*) v_{\vec{\beta}} = \frac{1}{2^N} A_{\vec{\alpha}\vec{\beta}} v_{\vec{\beta}}$ con

$$A = a^{\otimes N}, \quad a = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

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- ▶ Esto implica que los eigenvectores cumplen $\lambda_{\vec{\alpha}} = \frac{1}{2^N} A_{\vec{\alpha}\vec{\beta}} \tau_{\vec{\beta}}$.
- ▶ Diagonalizamos la matriz de Choi ($\propto \rho_{\mathcal{E}}$) para canales de Pauli.

Una relación entre indices de un canal

Basándonos en que

- ▶ A es casi idempotente: $A^{-1} = \frac{1}{4^N} A$
- ▶ Si $\lambda_{\vec{\alpha}} \geq 0$ (como debe ser para canales cuánticos), entonces $\sum_{\vec{\alpha} \in \Omega} \lambda_{\vec{\alpha}} = 0$ implica $\lambda_{\vec{\alpha}} = 0, \forall \vec{\alpha} \in \Omega,$

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Esta es una relación entre los indices que son preservados en un canal.

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Se puede reformular diciendo que si $\tau_{\vec{\alpha}} = \tau_{\vec{\beta}} = 1$, entonces $\tau_{\vec{\alpha} \oplus \vec{\beta}} = 1$ con la suma entre componentes definida por:

\oplus	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Un espacio vectorial *hermoso*

- ▶ Tenemos operación suma (\oplus) cerrada, y, si consideramos el campo $\{0, 1\}$, multiplicación por escalar.
- ▶ Es decir, tenemos un espacio vectorial (sin producto interior).

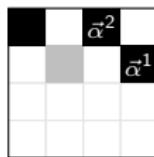
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Ejemplos:



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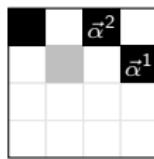
$$\vec{\alpha}^{(1)} = (0, 2), \vec{\alpha}^{(2)} = (1, 3)$$

pero $\vec{\alpha}^{(1)} \oplus \vec{\alpha}^{(2)} = (1, 1)$ y no se preserva. Entonces, este diagrama no corresponde a un canal.

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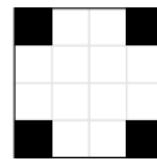
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Sus elementos son

$$\vec{\alpha}^{(0)} = (0, 0), \quad \vec{\alpha}^{(1)} = (0, 3)$$

$$\vec{\alpha}^{(2)} = (3, 0), \quad \vec{\alpha}^{(1)} = (3, 3)$$

que son cerrados bajo \oplus . Es decir *si* representa un canal.

Algunas consecuencias

Lo que habíamos observado, surge como consecuencia de la estructura vectorial de los canales.

- ▶ El numero de elementos de un espacio vectorial con campo escalar de q elementos tiene q^k vectores. Es decir nuestros canales preservan exactamente 2^k componentes.
- ▶ Analizando el numero de subespacios de dimensión k y las bases que lo pueden generar encontramos el número de canales que preservan 2^k componentes $S_{N,K} = \prod_{m=0}^{K-1} \frac{2^{2N-m}-1}{2^{K-m}-1}$.
- ▶ Verificamos la simetría $S_{N,K} = S_{N,2N-K}$
- ▶ Existen subespacios maximales (siguiente slide).

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

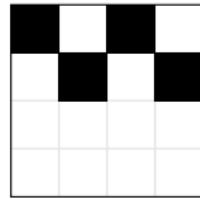
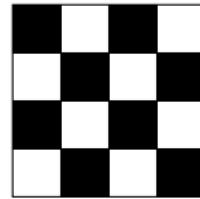
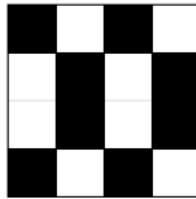
Aparición de los generadores

- ▶ Cualquier subespacio vectorial puede generarse como la intersección de subespacios maximales no triviales.
- ▶ Cualquier canal puede generarse como la composición de canales que preservan la mitad de componentes.

Aparición de los generadores

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Ejemplo:

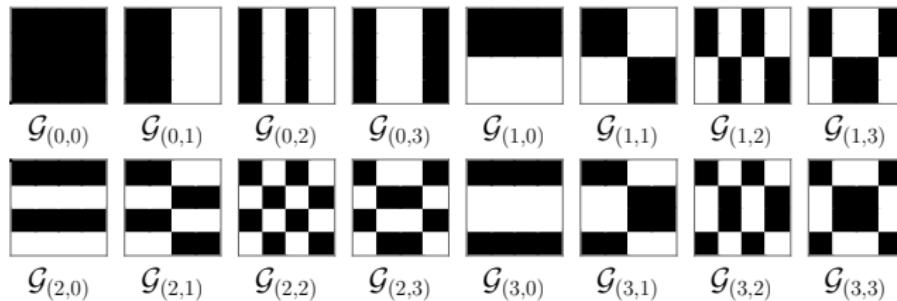


Cualquier par (de los primeros canales) genera el ultimo canal.

Algunas propiedades de los generadores

Algunas consecuencias:

- ▶ Las generadores tienen simetrías izquierda/derecha y arriba/abajo. Se puede comprobar de las propiedades mostradas anteriormente.
- ▶ Cada generador se puede clasificar mediante la acción sobre cada qubit.



Decoherencia generalizada

Podemos construir los procesos físicos que generan estos canales, notando que los operadores de Kraus que corresponden a los generadores son

$$\mathcal{G}_{\vec{\alpha}} \longrightarrow \left\{ \frac{\mathbb{1}}{\sqrt{2}}, \frac{\sigma_{\vec{\alpha}}}{\sqrt{2}} \right\}.$$

Este se puede interpretar muy fácilmente y generar su ecuación de Lindblad.

Otros PCEs se pueden ser creados e interpretados a partir de los generadores que lo produzcan.

Conclusions and outlook

- ▶ Se introducen los Pauli Component Erasing maps.
- ▶ Se logra diagonalizar de manera simple la matriz de Choi de cualquier canal de Pauli.
- ▶ Se asocian a los PCEs espacios vectoriales, lo que simplifica su tratamiento
- ▶ Se encuentran los generadores y sus operadores de Kraus, lo que permitirá asociarlos a procesos físicos existentes.

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Outlook

- ▶ qudits
- ▶ Allow $\tau = 0, 1, -1$
- ▶ Study the geometry of the set of channels