Numerical progress on Weyl component erasing operations

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1 3-level system

The results for Weyl component erasing (WCE) quantum channels claimed by Alejo have been reproduced by an independent party (me). Since Alejo has been using two indices for the components $\tau_{\mu\nu}$ to characterize a WCE operation we can represent the 3-level system WCE quantum channels as in Fig. 1. Note the existence of the additional quantum channel that leaves invariant α_{00}, $\alpha_{1,2}$ and $\alpha_{2,1}$ (fifth figure from left to right). Therefore, the so-called *rainbow rule* follows as 1 4 1 WCE channels that leave invariant 1 3 9 "Weyl" components of the density matrix.
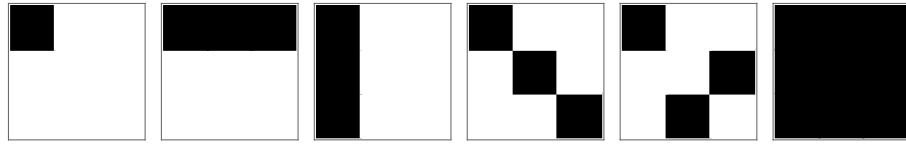

Figure 1: Representation of a 3-level system WCE quantum channels.

2 4-level system

In Fig. 2 I show the numerical results for 4-level system WCE quantum channels. It is worth noting that the *rainbow rule* is weirdly followed: 1 3 7 3 1 number of quantum channels that leave invariant 1 2 4 8 16 "Weyl components" of the density matrix.
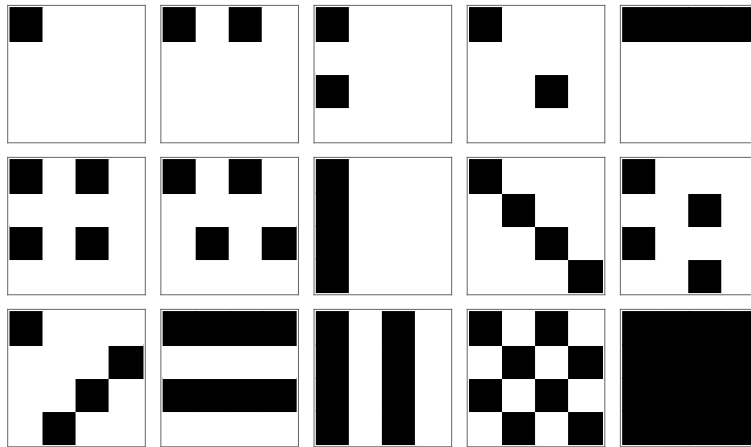

Figure 2: Representation of a 4-level system WCE quantum channels.

3 Generalization ⊕

3.1 New notation for $\vec{\alpha}$ and definition of ⊕

We will rewrite the multi-indices $\vec{\alpha}$ using base-d numeral system and will re-define the operation ⊕, previously associated by François with the Klein group. Every index in the multi-indices $\vec{\alpha}$ of a WCE quantum channel can be written as a length-2 list with its base-d numeral system digits. Therefore, every index α_i in $\vec{\alpha}$ can be written as

$$\alpha_i = (\alpha_{i_1}, \alpha_{i_2}). \tag{1}$$

The multi-indices $\vec{\alpha}$ then reads

$$\vec{\alpha} = \{(\alpha_{1_1}, \alpha_{1_2}), \ldots, (\alpha_{N_1}, \alpha_{N_2})\}. \tag{2}$$

We define the sum of two multi-indices $\vec{\alpha}$ and $\vec{\beta}$ as

$$\vec{\alpha} \oplus \vec{\beta} = \{(\alpha_{1_1} \oplus \beta_{1_1}, \alpha_{1_2} \oplus \beta_{1_2}), \ldots, (\alpha_{N_1} \oplus \beta_{N_1}, \alpha_{N_2} \oplus \beta_{N_2})\}, \tag{3}$$

where operation ⊕ is the sum modulo d.

3.2 Examples

Let us look at some examples. First, let us discuss a familiar example with a PCE quantum channel. The set of multi-indices associated with the PCE quantum channel of Fig. 3 are, in decimal basis, $\{(0,0), (1,1), (2,1), (3,0)\}$. The same multi-indices written in binary basis are

$$\{\{(0,0), (0,0)\}, \{(0,1), (0,1)\}, \{(1,0), (0,1)\}, \{(1,1), (0,0)\}\}.$$

Let us sum $(2,1) \oplus (1,1)$ as previously defined by François

$$(2,1) \oplus (1,1) = (2 \oplus 1, 1 \oplus 1) = (3,0). \tag{4}$$

Now, let us sum the same two multi-indices, written in binary basis, using the definition (3)

$$\{(1,0), (0,1)\} \oplus \{(0,1), (0,1)\} = \{(1 \oplus 0, 0 \oplus 1), (0 \oplus 0, 1 \oplus 1)\} = \{(1,1), (0,0)\}, \tag{5}$$

which is equal to $(3,0)$ in decimal basis. This is an example that shows how the operation ⊕ defined with the Klein group is equivalent with the new notation and definition (3).
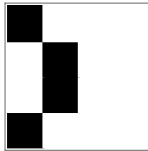

Figure 3: A random two-qubits PCE quantum channel.

Now, let us look at the set of multi-indices of the fifth 3-level-system WCE channel in Fig. 1: $\{\{(0,0)\}, \{(1,2)\}, \{(2,1)\}\}$. The only three different possible sums modulo 3 are

$$(0,0) \oplus (1,2) = (0 \oplus 1, 0 \oplus 2) = (1,2) \tag{6a}$$
$$(0,0) \oplus (2,1) = (0 \oplus 2, 0 \oplus 1) = (2,1) \tag{6b}$$
$$(1,2) \oplus (2,1) = (1 \oplus 2, 2 \oplus 1) = (0,0) \tag{6c}$$

Then, the set of multi-indices $\vec{\alpha}$ is closed. As the last example, let us check the closure of the set of multi-indices of the tenth 4-level-system WCE channel in Fig 2: $\{\{(0,0)\}, \{(1,2)\}, \{(2,0)\}, \{(3,2)\}\}$ The only six different possible sums modulo 4 are

$$(0,0) \oplus (1,2) = (1,2) \tag{7a}$$
$$(0,0) \oplus (2,0) = (2,0) \tag{7b}$$
$$(0,0) \oplus (3,2) = (3,2) \tag{7c}$$
$$(1,2) \oplus (2,0) = (3,2) \tag{7d}$$
$$(1,2) \oplus (3,2) = (0,0) \tag{7e}$$
$$(2,0) \oplus (3,2) = (1,2), \tag{7f}$$

then the set of multi-indices is closed.

3.3 Hypothesis

The set of multi-indices of a N-qudits WCE quantum channel, written in base-d numeral system as in (2), is closed under sum modulo d as defined in (3). There are numerical results that support this hypothesis. The method of brute force has been implemented to check if all sets of a multi-indices of 3, 4 and partially 5-level system WCE quantum channels are closed under $\oplus$ as defined in (3). The converse has been numerically analyzed with the method of brute force too. All sets of multi-indices that are closed under the operation $\oplus$ are associated with a WCE quantum channel. All of this shows clues that a vector structure exists as in the case of PCE quantum channels.