

Ojo, ocuparme de estos typos: Typos en diapositivas:

6 - entagled en lugar de mixed 8 - PEC en lugar de PCE 10 - mas en lugar de map 11 - actully 13 - con en lugar de where 14 - implica en lugar de implies 15 - opertation - asosciated 18 - título en español "algunas propiedades"

Otras notas: En la diapositiva 15 mencionaste que una figura está volteada. En la diapositiva 12, el itemize de las propiedades del conjunto podría ser más claro (el primer punto no lo caché hasta que se explicó).

Pauli component erasing operations

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¹Universidad Nacional Autónoma de México

²Universidad de San Carlos de Guatemala

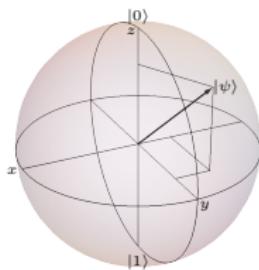
³Universidade Federal de Pernambuco (Brazil)

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arXiv:2205.05808

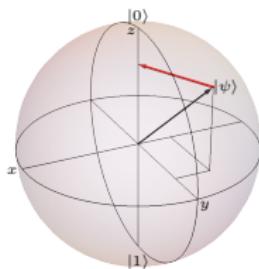
General goal

Quantum channels can project some components. But not all projections are allowed.



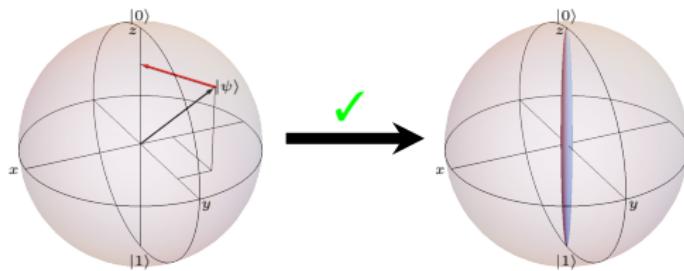
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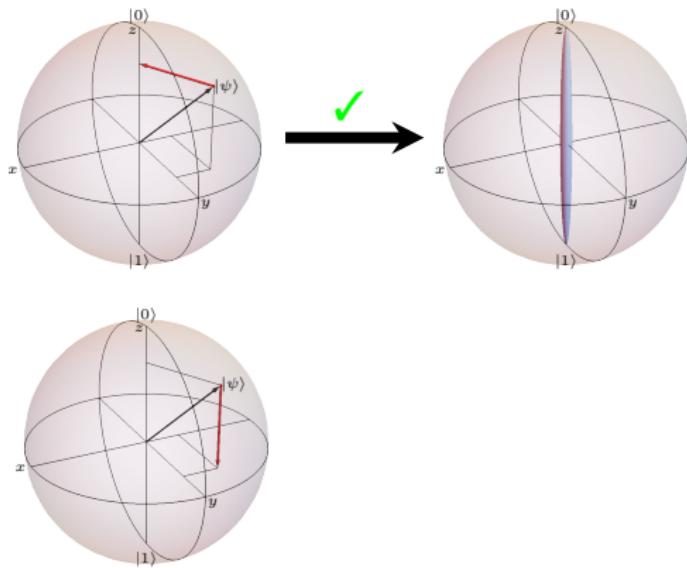
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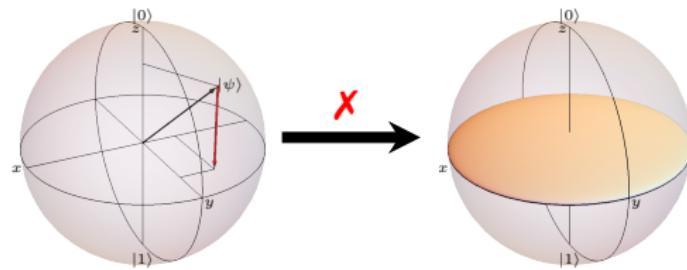
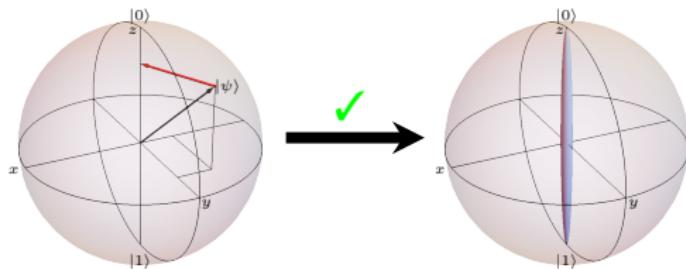
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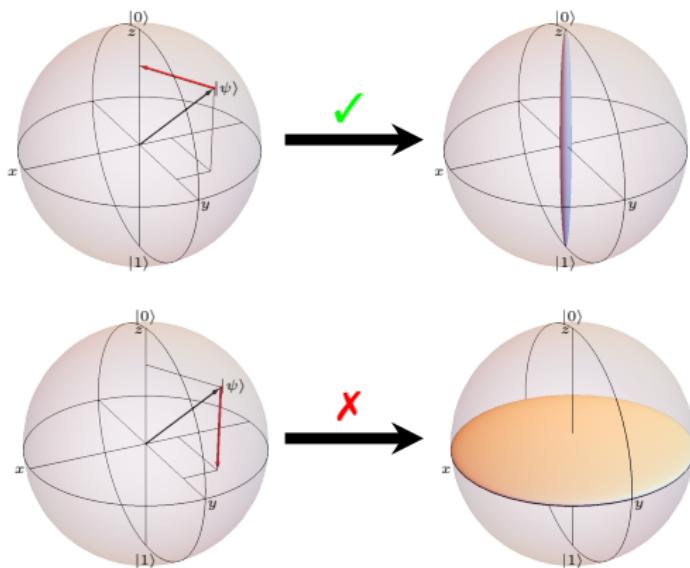
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We want to develop intuition about what's going on and generalize.

Outline

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

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Quantum channels

A quantum channel (Λ) generalizes conveniently unitary evolution, in a similar way as a density matrix generalizes a ket.

Quantum channels can describe:

- ▶ Unitary evolution $\Lambda(\rho) = U\rho U^\dagger$
- ▶ Reduced dynamics (decoherence) $\Lambda(\rho) = \text{tr}_{\text{env}} [U\rho \otimes \rho_{\text{env}} U^\dagger]$
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Quantum channels are linear operators acting on the spaces of operators, i.e. superoperators.

Quantum channels, some properties

Quantum channels map states into states $\mathcal{E}(\rho) = \rho'$

- ▶ They are linear $\mathcal{E}(\alpha\varsigma + \sigma) = \alpha\mathcal{E}(\varsigma) + \mathcal{E}(\sigma)$,
- ▶ preserve the trace $\text{tr}\mathcal{E}(\sigma) = \text{tr}\sigma$,
- ▶ preserve hermiticity $\mathcal{E}(\sigma) = \mathcal{E}(\sigma)^\dagger$ if $\sigma = \sigma^\dagger$, and
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They send extended states to extended states.

- ▶ Complete positivity, $(\mathcal{E} \otimes \mathbb{1}_k)(\sigma) \geq 0$ if $\sigma \geq 0$.

There are *maps* that send states to states, but not extended states to extended states, for example the transpose.

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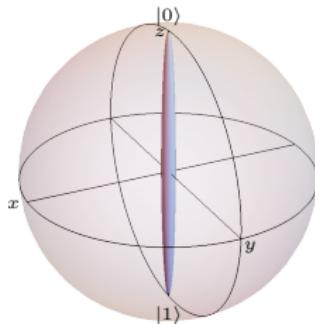
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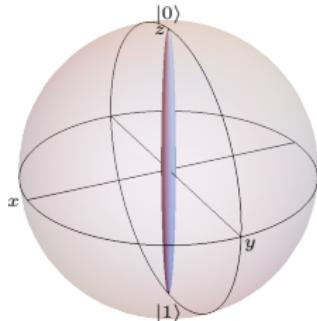
- ▶ We must check, diagonalizing $\rho_{\mathcal{E}}$, (which is proportional to the Choi matrix) if all its eigenvalues are semipositive.

Quantum channels



$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_z \sigma_z$$

Quantum channels

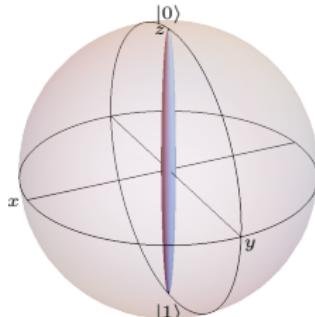


$$\mathcal{E}[1 + \vec{r} \cdot \vec{\sigma}] = 1 + r_z \sigma_z$$

$$\mathcal{E} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \rho_{\mathcal{E}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$\lambda_{\rho_{\mathcal{E}}} = (\frac{1}{2}, \frac{1}{2}, 0, 0)$
GOOD!

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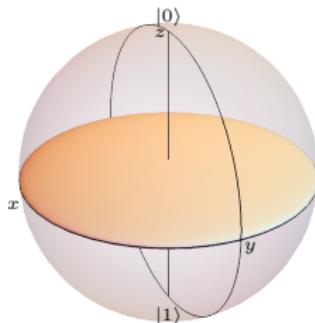


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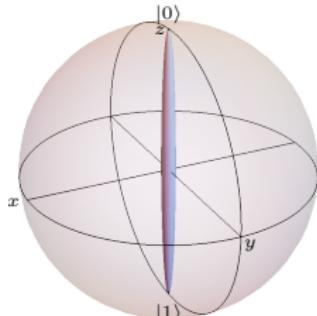
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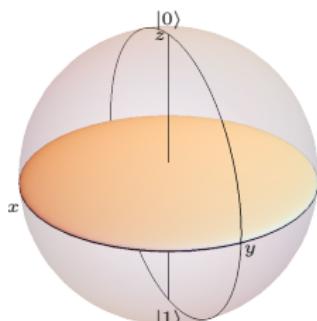


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$$\lambda_{\rho_{\mathcal{E}}} = (\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$$

BAD!

Some tools

PCE maps and PCE channels

The problem and its solution

Generators and decoherence

Single qubit PCE

A single qubit state can be expressed as

$$\rho = \frac{\mathbb{1} + \vec{r} \cdot \vec{\sigma}}{2} = \frac{\mathbb{1} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z}{2},$$

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PCE maps project over some of these components:

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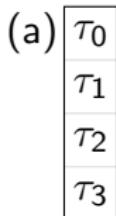
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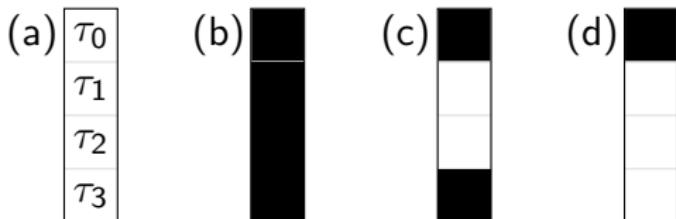
- ▶ They map density matrices to density matrices
- ▶ The previous maps are examples of PCE maps
- ▶ They are a subset of Pauli maps
- ▶ They are not always completely positive

Single qubit PCE and its diagrams



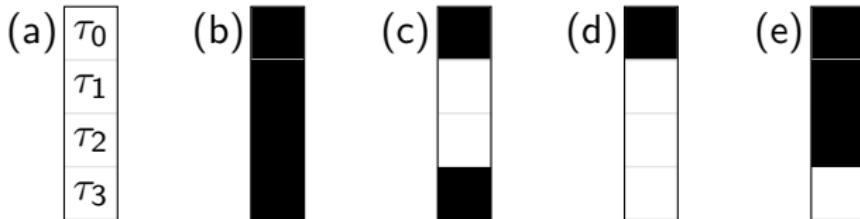
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Single qubit PCE and its diagrams



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- ▶ (b) is the identity channel, (c) is dephasing and (d) depolarizing.
- ▶ (e) is the map that projects to the xy plane

N qubit PCEs

- ▶ An N qubit state can be written as

$$\rho = \frac{1}{2^N} \sum_{\vec{\alpha}} r_{\vec{\alpha}} \sigma_{\vec{\alpha}},$$

with $\vec{\alpha} = (\alpha_1, \dots, \alpha_N)$, and $\sigma_{\vec{\alpha}} := \sigma_{\alpha_1} \otimes \dots \otimes \sigma_{\alpha_N}$.

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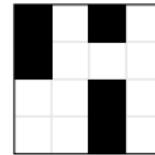
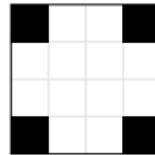
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- ▶ For 2 qubits, we have equivalent diagrams:

$\tau_{(0,0)}$	$\tau_{(0,1)}$	$\tau_{(0,2)}$	$\tau_{(0,3)}$
$\tau_{(1,0)}$	$\tau_{(1,1)}$	$\tau_{(1,2)}$	$\tau_{(1,3)}$
$\tau_{(2,0)}$	$\tau_{(2,1)}$	$\tau_{(2,2)}$	$\tau_{(2,3)}$
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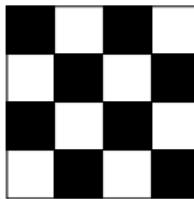
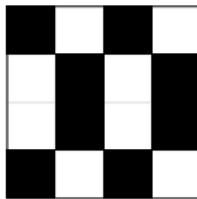
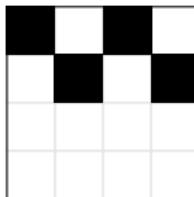
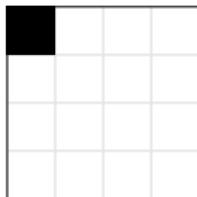
Generators and decoherence

Statement of the problem

Which PCE maps are actually quantum channels?

This will hopefully help us understand better the geometry of many-body quantum channels

Examples and rules



- ▶ Only PCEs that preserve 2^k components.
- ▶ Not all PCEs that preserve 2^k components are channels.
- ▶ The same number of channels that preserve 2^k and 2^{2N-k} components.
- ▶ There is a generating set that preserves half of the components.

Choi matrix diagonalization

- The channel can be expressed as $\hat{\mathcal{E}} = \frac{1}{2^N} \sum_{\vec{\alpha}} \tau_{\vec{\alpha}} |\sigma_{\vec{\alpha}}\rangle\langle\langle\sigma_{\vec{\alpha}}|$.

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- ▶ And the eigenvectors fulfill $(\sigma_{\vec{\alpha}} \otimes \sigma_{\vec{\alpha}}^*) v_{\vec{\beta}} = \frac{1}{2^N} A_{\vec{\alpha}\vec{\beta}} v_{\vec{\beta}}$ con

$$A = a^{\otimes N}, \quad a = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

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- ▶ This implies that the eigenvectors also fullfill $\lambda_{\vec{\alpha}} = \frac{1}{2^N} A_{\vec{\alpha}\vec{\beta}} \tau_{\vec{\beta}}$.
- ▶ So we just diagonalize the Choi matriz ($\propto \rho_{\mathcal{E}}$) for all Pauli channels

An important relation among indices of a channel

Based on the following

- ▶ A is almost idempotent: $A^{-1} = \frac{1}{4^N}A$
- ▶ If $\lambda_{\vec{\alpha}} \geq 0$ (as should be for channels), then $\sum_{\vec{\alpha} \in \Omega} \lambda_{\vec{\alpha}} = 0$ implies $\lambda_{\vec{\alpha}} = 0, \forall \vec{\alpha} \in \Omega$,

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This can be reformulated saying that if $\tau_{\vec{\alpha}} = \tau_{\vec{\beta}} = 1$, then $\tau_{\vec{\alpha} \oplus \vec{\beta}} = 1$ with the sum of the components defined by:

\oplus	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

A *beautiful* vector space

- ▶ We have an addition operation (\oplus) and, if we consider the field $\{0, 1\}$, multiplication by a scalar.
- ▶ That is, we have a vector space (but no interior product).

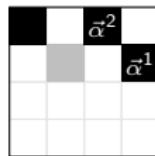
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Examples:



Two of its elements are

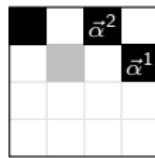
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but $\vec{\alpha}^{(1)} \oplus \vec{\alpha}^{(2)} = (1, 1)$ which is not preserved. Thus, this diagram does not correspond to a channel.

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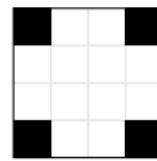
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Its elements are

$$\vec{\alpha}^{(0)} = (0, 0), \quad \vec{\alpha}^{(1)} = (0, 3)$$

$$\vec{\alpha}^{(2)} = (3, 0), \quad \vec{\alpha}^{(1)} = (3, 3)$$

which are closed under \oplus . Thus this diagram represents a channel.

Some consequences

What we have observed, are direct consequences of the subspace structure of the channels.

- ▶ The number of elements in a vector space with scalar field of q elements is q^k . That is, our channels preserve exactly 2^k components.
- ▶ We can count the number of subspaces of a dimension k , and it yields $S_{n,k} = \prod_{m=0}^{k-1} \frac{2^{2n-m}-1}{2^{k-m}-1}$.
- ▶ So we verify the symmetry $S_{N,K} = S_{N,2N-K}$.
- ▶ There are maximal proper subspaces (next slide).

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The problem and its solution

Generators and decoherence

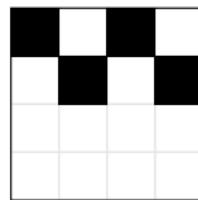
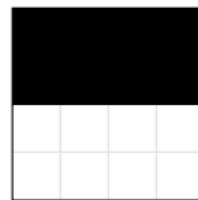
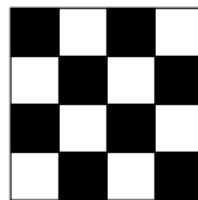
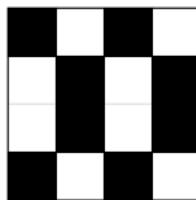
The generators

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Example:

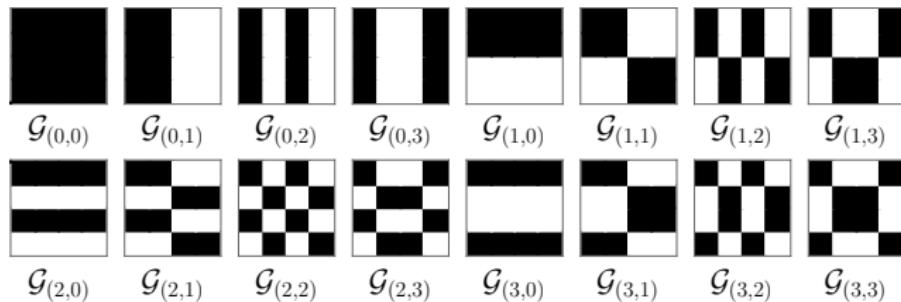


Any pair of the first channels generates the last one.

Algunas propiedades de los generadores

Some consequences:

- ▶ The generators have right/left and up/down symmetry. More generally reflection or anti-reflection for any number of particles.
- ▶ Any generator can be classified by the action on each qubit.



Decoherence

We can construct physical processes that generate the channels, by noticing that the Kraus operators corresponding to generators are

$$\mathcal{G}_{\vec{\alpha}} \longrightarrow \left\{ \frac{\mathbb{1}}{\sqrt{2}}, \frac{\sigma_{\vec{\alpha}}}{\sqrt{2}} \right\}.$$

This can be interpreted easily and we can also generate its Lindblad equation.

Other PCEs can be created and interpreted with the aid of the generators that produce them.

Conclusions and outlook

- ▶ We introduced Pauli component erasing maps.
- ▶ We managed to diagonalize the Choi matrix of any Pauli channel.
- ▶ We associate vector subspaces to PCE channels, and exploit this relation.
- ▶ We find generators, Kraus operators, Lindblad equations etc for PCEs, which allows to associate them to physical processes.

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Outlook

- ▶ qudits
- ▶ Allow $\tau = 0, 1, -1$
- ▶ Study the geometry of the set of channels