$$\mathcal{H}_T = \mathcal{H}_C \otimes \mathcal{H}_p = \mathbb{C}^2 \otimes \mathbb{C}^{2t+1}$$

- Probabilidad de encontrar al caminante en la posición x en el tiempo t:

$$P(X_t = x) = \mathrm{Tr}\left(|x\rangle\langle x|_p \, \mathrm{Tr}_C |\psi(t)\rangle\langle\psi(t)|\right), \quad |\psi(t)\rangle \in \mathcal{H}_T$$

$$= \sum_{x'} \underbrace{\langle x'|_p |x\rangle}_{\delta_{x,x'}} \langle x|_p \left(\mathrm{Tr}_C |\psi(t)\rangle\langle\psi(t)|\right) |x'\rangle_p$$

$$= {}_p\langle x| \mathrm{Tr}_C |\psi(t)\rangle\langle\psi(t)| \, |x\rangle_p$$

$$= {}_p\langle x| \sum_m \otimes {}_C\langle m|\psi(t)\rangle\langle\psi(t)|m\rangle_C \otimes |x\rangle_p \qquad \{|m\rangle\} = \text{base de } \mathcal{H}_C$$

$\in \mathcal{H}_p$

${}_C\langle m|\psi(t)\rangle \cancel{\text{es un número}}$

es un número

$\langle x,m|\psi(t)\rangle$ ✓

$$= \sum_m \langle x,m|\psi(t)\rangle\langle\psi(t)|m,x\rangle, \quad |m,x\rangle = |m\rangle_C \otimes |x\rangle_p$$

$$= \sum_m |\langle x,m|\psi(t)\rangle|^2 .$$

$${}_C\langle m|\psi(t)\rangle = {}_C\langle m| \otimes \underbrace{\left(\sum_x |x\rangle_p\langle x|_p\right)}_{\mathbb{1}_p} |\psi(t)\rangle$$

$$= \sum_x |x\rangle \left({}_C\langle m| \otimes {}_p\langle x|\right)|\psi(t)\rangle$$

$$\sum_x \langle m,\mathbb{1}| \, |\mathbb{1},x\rangle\langle\mathbb{1},x| \qquad = \sum_x \langle m,x|\psi(t)\rangle \, |x\rangle \quad ✓$$

Canal cuántico de la moneda

$\rho_0 = |c_0, x_0\rangle\langle c_0, x_0|$ (t_0) $\qquad \rho_0 = \rho_0^{(c)} \otimes \rho_0^{(p)}$

$\mathcal{E}(|c_0\rangle\langle c_0|) = \mathrm{Tr}_p\left(U|c_0, x_0\rangle\langle c_0, x_0|U^\dagger\right)$, Válido de t_0 a t

$\mathcal{E}$

$t_0 \qquad t_1 \qquad t_2 \qquad t_3 \qquad t_4$

Λ no necesariamente es CP

Representaciones de $\mathcal{E}$

1. $\mathcal{E}(\rho) = \mathrm{Tr}_E(U\rho\otimes\rho_E U^\dagger)$ $\qquad |\psi\rangle \in \mathcal{H}$
2. $\mathcal{E}(\rho) = \sum_j K_j \rho K_j^\dagger$ $\qquad L, H, \rho \in B(\mathcal{H})$
3. $(\mathcal{E}\otimes \mathbb{1})[|\phi^+\rangle\langle\phi^+|]$ $\qquad \rightarrow \mathcal{E} \in \mathcal{H}_S$
4. $\hat{\mathcal{E}}$ Superoperador

$$\rho \mapsto |\rho\rangle\rangle \in B(\mathcal{H})$$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \qquad |\rho\rangle\rangle = \begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix}$$

En el caso del $\mathcal{E}$ de la moneda: $\dim(\hat{\mathcal{E}}) = 4$

$\rightarrow K_j = {}_E\langle j|U|\psi_{0E}\rangle$ $\qquad \rho_0^{(E)} = |\psi_{0E}\rangle\langle\psi_{0E}|$

$$\mathrm{Tr}_E\left(U\, \rho_S \otimes |\psi_{0E}\rangle\langle\psi_{0E}|U^\dagger\right) = \sum_j K_j \rho K_j^\dagger$$

$2t+1$ $\quad U \quad (4)$

→ Teorema de dilatación de Stinespring

$$\mathcal{E} \qquad Tr_E(U\,\rho\otimes\rho_E\,U^{\dagger}) = \sum K_j \rho K_j^{\dagger}$$

$dim_{max}(entorno) = d^2 \qquad d=2 \qquad d^2=4$

→ Superoperador $\hat{\mathcal{E}}$.