

$$\mathcal{H}_T = \mathcal{H}_c \otimes \mathcal{H}_p = \mathbb{C}^2 \otimes \mathbb{C}^{2t+1}$$

- Probabilidad de encontrar al caminante en la posición x en el tiempo t :

$$P(X_t = x) = \text{Tr}(|x\rangle\langle x|_p \text{Tr}_c |\psi(t)\rangle\langle\psi(t)|), \quad |\psi(t)\rangle \in \mathcal{H}_T$$

$$= \sum_{x'} \underbrace{\langle x'|_p |x\rangle}_{\delta_{x,x'}} \langle x|_p (\text{Tr}_c |\psi(t)\rangle\langle\psi(t)| |x'\rangle_p)$$

$$= {}_p\langle x| \text{Tr}_c |\psi(t)\rangle\langle\psi(t)| |x\rangle_p$$

$$= {}_p\langle x| \sum_m \langle m| \psi(t)\rangle \langle\psi(t)| m\rangle \otimes |x\rangle_p \quad \{|m\rangle\} = \text{base de } \mathcal{H}_c$$

$$= \sum_m \langle x, m| \psi(t)\rangle \langle\psi(t)| m, x\rangle, \quad |m, x\rangle = |m\rangle_c \otimes |x\rangle_p$$

$$= \sum_m |\langle x, m| \psi(t)\rangle|^2.$$

$$\langle m| \psi(t)\rangle = \langle m| \otimes \left(\sum_x |x\rangle_p \langle x|_p \right) |\psi(t)\rangle$$

$\mathbb{1}_p$

$$= \sum_x |x\rangle \langle m| \otimes \langle x|_p |\psi(t)\rangle$$

$$\hookrightarrow \sum_x \langle m, \mathbb{1}| | \mathbb{1}, x\rangle \langle \mathbb{1}, x|$$

$$= \sum_x \langle m, x| \psi(t)\rangle |x\rangle \quad \checkmark$$

$\downarrow \in \mathcal{H}_p$
 $\langle m| \psi(t)\rangle$ no
 es un número

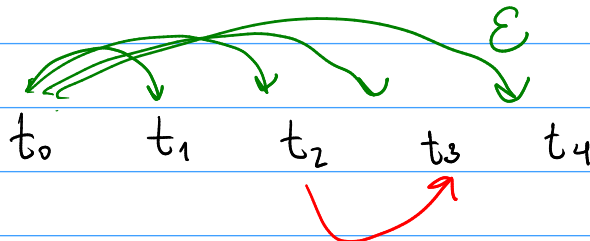
$\langle x, m| \psi(t)\rangle \checkmark$

Canal cuántico de la moneda

$$\rho_0 = |c_0, x_0\rangle\langle c_0, x_0| \quad (t_0)$$

$$\rho_0 = \rho_0^{(c)} \otimes \rho_0^{(p)}$$

$$\mathcal{E}(|c_0, x_0\rangle) = \text{Tr}_p(U |c_0, x_0\rangle\langle c_0, x_0| U^\dagger), \quad \text{Válido de } t_0 \text{ a } t$$



\wedge no necesariamente es CP

Representaciones de \mathcal{E}

$$1. \mathcal{E}(\rho) = \text{Tr}_E(U \rho \otimes p_E U^\dagger) \leftarrow$$

$$|\psi\rangle \in \mathcal{H}$$

$$2. \mathcal{E}(\rho) = \sum_j K_j \rho K_j^\dagger$$

$$L, H, \rho \in \mathcal{B}(\mathcal{H}) \quad \text{---}$$

$$3. (\mathcal{E} \otimes 1)[|\phi\rangle\langle\phi|]$$

$$\rightarrow \mathcal{E} \in \mathcal{HS} \quad \text{---}$$

$$4. \hat{\mathcal{E}} \text{ Superoperador } \leftarrow$$

$$\rho \mapsto |p\rangle\rangle \in \mathcal{B}(\mathcal{H})$$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

$$|p\rangle\rangle = \begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix}$$

En el caso del \mathcal{E} de la moneda: $\dim(\hat{\mathcal{E}}) = 4$

$$\rightarrow K_j = {}_E \langle j | U | \psi_{0E} \rangle \quad \rho_0^{(E)} = |\psi_{0E}\rangle\langle\psi_{0E}|$$

$$\text{Tr}_E(U \rho \otimes |\psi_{0E}\rangle\langle\psi_{0E}| U^\dagger) = \sum_j K_j \rho K_j^\dagger$$

$$(2t+1) \quad \text{---} \quad U \quad \text{---} \quad (4)$$

↙
→ Teorema de dilatación de Stinespring

$$\hat{\mathcal{E}} \quad \text{Tr}(U \rho U^\dagger) = \sum_k K_k \rho K_k^\dagger$$

$$\dim_{\max}(\text{entorno}) = d^2 \quad d=2 \quad d^2=4$$

→ Superoperador $\hat{\mathcal{E}}$.