The simplest discrete-time quantum walk producing Parrondo's paradox

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1. Introduction

Parrondo's Paradox, a counterintuitive phenomenon first observed in classical game theory, demonstrates that two individually losing strategies can combine in a specific sequence to yield an overall winning outcome. This intriguing effect, initially conceptualized as a pedagogical model for the Brownian ratchet, arises from the nonlinear interaction between the strategies. Its implications extend beyond games, offering insights into systems ranging from economics to biology.

Quantum mechanics offers a natural setting to revisit classical paradoxes, often revealing new behavior stemming from principles such as superposition and interference. Quantum walks, the quantum analogs of classical random walks, have gained more and more attention for this exploration. Unlike their classical counterparts, quantum walks exhibit ballistic spreading rather than diffusive, driven by quantum coherence. This fundamental difference suggests that Parrondo's Paradox, when translated to quantum walk systems, could manifest under distinct and perhaps simpler conditions than in its classical setting. A typical discrete-time quantum walk (DTQW) involves a quantum particle (the walker) on a lattice, equipped with an internal degree of freedom known as the coin state. At each step, a unitary operator acts on this internal state emulating *tossing a quantum coin*, creating a superposition, followed by a conditional shift operator that moves the walker based on its coin state.

Previous research has explored various avenues for realizing Parrondo's Paradox in quantum walks. Early theoretical studies demonstrated the paradox using alternating or combining different quantum coin operators, leveraging interference effects from phase factors to reverse bias [1, 2, 3]. The paradox has been shown to emerge from history-dependent coin operations [4] and has even been observed with higher-dimensional quantum coins, such as three-state (qutrit) or four-sided coins, sometimes requiring a classical ratcheting effect [5, 6, 7]. More recently, explorations have expanded to include time-dependent and spatially inhomogeneous coin operators [8, 9, 10], different shift operators [11], and even the unexpected role of noise [12] or spontaneous emission [13] in inducing or enhancing the paradox. The effect has also been theoretically extended to continuous-time quantum walks with defect modulation [14] and explored in the context of quantum search algorithms [15]. Furthermore, the interplay between Parrondo's Paradox and quantum entanglement has been a significant focus, with studies showing how Parrondo sequences can generate highly entangled states [16, 17] and, conversely, how entanglement dynamics are affected by the paradox [18, 12, 10]. Experimental realization of the paradox in 1D quantum walks has also been successfully achieved using optical setups [19], validating theoretical predictions.

Despite these diverse demonstrations, a systematic and simplified approach to inducing Parrondo's Paradox in discrete-time quantum walks using only the combination of two homogeneous coin rotations from SU(2), without recourse to position-dependent coins, complex higher-dimensional coins, or external noise, has remained elusive. Such a minimal setup would provide clearer insight into the fundamental quantum mechanisms underlying the paradox.

In this paper, we present the first systematic demonstration of Parrondo's Paradox in the simplest discrete-time quantum walk model. We show how to combine two individually losing games, each defined by a distinct homogeneous coin operation that is a rotation within SU(2), to produce an overall winning outcome. Our methodology meticulously outlines the conditions under which these two simple, unbiased coin operations, when applied in a specific sequence, lead to a rectified average position of the quantum walker. These results represent a significant simplification of prior realizations and offer a foundational understanding of the paradox's emergence from the interplay of quantum interference and unitary transformations alone.

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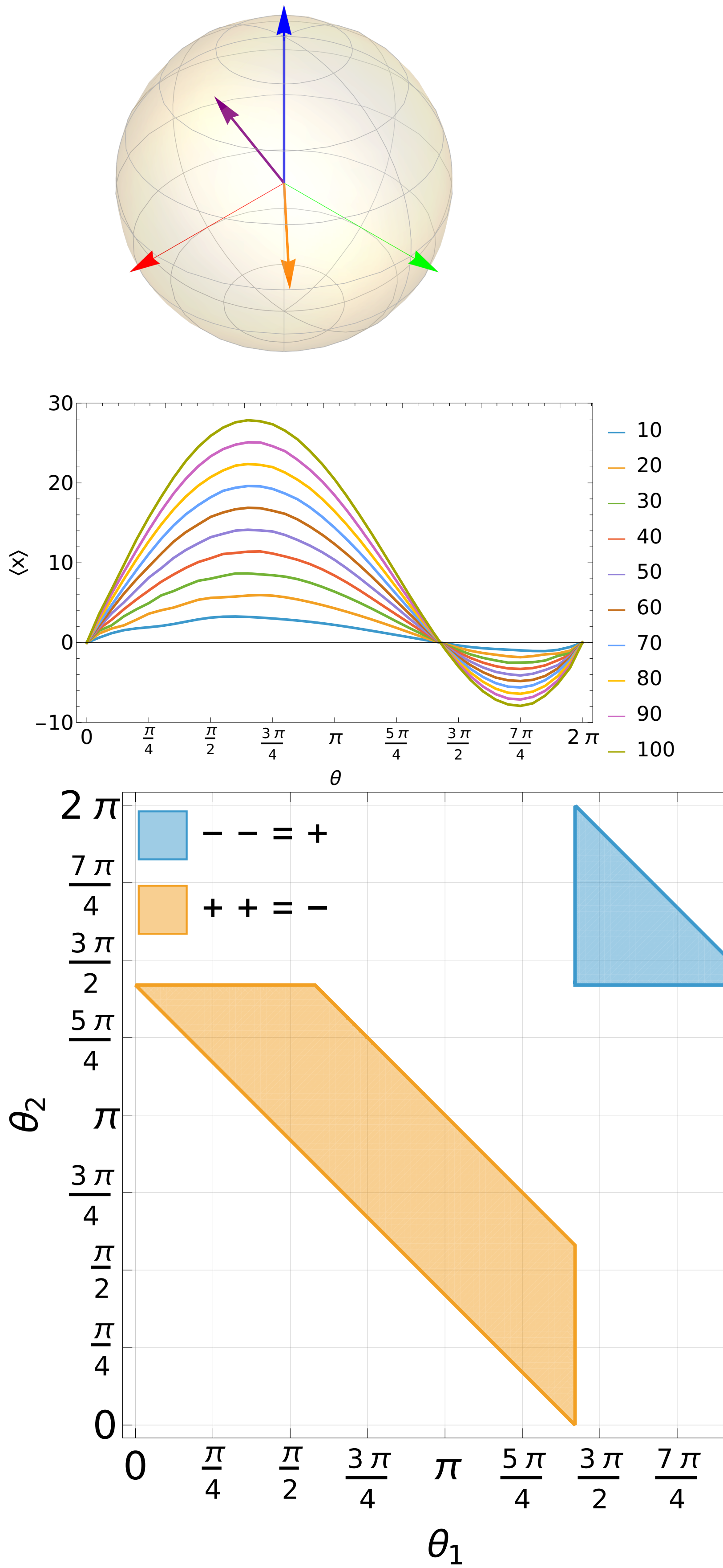

Figura 1: $|\psi_0(\theta,\phi)\rangle = |\psi_0(0.5\pi, 0.26\pi)\rangle$, eje de rotación: $(\alpha,\phi) = (0.18\pi, 0)$

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