$U = S(C\otimes \mathbb{1})$ $\qquad |\psi_0\rangle = |c_0\rangle\otimes|p_0\rangle$ $\qquad (AB)^\dagger = B^\dagger A^\dagger$

$$Tr_c\, U|\psi_0\rangle\langle\psi_0|U^\dagger = Tr_c\left(S C\otimes\mathbb{1}\; |c_0\rangle\langle c_0|\otimes|p_0\rangle\langle p_0|\; C^\dagger\otimes\mathbb{1}\; S^\dagger\right)$$

$$S = |0\rangle\langle 0|\otimes\sum_i |i+1\rangle\langle i| + |1\rangle\langle 1|\sum_i |i-1\rangle\langle i| \qquad S^\dagger = |0\rangle\langle 0|\otimes\sum_i|i\rangle\langle i+1| + |1\rangle\langle 1|\otimes\sum_i |i\rangle\langle i-1|$$

$$C = 1/\sqrt{2}\left(|0\rangle\langle0| + |0\rangle\langle1| + |1\rangle\langle0| - |1\rangle\langle1|\right)$$

$$C\otimes\mathbb{1} = C\otimes\sum_i|i\rangle\langle i|$$

$$S(C\otimes\mathbb{1}) = 1/\sqrt{2}\left\{(|0\rangle\langle0| + |0\rangle\langle1|)\otimes\sum_i|i+1\rangle\langle i| + (|1\rangle\langle0| - |1\rangle\langle1|)\otimes\sum_i|i-1\rangle\langle i|\right\}$$

$$[S(C\otimes\mathbb{1})]^\dagger = (C^\dagger\otimes\mathbb{1})S^\dagger = 1/\sqrt{2}\left\{(|0\rangle\langle0| + |1\rangle\langle0|)\otimes\sum_i|i\rangle\langle i+1| + (|0\rangle\langle1| - |1\rangle\langle1|)\otimes\sum_i|i\rangle\langle i-1|\right\}$$

$$Tr_c\, U|\psi_0\rangle\langle\psi_0|U^\dagger = \langle0|_c\, U|\psi_0\rangle\langle\psi_0|U^\dagger|0\rangle_c + \langle1|_c\, U|\psi_0\rangle\langle\psi_0|U^\dagger|1\rangle_c$$

$$\langle0|S(C\otimes\mathbb{1})|c_0\rangle = 1/\sqrt{2}\left(\langle0|c_0\rangle + \langle1|c_0\rangle\right)\sum_i|i+1\rangle\langle i| = K_0$$

$$\langle1|_c S(C\otimes\mathbb{1})|c_0\rangle = 1/\sqrt{2}\left(\langle0|c_0\rangle - \langle1|c_0\rangle\right)\sum_i|i-1\rangle\langle i| = K_1$$

$$\langle0|_c S(C\otimes\mathbb{1})|c_0\rangle|p_0\rangle\langle p_0|\langle c_0|(C^\dagger\otimes\mathbb{1})S^\dagger|0\rangle_c = 1/2\left(\langle0|c_0\rangle + \langle1|c_0\rangle\right)\sum_{i,j}|i+1\rangle\langle i|$$

$$K_0^\dagger K_0 = 1/2\left(\langle c_0|0\rangle + \langle c_0|1\rangle\right)\left(\langle0|c_0\rangle+\langle1|c_0\rangle\right)\sum_{i,j}|i\rangle\langle i+1|j+1\rangle\langle j|$$
$$= 1/2\left(|\langle0|c_0\rangle|^2 + |\langle1|c_0\rangle|^2 + \langle c_0|1\rangle\langle0|c_0\rangle + \langle c_0|0\rangle\langle1|c_0\rangle\right)\sum|i\rangle\langle i|$$

$$K_1^\dagger K_1 = 1/2\left(\langle c_0|0\rangle - \langle c_0|1\rangle\right)\left(\langle0|c_0\rangle - \langle1|c_0\rangle\right)\sum_{i,j}|i\rangle\langle i-1|j-1\rangle\langle j|$$
$$= 1/2\left(|\langle0|c_0\rangle|^2 + |\langle1|c_0\rangle|^2 - \langle c_0|1\rangle\langle0|c_0\rangle - \langle c_0|0\rangle\langle1|c_0\rangle\right)\sum_i|i\rangle\langle i|$$

$$K_0^\dagger K_0 + K_1^\dagger K_1 = \mathbb{1} \qquad K_i = \langle i|_c S(C\otimes\mathbb{1})|c_0\rangle$$

$$U|\psi_0\rangle = S(C\otimes\mathbb{1})\,1/\sqrt{2}\left(|0\rangle + i|1\rangle\right)\otimes|0\rangle = 1/2\, S\left(|0\rangle+|1\rangle + i|0\rangle - i|1\rangle\right)\otimes|0\rangle$$
$$= 1/2\left(|01\rangle + |1-1\rangle + i|01\rangle - i|1-1\rangle\right)$$
$$= 1/2\left\{(1+i)|01\rangle + (1-i)|1-1\rangle\right\} = 1/\sqrt{2}\left(e^{i\pi/4}|0,1\rangle + e^{-i\pi/4}|1,-1\rangle\right)$$
$$= e^{i\pi/4}/\sqrt{2}\left(|0,1\rangle + e^{-i\pi/2}|1,-1\rangle\right) = e^{i\pi/4}/\sqrt{2}\left(|0,1\rangle - i|1,-1\rangle\right)$$

$|\psi_1\rangle = e^{i\pi/4}/\sqrt{2}\,(|0,1\rangle - i|1,-1\rangle)$ $\quad |\psi_1\rangle\langle\psi_1| = 1/2\,(|0,1\rangle\langle 0,1| - i|1,-1\rangle\langle 0,1| + i|0,1\rangle\langle 1,-1| + |1,-1\rangle\langle 1,-1|)$

$\mathrm{Tr}_c|\psi_2\rangle\langle\psi_2| = \mathrm{Tr}_c\, U|\psi_1\rangle\langle\psi_1|U^\dagger$

$$\begin{aligned}|\psi_2\rangle &= S(C\otimes\mathbb{1})\,e^{i\pi/4}/\sqrt{2}\,(|0,1\rangle - i|1,-1\rangle)\\ &= e^{i\pi/4}/2\; S(|0,1\rangle + |1,1\rangle - i|0,-1\rangle + i|1,-1\rangle)\\ &= e^{i\pi/4}/2\,(|0,2\rangle + |1,0\rangle - i|0,0\rangle + i|1,-2\rangle)\end{aligned}$$

$\mathrm{Tr}_c(U|\psi_1\rangle\langle\psi_1|U^\dagger) = \Lambda(\rho_P^{(1)})$, $\quad \rho_P^{(1)} = \mathrm{Tr}_c|\psi_1\rangle\langle\psi_1| \quad$ ¿Λ?

$$\begin{tikzcd} \mathrm{Tr}_c|\psi_1\rangle\langle\psi_1| \arrow[r, "\Lambda"] & \mathrm{Tr}_c|\psi_2\rangle\langle\psi_2| \\ |\psi_1\rangle\langle\psi_1| \arrow[u] \arrow[r, mapsto, "U"'] & |\psi_2\rangle\langle\psi_2| \arrow[u] \end{tikzcd}$$

$|\psi_1\rangle = a|0,-1\rangle + b|0,0\rangle + c|0,1\rangle + d|1,-1\rangle + e|1,0\rangle + f|1,1\rangle$

$$\begin{aligned}U|\psi_1\rangle = 1/\sqrt{2}\,\{&a(|0,0\rangle + |1,-2\rangle) + b(|0,1\rangle + |1,-1\rangle) + c(|0,2\rangle + |1,0\rangle)\\ &+ d(|0,0\rangle - |1,-2\rangle) + e(|0,1\rangle - |1,-1\rangle) + f(|0,2\rangle - |1,0\rangle)) = |\psi_2\rangle\end{aligned}$$

$\mathrm{Tr}_c|\psi_2\rangle\langle\psi_2| =$

$|\psi_t\rangle = U^t|\psi_0\rangle$

$$\rho_0 \xmapsto{\mathcal{E}} \rho_1 \xmapsto{\mathcal{E}} \rho_2$$

$|\psi_0\rangle = 1/\sqrt{2}\,(|0\rangle + i|1\rangle)\otimes|0\rangle$

$$\begin{aligned}K_0 &= 1/\sqrt{2}\,(1/\sqrt{2} + i/\sqrt{2})\textstyle\sum_i |i+1\rangle\langle i|\\ &= 1/2\,(1+i)\textstyle\sum_i |i+1\rangle\langle i|\\ K_1 &= 1/2\,(1-i)\textstyle\sum_i |i-1\rangle\langle i|\end{aligned}$$

$|\psi_1\rangle = U|\psi_0\rangle = 1/2\,(|0,1\rangle + |1,-1\rangle + i|0,1\rangle - i|1,-1\rangle)$

$\mathrm{Tr}_c|\psi_1\rangle\langle\psi_1| = 1/2\,(|-1\rangle\langle -1| + |1\rangle\langle 1|)$

$K_0|0\rangle = 1/2(1+i)|1\rangle$ $\qquad K_1 = 1/2(1-i)|-1\rangle$

$\langle 0|K_0^{\dagger} = 1/2\,(1-i)\,\langle 1|$ $\qquad K_1 = 1/2\,(1+i)\,\langle -1|$

$$\mathcal{E}(|0\rangle\langle 0|) = 1/2\,(|-1\rangle\langle -1| + |1\rangle\langle 1|)$$

$$U|\psi_1\rangle = 1/2\,U\{(1+i)|0,1\rangle + (1-i)|1,-1\rangle\}$$
$$= 1/2\sqrt{2}\,\{(1+i)(|0,2\rangle + |1,0\rangle) + (1-i)(|0,0\rangle - |1,-2\rangle)\}$$
$$= 1/2\sqrt{2}$$

$K_0|-1\rangle = 1/2(1+i)|0\rangle$ $\qquad K_1|-1\rangle = 1/2(1-i)|-2\rangle$

$K_0|1\rangle = 1/2\,(1+i)\,|2\rangle$ $\qquad K_1|1\rangle = 1/2\,(1-i)\,|0\rangle$

$$1/2\,\{K_0|1\rangle\langle 1|K_0 + K_0|-1\rangle\langle -1|K_0 + K_1|1\rangle\langle 1|K_1^{\dagger} + K_1|-1\rangle\langle -1|K_1^{\dagger}\}$$
$$= 1/2\,(|2\rangle\langle 2| + |0\rangle\langle 0| + |0\rangle\langle 0| + |-2\rangle\langle -2|\}$$