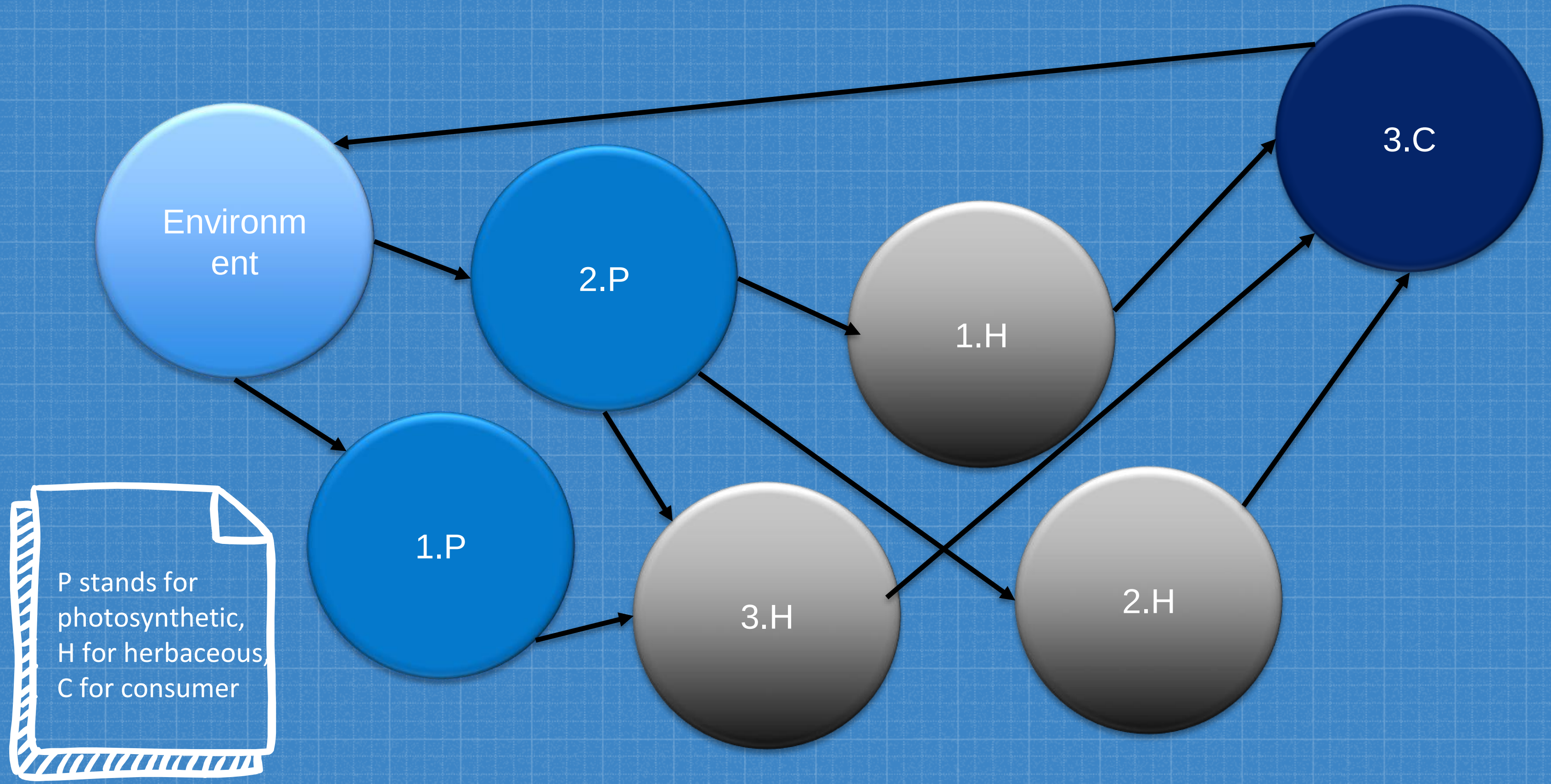
Who will survive?

(analysis of ecological models)

WHAT IS ECOLOGY?

That would be our first question

‘Ecology is the study of the relationships between living organisms, including humans, and their physical environment; it seeks to understand the vital connections between plants and animals and the world around them. Ecology also provides information about the benefits of ecosystems and how we can use Earth’s resources in ways that leave the environment healthy for future generations’



Graph representation of an Ecosystem

Adjacency Matrix Representation

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

i. Sum of rows:

Number of food sources

E.g. third specie only eats one specie

ii. Sum of columns:

Number of hunters

E.g. three species hunt the second one

iii. Dominant eigenvalue

the structural diversity of the ecosystem

E.g. with two ecosystem with same number of species the one with higher eigenvalue is connected better and number of paths with length of k will grow faster.

Adjacency Matrix Representation

$$A^3 = \begin{bmatrix} 1 & 3 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

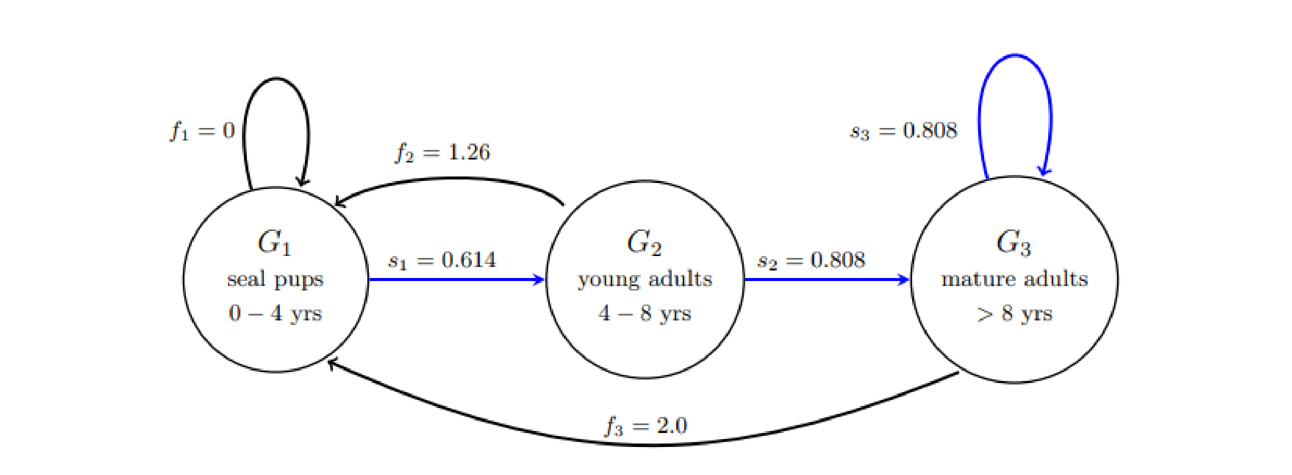
iv. Powers of the matrix:
Number of paths with length of k
E.g. There are 4 paths with length of 3 to go from the environment to species 7.

v. Dominant eigenvector
How much feeding opportunities each specie receives relatively.

i.e. $\lim_{k\to\infty} A^k u$ will show us the number of paths going to each specie of any length. And by power method we know that A^ku becomes the dominant eigenvector. Which would tell us, the flow of each node. E.g., in our case (.592 , .338,.338,.194,.194,.387,.443)t shows that the species 6 has twice the feeding opportunities that species 4 and 5 do.

Leslie Matrix

- G_i represents the group.
- f_i represents the fertility rate
- s_i represents the survival rate



Leslie Martix

$$L = \begin{bmatrix} f_1 & f_2 & \cdots & f_{n-1} & f_n \\ s_1 & 0 & \cdots & 0 & 0 \\ 0 & s_2 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & s_{n-1} & 0 \end{bmatrix}$$

We can assume $L_{nn} = S_n$ to be nonzero and it that case it is called a generalized Leslie matrix.

$$p_1(t+1) = \sum f_i p_i(t)$$

$$p_2(t+1) = s_1 p_1(t)$$

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$$P(t+1) = LP(t)$$

Leslie Matrix Properties

$$\begin{bmatrix} p_1^{(t+1)} \\ p_2^{(t+1)} \\ p_3^{(t+1)} \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & f_3 \\ s_1 & 0 & 0 \\ 0 & s_2 & s_3 \end{bmatrix} \begin{bmatrix} p_1^t \\ p_2^t \\ p_3^t \end{bmatrix}$$

- Dominant eigenvalue represent:
  - λ < 1 means the population will decline exponentially.
  - λ > 1 means the population will grow exponentially.
  - λ = 1 means the population is stable, it does not change.
- For our sheepy situation λ = 1.75

- A theorem states that, there exists a real and positive eigenvalue, which is greater than any other eigenvalues in modulus.
- As the steps become greater $(k\ becomes\ larger)\frac{p(k+1)}{p(k)} = \lambda$
- In our case eigenvalue is 1.49. which shows population is growing rapidly.
- The corresponding eigenvector provides the stable age distribution. When a population reaches a stable age distribution, the population continues to grow in the rate of the dominant eigenvalue and remains in the same stable age distribution structure

Harvest

We need to make money!

How to harvest and have a growing population?

The answer is again a matrix.

Harvest

$$\mathbf{H} = \begin{bmatrix} H_1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & H_2 & \ddots & & & \vdots \\ \vdots & \ddots & H_3 & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & H_{A-1} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & H_A \end{bmatrix},$$

- Harvesting and natural death can be considered as independent events. So, the chance of surviving both would be $h_i s_i$.
- Now our equation changes to $P(t+1) = HLP(t)$. Which is the population that lives a whole cycle and survives harvesting. For the population to be sustainably harvested (λ = 1), M = HL must satisfy Mu = u.

Harvesting

Same amount from every group

$$h_1 = h_2 = \cdots = h_n = 1 - {}^{1}\!/_{\lambda}$$

Only from one group

Lambs are better!

$$(1 - h)(f_1 + f_2 s_1 + f_3 s_1 s_2 + \cdots) = 1$$

$R = (f_1 + f_2 s_1 + f_3 s_1 s_2 + \cdots)$ is called **net reproduction rate.**

Harvest as much as you can

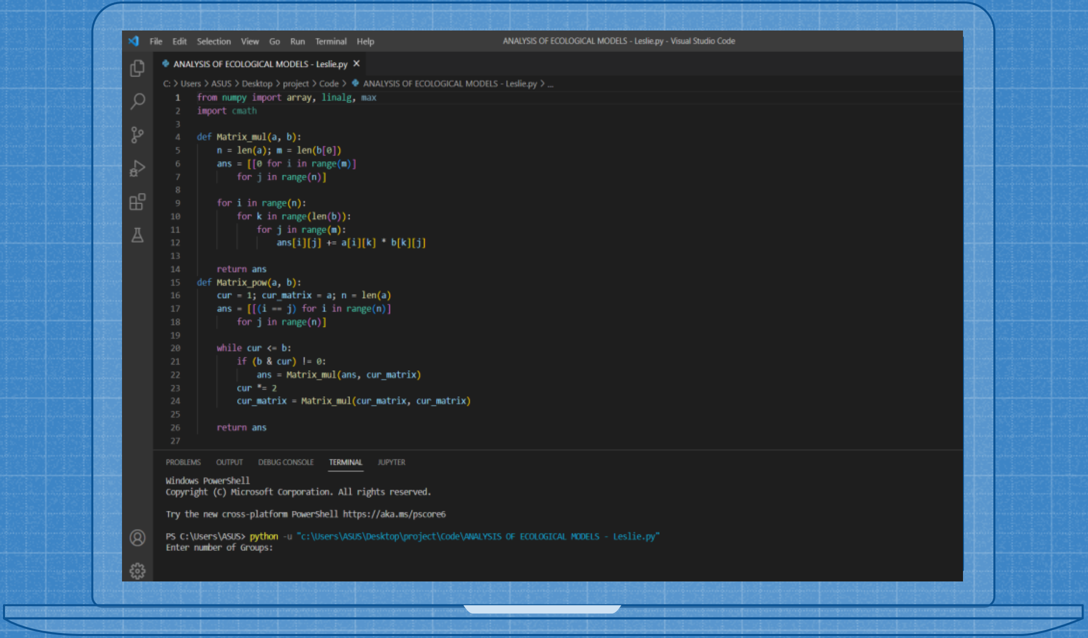
There's a bound to it.

If a sustainable harvesting policy is optimal, it harvests only from one or two age classes. If two age classes are harvested, then the older class is completely harvested

The codes!

All we said, is constructed in a python code!

Food webs, Leslie matrix and different harvestings.



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Thanks!