THERMO FINAL

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FIRST LAW

For a control mass: $\underline{\Delta U} = \underline{Q - W}$ — rev. and irrev.

$u(T)$ ← not state

$du = Tds - pdv$

$du = \delta Q - \delta W$

reversible: $\delta w = pdV$, $\delta Q = TdS$

$du = Tds - \delta w$

$du = \delta Q - pdv$

Steady State, control vol:

$\dot{m}_{out} h_{tout} - \dot{m}_{in} h_{tin} = \dot{Q} - \dot{W}_{shaft}$

$h_{tout} - h_{tin} = \dot{q} - w_{shaft}$

SECOND LAW

$dS = \frac{dQ_{rev}}{T}$ $\quad \Delta S = \int_A^B \frac{dQ_{rev}}{T}$ (system + surrounding)

for real processes, $\Delta S > 0$

ideal/rev proc., $\Delta S = 0$

$du = TdS - pdv$

$dh = Tds + vdp = Tds + \frac{dp}{\rho}$ — GIBBS

Lost Work Example

real $\quad u_2 - u_1 = q - w$ $\quad$ (constant T, T_0)

ideal $\quad u_2 - u_1 = q_{rev} - w_{rev}$

$0 = q - q_{rev} - (w - w_{rev})$

$w_{lost} \rightarrow (w_{rev} - w) = q - q_{rev}$

$\Delta S_{total} = \Delta S_{syst} + \Delta S_{surr} = 0 + \frac{q_{rev}}{T_0} - \frac{q}{T_0} = \frac{w_{lost}}{T_0}$

$w_{lost} = \Delta S_{total} \cdot T_0$

TWO PHASE MEDIA

a b c P=P(T)

$X = \frac{m_g}{m_f + m_g} = \frac{ab}{ac}, \bar{v} = \frac{m_f v_f + m_g v_g}{m_f + m_g}$

Transition Characteristics

$dW = PdV = P(v_g - v_f)\, dm_{fg}$

$U = u_f m_f + u_g m_g$

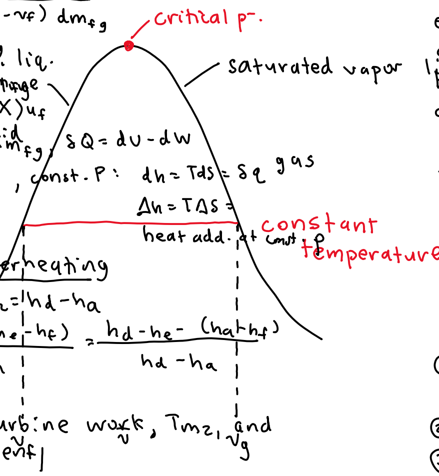
$dU = (u_g - u_f)\, dm_{fg}$

$u = X u_g + (1-X) u_f$

$dQ = (h_g - h_f)\, dm_{fg}$, $\delta Q = du - dw$

$dh = Tds + \frac{1}{\rho} dP$, const. P: $dh = Tds = \delta q$

$\Delta h = T\Delta S$ = heat add. at const P

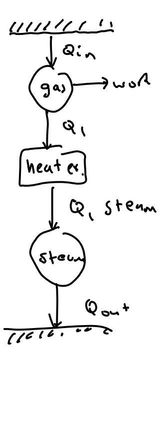


Rankine w/ Superheating

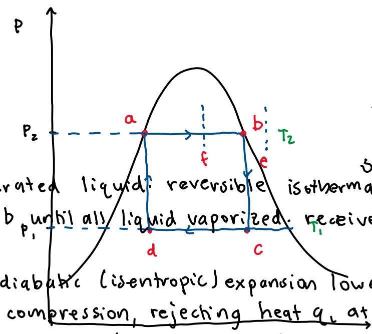
isobaric @ $q_2 = h_d - h_a$

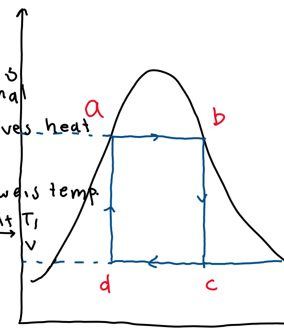
$\eta_{th} = \frac{h_d - h_a - (h_e - h_f)}{h_d - h_a} = \frac{h_d - h_e - (h_a - h_f)}{h_d - h_a}$

- increased turbine work, T_{m2}, and cycle efficient



TWO PHASE CARNOT





1. a with saturated liquid: reversible isothermal expansion to b until all liquid vaporized. receives heat q_H.
2. reversible, adiabatic (isentropic) expansion lowers temp
3. Isothermal compression, rejecting heat q_L at T_1
4. Isentropic compression (vapor → liquid)

$\eta = \frac{W_{net}}{q_H} = 1 - \frac{T_1}{T_2}$

$TdS = dh - \frac{dp}{\rho}$ → isobar $dh = Tds$ $\quad \left(\frac{\partial h}{\partial s}\right)_p = T =$ constant

$q_H = h_B - h_A$ $\quad \eta = \frac{q_H + q_L}{q_H} = \frac{(h_B - h_A) + (h_D - h_C)}{h_B - h_A}$

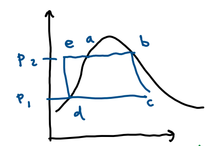
$q_L = h_d - h_c$

Clausius - Clapeyron

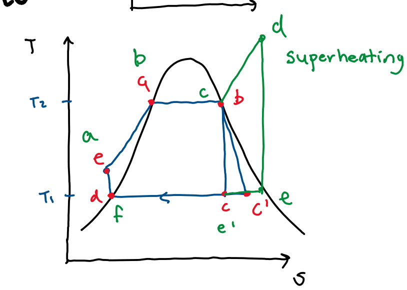
- gives slope of vapor pressure curve

$\ln\left(\frac{P_2}{P_1}\right) = \frac{h_{fg}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\frac{dP}{dT} = \frac{h_{fg}}{T(v_g - v_f)}$



RANKINE POWER CYCLES



d-e: cold liquid at temp T_1 pressurized reversibly to high pressure by pump

e→a: reversible constant pressure heating to boiler at temp T_2

a→b: heat added at const temp. T_2 with transition to vapor

b→c: isentropic expansion through a turbine

c→d: liquid-vapor mixture condensed at temp T_1 by extracting heat

T_m = mean effective temp

$q_H = T_{m2} \Delta S_2$ $\quad s_b - s_e = s_c - s_d$

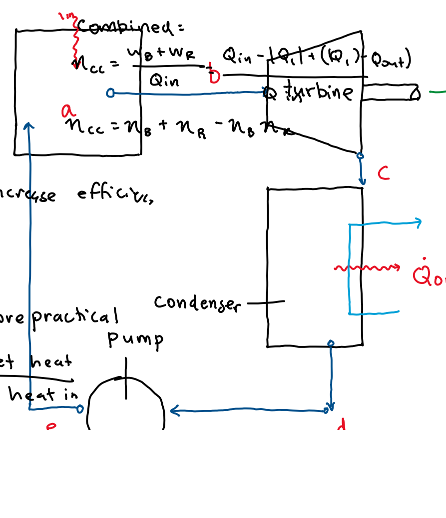
$q_L = T_{m1} \Delta S_1$

$\eta_{th} = \frac{T_{m2}(s_b - s_e) - T_{m1}(s_c - s_d)}{T_{m2}(s_b - s_e)}$

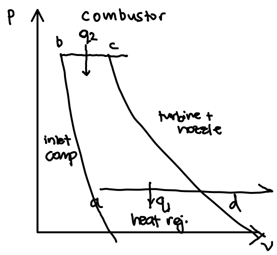
$T_{m1} \simeq T_1$, $T_{m2} < T_2$ increase T_{m2} → increase efficiency

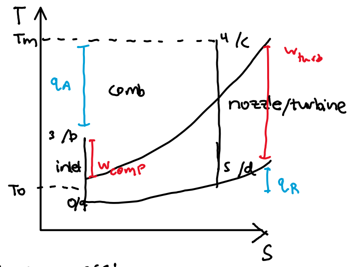
1. T-s and h-s not similar, $\left(\frac{\partial h}{\partial s}\right)_p = T$
2. irreversibilities to c'
3. less efficient than Carnot but more practical

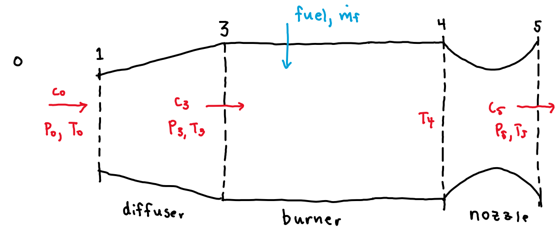
$\eta_{th} = \frac{\text{turbine work} - \text{pump work}}{\text{heat in}} = \frac{\text{net heat}}{\text{heat in}}$



CYCLES (1 PHASE)







isentropic diffusion, $M_3 \approx 0$

constant pressure combustion

isentropic expansion

$$\eta_{Brayton} = 1 - \frac{T_0}{T_{comp,\,exit}} = 1 - \frac{T_0}{T_3} = 1 - \frac{T_0}{T_{t0}}$$

$$\frac{T_0}{T_{t0}} = \frac{1}{\left(1 + \frac{\gamma-1}{2} M_0^2\right)}$$

$$\eta_{Brayton} = 1 - \frac{\frac{\gamma-1}{2} M_0^2}{1 + \frac{\gamma-1}{2} M_0^2}$$

a-b: adiabatic compression

b-c: isobaric combustion /expansion

c-d: adiabatic expansion

d-a: isobaric compression

ideal gas: $dh = c_p dT$, $dh = Tds + dP/\rho$

$dU = c_v dT$

$$\eta = \frac{\text{work}}{\text{heat}} = \frac{T_H \Delta S - T_0 \Delta S}{T_H \Delta S}$$

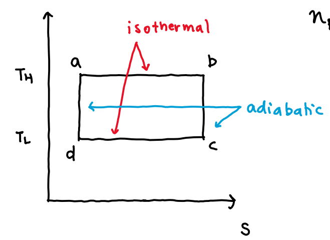
$$\eta_i = 1 - \frac{T_c}{T_h}$$

$$\left(\frac{P_2}{P_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_0} \rightarrow \eta_B = 1 - \frac{T_0}{T_2}$$ max. efficiency → high TR

$w_{comp} = -\Delta h_{03} = \Delta h_{comp}$

$w_{turb} = -\Delta h_{turb}$

CARNOT CYCLE



$$= 1 - \frac{T_L}{T_H}$$

a→b: contacts heat reservoir, isothermal expansion w/ Q_2 absorbed

b→c: adiabatic expansion to T_L

c→d: contact w/ heat reservoir, isothermal compression rejecting Q_1

d→a: adiabatic compression to T_H

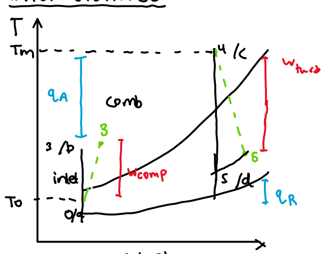
$$Q_H = \int_a^b T\,dS = T_H (S_b - S_a) = T_H \Delta S$$

$dU = dQ - dW$ dQ

$= TdS - dW$ (only good for rev.)

DEPARTURES FROM IDEAL

Inefficiencies



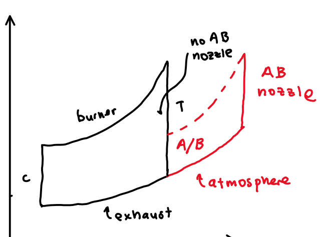
$$\eta_{turb} = \frac{w_{actual}}{w_{ideal}} = \frac{h_4 - h_5}{h_4 - h_{5s}} , \quad \eta_c = \frac{w_{ideal}}{w_{actual}} \quad \text{(given PR)}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 + \frac{Q_L}{Q_H} = 1 - \frac{T_5 - T_0}{T_4 - T_3}$$

$$\eta_{th} = \frac{\left[1 - \frac{1}{\tau_s}\right]\left[\eta_c \eta_t \frac{T_4}{T_0} - \tau_s\right]}{1 + \eta_c \left[\frac{T_4}{T_0} - 1\right] - \tau_s} , \quad \tau_s = PR^{\frac{\gamma-1}{\gamma}}$$

$$\frac{W_{net}}{\dot{m} c_p T_0} = (\tau_s - 1)\left[\frac{\eta_t \frac{T_4}{T_0}}{\tau_s} - \frac{1}{\eta_c}\right]$$

Afterburner



$$\text{Work} = \oint T\,dS = q_{in} - q_{out}$$

$$\eta_B = \text{work}/q_{in}$$

$$\text{max work: } \frac{T_{comp,\,exit}}{T_{inlet}} = \sqrt{\frac{T_{turbine\ entry}}{T_{inlet}}}$$

$\eta_{overall} = \eta_{th}\, \eta_{prop}$

thrust $= F c_0$, $\eta_{prop} = \dfrac{\text{thrust power}}{\text{mechanical power}}$

$$= \frac{\dot{m}(c_e - c_0)c_0}{\dot{m}(c_e^2 - c_0^2)} = \frac{2}{1 + c_e/\ldots}$$

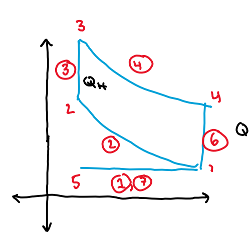
Otto Cycle

1. Intake (5→1)
2. compression (1→2)
3. Combustion (const. volume) (2→3)
4. Power stroke: expansion (3→4)
5. Valve exhaust
6. Model: rejection of heat to res. (4→1)
7. Exhaust (1-5)

$$\eta = 1 + \frac{Q_L}{Q_H}$$

$$\eta_{otto} = 1 - \frac{1}{(v_1/v_2)^{\gamma-1}} = 1 - \frac{1}{r^{\gamma}-1}$$

$$\text{Power/enthalpy flux} = \frac{\dot{w}}{\dot{m} c_p T_1}$$



THERMOCHEMISTRY

FUEL/AIR RATIO

Theoretical air: $3.72 \times$ moles O_2 -

$CH_4 + 2O_2 + 2(3.76)N_2 \longrightarrow CO_2 + $ ~~2H₂O~~ $7.52N_2$

Reaction for aeroengine fuel at stoichiometric:

$C_8H_{18} + 12.5O_2 + 12.5(3.76)N_2 \longrightarrow 8CO_2 + 9H_2O + 47.0\ N_2$

Molar ratio of fuel/air =

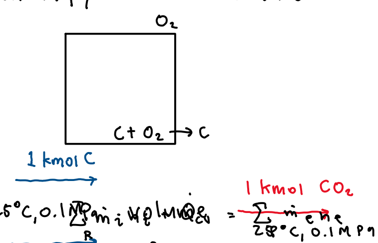
$\frac{1}{12.5+47.0} = 0.0167$, but must multiply by molar mass

$\frac{1\text{mol} \cdot 114\ g/\text{mol (fuel)}}{12.5 \times 32\ g/\text{mol} + 12.5 \times 3.76 \times 28\ g/\text{mol}} = 0.0664$

ENTHALPY OF FORMATION

Reference state temp: 25°C (298 K), 0.1 MPa

enthalpy = 0 for elements



$\sum_R \dot{m}_i h_i + \dot{Q}_{cv} = \sum \dot{m}_e h_e$

$\dot{n}_i$ = molar mass flow rate, M_i = molecular weight,

$\dot{n}_i = \dot{m}_i / M_i$ = mass flow kmol/sec $\quad \bar{h}_i$ = mol. enthalpy

$\bar{h}_i = M_i h_i$ = enthalpy / kmol

$Q_{cv} = -393{,}522$ kJ (out of C.V.)

$\sum_R \dot{n}_i \bar{h}_i + \dot{Q}_{cv} = \sum_P \dot{n}_e \bar{h}_e$

or

$$Q_{cv} + \sum_R n_i(\underbrace{\bar{h}^\circ_f + \Delta\bar{h}}_{T_i, P_i \text{ and ref}}) = \sum_P n_e(\underbrace{\bar{h}^\circ_f + \Delta\bar{h}}_{T_e, P_e \text{ and ref}})$$

Example

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O(l)$

$\sum_R n_i \bar{h}_i = (\bar{h}^\circ_f)_{CH_4} + 0$ (from O_2) $\longrightarrow -74{,}873$ kJ

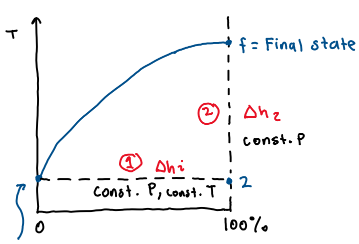
$\sum_P n_e \bar{h}_e = (\bar{h}^\circ_f)_{CO_2} + 2(\bar{h}^\circ_f)_{H_2O(l)} \longrightarrow -965{,}198$ kJ

$Q_{cv} = -965{,}198 - (-74{,}873)$

ADIABATIC FLAME TEMP

- adiabatic, no shaft work reaction: temperature of prod. is the adiabatic flame temperature

* stoichiometric = max temp



1. extract heat q_1

$h_2 - h_i = -q_1 = (h^\circ_f)_{\text{unit mass}}$

2. input heat q_1

$h_f - h_2 = q_1$

$c_{p,avg}(T_f - T_2) = q_1$

$\Delta h_1 + \Delta h_2 = \Delta h_{adiabatic} = 0$

$(T_f - T_2) = \frac{q_1}{c_{p,avg}} = \frac{|(h^\circ_f)_{\text{unit mass}}|}{c_{p,avg}}$

Example

$C_8H_{18}(l) + 12.5O_2 + 12.5(3.76N_2) + 3(12.5O_2 + 12.5(3.76)N_2 \longrightarrow$

$8CO_2 + 9H_2O_{(g)} + 37.5O_2 + 188N$

$\sum_R n_i(\bar{h}^\circ_f + \Delta h)_i = \sum_P n_e(\bar{h}^\circ_f + \Delta h)_e$

adiabatic flame temp.

1. Bring reactants to 25°C (Δh_i) using heat transfer.
2. Reaction at 25°C $(\bar{h}^\circ_f)_{R \to P}$. Some heat transfer q_B.
3. $q_A + q_B$ = adiabatic flame temp.

$(\bar{h}^\circ_f)_{C_8H_{18}(l)} = 8(\bar{h}_f)_{CO_2} + 9(\bar{h}_f)_{H_2O} + \{\Delta\bar{h}_{CO_2} + 9\Delta\bar{h}_{H_2O} + 37.5\Delta\bar{h}_{O_2} + 188\Delta\bar{h}_{N_2}\}$

$= 8(\bar{h}_f)_{CO_2} + 9(\bar{h}^\circ_f)_{H_2O} - (\bar{h}^\circ_f)_{C_8H_{18}(l)}$

HEAT TRANSFER

Problem Solving

1. Define energy balance.

heat in - heat out $= \frac{dU}{dT}$

$(A\dot{q})_{in} - (A\dot{q})_{out} = \frac{dU}{dT}$

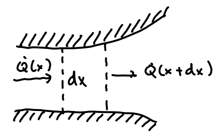
2. Apply heat conduction or convection description.

$q = -k\Delta T$ or $q = -k\frac{dT}{dx}$, $\quad \dot{Q} = -k\left(A\frac{dT}{dx}\right)_x$

$\dot{q} = h(T_w - T_\infty)$ (convection)

3. Determine differential equations.
4. Define boundary conditions
5. Solve equations using condition

Steady-State 1D Conduction

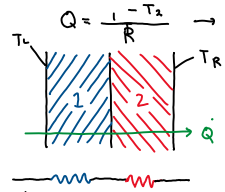


$$\frac{d^2T}{dx^2} + \left(\frac{1}{A}\frac{dA}{dx}\right)\frac{dT}{dx} = 0$$

Thermal Resistance Circuits

$T_1 - T_2$ = voltage diff, $\dot{Q}$ = current

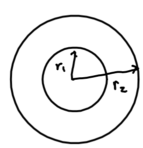
$Q = \frac{T_1 - T_2}{R} \rightarrow$ a slab, $R = \frac{L}{kA}$



$R_t = R_1 + R_2$

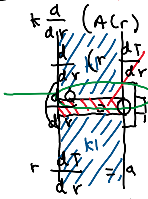
$\dot{Q} = \frac{T_L - T_R}{R} = \frac{T_L - T_R}{R_1 + R_2}$

Cylindrical Shell



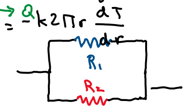
$k\frac{d}{dr}\left(A(r)\frac{dT}{dr}\right) = 0$

$\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$



$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$

$Q = -k2\pi r\frac{dT}{dr}$



$dT = a\frac{dr}{r}$

$T = a\ln\left(\frac{r}{r_1}\right) + b \rightarrow T_2 = a\ln\left(\frac{r_2}{r_1}\right) + 1$

$\frac{1 - T_1}{T_2 - T} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}$, $\quad \dot{Q} = 2\pi\frac{k(T_1 - T_2)}{\ln(r_2/r_1)}$

$R = \frac{\ln(r_2/r_1)}{2\pi k}$ $\quad \frac{r_2 - r_1}{r_1} \ll 1$: plane slab thing

$T = T_1 + (T_2 - T_1)\frac{x}{L} \quad (r_2 - r_1) = L,\ r - r_1 = x$

Spherical Shell

$A = 4\pi r^2$ $\quad \frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$

$T = -\frac{a}{r} + b$

$T = \frac{a'}{(r/r_1)} + b$

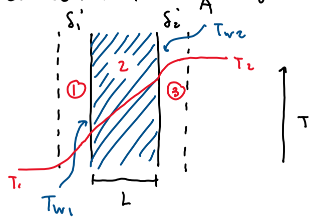
$T(r_2) = T_2 = \frac{a'}{(r_2/r_1)} + b$

$\frac{T_1 - T}{T_1 - T_2} = \frac{1 - (r_1/r)}{1 - (r_1/r_2)}$

HEAT TRANSFER

CONVECTION + CONDUCTION

Convective HT: $\frac{\dot{Q}}{A} = \dot{q} = h(T_w - T_\infty)$



$$\frac{\dot{Q}}{A} = h_1 (T_{w1} - T_1)$$

$$\frac{\dot{Q}}{A} = h_2 (T_2 - T_{w2})$$

$$\frac{\dot{Q}}{A} = \frac{k}{L} (T_{w2} - T_{w1})$$

$$T_2 - T_1 = (T_2 - T_{w2}) + (T_{w2} - T_{w1}) + (T_{w1} - T_1) = \frac{\dot{Q}}{A}\left[\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}\right]$$

$$R = \frac{1}{h_1 A} + \frac{L}{Ak} + \frac{1}{h_2 A}$$

DIMENSIONLESS NUMBERS

Biot number = $\frac{h r_2}{k}$, $\frac{hL}{k}$: $Bi \gg 1$, little resistance in convection

$Bi \ll 1$, conduction has little res.

$Bi \gg 1$: $\frac{T - T_1}{T_\infty - T} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}$

$Bi \ll 1$: $\frac{T - T_1}{T_\infty - T_1} = Bi \ln\left(\frac{r}{r_1}\right)$

$$\frac{T - T_1}{T_\infty - T_1} = \frac{\ln(r/r_1)}{\frac{1}{Bi} + \ln\left(\frac{r_2}{r_1}\right)}$$