Unit 1 Review

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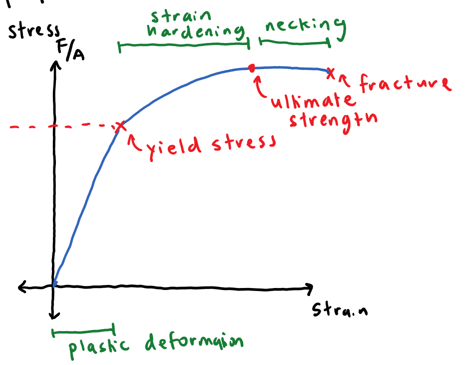
Covered on this exam:

1. Intro to materials ✓
2. Breguet Range Equation ✓
3. Structural Equilibrium ✓
   a. Equipollent Force Systems ✓
   b. Stability ✓
   c. Static Determinant/Indeterminant ✓
4. Trusses ✓
   a. Method of Joints ✓
   b. Method of Sections ✓
   c. More Strategies (R03) ✓
5. Statically Indeterminate Systems ✓
   a. Constitutive Responses under Load/Temperature ✓
6. Stress
   a. Stress Tensors
   b. Stress Transformations

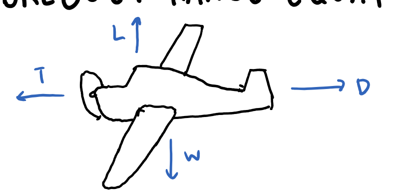
INTRO TO MATERIALS

The most important takeaway here is that different materials have different properties.



another way to see these properties are things called Ashby charts

BREGUET RANGE EQUATION



<u>Equilibrium</u>

$L = W$ (for cruise)

$T = D$

$W = \left(\frac{L}{D}\right) T$ aerodynamic coefficient of the aircraft

<u>Conservation of Energy</u>

Power $= T \cdot \Delta x / \Delta t$

$= T \cdot v$ power provided by fuel

$\eta \cdot \dot{m}_f \cdot h_f = T \, \Delta x / \Delta t$

total efficiency

$h_f =$ fuel energy / unit mass $= J/kg$

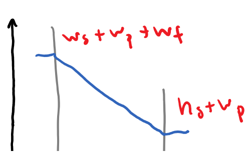
$\dot{m}_f =$ mass of fuel / second $= kg/s$

$\dot{m}_f \cdot h_f =$ propulsive power

<u>Conservation of Mass</u>

$\frac{dW}{dt} = \frac{dW_f}{dt}$ → total change in mass provided by a change of mass of fuel

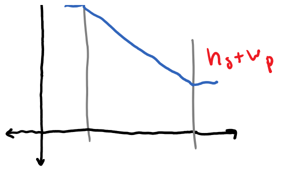
$= -\dot{m}_f g$



$\frac{dW}{dt} = \frac{dW}{dR} \cdot \frac{dR}{dt} = -\dot{m}_f g$

$\frac{dW_R}{dt} \cdot v = -\dot{m}_f g$

$\frac{\dot{m}_f}{v} = \frac{T}{\eta h_f}$



$$\frac{\quad}{dt} \cdot \frac{\quad}{dR}\,\frac{\quad}{dt} = -\dot{m}_f g$$

$$\frac{dW_R}{dt} \cdot v = -\dot{m}_f g \qquad \frac{\dot{m}_f}{v} = \frac{T}{\eta h_f}$$

$$\frac{dW_R}{dt} = \frac{-\dot{m}_f g}{v}$$

$$= -\frac{Tg}{\eta h_f}$$

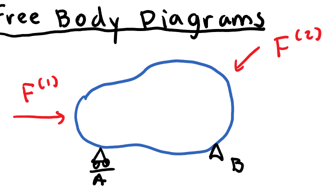
$$\frac{dW_R}{dt} = -\frac{Wg}{\eta h_f \left(\frac{L}{D}\right)} \qquad \text{diff eq}$$

$$R(w) = \eta_0 \frac{h_f}{g}\left(\frac{L}{D}\right) \ln\left(\frac{W_{init}}{W}\right)$$

STRUCTURAL ANALYSIS

1. Equilibrium
2. compatibility - internal connectivity
3. constitutive Relations - stiffness, strength, etc.

Free Body Diagrams



1. FBD
2. sum of forces / moments

1. Draw FBD of complete structure
2. Apply equilibrium to determine external forces
3. Draw FBD of structures / members to determine internal forces

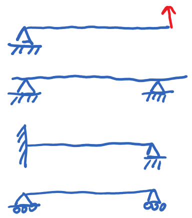
$$\underline{M} = \underline{r} \times \underline{F} = \begin{vmatrix} e_1 & e_2 & e_3 \\ r_1 & r_2 & r_3 \\ F_1 & F_2 & F_3 \end{vmatrix}$$

couple - two parallel forces of equal magnitudes bu

equipollent force systems - produce same moment and force but different deformational internal forces

roller	$u_2 = 0$	R_2
roller	$u_1 = 0$	R_1
pin	$u_1, u_2 = 0$	R_1, R_2
fixed	$u_1, u_2 = 0$, $\theta = 0$	R_1, R_2, M_3

statically indeterminate - too many constraints



unstable

stable, statically determinant

stable, statically indeterminate

unstable

TRUSSES

structure formed by combining linear structures in triangular pattern

Idealizations

- external loads only applied at joints
- no moments generated at ends of members
- straight members

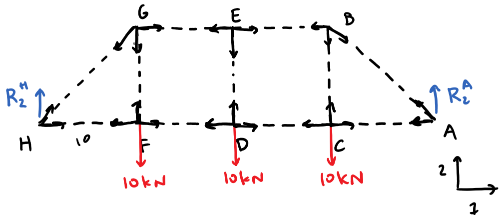
- weight small compared

Method of Joints

⓪ Solve for reactions at supports using free body diagram of truss as a whole.

① Isolate a joint, representing member forces converging to joint and external load

② Write equations of equilibrium



① $\sum F_z = R_z^H + R_z^A - 30 = 0$

By symmetry, $R_z^H = R_z^A = 15\,kN$

$\sum M^H = -100 - 200 - 300 + R_z^A\,(40) = 0$

$R_z^A = \frac{+600}{40}$

$R_z^A = 15\,kN$

② $F_1^H = F_{GH}\cos(\theta) + F_{FH} = 0$

$-15\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) + F_{FH} = 0$

$F_{FH} = 15\,kN$

$F_z^H = F_{GH}\sin\theta + R_z^H = 0$

$F_{GH}\left(\frac{\sqrt{2}}{2}\right) + 15 = 0$

$F_{GH} = -15\sqrt{2}$ (expansive)

continue until have all joints

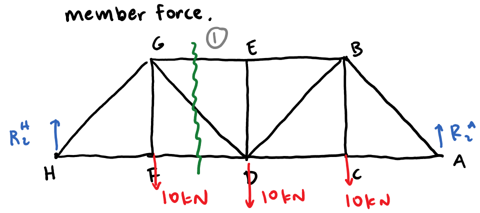
- not good for finding middle joints because you have to calculate everything

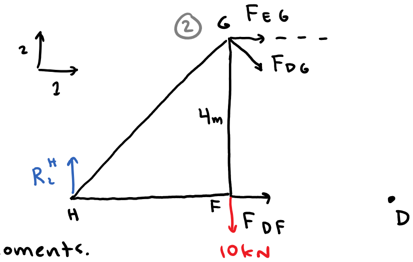
Method of Sections

① Pass a plane through truss, cutting member

② Draw a FBD of either side of the structure.

③ Use equilibrium equations (forces AND moments) to determine desired member force.





③ Helpful to pick points that have intersecting forces to calculate moments.

$\sum M_G = F_{DF}(4) - R_z^H \cdot 4 = 0$

$F_{DF} = R_z^H$ (From above, $R_z^H = 15\,kN$)

$F_{DF} = 15\,kN$

$\sum M_D = -F_{EG}\cdot 4 + 10kN\cdot 4 - 15kN\cdot 8 = 0$

$-4F_{EG} = 80\,kN$

$F_{EG} = -20\,kN$

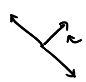
$\sum F_z = -F_{DG}\sin(45) - 10kN + 15kN = 0$

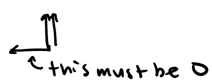
$F_{DG}\sin(45) = 5\,kN$

$F_{DG} = 5\sqrt{2}\,kN$

$\begin{array}{r} \overset{4}{1}5 \\ \times\ 8 \\ \hline 120 \end{array}$

More Strategies



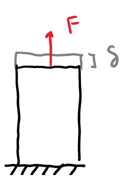


- get rid of reactions in contilevered structures
- take moments around convenient points

STATICALLY INDETERMINATE SYSTEMS

Too many unknowns, not enough equations → look at constitutive responses of

Constitutive Responses

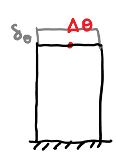


L = Length of Bar

A = Area of Cross-Section

E = Young's Modulus

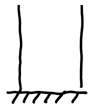
δ_m = elongation



δ_θ = elongation

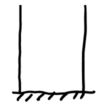
α = coefficient of thermal expansion

$\Delta\theta$ = temperature change



A = Area of Cross-Section
E = Young's Modulus
δ_m = elongation
F = Force

$$\delta_m = \frac{FL}{EA}$$



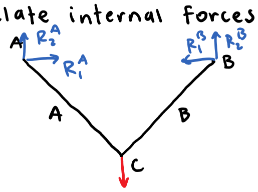
α = coefficient of thermal expansion
$\Delta\theta$ = temperature change

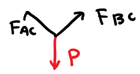
$$\delta_\theta = \alpha \Delta\theta L$$

$$\delta_T = \delta_m + \delta_\theta = \frac{FL}{EA} + \alpha\Delta\theta L$$

<u>Solution Procedure</u>

① Calculate internal forces in the bars





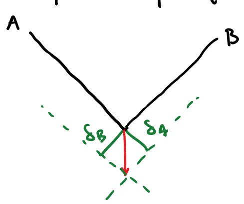
$$\sum F_1^c = -F_{AC}\cos\theta + F_{BC}\cos\theta = 0$$
$$F_{AC} = F_{BC}$$
$$\sum F_2^c = F_{AC}\sin\theta + F_{BC}\sin\theta - P = 0$$
$$\cancel{2} F_{AC}\left(\frac{\sqrt{2}}{\cancel{2}}\right) = P$$
$$F_{AC} = P\frac{\sqrt{2}}{2} = F_{BC}$$

② Find constitutive relations of each bar.

$$\delta_A = \frac{F_A L}{EA}, \quad \delta_B = \frac{F_B L}{EA}$$

$$\delta_A = \frac{PL}{EA\sqrt{2}} \qquad \delta_B = \frac{PL}{EA\sqrt{2}}$$

③ Apply compatibility equations (approximating circles as lines)



Elongation projection of u_c onto the bar.

$$u_1^c \cos\theta + u_2^c \sin\theta = \delta^{AC}$$