$$\mu(v) = \frac{v^T A v}{v^T v}$$ → $\mu(v)$ gives us the associated eigenvalue with v.

$$v = \begin{bmatrix} h \\ 1 \end{bmatrix}$$

$$\frac{\begin{bmatrix} h & 1 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} h \\ 1 \end{bmatrix}}{h^2+1} = \frac{[(5h+4)\ (2h+7)] \begin{bmatrix} h \\ 1 \end{bmatrix}}{h^2+1} = \frac{5h^2+6h+7}{h^2+1}$$

Answer

Rayleigh quotient

$$\begin{bmatrix} 5-\lambda & 2 \\ 4 & 7-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(5-\lambda)(7-\lambda) - 8 = 0$$

$$\lambda^2 - 12\lambda - 27 = 0$$

$$(\lambda - 9)(\lambda - 3) = 0$$

$$\lambda_1 = 9$$

$$\lambda_2 = 3$$

$$(5h+4)\ (2h+7)$$

$$5h^2+4h+2h+7 = 5h^2+6h+7$$

$$\begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 = 0$$

$$x_1 = \frac{1}{2} x_2$$

$$x_2 = k$$

$$v = \begin{bmatrix} \frac{1}{2}k \\ k \end{bmatrix}$$

$k=1$ dersek

$$v = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$h = 1/2$, the vectors we get with power method approaching this eigenvector and its eigenvalue is 9.

$$\frac{\frac{5}{4} + \frac{12}{4} + \frac{28}{4}}{\frac{5}{4}} = \frac{\frac{45}{4}}{\frac{5}{4}} = 9$$

But h is not important we should find largest value in terms of h.

1

i	x_i	f[·]	f[·,·]	f[·,·,·]	f[·,·,·,·]	f[·,·,·,·,·]
0	0	1				
1	1	9	8			
2	2	23	14	3		
3	4	93	35	7	1	
4	6	259	83	12	1	0

(a)

Answer

Not too important

$\frac{166}{2} = 83$ $\quad \frac{14-8}{2} = \frac{6}{2} = 3$ $\quad \frac{35-14}{4-1} = \frac{21}{3} = 7$

$\frac{83-35}{6-2} = \frac{48}{4} = 12$ $\quad \frac{7-3}{2} = 1$ $\quad \frac{12-7}{6-1}$

$c_0 = 1 \quad c_3 = 1$
$c_1 = 8 \quad c_4 = 0$
$c_2 = 3$

$$P(x) = c_0 + c_1(x-x_0) + c_2(x-x_0)(x-x_1) + c_3(x-x_0)(x-x_1)(x-x_2) + c_4(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

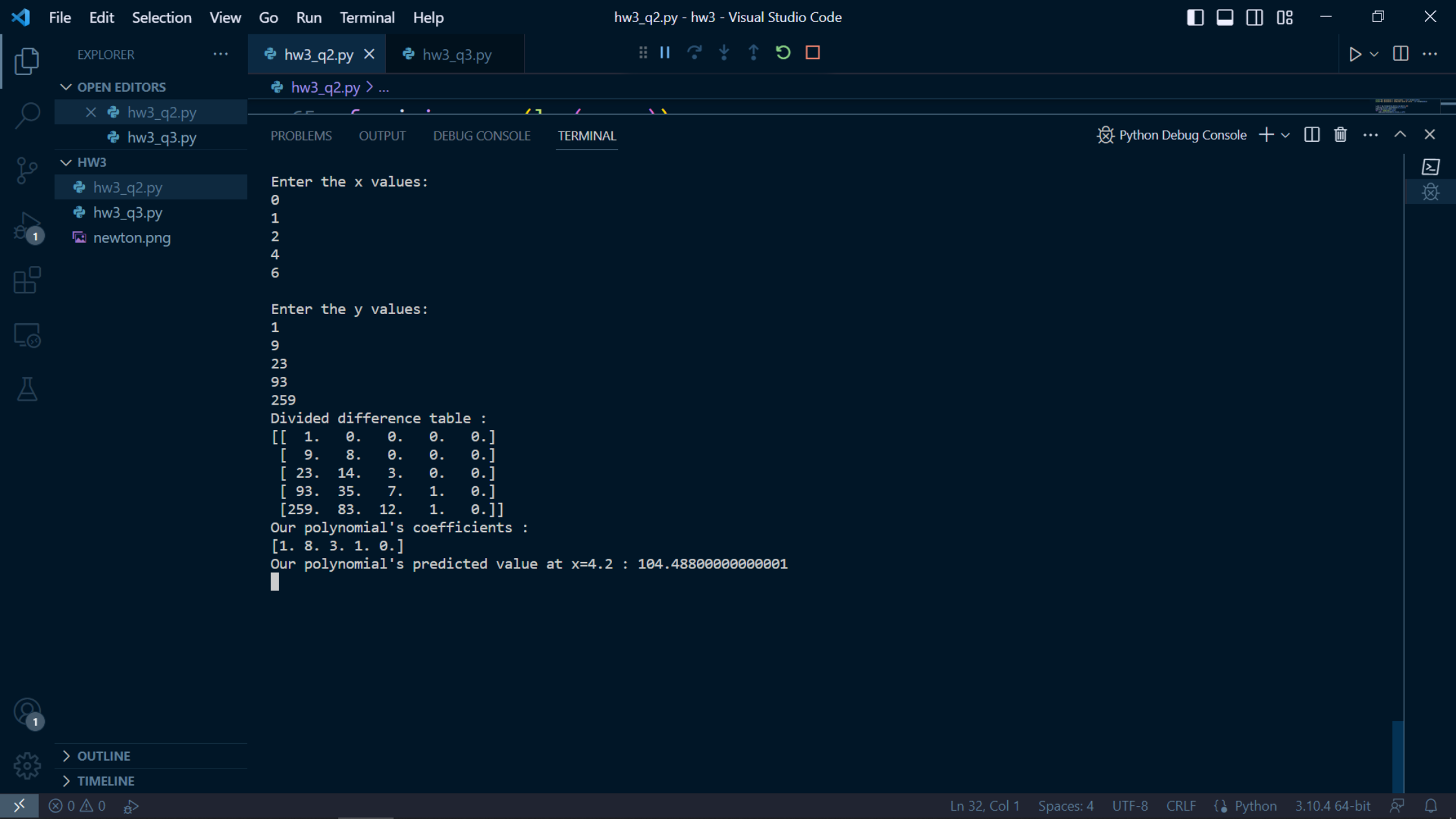
(c)

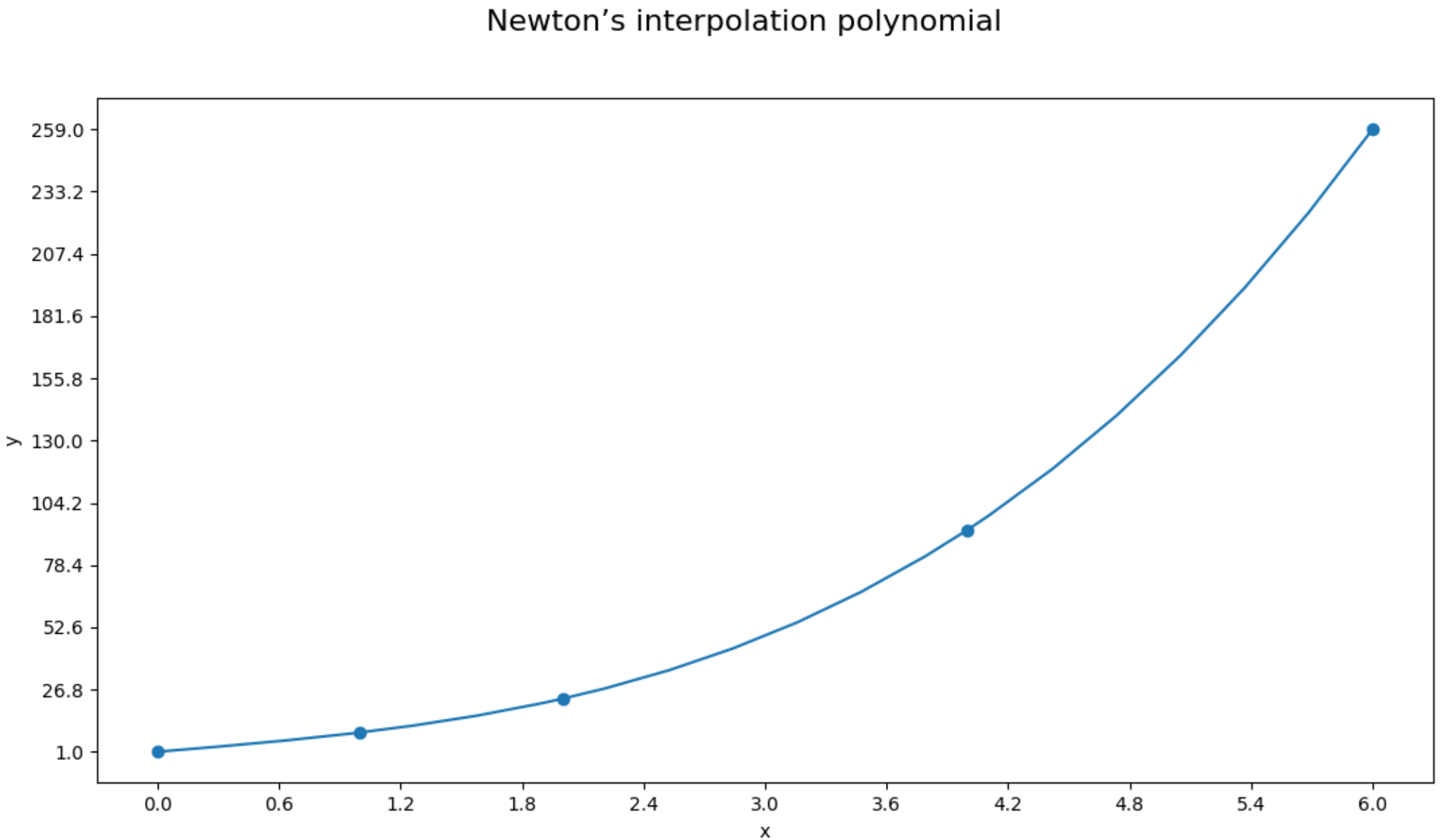
$$P(x) = 1 + 8(x-0) + 3(x-0)(x-1) + 1(x-0)(x-1)(x-2)$$

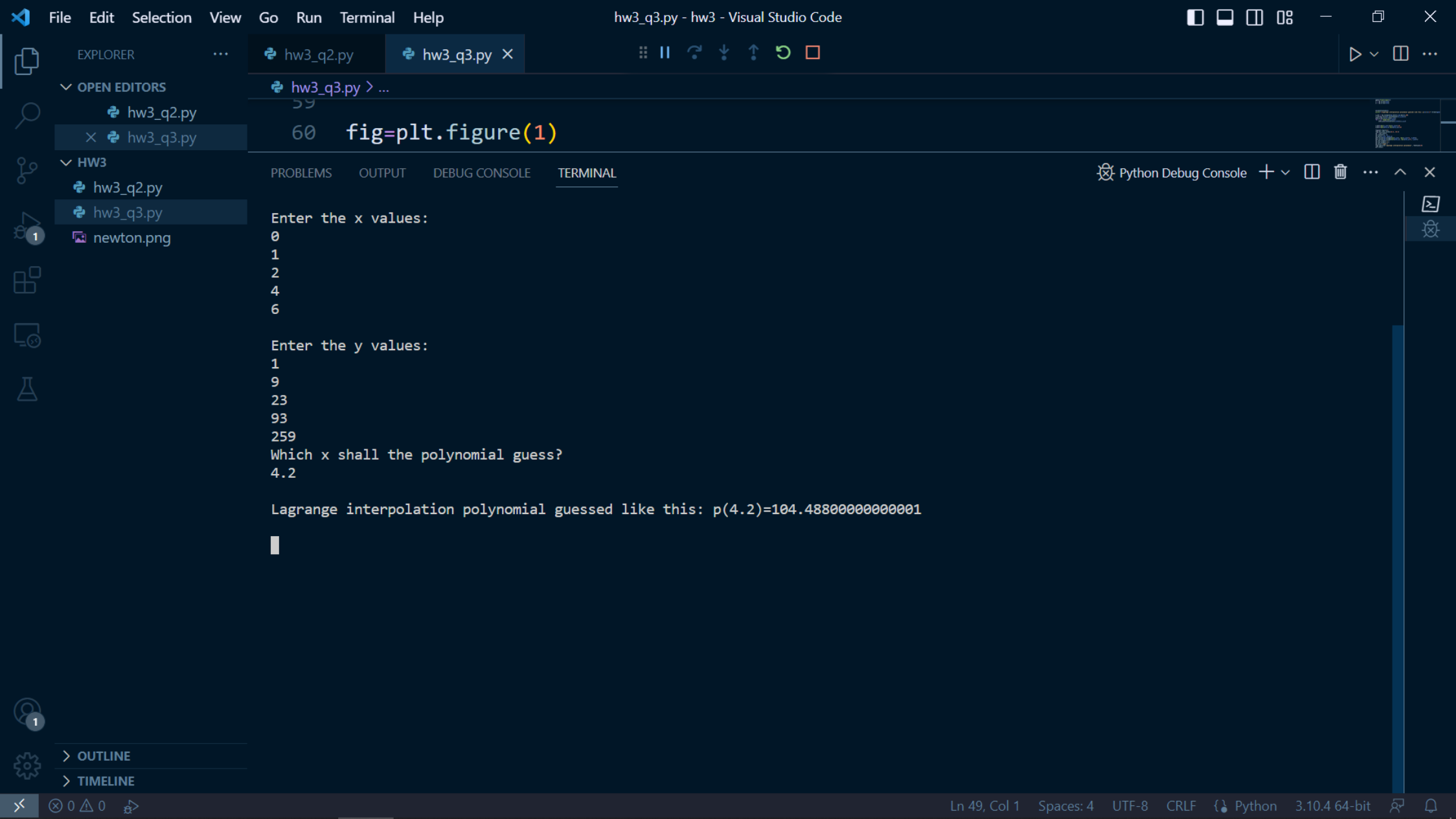
$$P(4.2) = 1 + 8 \times 4.2 + 3 \times 4.2 \times 3.2 + 4.2 \times 3.2 \times 2.2$$

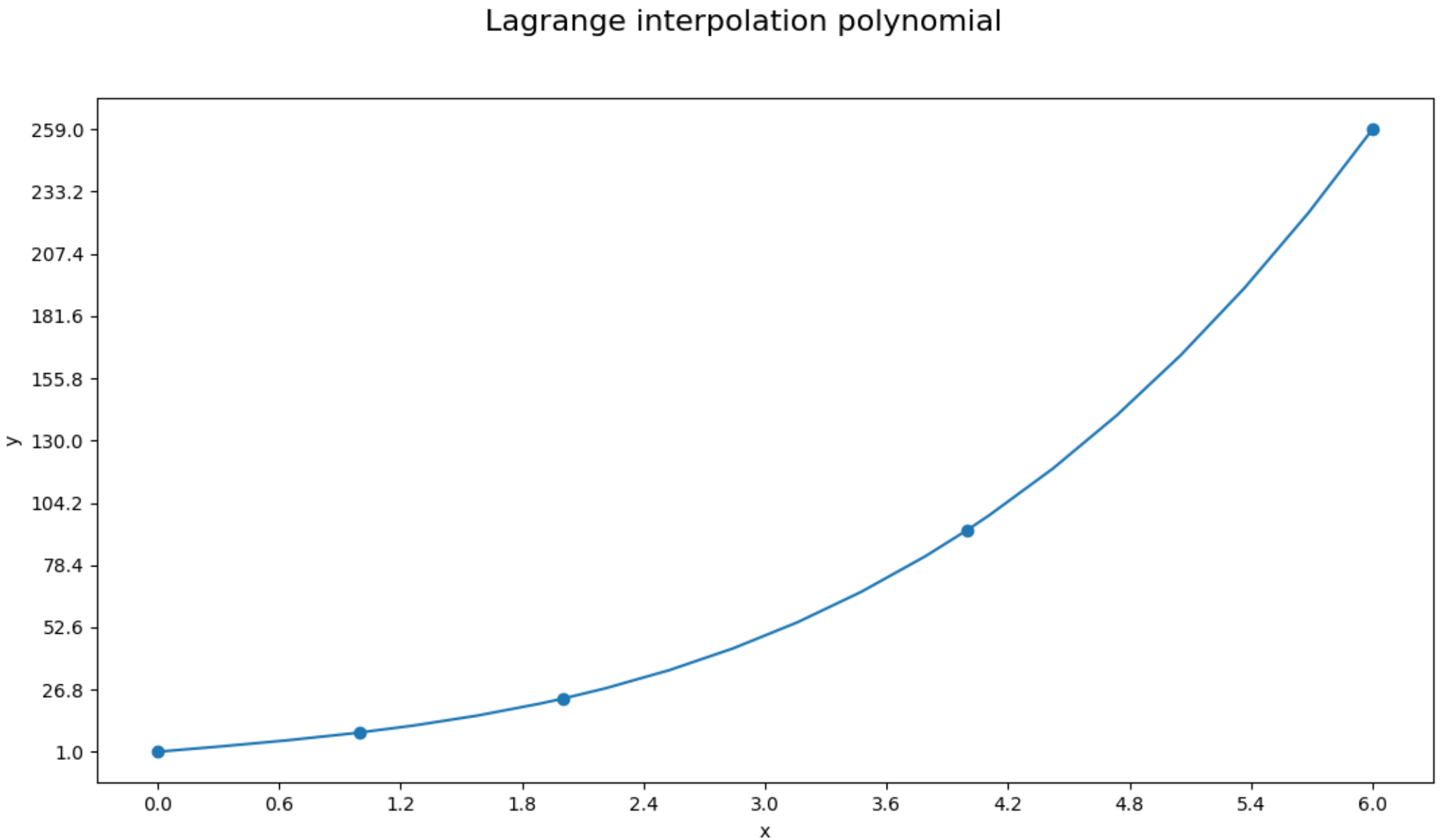
$P(4.2) = 104.488$ answer

2


File Edit Selection View Go Run Terminal Help
hw3_q2.py - hw3 - Visual Studio Code
EXPLORER
hw3_q2.py
hw3_q3.py
OPEN EDITORS
hw3_q2.py
hw3_q3.py
HW3
hw3_q2.py
hw3_q3.py
newton.png
hw3_q2.py > ...
PROBLEMS
OUTPUT
DEBUG CONSOLE
TERMINAL
Python Debug Console
Enter the x values:
0
1
2
4
6
Enter the y values:
1
9
23
93
259
Divided difference table :
[[1. 0. 0. 0. 0.]
[9. 8. 0. 0. 0.]
[23. 14. 3. 0. 0.]
[93. 35. 7. 1. 0.]
[259. 83. 12. 1. 0.]]
Our polynomial's coefficients :
[1. 8. 3. 1. 0.]
Our polynomial's predicted value at x=4.2 : 104.48800000000001
OUTLINE
TIMELINE
Ln 32, Col 1
Spaces: 4
UTF-8
CRLF
Python
3.10.4 64-bit


Newton’s interpolation polynomial
259.0
233.2
207.4
181.6
155.8
130.0
104.2
78.4
52.6
26.8
1.0
y
0.0
0.6
1.2
1.8
2.4
3.0
3.6
4.2
4.8
5.4
6.0
x


File Edit Selection View Go Run Terminal Help
hw3_q3.py - hw3 - Visual Studio Code
EXPLORER
OPEN EDITORS
hw3_q2.py
hw3_q3.py
HW3
hw3_q2.py
hw3_q3.py
newton.png
OUTLINE
TIMELINE
hw3_q2.py
hw3_q3.py
hw3_q3.py > ...
60 fig=plt.figure(1)
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL
Python Debug Console
Enter the x values:
0
1
2
4
6
Enter the y values:
1
9
23
93
259
Which x shall the polynomial guess?
4.2
Lagrange interpolation polynomial guessed like this: p(4.2)=104.48800000000001
Ln 49, Col 1 Spaces: 4 UTF-8 CRLF Python 3.10.4 64-bit


Lagrange interpolation polynomial
259.0
233.2
207.4
181.6
155.8
130.0
104.2
78.4
52.6
26.8
1.0
y
0.0
0.6
1.2
1.8
2.4
3.0
3.6
4.2
4.8
5.4
6.0
x

(4) (i)

$$\begin{bmatrix} 1 & -1.2 & 1.44 \\ 1 & 0.8 & 0.09 \\ 1 & 1.1 & 1.21 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} -5.76 \\ -5.61 \\ -3.69 \end{bmatrix}$$

Thus we have set a linear system.

We are trying to find $P_2(x) = a + bx + cx^2$ with monomial interpolation.

$$\left[\begin{array}{ccc|c} 1 & -1.2 & 1.44 & -5.76 \\ 0 & 1.5 & -1.35 & 0.15 \\ 0 & 2.3 & -0.23 & 2.07 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1.2 & 1.44 & -5.76 \\ 0 & 1 & -0.9 & 0.1 \\ 0 & 1 & -0.1 & 0.9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} a & b & c & \\ 1 & -1.2 & 1.44 & -5.76 \\ 0 & 1 & -0.9 & 0.1 \\ 0 & 0 & 0.8 & 0.8 \end{array}\right]$$

$a = 1$
$b = 1$
$c = -6$

$$\begin{array}{r} -5.76 \\ 0.24 \\ \hline 0.00 \end{array}$$

We found the coefficients!

$p(x) = c + bx + ax^2$

According to linear system method $P(x) = -6 + x + x^2$

3

(a) (ii) $p(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0.3)(x-1.1)}{(-1.5)(-2.3)}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x+1.2)(x-1.1)}{(1.5)(-0.8)}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x+1.2)(x-0.3)}{(2.3)(0.8)}$$

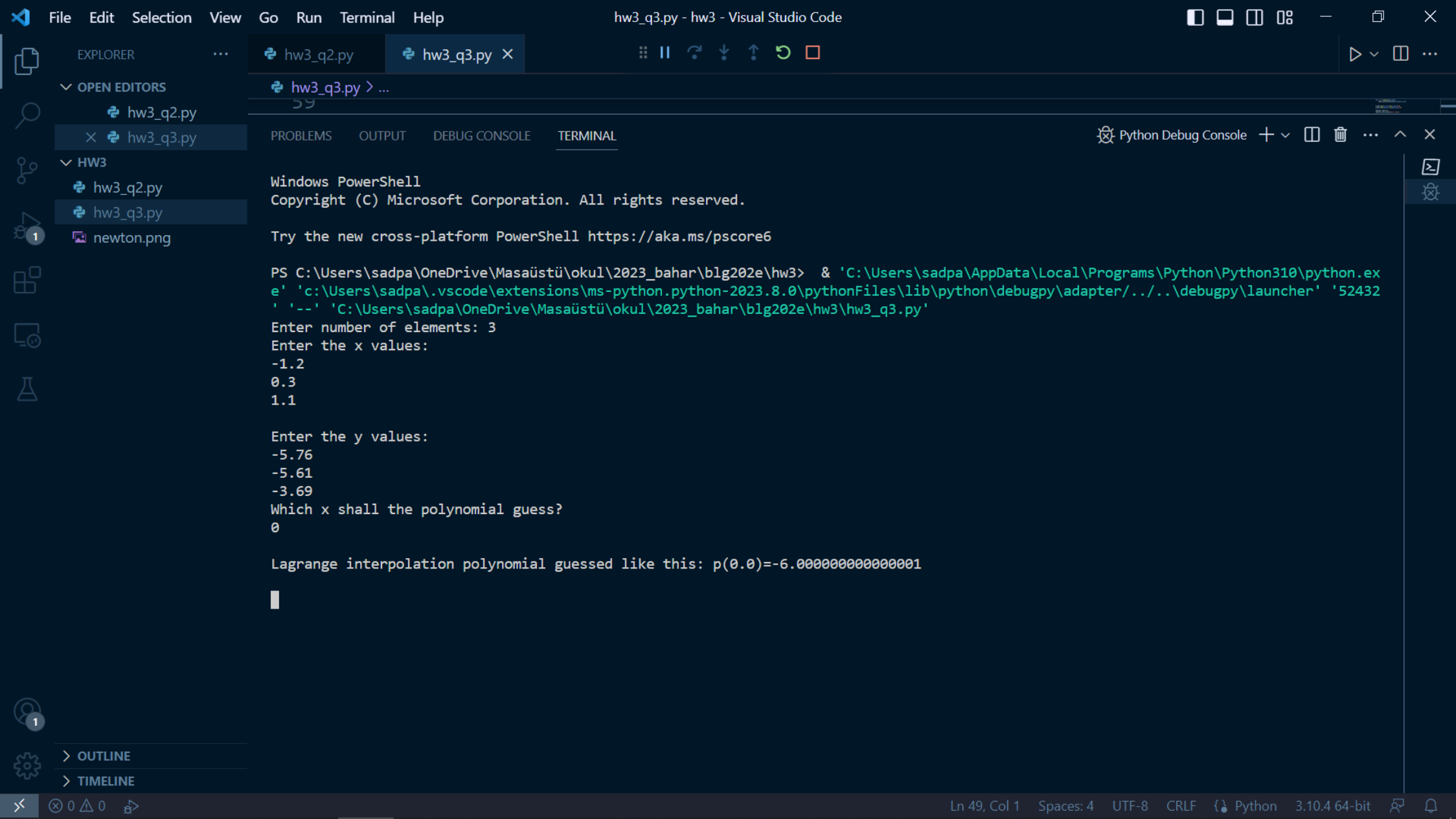
$$\begin{array}{r} 12 \\ \times 11 \\ \hline 12 \\ 120 \\ \hline 132 \end{array}$$

$$p(x) = \frac{-5.76}{(+1.5)(+2.3)}(x^2 - 1.4x + 0.33) + \frac{+5.61}{(1.5)(+0.8)}(x^2 + 0.1 + 1.32) = \text{Answer}$$

$$+ \frac{-3.69}{(2.3)(0.8)}(x^2 + 0.9x - 0.36)$$ proof by calculator

$$= -6 + x + x^2$$

(c) $p(x) = -6 + x + x^2$ $p(0) = -6$


File Edit Selection View Go Run Terminal Help
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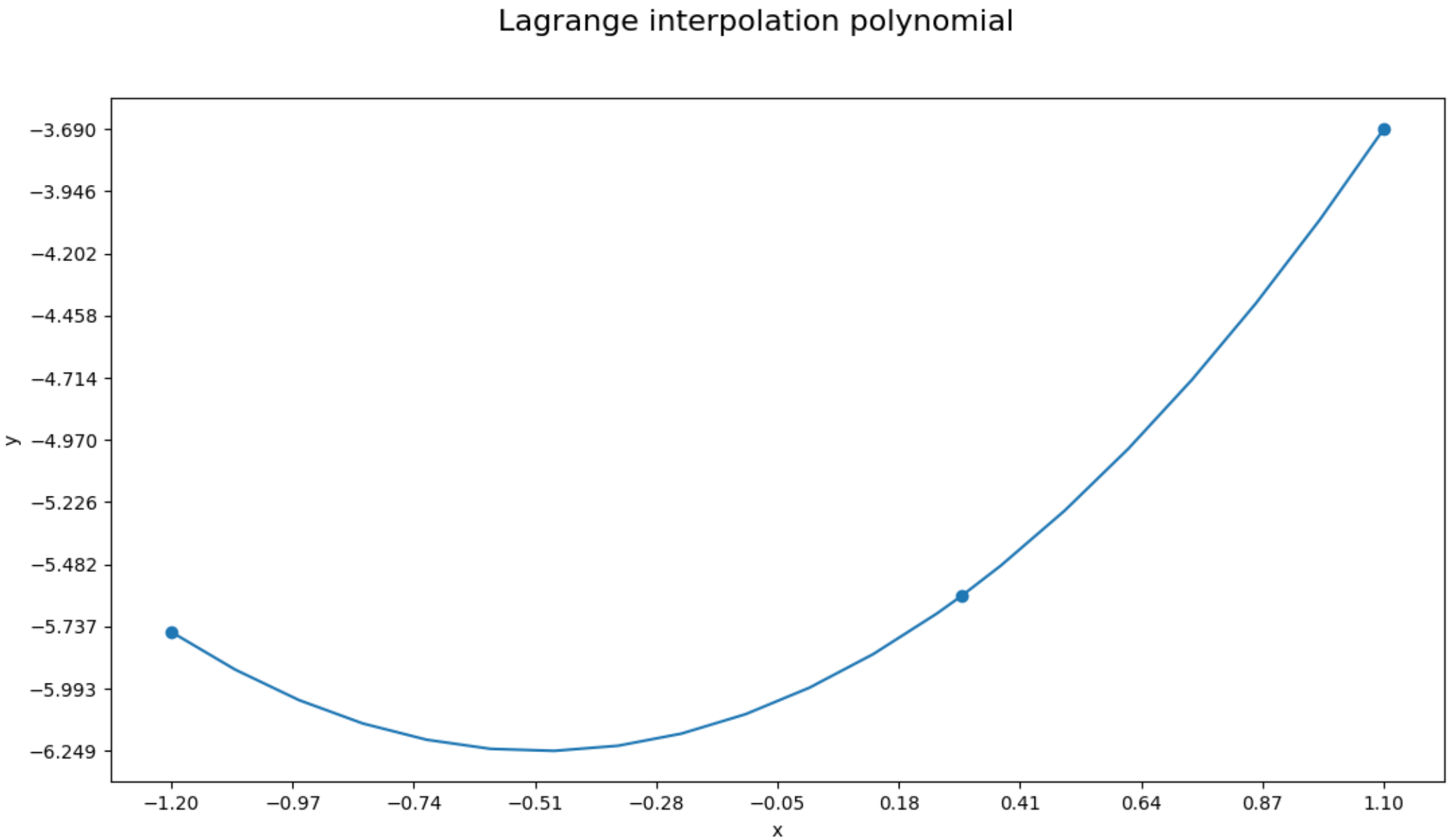
hw3_q3.py
hw3_q3.py > ...
PROBLEMS
OUTPUT
DEBUG CONSOLE
TERMINAL
Python Debug Console
Windows PowerShell
Copyright (C) Microsoft Corporation. All rights reserved.
Try the new cross-platform PowerShell https://aka.ms/pscore6
PS C:\Users\sadpa\OneDrive\Masaüstü\okul\2023_bahar\blg202e\hw3> & 'C:\Users\sadpa\AppData\Local\Programs\Python\Python310\python.exe' 'c:\Users\sadpa\.vscode\extensions\ms-python.python-2023.8.0\pythonFiles\lib\python\debugpy\adapter/../..\debugpy\launcher' '52432' '--' 'C:\Users\sadpa\OneDrive\Masaüstü\okul\2023_bahar\blg202e\hw3\hw3_q3.py'
Enter number of elements: 3
Enter the x values:
-1.2
0.3
1.1
Enter the y values:
-5.76
-5.61
-3.69
Which x shall the polynomial guess?
0
Lagrange interpolation polynomial guessed like this: p(0.0)=-6.000000000000001
Ln 49, Col 1
Spaces: 4
UTF-8
CRLF
Python
3.10.4 64-bit


Lagrange interpolation polynomial
−3.690
−3.946
−4.202
−4.458
−4.714
−4.970
−5.226
−5.482
−5.737
−5.993
−6.249
y
−1.20
−0.97
−0.74
−0.51
−0.28
−0.05
0.18
0.41
0.64
0.87
1.10
x

i	x_i	$f[\cdot]$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$
0	0.0	1		
1	0.4	3	5	
2	0.7	6	10	5017

(a) $f[x_0] = 1$
$f[x_1] = 3$
$f[x_0, x_1] = 5$ → Answer

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2] - f[x_0,x_1]}{x_2 - x_0} = \frac{10 - ?}{0.7} = \frac{50}{7} = \frac{5}{0.7} \quad ? = 5 = f[x_0,x_1]$$

$$f[x_1,x_2] = 10 = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{6 - f[x_1]}{0.3} \quad 6 - f[x_1] = 3 \quad f[x_1] = 3$$

$$\frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{3 - f[x_0]}{0.4} = 5 = f[x_0,x_1] \quad 3 - f[x_0] = 2 \quad f[x_0] = 1$$

$\frac{3}{0.3} = 10$ $\quad \frac{5}{0.7} = \frac{50}{7}$

Every time we get a new data, we add new row and column to our divided difference table; we add new term to our polynomial.

(b) First we get 0.0

0.0	1

divided difference table

$P_0(x) = 1$ ← the polynomial

Second we get 0.4

0.0	1	
0.4	3	5

updated

$P_1(x) = 1 + 5(x - 0.0)$ ← gets updated

Third we get 0.7

0.0	1		
0.4	3	5	
0.7	6	10	5017

$P_2(x) = 1 + 5(x - 0.0) + \frac{50}{7}(x - 0.0)(x - 0.4)$

Finally, we got this difference table and polynomial.

(c) First we get $x_2 = 0.7$

0.7	6

$P_0(x) = 6$

Second we get $x_1 = 0.4$

0.7	6	
0.4	3	10

$\frac{-3}{-0.3} = 10$

$P_1(x) = 6 + 10(x - 0.7)$

Third we get $x_0 = 0.0$

0.7	6		
0.4	3	10	
0.0	1	5	50/7

$\frac{-2}{-0.4} = 5$ part (b) polynomial

$\frac{-5}{-0.7} = \frac{50}{7}$

Our final polynomial and divided difference table.

$P_2(x) = 6 + 10(x - 0.7) + \frac{50}{7}(x - 0.7)(x - 0.4)$

(d) $P_b(x) = 1 + 5x + \frac{50}{7}x(x - 0.4)$

$= \frac{50}{7}x^2 - \frac{20}{7}x + 5x + 1$

$P_b(x) = \frac{50}{7}x^2 + \frac{15}{7}x + 1$

$P_c(x) = 6 + 10(x - 0.7) + \frac{50}{7}(x - 0.7)(x - 0.4)$

↓ part (c) polynomial

$\frac{50}{7}(x^2 - 1.1x + 0.28)$

$\frac{50}{7}x^2 - \frac{55}{7}x + 2$

$10x - 1$

$70 - 55 = 15$

$P_c(x) = \frac{50}{7}x^2 + \frac{15}{7}x + 1$

$2.8 \times 5 = 140$

Yes, our final polynomials we have found in part (b) and (c), are the same.

Answer

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