Data Summary

STAT 211 - 509

2018/09/11

Descriptive statistics

- We have a sample of data, drawn from some distribution
- How to compute numerical summaries of the data?
- How to visualize the data?

Variables

- **Variable**: any characteristic or quantity to be measured on units in a study
- **Categorical variable**: places a unit into one of several categories
- **Quantitative variable**: takes on numerical values
- **Univariate**: data with one variable
- **Bivariate**: data with two variables
- **Multivariate**: data with three or more variables

Example: US cereal

```
dat <- MASS::UScereal
str(MASS::UScereal)
```

```
## 'data.frame':    65 obs. of  11 variables:
##  $ mfr      : Factor w/ 6 levels "G","K","N","P",..: 3 2 2 1 2 1 6 4 5 1 ...
##  $ calories : num  212 212 100 147 110 ...
##  $ protein  : num  12.12 12.12 8 2.67 2 ...
##  $ fat      : num  3.03 3.03 0 2.67 0 ...
##  $ sodium   : num  394 788 280 240 125 ...
##  $ fibre    : num  30.3 27.3 28 2 1 ...
##  $ carbo    : num  15.2 21.2 16 14 11 ...
##  $ sugars   : num  18.2 15.2 0 13.3 14 ...
##  $ shelf    : int  3 3 3 1 2 3 1 3 2 1 ...
##  $ potassium: num  848.5 969.7 660 93.3 30 ...
##  $ vitamins : Factor w/ 3 levels "100%","enriched",..: 2 2 2 2 2 2 2 2 2 2 ...
```

Summarizing categorical variable

- **Frequency**: number of times a value occurs in data
- **Relative frequency**: proportion of data that has a value

```
freqs <- table(dat$mfr)
freqs
```

```
##
##  G  K  N  P  Q  R
## 22 21  3  9  5  5
```

```
props <- freqs/nrow(dat)
props
```

```
##
##          G          K          N          P
## 0.33846154 0.32307692 0.04615385 0.13846154
##          Q          R
## 0.07692308 0.07692308
```

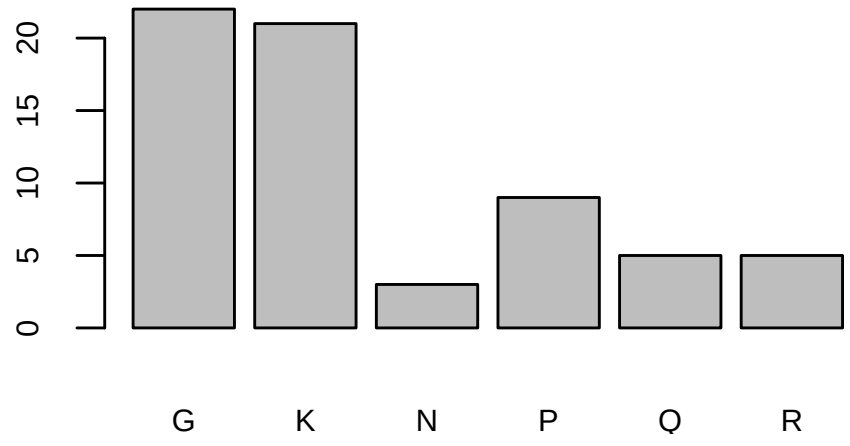
```
sum(props)
```

```
## [1] 1
```

Bar chart

- Compares frequencies
- Unordered

```
barplot(freqs, cex.axis = 0.7, cex.lab = 0.7,
    cex = 0.7)
```



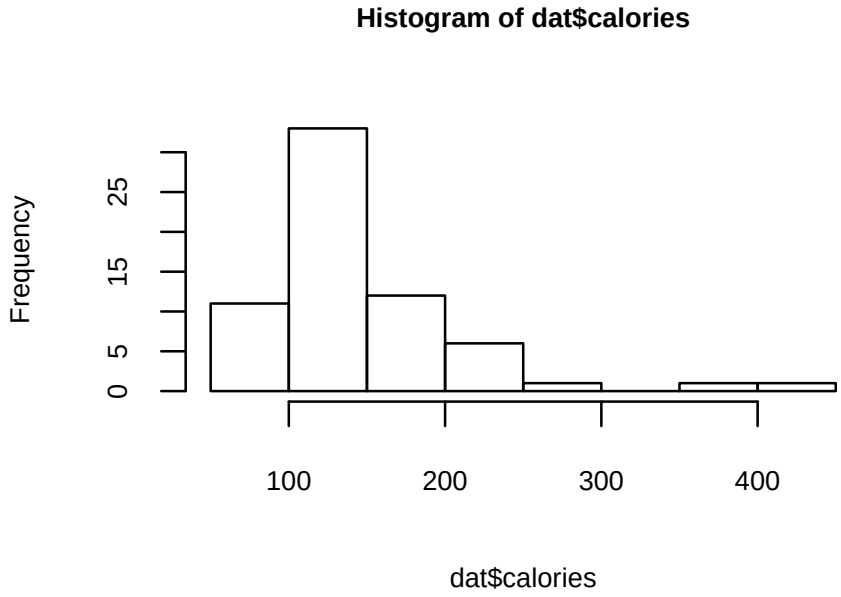
Summarizing quantitative variable

- What is the typical value of the variable?
- What is the spread of the variable?

Histogram

- **Histogram**: bar graph of binned data where height of bar above each bin denotes frequency or relative frequency of values in the bin

```
hist(dat$calories, cex.axis = 0.7, cex.lab = 0.7,
    cex.main = 0.7)
```



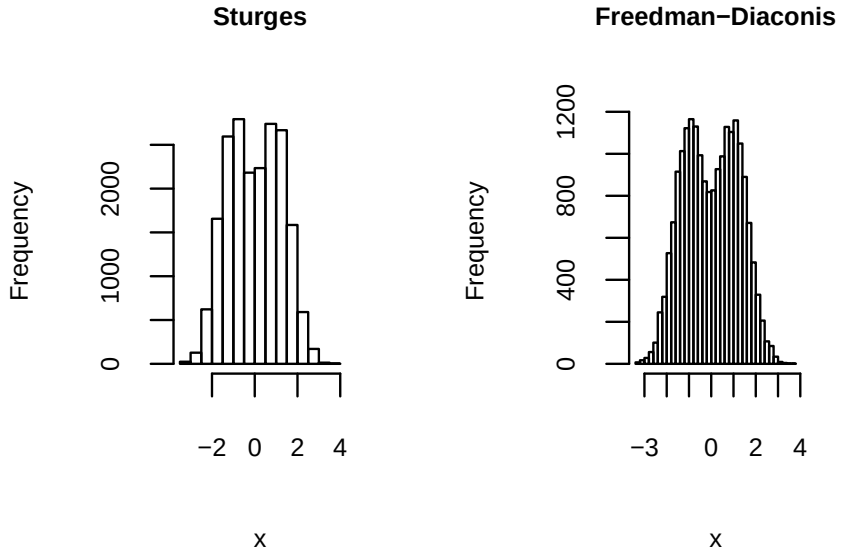
- Need to choose number of bins among which we divide the n data points
- General rule: number of bins $\approx \sqrt{n}$
- `breaks` argument in `hist()`. Can be a string that specifies a built in algorithm for binning. Good default is "FD", for Freedman-Diaconis rule

Breaks example

- Data drawn from a distribution with two modes

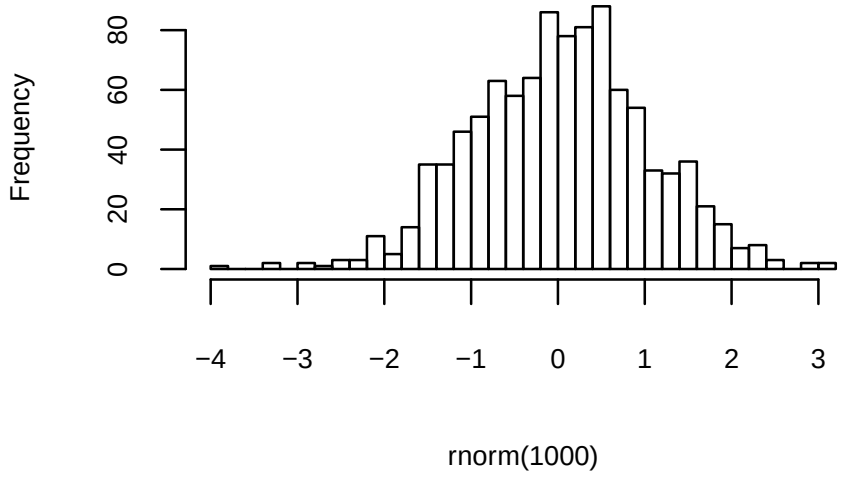
```
set.seed(1)
z <- rbinom(20000, 1, 0.5) + 1
means <- c(-1, 1)
```

```
x <- rnorm(20000, mean = means[z], 0.7)
par(mfrow = c(1, 2))
hist(x, main = "Sturges", cex.main = 0.7, cex.axis = 0.7,
    cex.lab = 0.7)
hist(x, breaks = "FD", main = "Freedman-Diaconis",
    cex.main = 0.7, cex.axis = 0.7, cex.lab = 0.7)
```

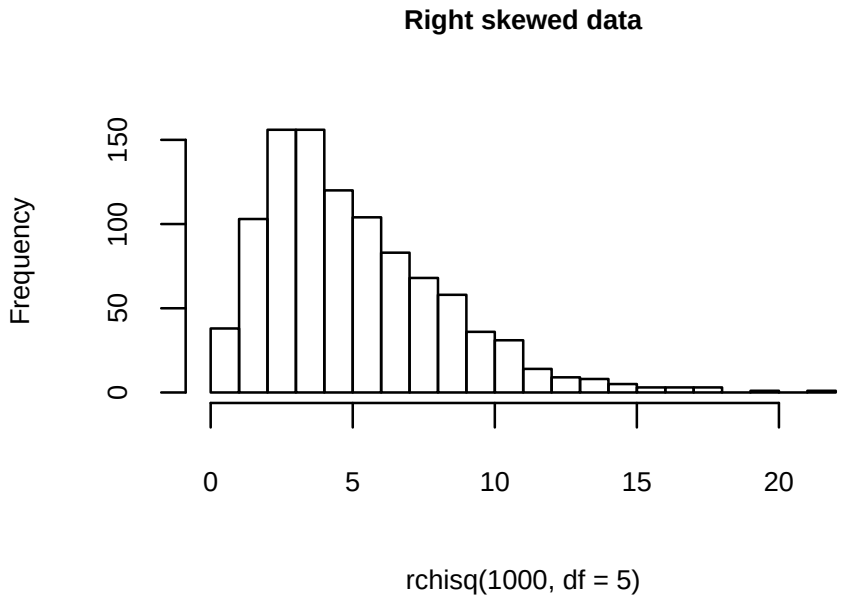


The shape of quantitative data

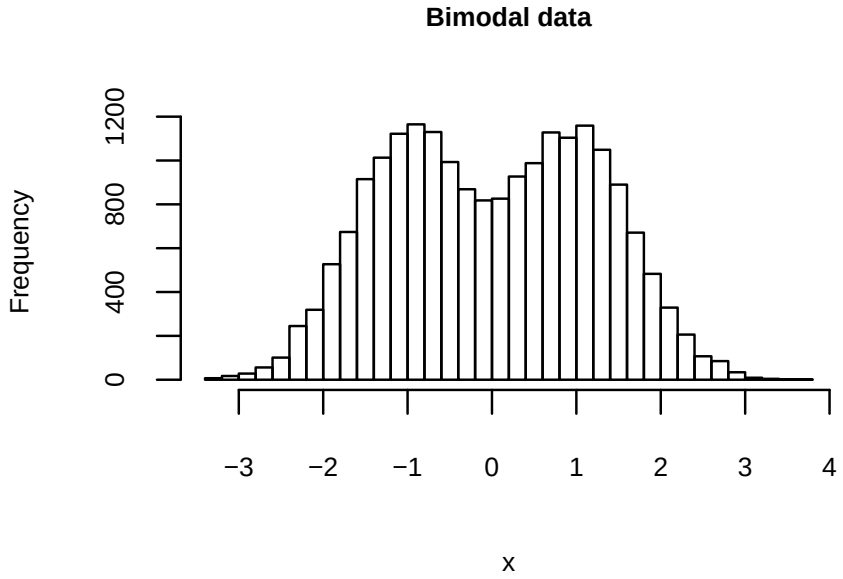
- **Symmetric** data is mirrored about each side of a center value



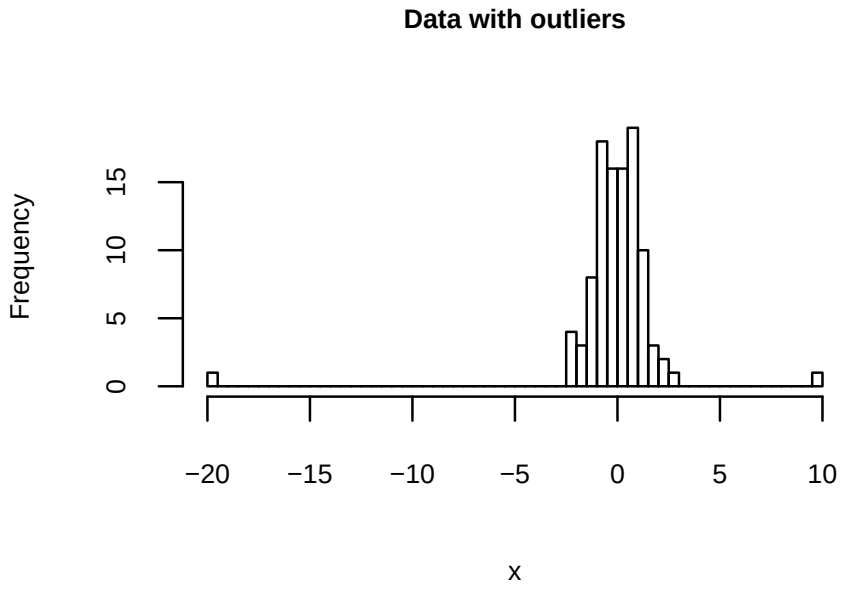
- **Skewed** data has one side much longer than the other



- The **mode** is the peak value of the distribution
- **Multimodal** data has multiple modes



- **Outliers** are data points "far" from most other data
- Determination of outliers is subjective
- *Do not* remove outliers if you don't know for sure that the data is erroneous



Summary statistics for quantitative data

Measures of central tendency

- **Sample median**: value separating lower 50% of data from upper 50% of sample
  - For finite set of numbers, the middle value
  - If even number of values, then mean of middle two numbers
- **Sample mean**: Given sample values $x_1, \ldots, x_n$,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

```
mean(dat$calories)
```

```
## [1] 149.4083
```

Percentiles

- **Percentile**: the pth percentile is the value such that $p \times 100\%$ of sample data is below it and $(1 - p) \times 100\%$ are above it.
- **First quartile (Q1)** is 25th percentile
- **Second quartile (Q2)** is 50th percentile
- **Third quartile (Q3)** is 75th percentile
- **Five-number summary**

```
fivenum(dat$calories)
```

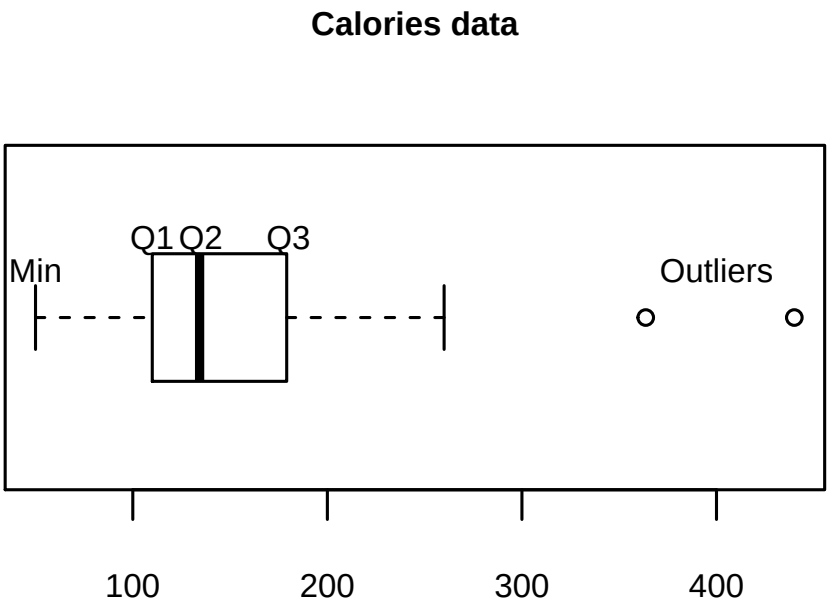
```
## [1]  50.0000 110.0000 134.3284 179.1045
## [5] 440.0000
```

```
summary(dat$calories)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.
##    50.0   110.0   134.3   149.4   179.1
##    Max.
##   440.0
```

Boxplot

- Visualize the 5 number summary
- In R: `boxplot()`
- **Interquartile range**: $IQR = Q3 - Q1$
- Outliers: values greater than $Q3 + IQR$ or less than $Q1 - IQR$ are represented with a point



Measures of spread

- **IQR**: $Q3 - Q1$, the range of the middle 50% of the data
- **Sample variance,** s^2: sum of squared deviations from the mean divided by $n - 1$:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- **Sample standard deviation, s**: square root of sample variance. Has same units as data

```
var(dat$calories)
```

```
## [1] 3895.242
```

```
sd(dat$calories)
```

```
## [1] 62.41187
```

Choosing measure of central tendency and spread

- Sample mean and sample standard deviation good for symmetric data
- For skewed data or data with outliers, sample median and interquartile range may be more appropriate