Exam 1 Review

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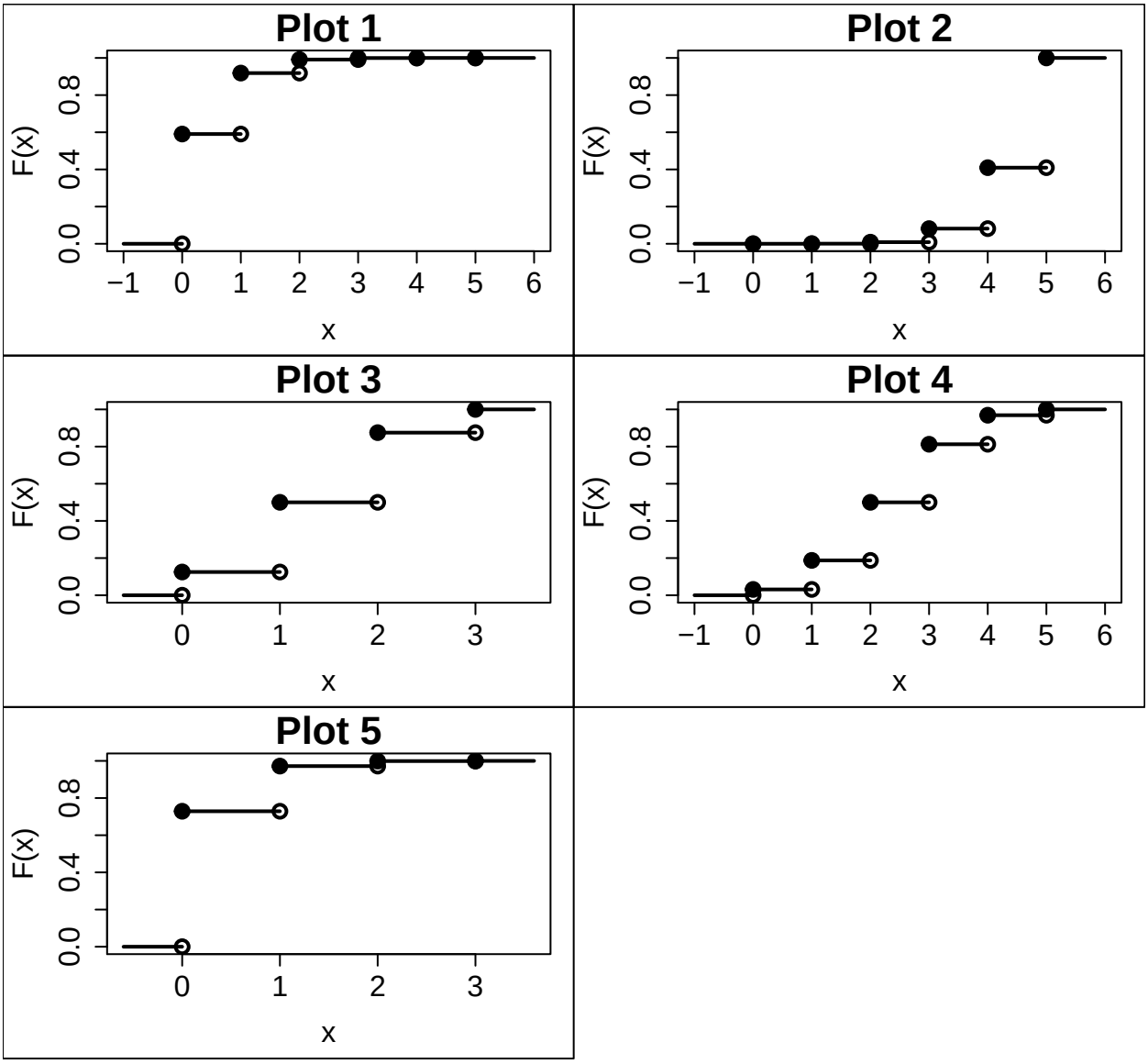
1



Which of the above plots is the cumulative distribution function for $X \sim Binomial(n = 3, p = 0.1)$?

2

If $B \subset A$, then $P(A \cup B)$ is

a) $P(A)$

b) $P(A) - P(A \cap B)$

c) $P(A|B)$

d) 1

e) 0

3

A plane has 4 engines. Let X represent the number of engines that are working at any point in time. Consider the probability model shown below for X:

X	0	1	2	3	4
Probability	0.23	0.27	0.11	0.05	0.34

What is the expected number of engines that are working at any point in time?

a) 0.4

b) 0.5

c) 1.5

d) 2

e) 2.23

4

On an exam, the probability that a particular student will make a mistake on any single question is 0.25. If there are 8 questions, and the student's answers are mutually independent of each other, then the probability that the student makes exactly two mistakes is

a) $\sum_{x=0}^{2} \binom{8}{x}(.25)^x(.75)^{8-x}$

b) .689

c) .311

d) $\sum_{x=2}^{8} \binom{8}{x}(.25)^x(.75)^{8-x}$

e) 1, since 2 is the expected value in this case

The probability mass function of the random variable X is:

x	f_X
0	0.2
1	0.8

Use this table to answer the next **two questions**.

5

What is $E[X]$?

a) 0.8

b) 0.16

c) 1.0

d) 0.5

e) 0.64

6

What is $Var(X)$?

a) 0.20

b) 0.16

c) 0.10

d) 0.25

e) 0.64

A test has 20 questions with 5 possible answers each. A student who has not studied at all decides to randomly guess an answer for each question (with equal probability for each of the five answer choices). Answer the following **three questions**.

7

What is the expected number of correct answers?

a) 3

b) 4

c) 5

d) 6

e) 7

8

What is the standard deviation of the number of correct answers?

a) 1.8

b) 4

c) 3.2

d) 2

e) 10

9

What is the probability that the student gets more than one answer correct on the test?

a) 0.07

b) 0.25

c) 0.75

d) 0.93

e) 0.97

In a batch of voltage regulators, 60% came from supplier 1, 30% from supplier 2 and 10% from supplier 3.

- 95% of regulators from supplier 1 work
- 80% of regulators from supplier 2 work
- 65% of regulators from supplier 3 work

Use this information to answer the following **two questions**.

10

What is the probability a randomly selected regulator from the batch works?

a) 0.667

b) 0.875

c) 0.615

d) 0.125

e) 0.960

11

If a randomly selected regulator from the batch works, what is the probability it came from supplier 3?

a) 0.65

b) 0.10

c) 0.05

d) 0.50

e) 0.07

12

Consider randomly selecting a student at TAMU. Let A denote the event that the student has a MasterCard and B denote the event that the student has a Visa card. Suppose that $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.4$. The probability that a randomly selected student at TAMU has exactly one of the two types of cards is

a) 0.3

b) 0.7

c) 0.1

d) 0.4

e) 0.2

13

A discrete rv X has a pmf given by

$$p(x) = \begin{cases} C(x+1), & \text{x=0,1,2,3} \\ 0, & otherwise. \end{cases}$$

The value of C that makes $p(x)$ a pmf is

a) 10

b) $\frac{1}{6}$

c) $\frac{1}{10}$

d) 6

e) $\frac{1}{30}$

14

Suppose that X is a discrete rv with $E(X) = 5$ and $V(X) = 10$. Suppose that $Y = 5X - 2$. Then the mean and variance of Y are

a) $E(Y) = 23,\ V(Y) = 250$

b) $E(Y) = 23,\ V(Y) = 15.811$

c) $E(Y) = 23,\ V(Y) = 50$

d) $E(Y) = 23,\ V(Y) = 246$

e) $E(Y) = 23,\ V(Y) = 48$

15

Suppose $X \sim Binomial(10, .72)$. Which of the following would compute $P(X = 9)$ in R?

a) rbinom(n = 10, size = 10, prob = .72)

b) pbinom(q = 9, size = 10, prob = .72)

c) dbinom(x = 9, size = 9, prob = .72)

d) dbinom(x = 10, size = 9, prob = .72)

e) dbinom(x = 9, size = 10, prob = .72)

16

Suppose $W \sim Binomial(10, .3)$. Which of the following would compute $P(3 \leq W < 8)$ in R?

a) pbinom(q = 8, size = 10, prob = .3) - pbinom(q = 3, size = 10, prob = .3)

b) dbinom(x = 3:7, size = 10, prob = .3)

c) pbinom(q = 7, size = 10, prob = .3) - pbinom(q = 2, size = 10, prob = .3)

d) pbinom(q = 7, size = 10, prob = .3)

e) sum(dbinom(x = 3:8, size = 10, prob = .3))

Consider $x_1, \ldots, x_n \sim iid\ geometric(\gamma)$, where the pmf is

$$f_\gamma(x_i) = (1-\gamma)^{x_i - 1}\gamma$$

and $x = 1, 2, \ldots$, $0 < \gamma < 1$. Answer the following **two questions**.

17

You construct the likelihood function by multiplying the pmfs evaluated at the data together: $\prod_{i=1}^{n} f_\gamma(x_i)$. So the likelihood function is of the form: $L(\gamma) = (1-\gamma)^A \gamma^B$. What is A and B?

a) $A = x_1, \quad B = 1$

b) $A = \sum_{i=1}^{n} x_i - n, \quad B = 1$

c) $A = \prod_{i=1}^{n} x_i, \quad B = n$

d) $A = \sum_{i=1}^{n} x_i - n, \quad B = n$

e) $A = \sum_{i=1}^{n} x_i - 1, \quad B = n$

18

What is the maximum likelihood estimate of p?

a) $\hat{p} = \frac{A}{B}$

b) $\hat{p} = \frac{A}{B} + 1$

c) $\hat{p} = \left(\frac{A}{B} - 1\right)^{-1}$

d) $\hat{p} = \left(\frac{A}{B} + 1\right)^{-1}$

e) $\hat{p} = A + B$

Consider the following joint probability distribution:

	X = -2	X = 2
Y = 3	.3	.1
Y = 4	.4	.2

Answer the following **two questions**.

19

What is the marginal PMF of X?

a)

$$f_X(x) = \begin{cases} .3 & x = -2 \\ .1 & x = 2 \end{cases}$$

b)

$$f_X(x) = \begin{cases} .4 & x = -2 \\ .2 & x = 2 \end{cases}$$

c)

$$f_X(x) = \begin{cases} .7 & x = -2 \\ .3 & x = 2 \end{cases}$$

d)

$$f_X(x) = \begin{cases} .3 & x = -2 \\ .7 & x = 2 \end{cases}$$

e)

$$f_X(x) = \begin{cases} .4 & x = -2 \\ .6 & x = 2 \end{cases}$$

20

What is $E[Y|X = 2]$?

a) $\frac{1}{3}$
b) $\frac{11}{3}$
c) $\frac{2}{3}$
d) $\frac{25}{7}$
e) $\frac{1}{4}$

Solutions

1. Plot 5
2. a: $P(A)$
3. d: 2
4. c: .311
5. a: 0.8
6. b: 0.16
7. b: 4. If X is the number of correct answers (guesses), then $X \sim Binomial(n = 20, p = 1/5)$. Therefore, $E(X) = np = 4$
8. a: 1.8. $Var(X) = np(1-p) = 20(1/5)(4/5) = 3.2 \Rightarrow sd(X) = \sqrt{3.2} = 1.8$
9. d: 0.93. $P(X > 1) = 1 - \Big(P(X = 0) + P(X = 1)\Big) = 1 - \left(\binom{20}{0}(1/5)^0(4/5)^{20} + \binom{20}{1}(1/5)^1(4/5)^{19}\right) = 0.93$
10. b: $0.875 = .6 \times .95 + .3 \times .8 + .1 \times .65$
11. e: $0.07 = (.1 \times .65)/.875$
12. a: 0.3
13. c: $\frac{1}{10}$
14. a: $E(Y) = 23,\ V(Y) = 250$
15. e: dbinom(x = 9, n = 10, p = .72)

16. c: pbinom(q = 7, size = 10, prob = .3) - pbinom(q = 2, size = 10, prob = .3)
17. d: $A = \sum_{i=1}^{n} x_i - n, \quad B = n$
18. d: $\hat{p} = \left(\frac{A}{B} + 1\right)^{-1}$
19. c: $f_X(x) = \begin{cases} .7 & x = -2 \\ .3 & x = 2 \end{cases}$
20. b: $\frac{11}{3}$