Exam 1

Version 2

STAT 211 - 509

10-4-2018

Name: ________________________________

Please sign the following pledge and read all instructions carefully before starting the exam.

Pledge: I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code. Signature: ________________________________

Instructions

- This test has 15 multiple choice problems. Record all answers on your large Scantron with the appropriate pencil.
- Total time is 75 minutes (12:45 P.M to 2:00 P.M.).
- You are permitted one 8.5in by 11in cheatsheet and a calculator. No other resources (phones, laptop, tablet, etc.) are allowed.
- If you are wearing a hat and/or smartwatch, please remove them while taking the test.
- There is no penalty for incorrect answers; have an answer for every question.
- When you are done, turn in your cheatsheet and exam booklet along with the Scantron.

Let $X \sim Poisson(1)$ and $Y \sim Binomial(8, 1/2)$. Assume that X and Y are independent. Answer the following **three** questions:

Question 1

What is $E[aX + bY]$?

a) a + 4b

b) a + 8b

c) a + b

d) 2a + b

e) 2a + 4b

SOLUTION: a

Question 2

What is $Var(aX + bY)$?

a) a^2

b) $a^2 + 2b^2$

c) $a^2 + 4b^2$

d) $2b^2$

e) $a^2 + b^2$

SOLUTION: b

Question 3

What is $E[(aX + bY)^2]$?

a) $a^2 + 2b^2 - (a + 4b)^2$

b) $a^2 + 2b^2 + (a + 4b)^2$

c) $a^2 + 2b^2 - a - 4b$

d) $a^2 + 2b^2 + a + 4b$

e) $(a + 4b)^2$

SOLUTION: b

Question 4

Let L be the event that your bus is late and T be the event that it is on time. Suppose $P(L) = 0.1$ and $P(T) = 0.3$. What is $P(L \cup T)$?

a) 0.03

b) 0.45

c) 0.4

d) Can't answer without knowing $P(A \cap B)$

e) 0.27

SOLUTION: c

Question 5

Suppose for two events A and B we have that $B \subset A$, meaning B is completely contained in A. This means every outcome in B is also in A. Then $P(A \cap B)$ is:

Hint: draw a Venn diagram. For an example of an event completely contained in another, let P be the event you pass this exam and A be the event you get an A. $A \subset P$, since all the scores you get that lead to an A also lead to you passing.

a) P(A) + P(B)

b) P(B)

c) P(A)

d) 0

e) 1

SOLUTION: b

3% of a population is infected with HIV. For a certain HIV test there is a 98% chance the test will be positive given that the tested individual is actually infected with HIV. If the person is not infected, there is a 4% chance the test will be positive. Answer the following **two** questions.

Question 6

What is the probability that the test is positive for a randomly selected person from the population?

a) 0.0682

b) 1

c) 0.0294

d) 0.03

e) 0.0388

SOLUTION: a

Question 7

If a randomly selected person from the population is tested positive, what is the probability that the person is actually infected?

a) 0.04

b) 0.0294

c) 0.4311

d) 0.98

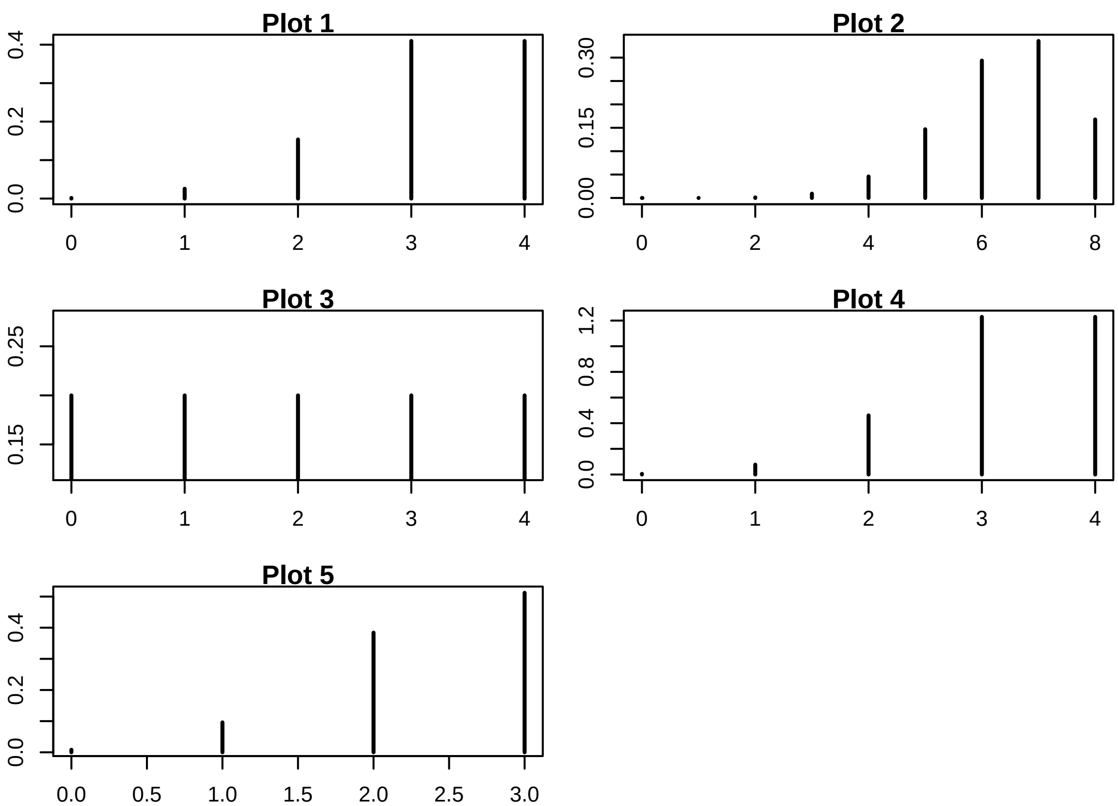
e) 0.03

SOLUTION: c

Let $X \sim Binomial(n = 4, p = 0.8)$. Answer the following **three** questions.

Question 8

Which of the following plots is of the pmf for X? The y axis is $f(x)$ for each plot.



a) Plot 1

b) Plot 2

c) Plot 3

d) Plot 4

e) Plot 5

SOLUTION: a

Question 9

Which of the following correctly computes $P(X \leq 2)$ in R?

a) dbinom(x = 2, size = 4, prob = 0.8)

b) pbinom(q = 2, size = 4, prob = 0.8)

c) rbinom(n = 2, size = 4, prob = 0.8)

d) pgeom(q = 2, prob = 0.8)

e) pbinom(q = 2, size = 3, prob = 1)

SOLUTION: b

Question 10

What is $P(X \geq 1)$?

a) 0.5888

b) 0.0016

c) 0.1536

d) 0.0256

e) 0.9984

SOLUTION: e

Let $X \sim Binomial(N, p)$, where $N \sim Poisson(\lambda)$. Note that N is a random variable, not a number. Answer the following **two** questions.

Question 11

What is $E[X|N = n]$?

Hint: read this as, if N were not a random variable but an integer n, then what is the mean of X?

a) np

b) p

c) n

d) N

e) Np

SOLUTION: a

Question 12

What is $E[X]$?

Hint: use the law of total expectation. For random variables A and B, $E[A] = \sum_b E[A|B = b]P(B = b)$.

a) λ

b) $n\lambda$

c) np

d) Np

e) λp

SOLUTION: e

Consider the following joint probability distribution:

	X = 0	X = 1
Y = -3	0.1	0.2
Y = -2	0.4	0.1
Y = -1	0.1	0.1

Answer the following **two** questions.

Question 13

What is the conditional distribution of Y given $X = 0$, $f_{Y|X}(y)$?

a)

$$f_{Y|X}(y) = \begin{cases} 0.2, & y = -3 \\ 0.1, & y = -2 \\ 0.1, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

b)

$$f_{Y|X}(y) = \begin{cases} 0.1667, & y = -3 \\ 0.6667, & y = -2 \\ 0.1667, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

c)

$$f_{Y|X}(y) = \begin{cases} 0.5, & y = -3 \\ 0.25, & y = -2 \\ 0.25, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

d)

$$f_{Y|X}(y) = \begin{cases} 0.3, & y = -3 \\ 0.5, & y = -2 \\ 0.2, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

e)

$$f_{Y|X}(y) = \begin{cases} 0.1, & y = -3 \\ 0.4, & y = -2 \\ 0.1, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

SOLUTION: b

Question 14

What is $E[Y|X = 0]$?

a) -2

b) -2.1

c) -1.5

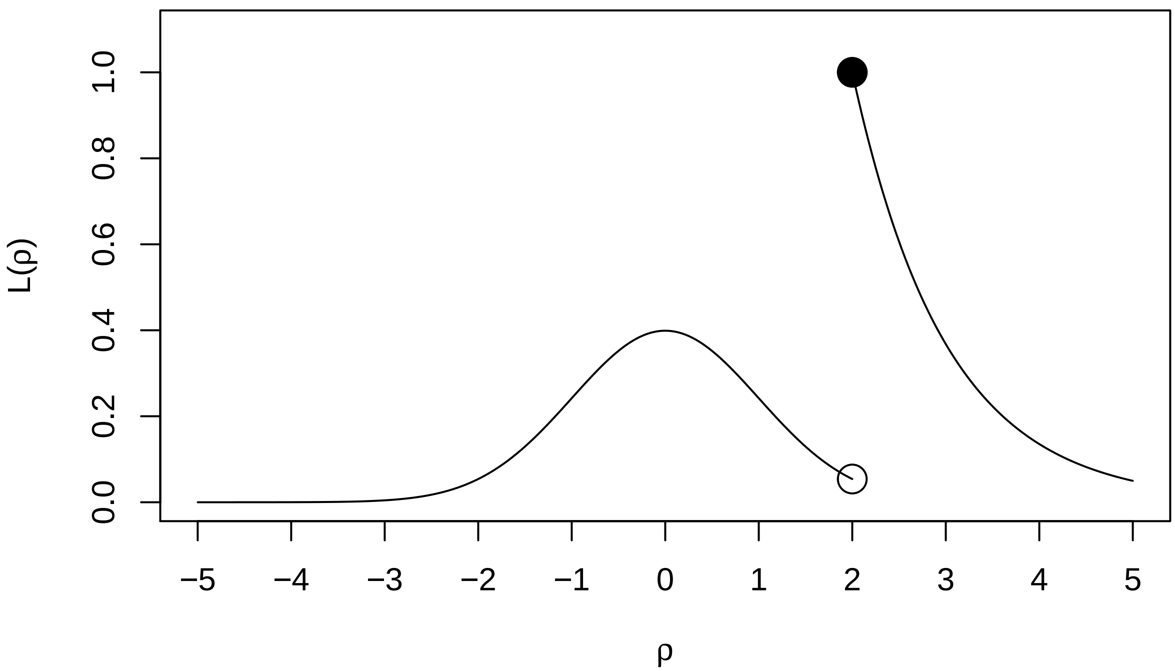
d) -1.2

e) -0.9

SOLUTION: a

Question 15

Suppose we have $X_1, \ldots, X_n \sim f_\rho$, an iid sample of random variables from a discrete distribution with pmf $f_\rho(x)$ and parameter ρ. We construct the likelihood function and plot it:



What is the maximum likelihood estimate for ρ?

a) 5

b) 3

c) 0

d) -5

e) 2

SOLUTION: e