Continuous Random Variables

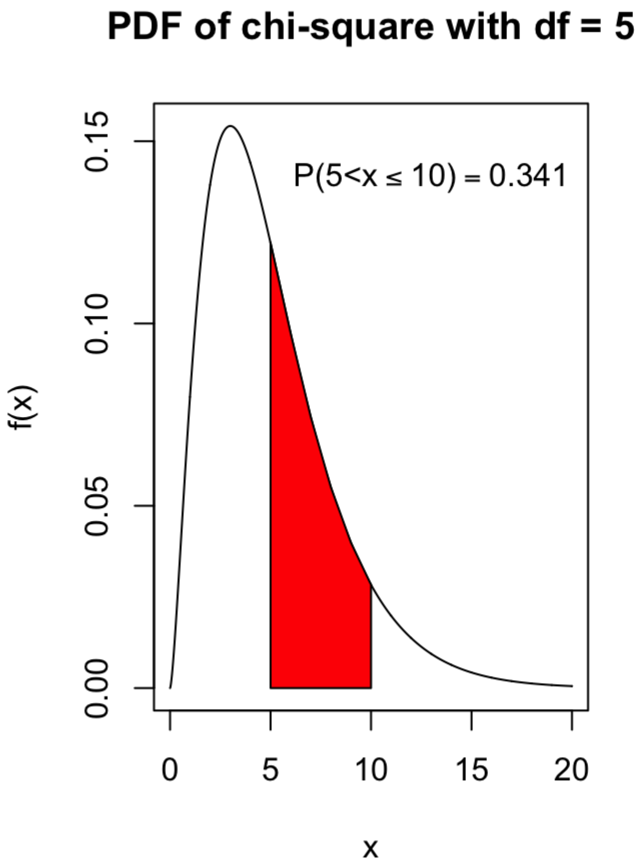
STAT 211 - 509

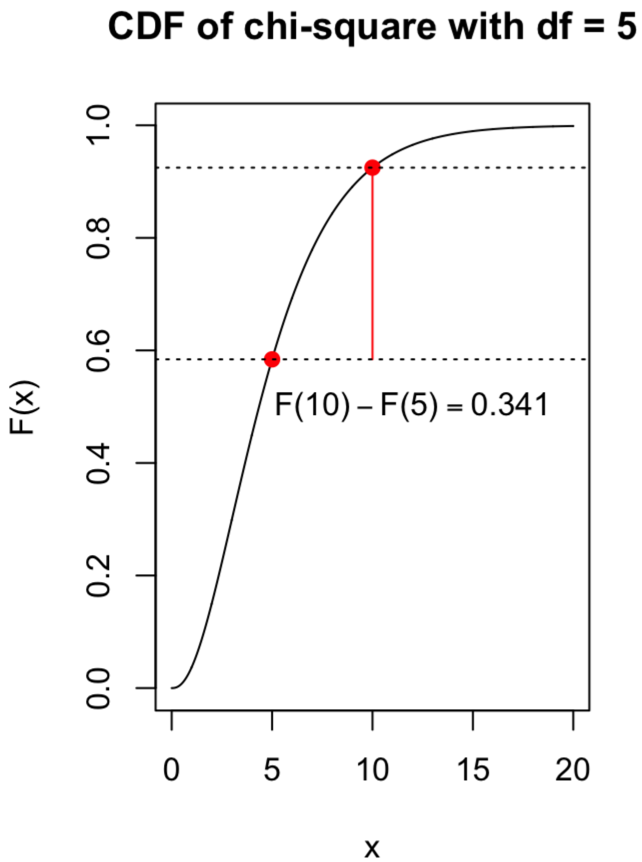
2018-10-14

Continuous Random Variables

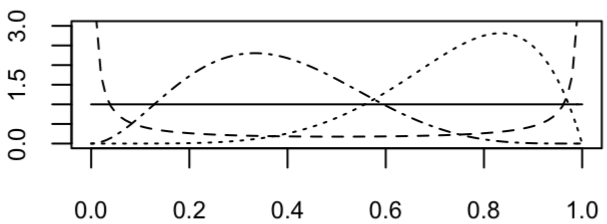
Let be a continuous random variable:

- The **probability density function (pdf)** is defined as
- The **cumulative distribution function (cdf)** is defined as
- At every point at which is continuous,

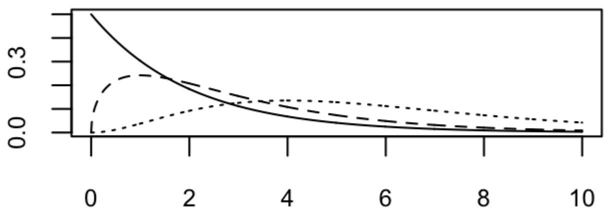

PDF of chi-square with df = 5
P(5<x ≤ 10) = 0.341
f(x)
x
0.00
0.05
0.10
0.15
0
5
10
15
20


CDF of chi-square with df = 5
F(10) − F(5) = 0.341
F(x)
x
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20

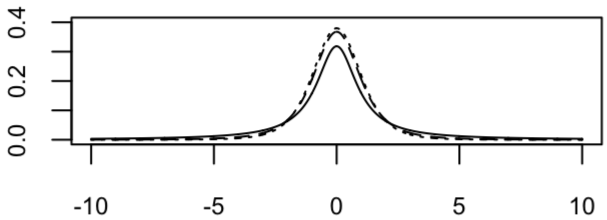
- Beta
  - conjugate prior for binomial distribution parameter
  - In R: (d|p|q|r)beta



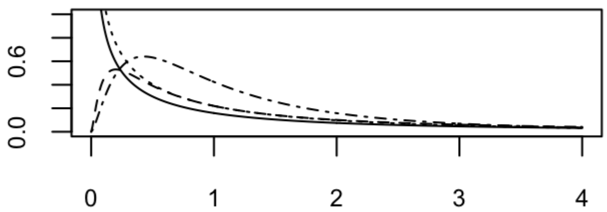
- Chi-square
    -
    - Chi-square test for goodness of fit
    - In R: (d|p|q|r)chisq



- t
    -
    - t-test for difference in means
    - In R: (d|p|q|r)t



- F
    -
    - F-test for equality of means in ANOVA
    - In R: (d|p|q|r)f



Continuous distributions in R

Suppose .

:

```
pchisq(3, 2) - pchisq(1, 2)
```

```
## [1] 0.3834005
```

:

```
dchisq(3, 2)
```

```
## [1] 0.1115651
```

Sample 10 numbers from :

```
rchisq(10, 2)
```

```
##  [1] 0.3102827 3.7648032 3.6090250 1.6723553 2.4450873 2.3167105 1.9800399
##  [8] 0.6147466 0.1892382 0.3144031
```

Expected Values With Continuous RVs

Let be a continuous random variable:

- The expected value (*mean*) of is
- The expected value of is
- The variance of is
- The moment generating function of is

Joint and Conditional Distributions

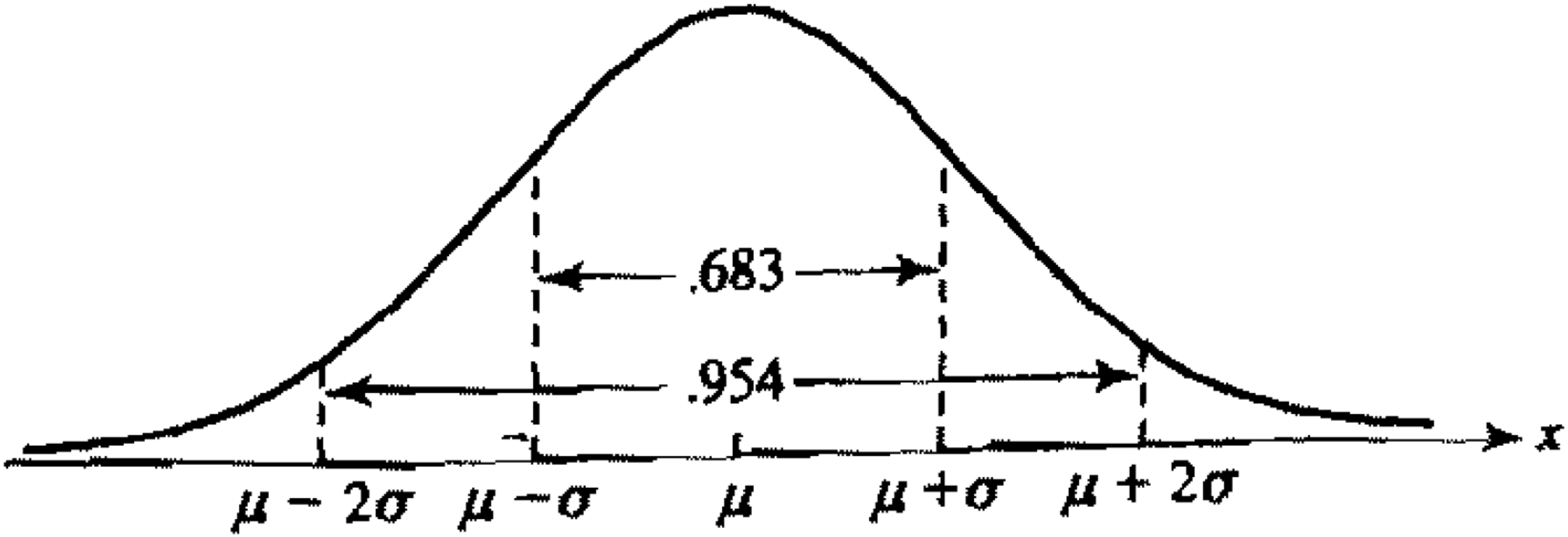
Let and be continuous random variables with marginal pdfs and and **joint probability density function** :

- Joint probabilities are double integrals of the joint pdf:
- The **conditional probability density function** of given that is:
- and are **independent** if

The Normal Distribution

The random variable has a Normal distribution with parameters and (written) if its pdf is

- ,
- Let . Then .
- The 68 / 95 / 99.7 rule:


.683
.954
x
$\mu - 2\sigma$
$\mu - \sigma$
μ
$\mu + \sigma$
$\mu + 2\sigma$

Bivariate normal

If are bivariate normal, then:

where , are the means of and , , are the variances, and is the correlation between and .

Sums of Independent Normal RVs

Let be iid Normal random variables with , . Let be arbitrary constants. Then

Special case: let . Then sample mean

—

Central Limit Theorem

Let be an IID sample from a distribution with mean and variance . This is not necessarily a Normal distribution. Assume that and .

- As , the (sampling) distribution of converges to the Normal distribution with mean and variance , regardless of the distribution from which the sample was drawn.

- If we have a “large enough” sample size, we can use a Normal distribution to approximate the sampling distribution of , regardless of the form of the population distribution.

    - This facilitates mean-based statistical inference.

    - Common rule of thumb for “large enough” is 30, although the required sample size varies depending on the situation.