

# Exam 1

Version 2

*STAT 211 - 509*

*10-4-2018*

**Name:** \_\_\_\_\_

Please sign the following pledge and read all instructions carefully before starting the exam.

Pledge: I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code. Signature: \_\_\_\_\_

## Instructions

- This test has 15 multiple choice problems. Record all answers on your large Scantron with the appropriate pencil.
- Total time is 75 minutes (12:45 P.M to 2:00 P.M.).
- You are permitted one 8.5in by 11in cheatsheet and a calculator. No other resources (phones, laptop, tablet, etc.) are allowed.
- If you are wearing a hat and/or smartwatch, please remove them while taking the test.
- There is no penalty for incorrect answers; have an answer for every question.
- When you are done, turn in your cheatsheet and exam booklet along with the Scantron.

Let  $X \sim \text{Poisson}(1)$  and  $Y \sim \text{Binomial}(8, 1/2)$ . Assume that  $X$  and  $Y$  are independent. Answer the following **three** questions:

### Question 1

What is  $E[aX + bY]$ ?

- a)  $a + 4b$
- b)  $a + 8b$
- c)  $a + b$
- d)  $2a + b$
- e)  $2a + 4b$

### Question 2

What is  $\text{Var}(aX + bY)$ ?

- a)  $a^2$
- b)  $a^2 + 2b^2$
- c)  $a^2 + 4b^2$
- d)  $2b^2$
- e)  $a^2 + b^2$

### Question 3

What is  $E[(aX + bY)^2]$ ?

- a)  $a^2 + 2b^2 - (a + 4b)^2$
  - b)  $a^2 + 2b^2 + (a + 4b)^2$
  - c)  $a^2 + 2b^2 - a - 4b$
  - d)  $a^2 + 2b^2 + a + 4b$
  - e)  $(a + 4b)^2$
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#### Question 4

Let  $L$  be the event that your bus is late and  $T$  be the event that it is on time. Suppose  $P(L) = 0.1$  and  $P(T) = 0.3$ . What is  $P(L \cup T)$ ?

- a) 0.03
  - b) 0.45
  - c) 0.4
  - d) Can't answer without knowing  $P(A \cap B)$
  - e) 0.27
- 

#### Question 5

Suppose for two events  $A$  and  $B$  we have that  $B \subset A$ , meaning  $B$  is completely contained in  $A$ . This means every outcome in  $B$  is also in  $A$ . Then  $P(A \cap B)$  is:

*Hint: draw a Venn diagram. For an example of an event completely contained in another, let  $P$  be the event you pass this exam and  $A$  be the event you get an A.  $A \subset P$ , since all the scores you get that lead to an A also lead to you passing.*

- a)  $P(A) + P(B)$
  - b)  $P(B)$
  - c)  $P(A)$
  - d) 0
  - e) 1
-

3% of a population is infected with HIV. For a certain HIV test there is a 98% chance the test will be positive given that the tested individual is actually infected with HIV. If the person is not infected, there is a 4% chance the test will be positive. Answer the following **two** questions.

### Question 6

What is the probability that the test is positive for a randomly selected person from the population?

- a) 0.0682
- b) 1
- c) 0.0294
- d) 0.03
- e) 0.0388

### Question 7

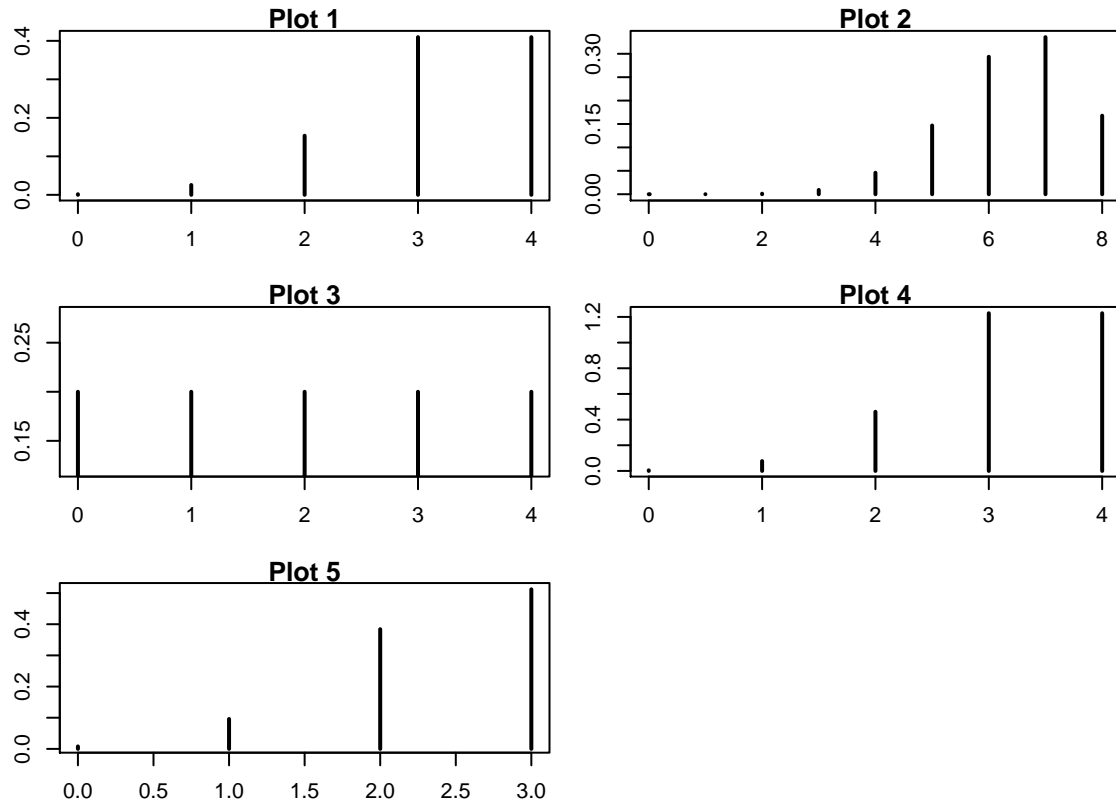
If a randomly selected person from the population is tested positive, what is the probability that the person is actually infected?

- a) 0.04
  - b) 0.0294
  - c) 0.4311
  - d) 0.98
  - e) 0.03
-

Let  $X \sim \text{Binomial}(n = 4, p = 0.8)$ . Answer the following **three** questions.

### Question 8

Which of the following plots is of the pmf for  $X$ ? The  $y$  axis is  $f(x)$  for each plot.



- a) Plot 1
- b) Plot 2
- c) Plot 3
- d) Plot 4
- e) Plot 5

### Question 9

Which of the following correctly computes  $P(X \leq 2)$  in R?

- a) `dbinom(x = 2, size = 4, prob = 0.8)`
- b) `pbinom(q = 2, size = 4, prob = 0.8)`
- c) `rbinom(n = 2, size = 4, prob = 0.8)`
- d) `pgeom(q = 2, prob = 0.8)`
- e) `pbinom(q = 2, size = 3, prob = 1)`

### Question 10

What is  $P(X \geq 1)$ ?

- a) 0.5888
  - b) 0.0016
  - c) 0.1536
  - d) 0.0256
  - e) 0.9984
- 

Let  $X \sim \text{Binomial}(N, p)$ , where  $N \sim \text{Poisson}(\lambda)$ . Note that  $N$  is a random variable, not a number. Answer the following **two** questions.

### Question 11

What is  $E[X|N = n]$ ?

*Hint: read this as, if  $N$  were not a random variable but an integer  $n$ , then what is the mean of  $X$ ?*

- a)  $np$
- b)  $p$
- c)  $n$
- d)  $N$
- e)  $Np$

### Question 12

What is  $E[X]$ ?

*Hint: use the law of total expectation. For random variables  $A$  and  $B$ ,  $E[A] = \sum_b E[A|B = b]P(B = b)$ .*

- a)  $\lambda$
  - b)  $n\lambda$
  - c)  $np$
  - d)  $Np$
  - e)  $\lambda p$
-

Consider the following joint probability distribution:

	X = 0	X = 1
Y = -3	0.1	0.2
Y = -2	0.4	0.1
Y = -1	0.1	0.1

Answer the following **two** questions.

### Question 13

What is the conditional distribution of  $Y$  given  $X = 0$ ,  $f_{Y|X}(y)$ ?

a)

$$f_{Y|X}(y) = \begin{cases} 0.2, & y = -3 \\ 0.1, & y = -2 \\ 0.1, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

b)

$$f_{Y|X}(y) = \begin{cases} 0.1667, & y = -3 \\ 0.6667, & y = -2 \\ 0.1667, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

c)

$$f_{Y|X}(y) = \begin{cases} 0.5, & y = -3 \\ 0.25, & y = -2 \\ 0.25, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

d)

$$f_{Y|X}(y) = \begin{cases} 0.3, & y = -3 \\ 0.5, & y = -2 \\ 0.2, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

e)

$$f_{Y|X}(y) = \begin{cases} 0.1, & y = -3 \\ 0.4, & y = -2 \\ 0.1, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

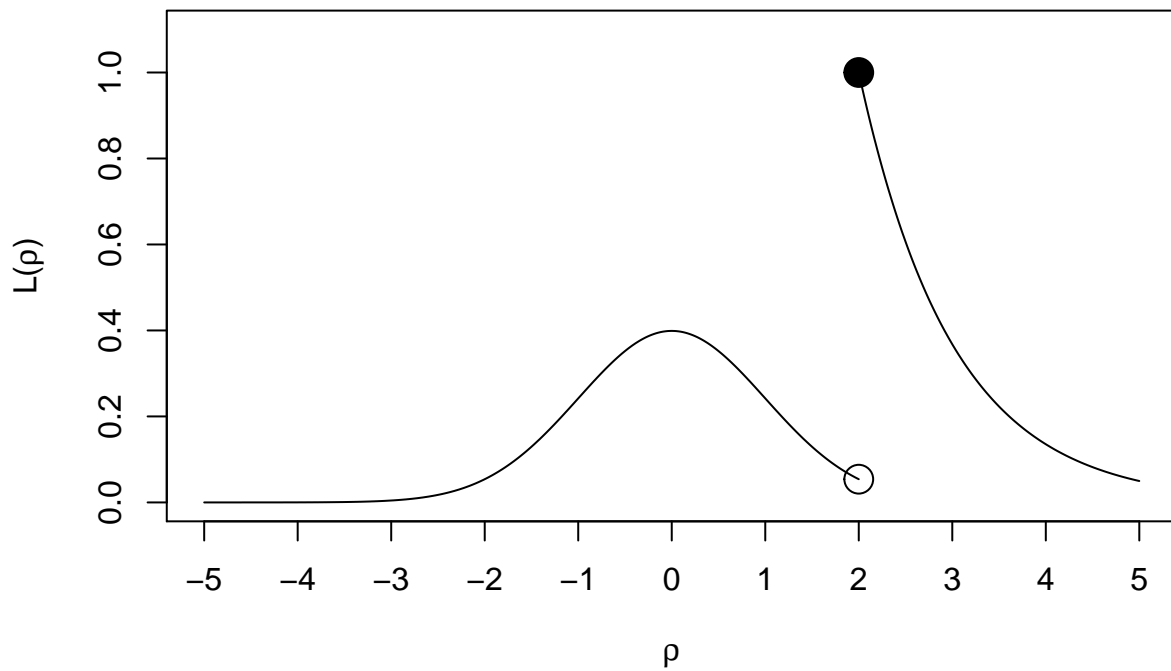
### Question 14

What is  $E[Y|X = 0]$ ?

- a) -2
  - b) -2.1
  - c) -1.5
  - d) -1.2
  - e) -0.9
- 

### Question 15

Suppose we have  $X_1, \dots, X_n \sim f_\rho$ , an iid sample of random variables from a discrete distribution with pmf  $f_\rho(x)$  and parameter  $\rho$ . We construct the likelihood function and plot it:



What is the maximum likelihood estimate for  $\rho$ ?

- a) 5
- b) 3
- c) 0
- d) -5
- e) 2