

Exam 1

Version 1

STAT 211 - 509

10-4-2018

Name: _____

Please sign the following pledge and read all instructions carefully before starting the exam.

Pledge: I have neither given nor received any unauthorized aid in completing this exam, and I have conducted myself within the guidelines of the University Honor Code. Signature: _____

Instructions

- This test has 15 multiple choice problems. Record all answers on your large Scantron with the appropriate pencil.
- Total time is 75 minutes (12:45 P.M to 2:00 P.M.).
- You are permitted one 8.5in by 11in cheatsheet and a calculator. No other resources (phones, laptop, tablet, etc.) are allowed.
- If you are wearing a hat and/or smartwatch, please remove them while taking the test.
- When you are done, turn in your cheatsheet and exam booklet along with the Scantron.

Let $X \sim \text{Poisson}(1)$ and $Y \sim \text{Binomial}(8, 1/2)$. Assume that X and Y are independent. Answer the following **three** questions:

Question 1

What is $E[aX + bY]$?

- a) $a + b$
- b) $2a + b$
- c) $2a + 4b$
- d) $a + 8b$
- e) $a + 4b$

SOLUTION: e

Question 2

What is $\text{Var}(aX + bY)$?

- a) a^2
- b) $2b^2$
- c) $a^2 + b^2$
- d) $a^2 + 4b^2$
- e) $a^2 + 2b^2$

SOLUTION: e

Question 3

What is $E[(aX + bY)^2]$?

- a) $(a + 4b)^2$
- b) $a^2 + 2b^2 + (a + 4b)^2$
- c) $a^2 + 2b^2 - (a + 4b)^2$
- d) $a^2 + 2b^2 + a + 4b$
- e) $a^2 + 2b^2 - a - 4b$

SOLUTION: b

Question 4

Let L be the event that your bus is late and T be the event that it is on time. Suppose $P(L) = 0.1$ and $P(T) = 0.3$. What is $P(L \cup T)$?

- a) 0.45
- b) 0.03
- c) 0.27
- d) Can't answer without knowing $P(A \cap B)$
- e) 0.4

SOLUTION: e

Question 5

Suppose for two events A and B we have that $B \subset A$, meaning B is completely contained in A . This means every outcome in B is also in A . Then $P(A \cap B)$ is:

Hint: draw a Venn diagram. For an example of an event completely contained in another, let P be the event you pass this exam and A be the event you get an A. $A \subset P$, since all the scores you get that lead to an A also lead to you passing.

- a) 0
- b) $P(B)$
- c) $P(A) + P(B)$
- d) 1
- e) $P(A)$

SOLUTION: b

4% of a population is infected with HIV. For a certain HIV test there is a 98% chance the test will be positive given that the tested individual is actually infected with HIV. If the person is not infected, there is a 6% chance the test will be positive. Answer the following **two** questions.

Question 6

What is the probability that the test is positive for a randomly selected person from the population?

- a) 0.0392
- b) 1
- c) 0.04
- d) 0.0968
- e) 0.0576

SOLUTION: d

Question 7

If a randomly selected person from the population is tested positive, what is the probability that the person is actually infected?

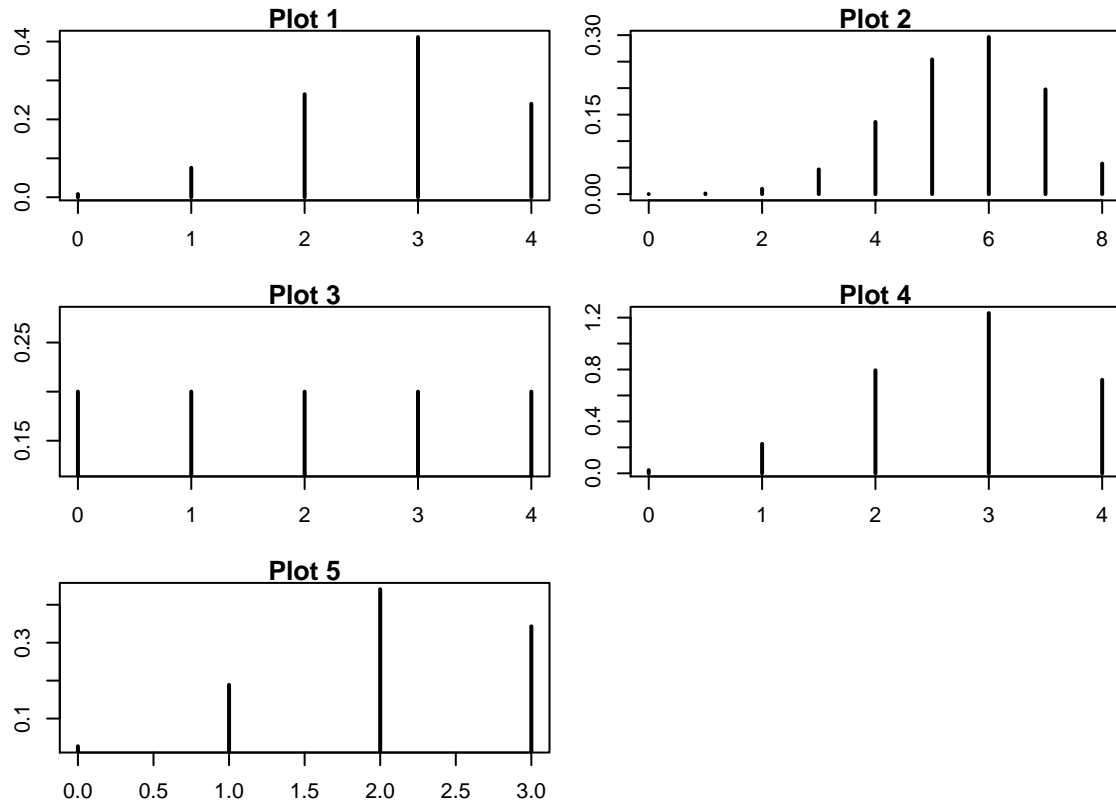
- a) 0.06
- b) 0.0392
- c) 0.405
- d) 0.98
- e) 0.04

SOLUTION: c

Let $X \sim \text{Binomial}(n = 4, p = 0.7)$. Answer the following **three** questions.

Question 8

Which of the following plots is of the pmf for X ? The y axis is $f(x)$ for each plot.



- a) Plot 1
- b) Plot 2
- c) Plot 3
- d) Plot 4
- e) Plot 5

SOLUTION: a

Question 9

Which of the following correctly computes $P(X \leq 2)$ in R?

- a) `rbinom(n = 2, size = 4, prob = 0.7)`
- b) `pgeom(q = 2, prob = 0.7)`
- c) `dbinom(x = 2, size = 4, prob = 0.7)`
- d) `pbinom(q = 2, size = 3, prob = 0.9)`
- e) `pbinom(q = 2, size = 4, prob = 0.7)`

SOLUTION: e

Question 10

What is $P(X \geq 1)$?

- a) 0.9919
- b) 0.0081
- c) 0.0756
- d) 0.2646
- e) 0.7518

SOLUTION: a

Let $X \sim \text{Binomial}(N, p)$, where $N \sim \text{Poisson}(\lambda)$. Note that N is a random variable, not a number. Answer the following **two** questions.

Question 11

What is $E[X|N = n]$?

Hint: read this as, if N were not a random variable but an integer n , then what is the mean of X ?

- a) N
- b) Np
- c) np
- d) p
- e) n

SOLUTION: c

Question 12

What is $E[X]$?

Hint: use the law of total expectation. For random variables A and B , $E[A] = \sum_b E[A|B = b]P(B = b)$.

- a) np
- b) Np
- c) $n\lambda$
- d) λp
- e) λ

SOLUTION: d

Consider the following joint probability distribution:

	X = 0	X = 1
Y = -3	0.1	0.4
Y = -2	0.2	0.1
Y = -1	0.1	0.1

Answer the following **two** questions.

Question 13

What is the conditional distribution of Y given $X = 0$, $f_{Y|X}(y)$?

a)

$$f_{Y|X}(y) = \begin{cases} 0.6667, & y = -3 \\ 0.1667, & y = -2 \\ 0.1667, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

b)

$$f_{Y|X}(y) = \begin{cases} 0.25, & y = -3 \\ 0.5, & y = -2 \\ 0.25, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

c)

$$f_{Y|X}(y) = \begin{cases} 0.4, & y = -3 \\ 0.1, & y = -2 \\ 0.1, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

d)

$$f_{Y|X}(y) = \begin{cases} 0.5, & y = -3 \\ 0.3, & y = -2 \\ 0.2, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

e)

$$f_{Y|X}(y) = \begin{cases} 0.1, & y = -3 \\ 0.2, & y = -2 \\ 0.1, & y = -1 \\ 0, & \text{otherwise} \end{cases}$$

SOLUTION: b

Question 14

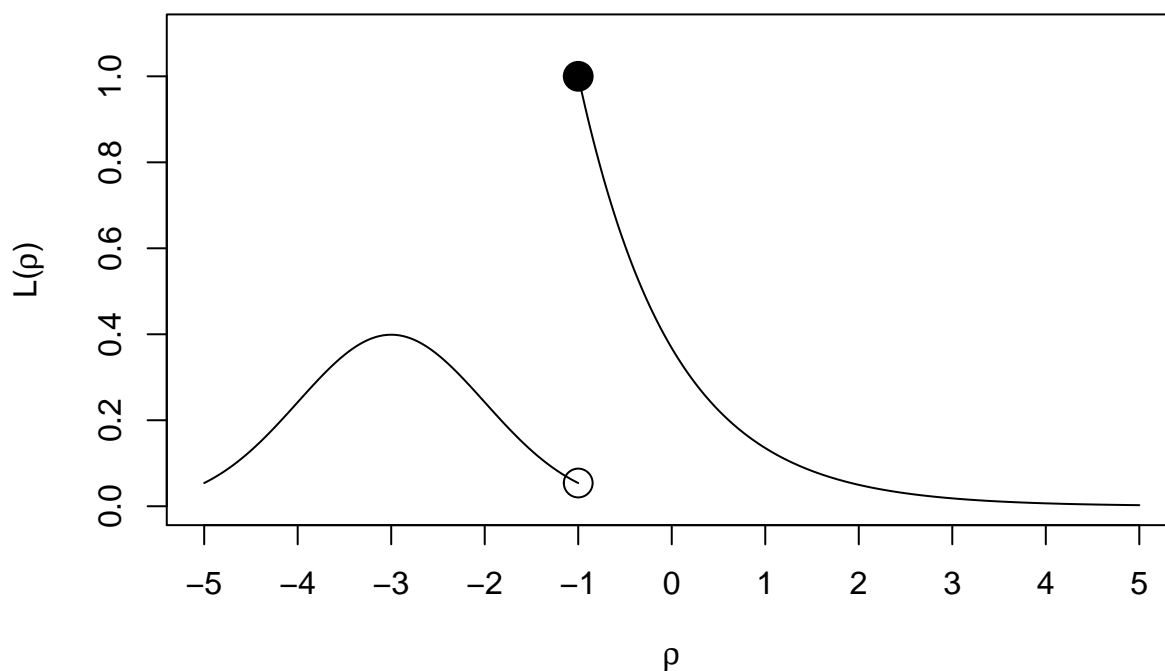
What is $E[Y|X = 0]$?

- a) -0.8
- b) -2.3
- c) -3.75
- d) -1.5
- e) -2

SOLUTION: e

Question 15

Suppose we have $X_1, \dots, X_n \sim f_\rho$, an iid sample of random variables from a discrete distribution with pmf $f_\rho(x)$ and parameter ρ . We construct the likelihood function and plot it:



What is the maximum likelihood estimate for ρ ?

- a) -5
- b) 0
- c) -3
- d) 5
- e) -1

SOLUTION: e