

**Statistics 211**  
**In-Class Assessments**

Topic: Chapter 5

Date: Oct. 11, 2016

1. Consider a Binomial experiment, with  $n$  independent Bernoulli trials each with probability  $p$  of success.
  - (a) Suppose we wish to test the null hypothesis  $H_0 : p = p_0$  vs. the two-sided alternative  $H_a : p \neq p_0$ . We will test at *significance level*  $\alpha$  (what we have also called inference with  $(1 - \alpha) \times 100\%$  *confidence*).
    - i. Suppose  $H_0$  is true. If we use significance level  $\alpha = 0.05$ , what is the probability we will commit a Type I error due to a p-value less than 0.05?
    - ii. Suppose  $H_0$  is true. If we use significance level  $\alpha = 0.01$ , what is the probability we will commit a Type I error due to a p-value less than 0.01?
    - iii. For a given alternative value  $p_1 \neq p_0$ , the *power* of a test (e.g., a bootstrap-based test) is the probability of rejecting  $H_0$  due to a p-value less than  $\alpha$ . Consider two possible alternative values  $p_1$  and  $p_2$  such that  $p_0 < p_1 < p_2$ . Which of the following would you expect to be true?
      - A. Power to detect  $p_1$  = power to detect  $p_2$  =  $\alpha$
      - B. Power to detect  $p_1$  < power to detect  $p_2$
      - C. Power to detect  $p_1$  > power to detect  $p_2$- 2. Consider a random sample  $x_1, x_2, \dots, x_n$  from a population with median  $\theta$ . Suppose we wish to test the null hypothesis  $H_0 : \theta = 0$  vs. the one-sided alternative  $H_a : \theta > 0$ . Which of the following shows how to use the bootstrap to compute a p-value? In each, let  $\mathbf{x}$  be the R variable containing our  $n$  observations.

(a) Code:

```
med_obs <- median(x)
x_0 <- x - median(x)
med_b <- numeric(B)
for(b in 1:B) {
  med_b[b] <- median(sample(x, replace = TRUE))
}
p_val <- sum(med_b > med_obs) / B
```

(b) Code:

```
med_obs <- median(x)
x_0 <- x - median(x)
med_b <- numeric(B)
for(b in 1:B) {
  med_b[b] <- mean(sample(x_0, replace = FALSE))
}
```

```
}  
p_val <- sum(med_b > med_obs) / B
```

(c) Code:

```
med_obs <- median(x)  
x_0 <- x - median(x)  
med_b <- numeric(B)  
for(b in 1:B) {  
  med_b[b] <- mean(sample(x, replace = TRUE))  
}  
p_val <- sum(med_b > med_obs) / B
```

(d) Code:

```
med_obs <- median(x)  
x_0 <- x - median(x)  
med_b <- numeric(B)  
for(b in 1:B) {  
  med_b[b] <- median(sample(x_0, replace = TRUE))  
}  
p_val <- sum(med_b > med_obs) / B
```