TN3125
Information and Computation

Lecture 1

1B - Introduction to information theory

The abstract communications model

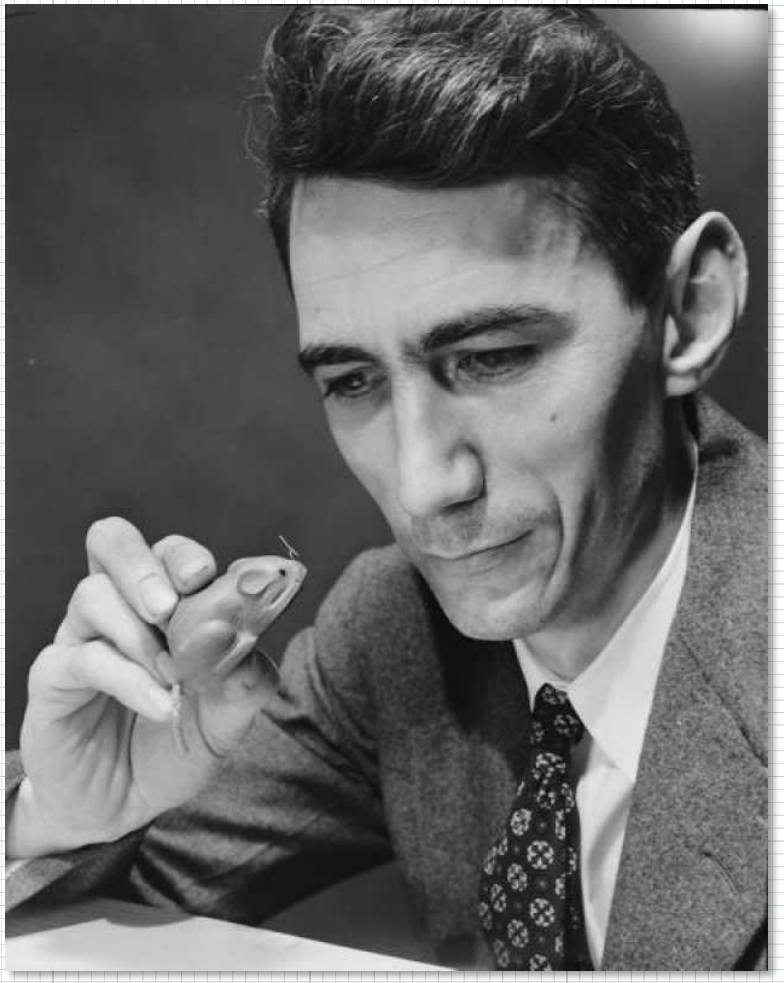
The fundamental problem of communication is that of reliably transmitting information over a channel

Examples:

VOICE → EARS

EARS → BRAIN

ANTENA → SATELLITE



A mathematical theory of communications.
Claude Shannon. Bell systems technical journal (1948)

What is information?

0

1

	Days with no rain	Days with rain
Rotterdam	212	153
Atacama desert	360	5

Exercise 1.1 - Let us assume that you are living in the Atacama desert where it rarely rains. You receive a 0. How much information does this message carry?

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Exercise 1.1 - Let us assume that you are living in the Atacama desert where it rarely rains. You receive a 0. How much information does this message carry?

Exercise 1.2 - Now let us assume that you live in the Netherlands where it does rain quite often, but certainly not every day. You receive a 0. Does the message contain information?

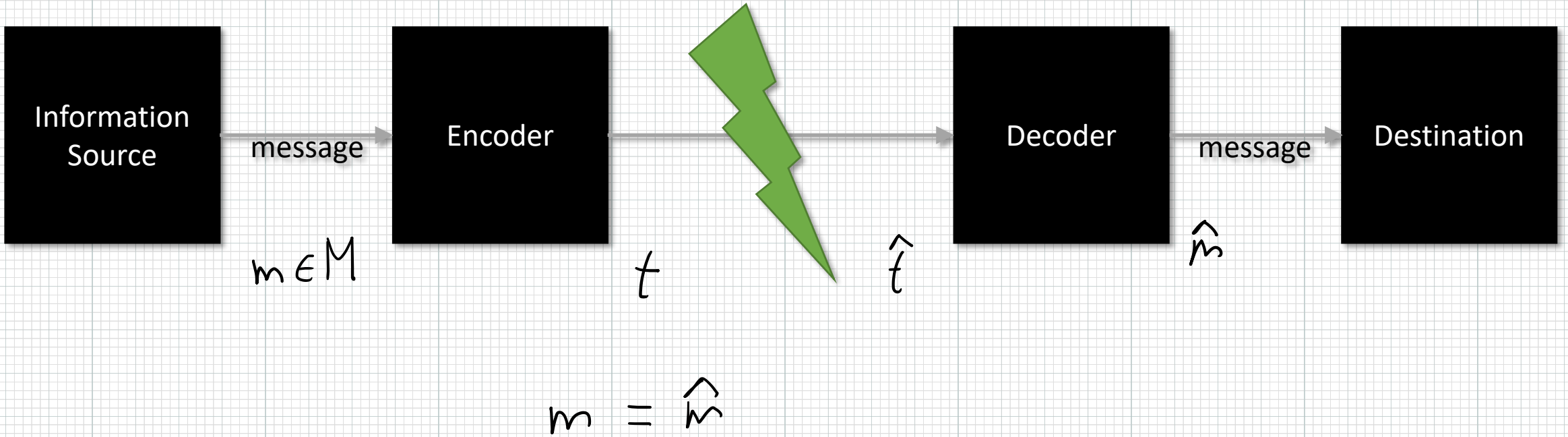
	Days with no rain	Days with rain
Summer	182	1
Winter	30	152

Exercise 1.1 - Let us assume that you are living in the Atacama desert where it rarely rains. You receive a 0. How much information does this message carry?

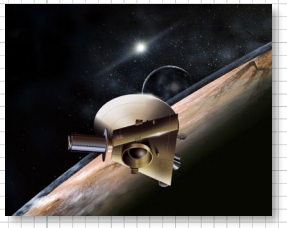
Exercise 1.2 - Now let us assume that you live in the Netherlands where it does rain quite often, but certainly not every day. You receive a 0. Does the message contain information?

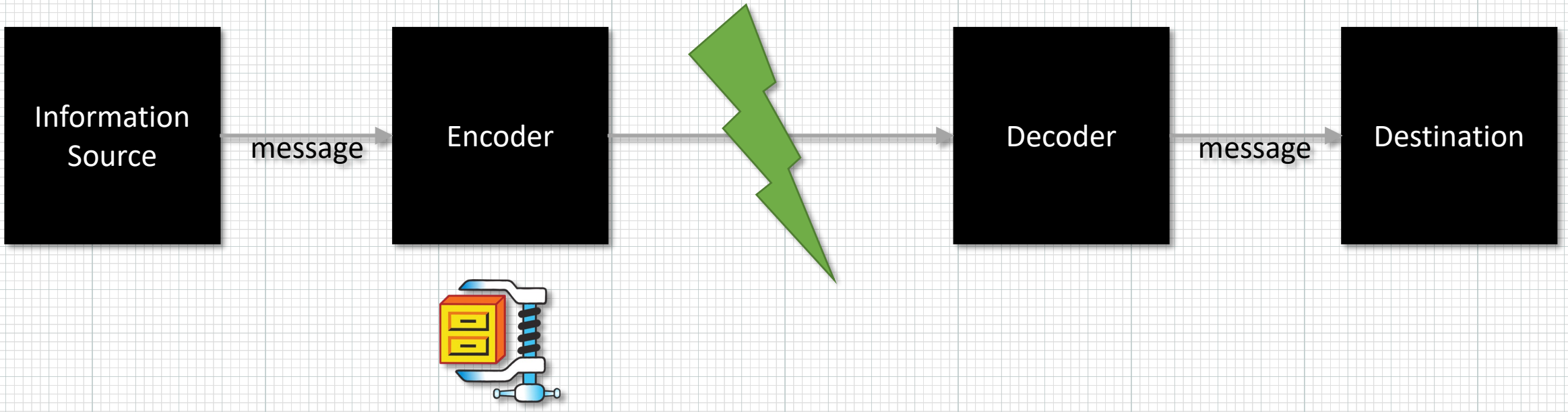
Exercise 1.3 - Finally, let us assume that you live in the Netherlands but (boldly) also that you are aware of the current season. You receive a 0. Does the message 0 carry the same information in summer and in winter?

The abstract communications model



The abstract communications model





Lecture 1: Learning goals

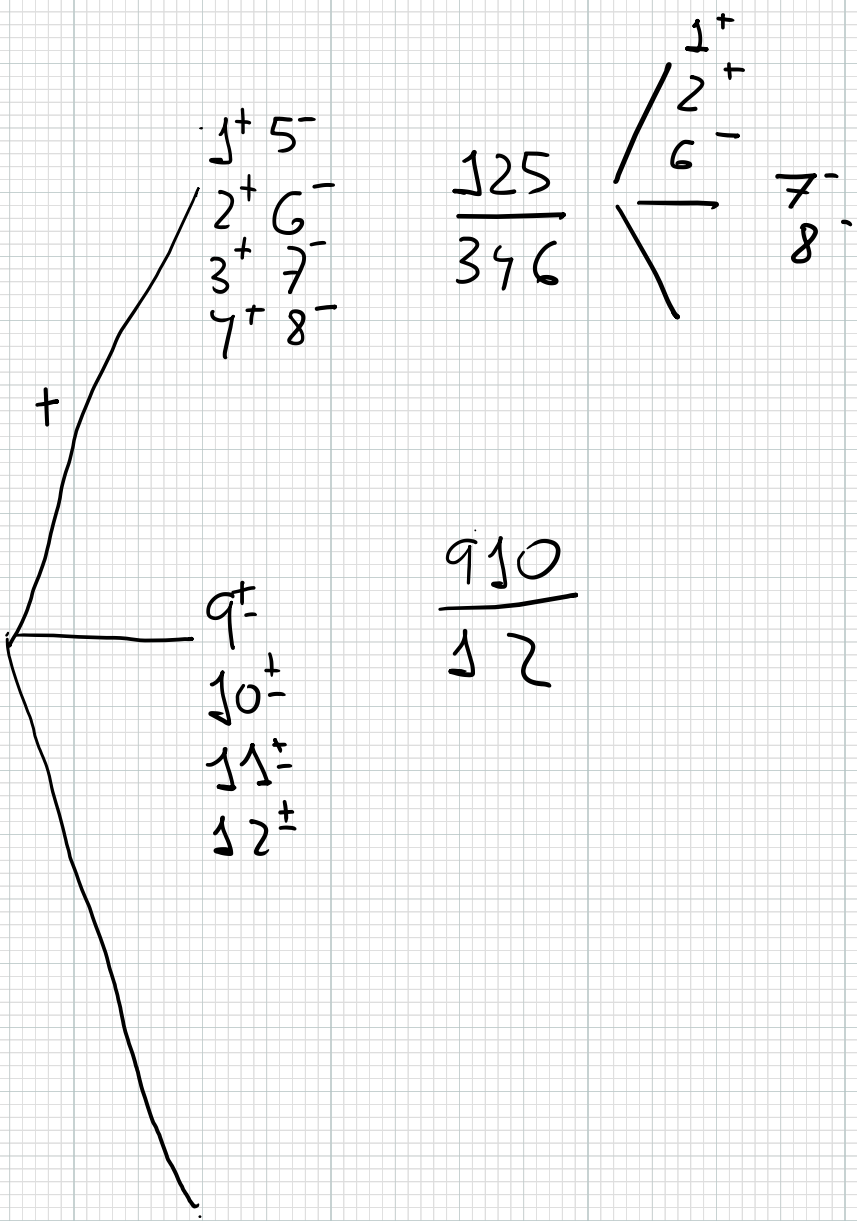
- Quantify the information of an event
- Quantify the average information of a probability distribution
- Quantify conditional information
- Derive basic properties of the entropy function

Exercise. There are 12 coins, one of which is a counterfeit. The counterfeit is either lighter of heavier than normal coins but not of the same weight. You are given a two plate scale where you can compare weights.

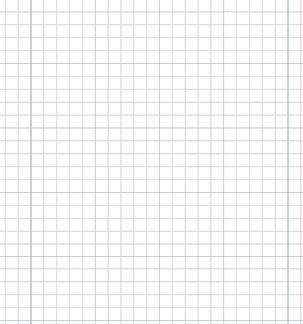
- Devise a scheme that detects the counterfeit coin when it is heavier
- Devise a scheme that detects the counterfeit coin in the general case

$1^{\pm}$
$2^{\pm}$
$3^{\pm}$
$4^{\pm}$
$5^{\pm}$
$6^{\pm}$
$7^{\pm}$
$8^{\pm}$
$9^{\pm}$
$10^{\pm}$
$11^{\pm}$
$12^{\pm}$

$\frac{1234}{5678}$



$\frac{125}{346}$



$\frac{910}{12}$

Exercise. There are 12 coins, one of which is a counterfeit. The counterfeit is either lighter of heavier than normal coins but not of the same weight. You are given a two plate scale where you can compare weights.

- Devise a scheme that detects the counterfeit coin when it is heavier
- Devise a scheme that detects the counterfeit coin in the general case
- Is it possible to detect the counterfeit coin with three weighings? And with two?