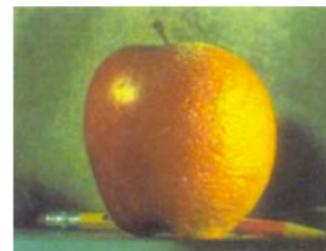


2. Image Formation



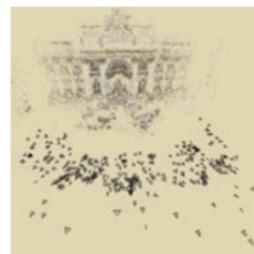
3. Image Processing



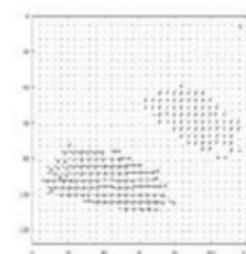
4. Features



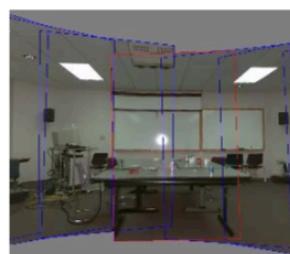
5. Segmentation



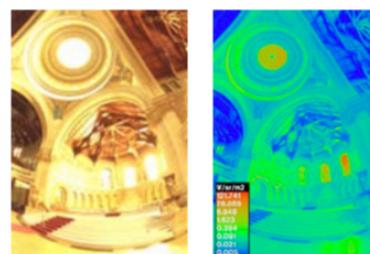
6-7. Structure from Motion



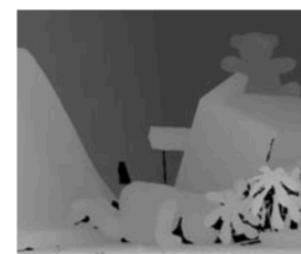
8. Motion



9. Stitching



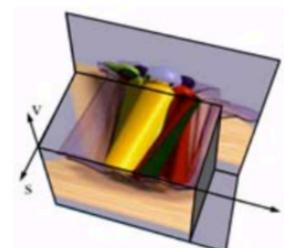
10. Computational Photography



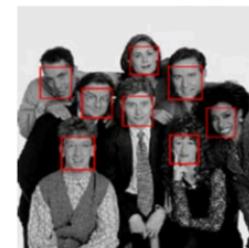
11. Stereo



12. 3D Shape

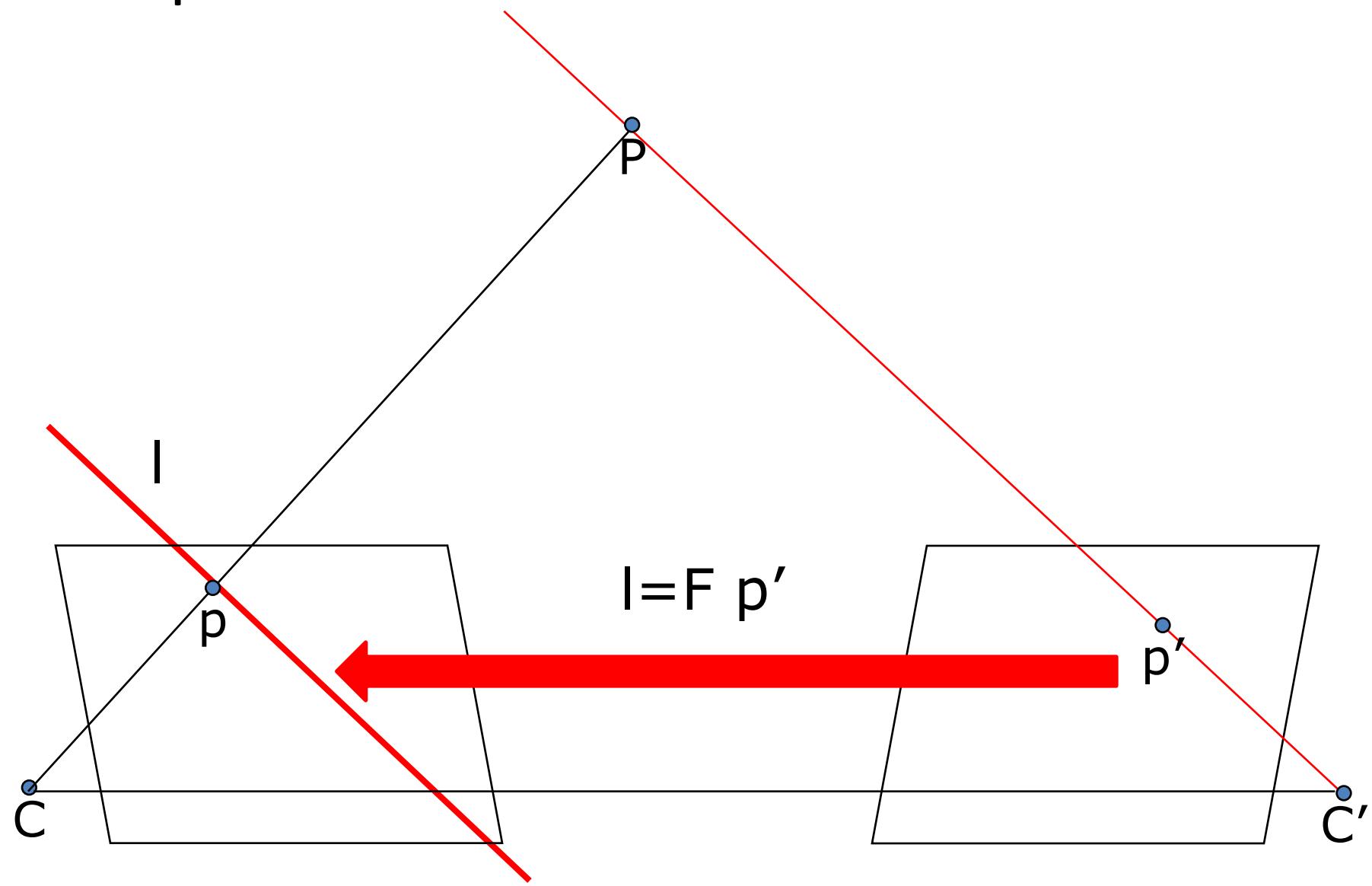


13. Image-based Rendering



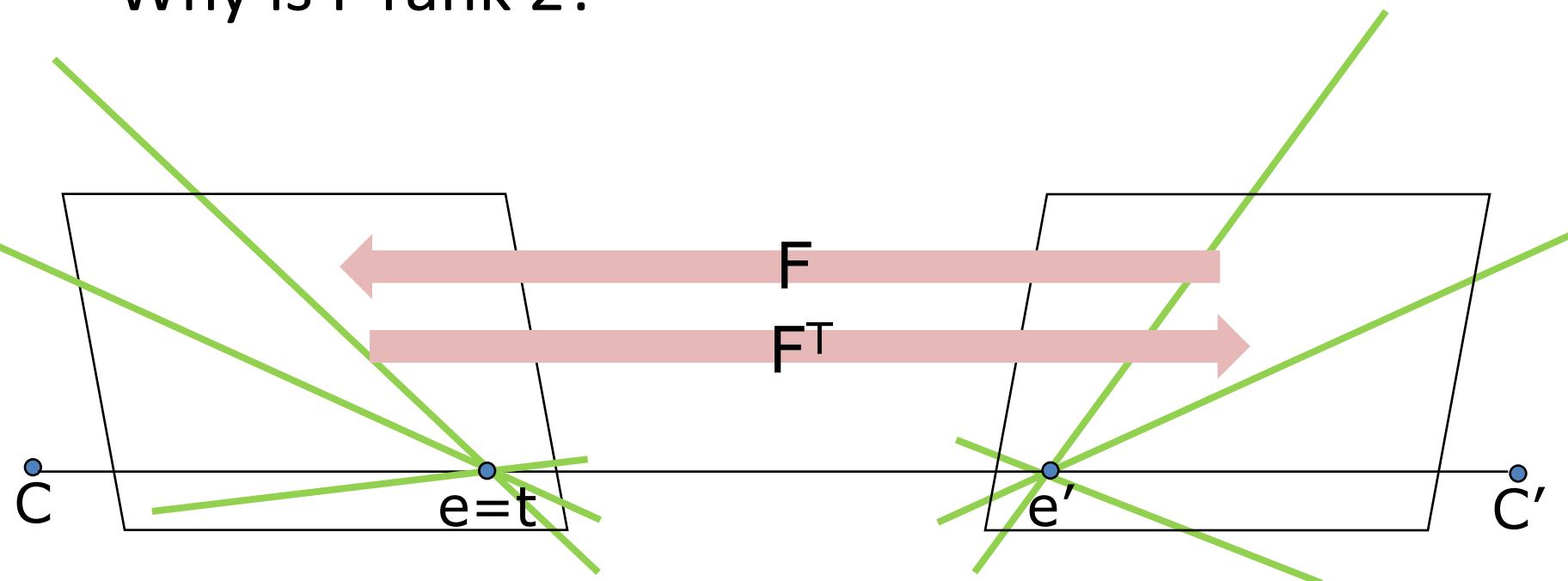
14. Recognition

Recap: Two views and Fundamental Matrix F



Rank 2 Constraint

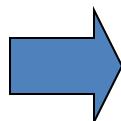
- Why is F rank 2?



- Not invertible! Collection of points is mapped to a pencil of lines. Epipoles map to zero.
- What would it mean to be rank 1?

The Eight-Point Algorithm (Longuet-Higgins, 1981)

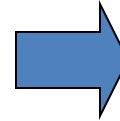
$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$



$$(uu', uv', u, vu', vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$



$$\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$



Minimize:

$$\sum_{i=1}^n (\mathbf{p}_i^T \mathcal{F} \mathbf{p}_i')^2$$

under the constraint

$$|\mathcal{F}|^2 = 1.$$

The Normalized Eight-Point Algorithm (Hartley, 1995)

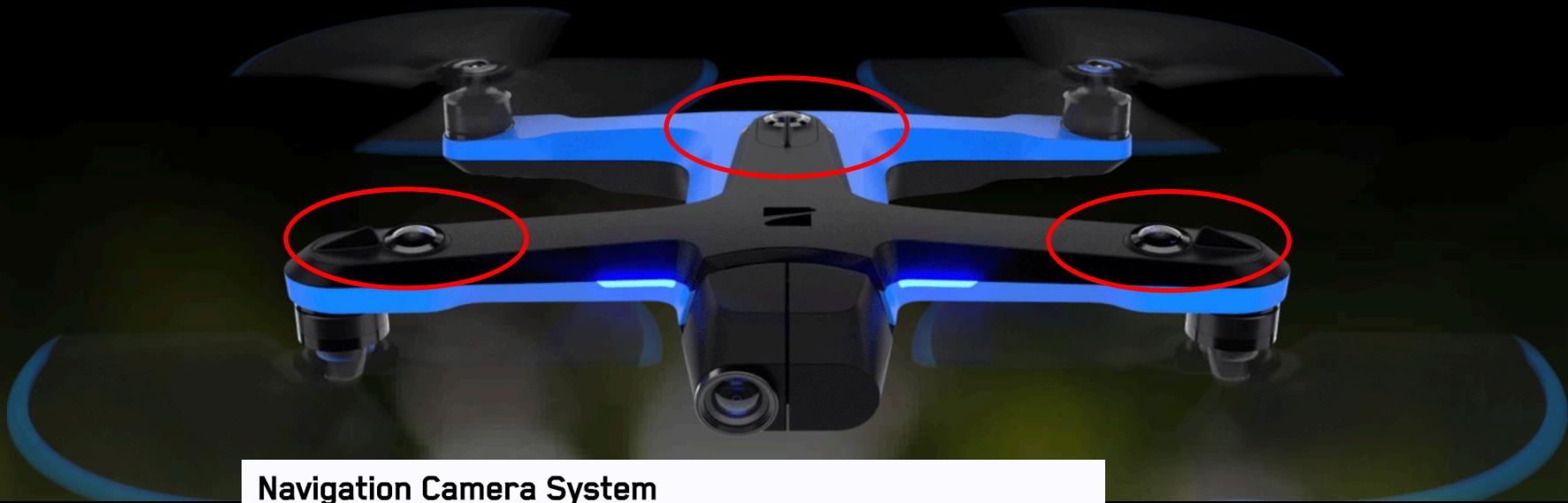
- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels:

$$q_i = T p_i \quad q'_i = T' p'_i.$$

- Use the eight-point algorithm to compute \mathcal{F} from the points q_i and q'_i .
- Enforce the rank-2 constraint.
- Output $T^{-1} \mathcal{F} T'$.

Trinocular Camera rigs

Skydio 2



Navigation Camera System

CONFIGURATION

6x cameras in trinocular configuration top
and bottom

SENSOR TYPE

Sony 1/3" 4K color CMOS

LENS APERTURE

f/1.8

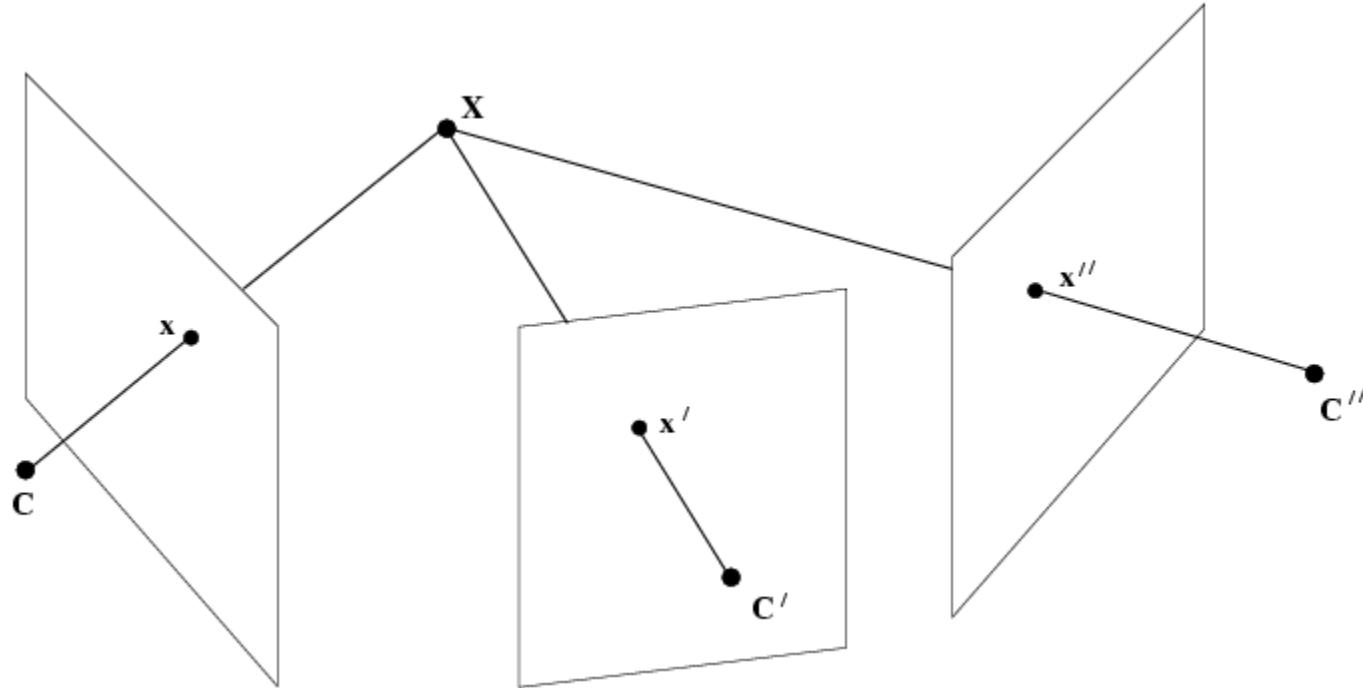
FIELD-OF-VIEW

200°

ENVIRONMENT COVERAGE

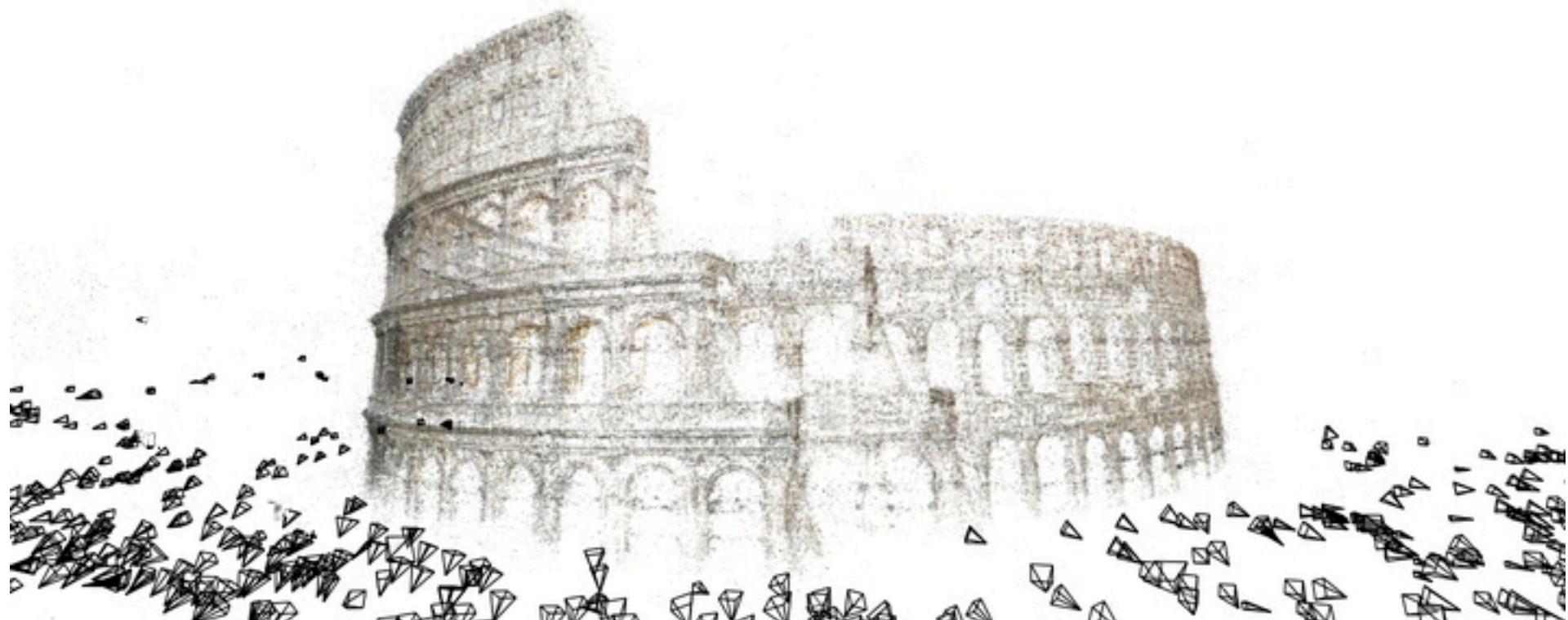
True 360°

Trifocal Geometry



$$[\mathbf{x}'] \times \left(\sum_i x^i \mathbf{T}_i \right) [\mathbf{x}''] \times = \mathbf{0}_{3 \times 3}$$

Structure from Motion



Building Rome in a Day
Agarwal et al

Frank Dellaert Fall 2019

Motivation

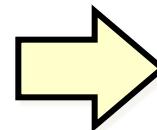
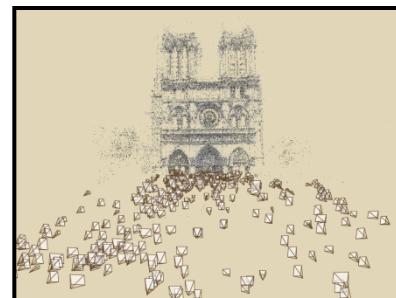
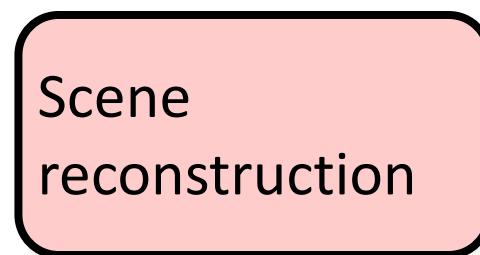
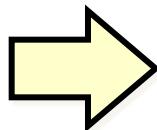
- Photo Tourism
- Photosynth
- Multi-view stereo
- Building Rome in a Day
- Rome on a Cloudless Day

Photo Tourism

Noah Snavely, Steven M. Seitz, Richard Szeliski, [Photo tourism: Exploring photo collections in 3D," ACM Transactions on Graphics \(SIGGRAPH Proceedings\), 25\(3\), 2006, 835-846.](#)



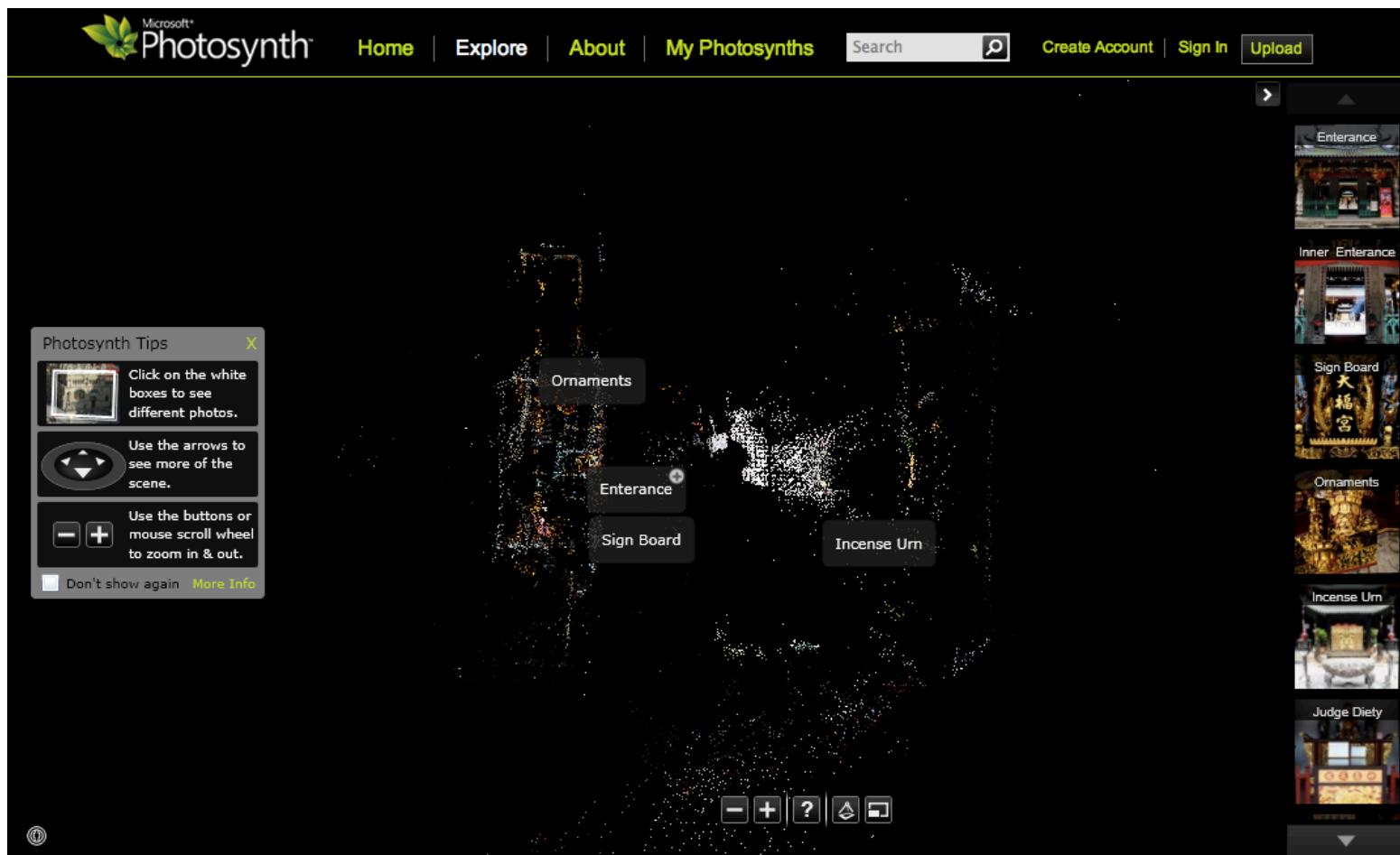
Input photographs



<http://phototour.cs.washington.edu/>

Photosynth

photosynth.net



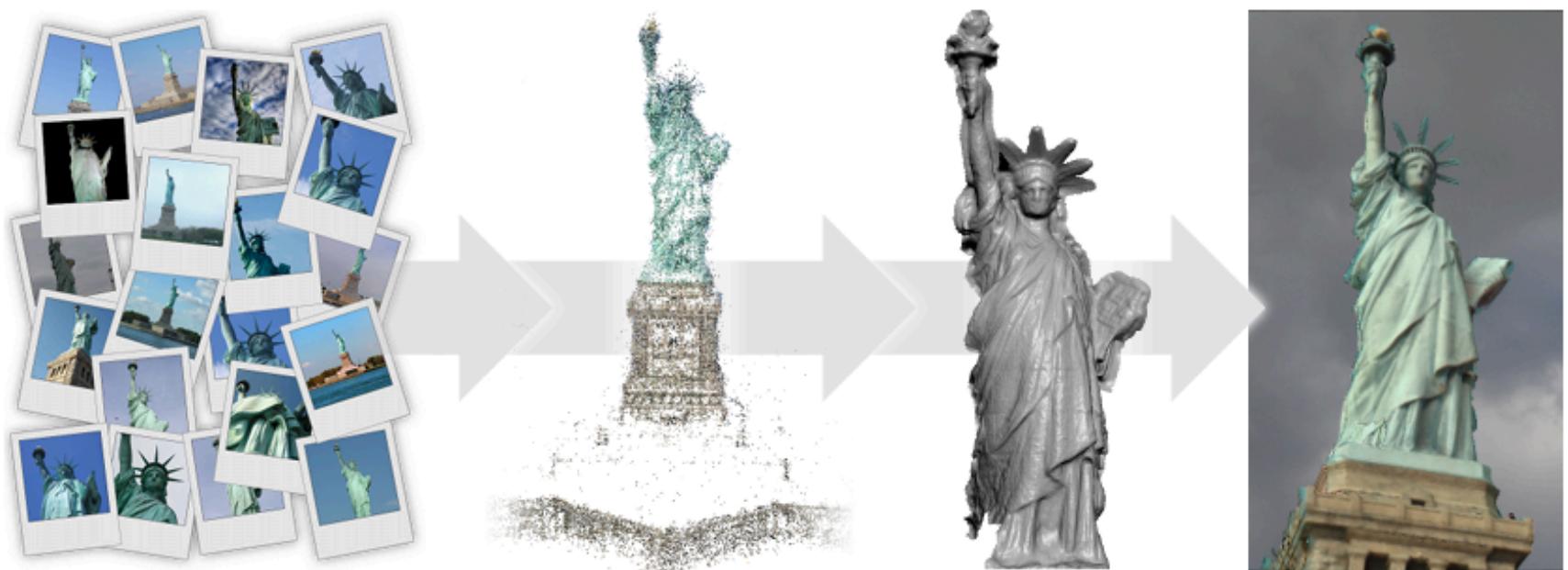
- <http://photosynth.net/view.aspx?cid=29aa8616-a43a-43e4-9d6e-b8ad9b50483e>

Multi-view Stereo

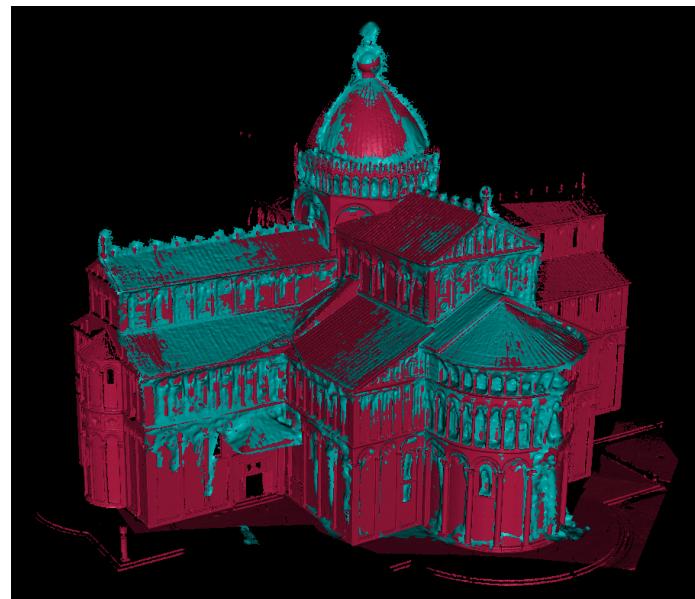
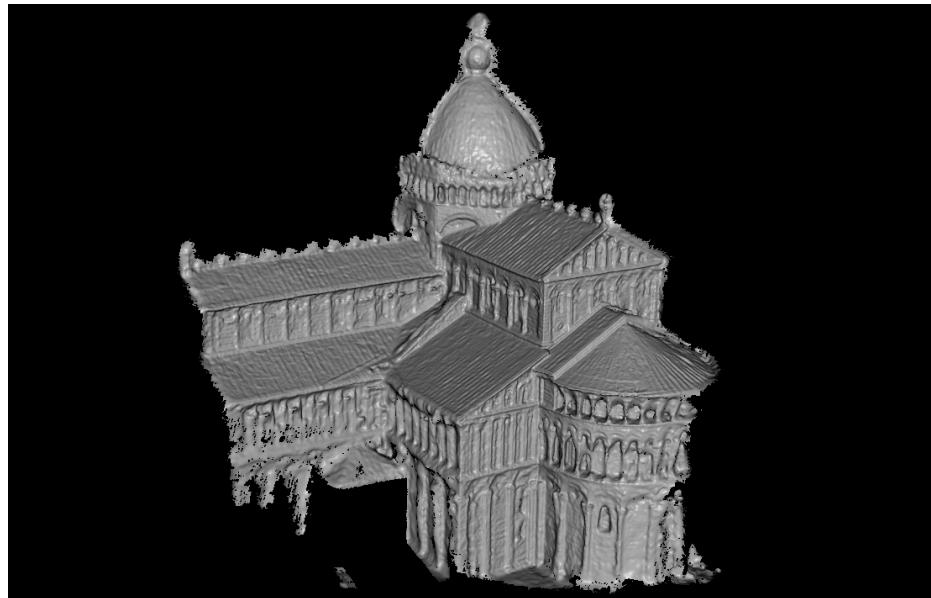
Multi-View Stereo for Community Photo Collections

Michael Goesele, Noah Snavely, Brian Curless, Hugues Hoppe, and Steven M. Seitz

ICCV 2007



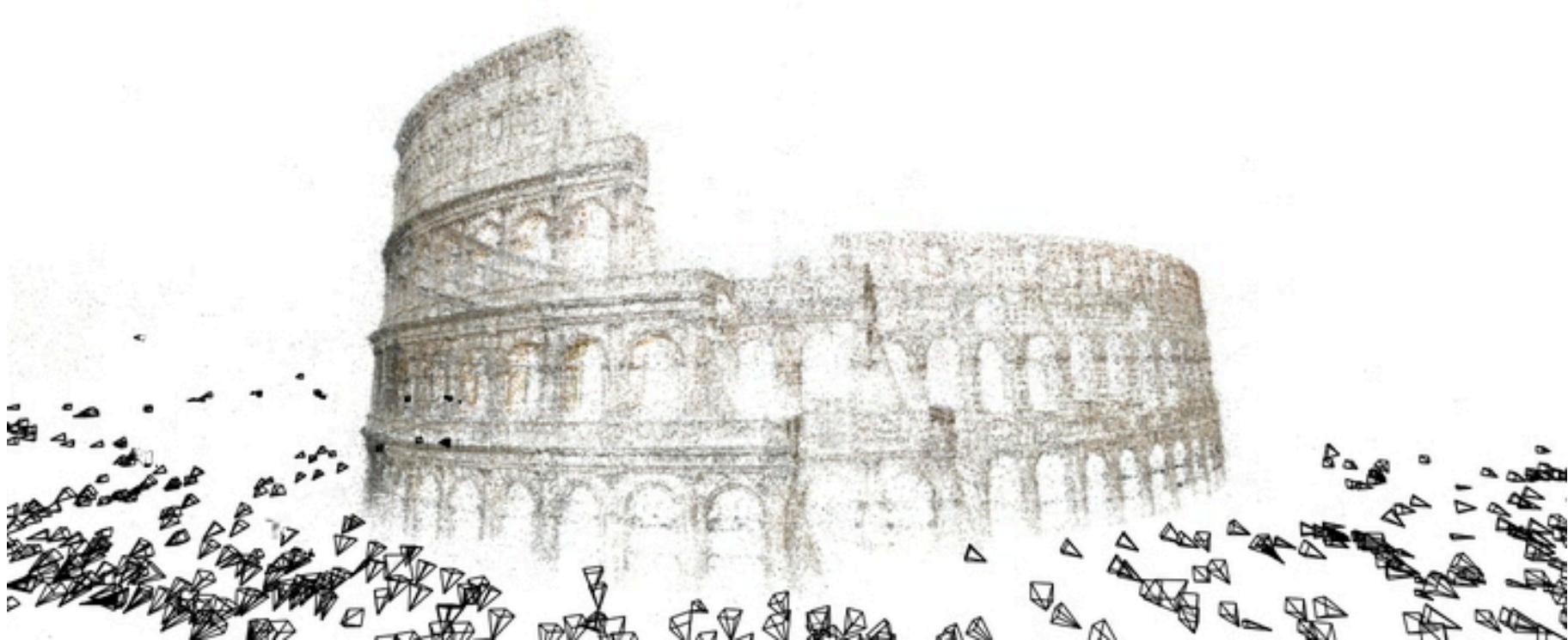
Multi-view Stereo



Compared with Laser-Scanner

Building Rome in a Day

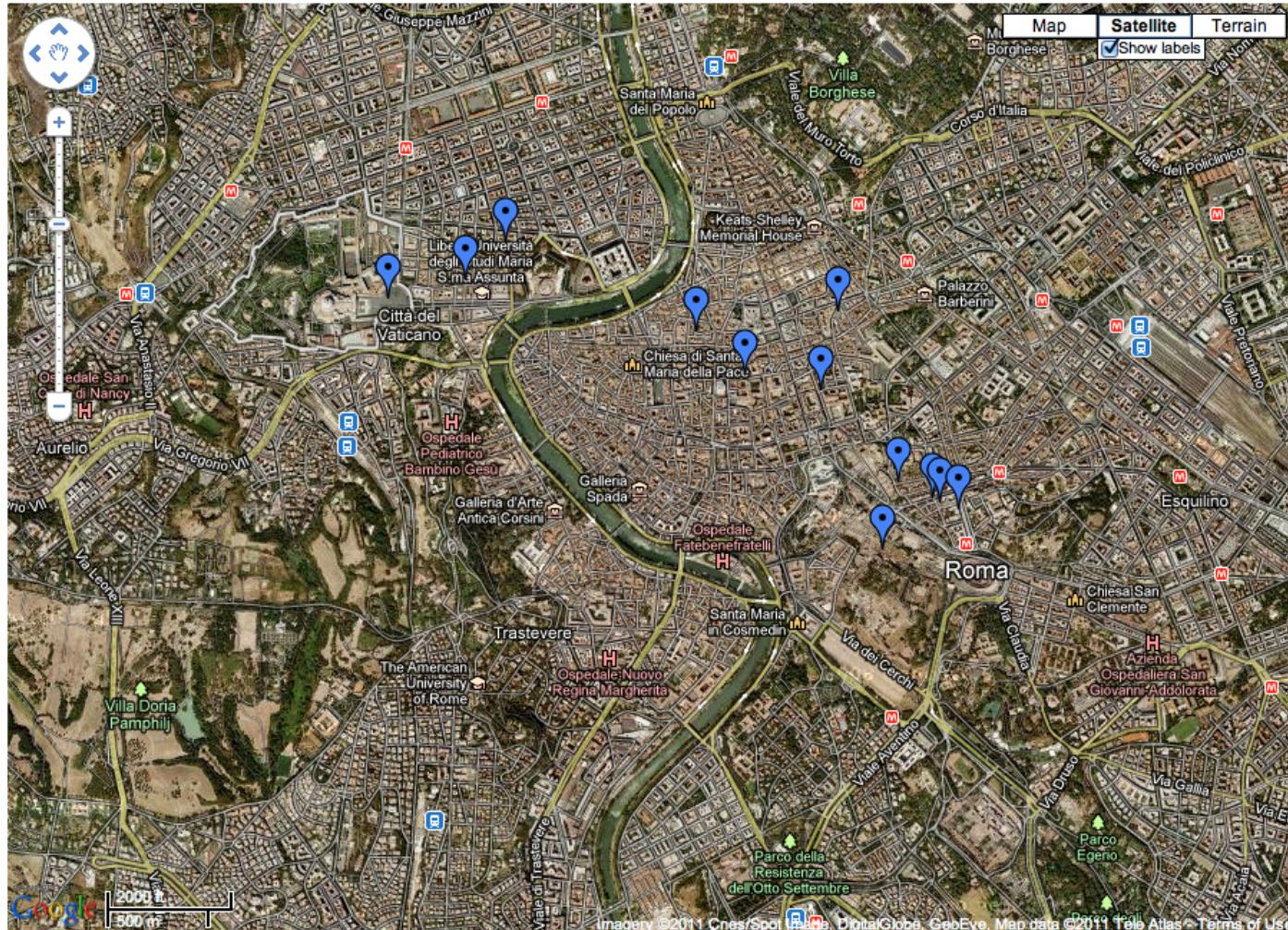
[Building Rome in a Day](#) Sameer Agarwal, Noah Snavely, Ian Simon, Steven M. Seitz and Richard Szeliski International Conference on Computer Vision, 2009, Kyoto, Japan.



<http://grail.cs.washington.edu/rome/>

Rome on a Cloudless Day

Jan-Michael Frahm, Pierre Georgel, David Gallup, Tim Johnson, Rahul Raguram, Changchang Wu, Yi-Hung Jen, Enrique Dunn, Brian Clipp, Svetlana Lazebnik, Marc Pollefeys, *ECCV 2010*



http://www.cs.unc.edu/~jmf/rome_on_a_cloudless_day/

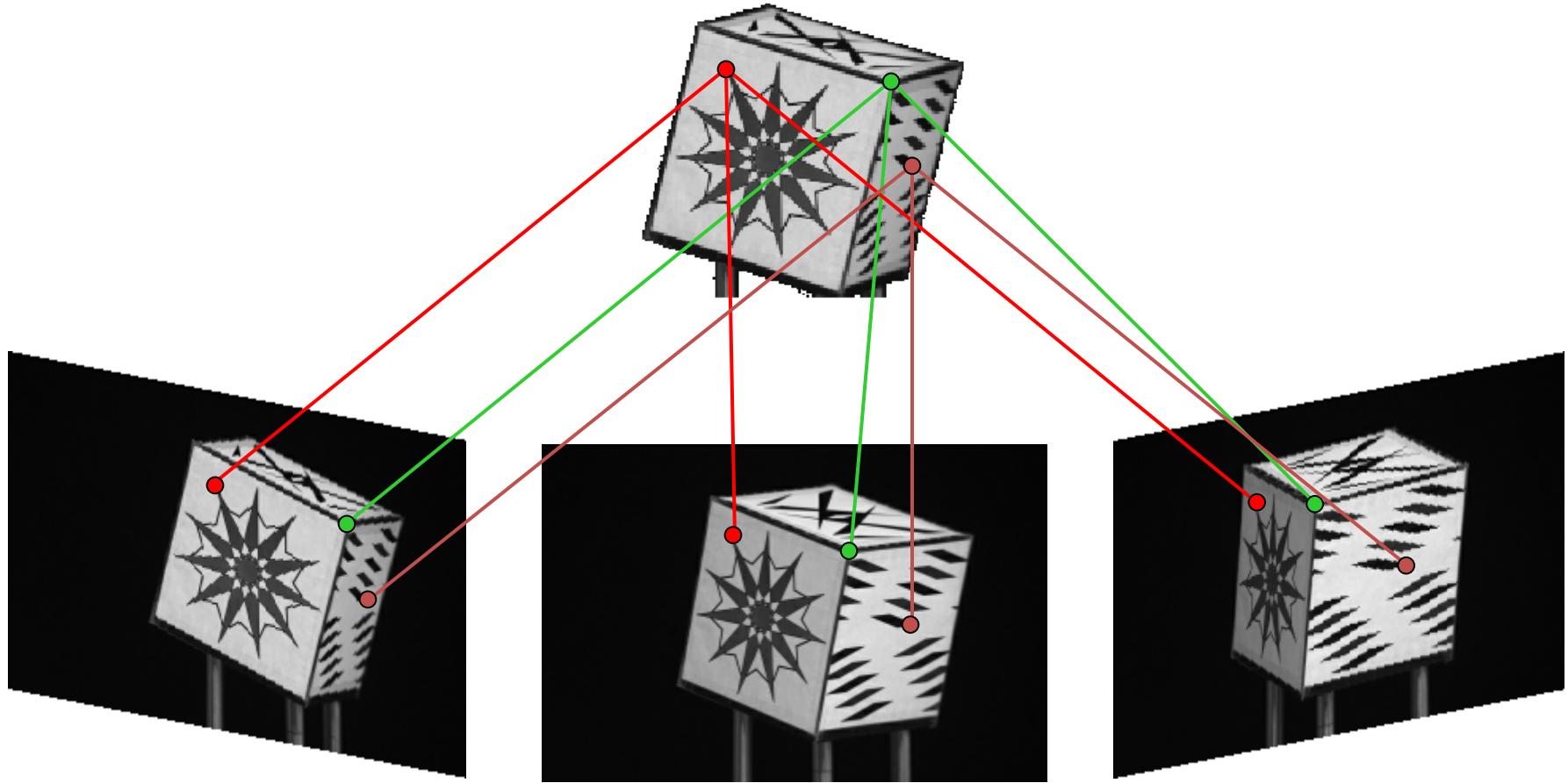
Frank Dellaert Fall 2019

2 Problems !

Correspondence

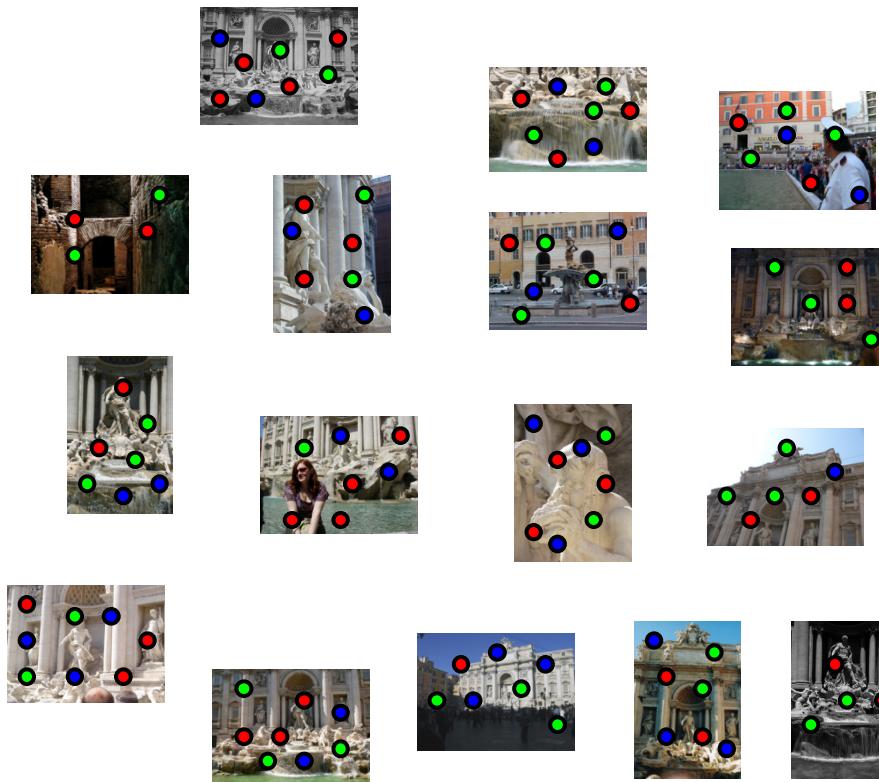
Optimization

A Correspondence Problem



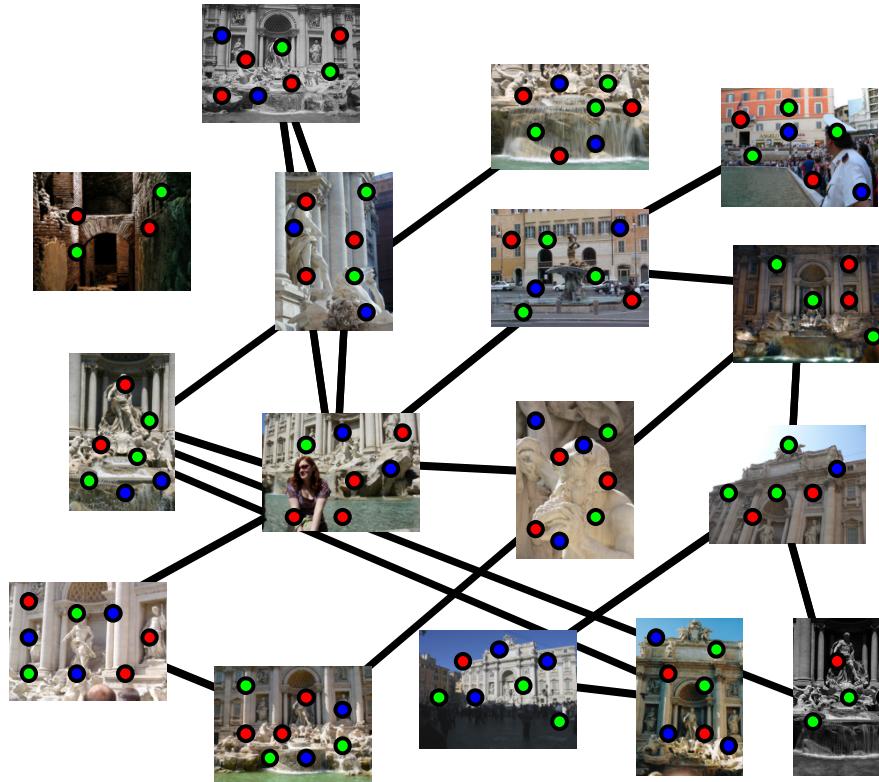
Feature detection

- Detect features using SIFT [Lowe, IJCV 2004]



Feature matching

Refine matching using RANSAC [Fischler & Bolles 1987] to estimate fundamental matrices between pairs



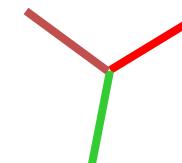
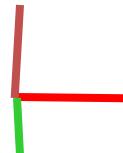
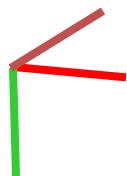
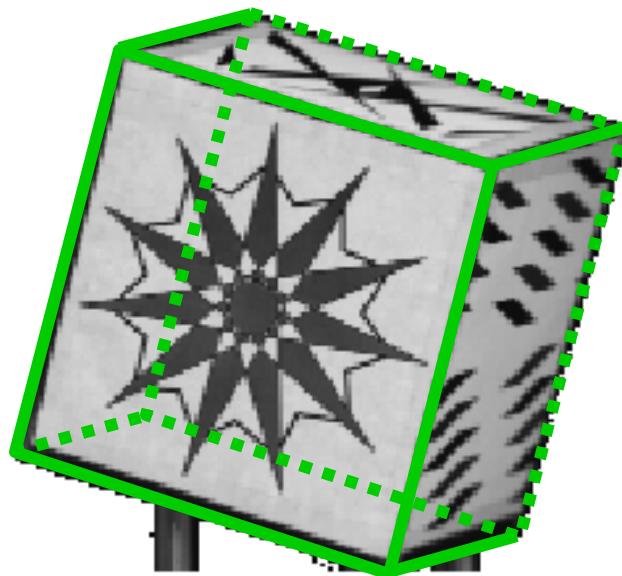
2 Problems !

Correspondence

Optimization

An Optimization Problem

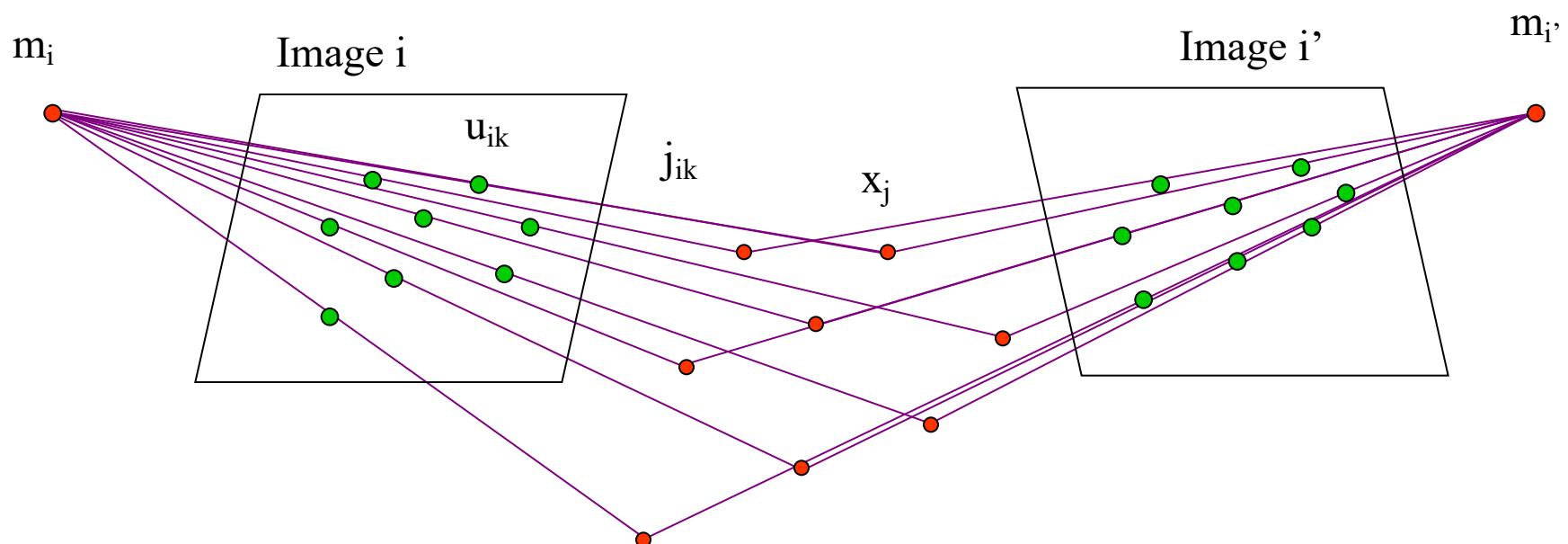
- Find the **most likely** structure and motion Θ



Optimization

=Non-linear Least-Squares !

$$\sum_{i=1}^m \sum_{k=1}^{K_i} \|\mathbf{u}_{ik} - \mathbf{h}(\mathbf{m}_i, \mathbf{x}_{j_{ik}})\|^2$$



Recall: Nonlinear Least Squares

$$E_{NLS} = \sum_i \|f(x_i; p) - x'_i\|^2$$

Linearize around a current guess p :

$$f(x; p + \Delta p) = f(x; p) + J(x; p)\Delta p$$

$$r = x' - f(x; p) = J(x; p)\Delta p$$

$$E_{NLS} = \sum_i \|f(x; p) + J(x; p)\Delta p - x'_i\|^2 = \sum_i \|J(x; p)\Delta p - r_i\|^2$$

Differentiate and set to 0:

$$2 \sum_i J^T(x_i; p) (J(x_i; p)\Delta p - r_i) = 0$$

$$\text{Normal equations} — \left[\sum_i J^T(x_i; p) J(x_i; p) \right] \Delta p = \sum_i J^T(x_i; p) r_i$$

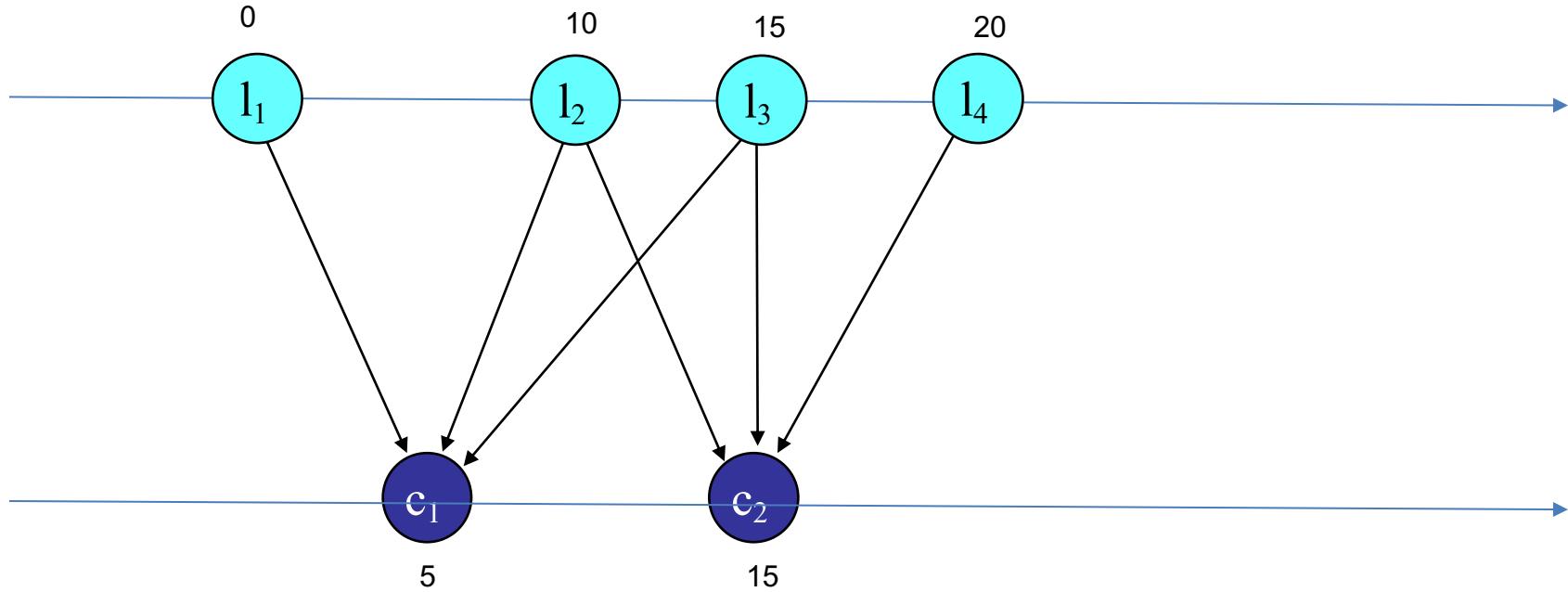
$$A\Delta p = b$$

$$\Delta p^* = A^{-1}b$$

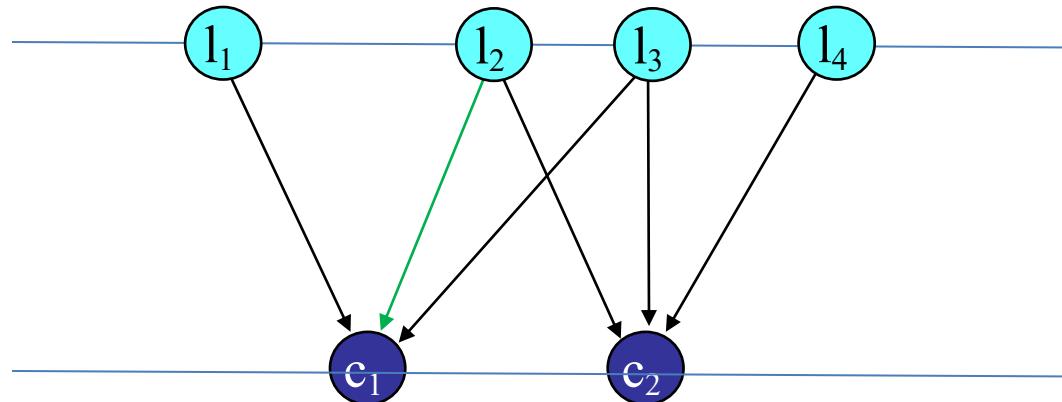
Hessian

Sparse nonlinear least squares

- Simple 1-Dimensional Example
- $p = 2$ cameras and 4 points $\{c_1 \ c_2 \ |l_1 \ l_2 \ l_3 \ l_4\}$
- $f(u_{ik}; p) = \text{difference in } x \text{ position} = l_{j(ik)} - c_i$



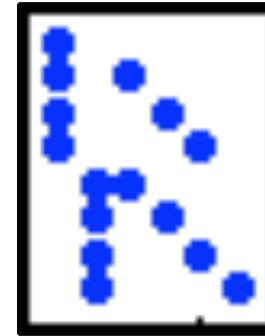
Sparse Jacobian and Hessian



$$A = \begin{bmatrix} c_1 & c_2 & l_1 & l_2 & l_3 & l_4 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$b =$

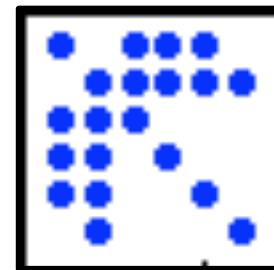
5
-5
5
10
-15
-5
0
5



$$A' * A = \begin{bmatrix} c_1 & c_2 & l_1 & l_2 & l_3 & l_4 \\ 4 & 0 & -1 & -1 & -1 & 0 \\ 0 & 4 & -1 & -1 & -1 & -1 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -1 & -1 & 0 & 2 & 0 & 0 \\ -1 & -1 & 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

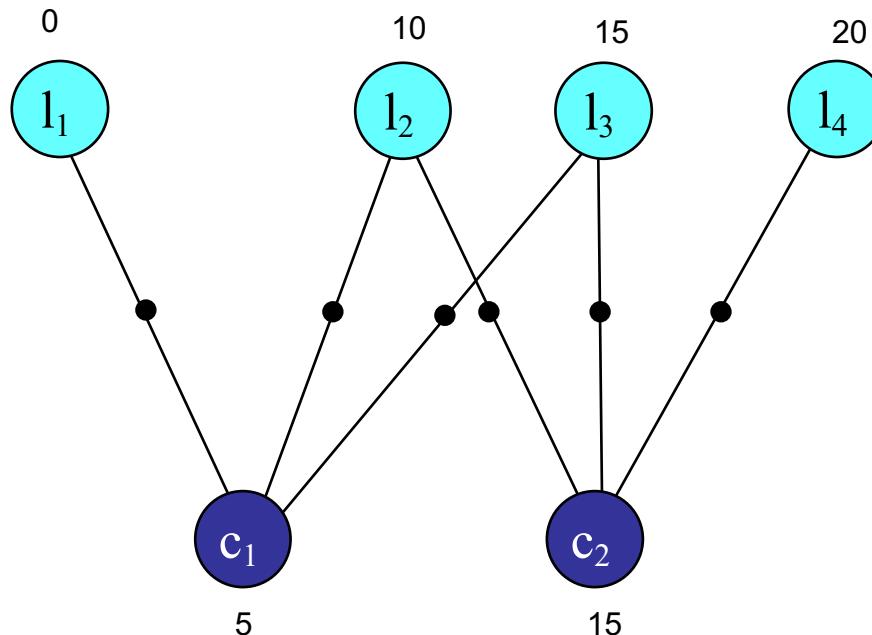
$(A' * A) \setminus A' * b =$

5.0000
15.0000
0.0000
10.0000
15.0000
20.0000



A general formalism: Factor Graphs

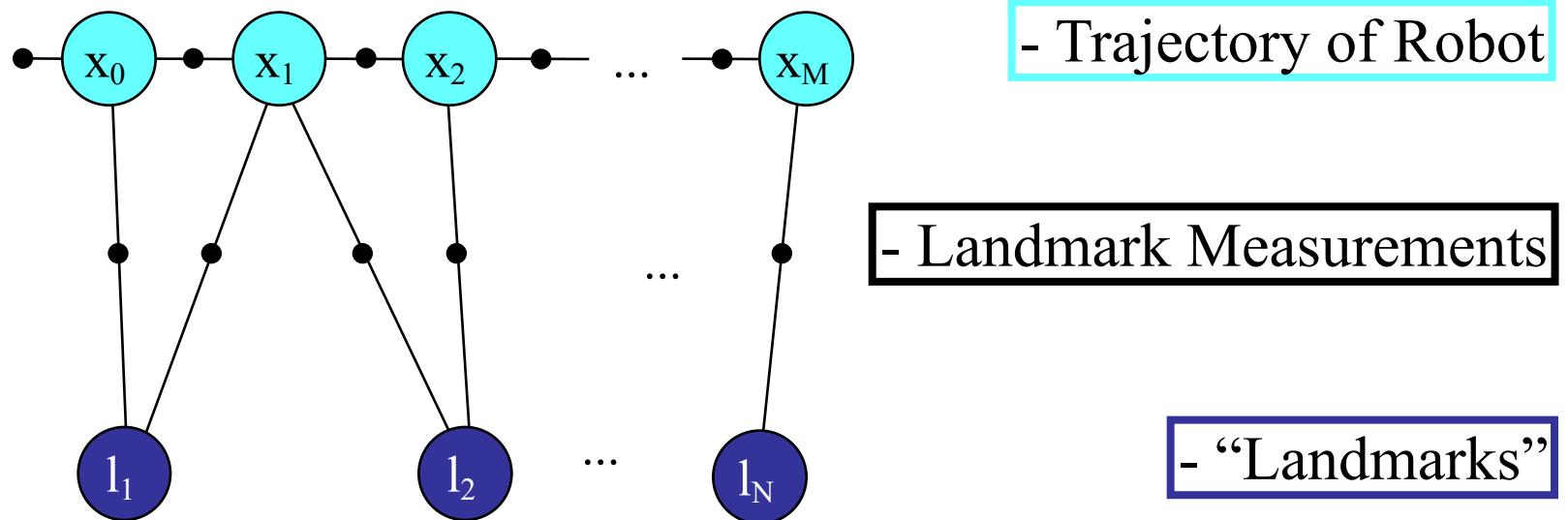
- Bipartite graph
- Two types of nodes:
 - Unknowns
 - Factors: correspond to squared errors
- Connectivity = sparsity! Factor is function of small set.



SLAM: Simultaneous Localization and Mapping

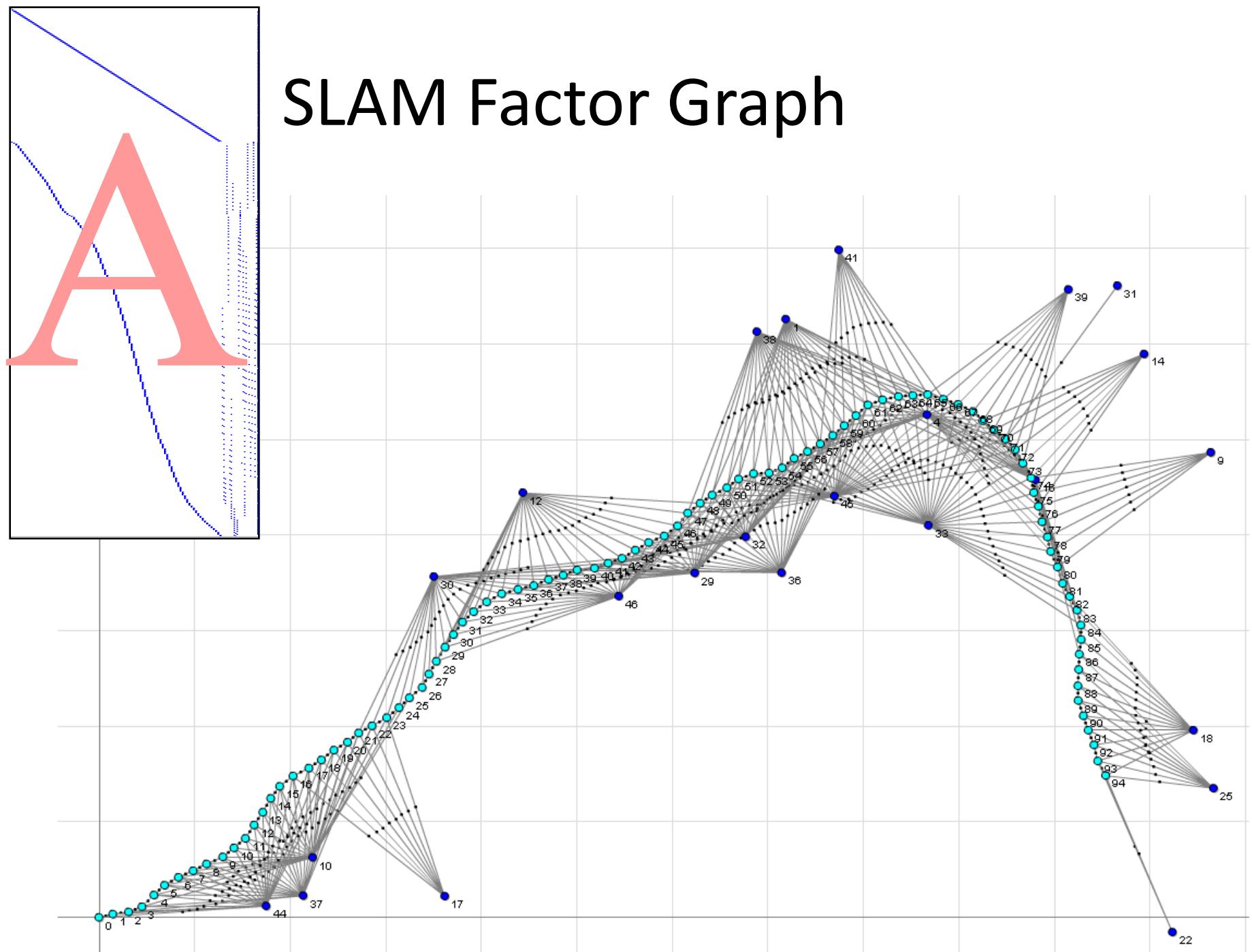


SLAM Factor Graph

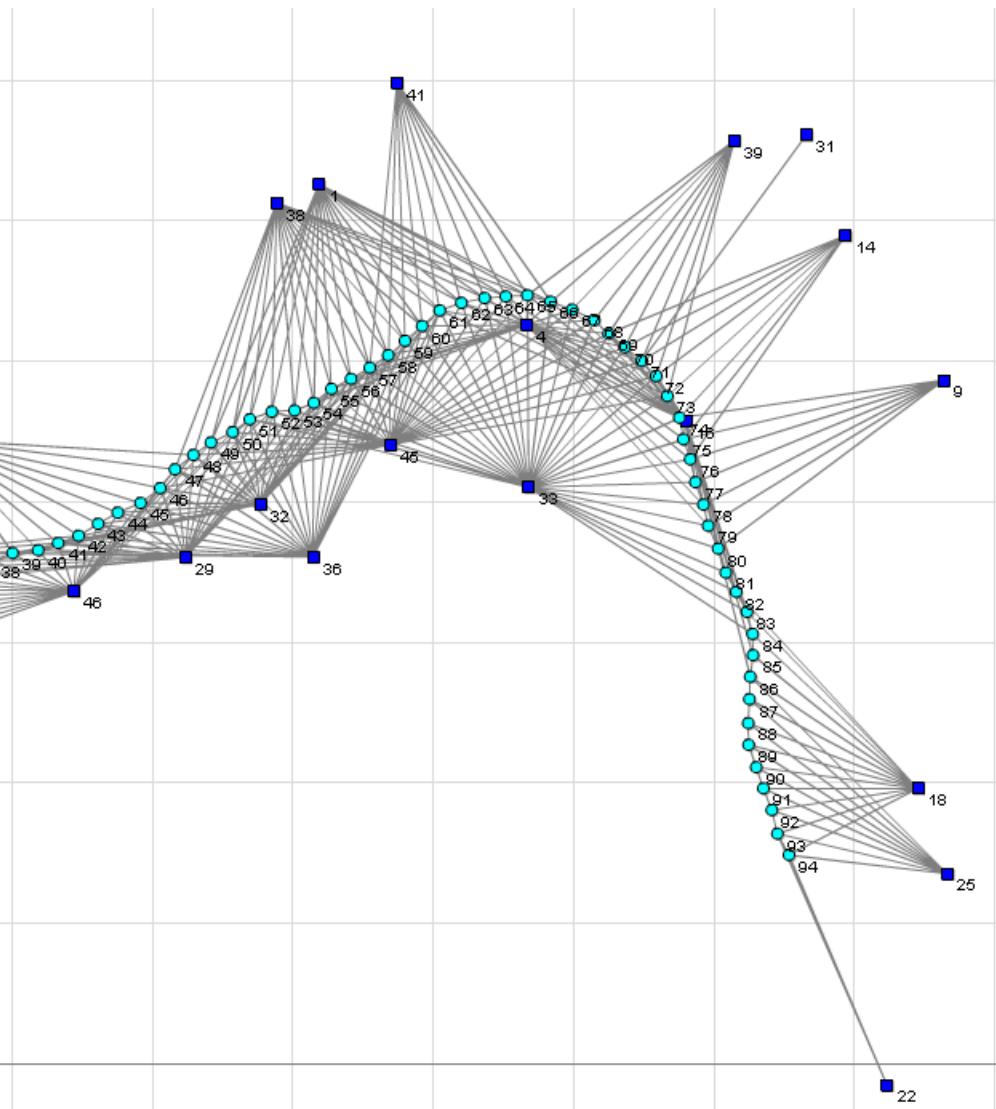
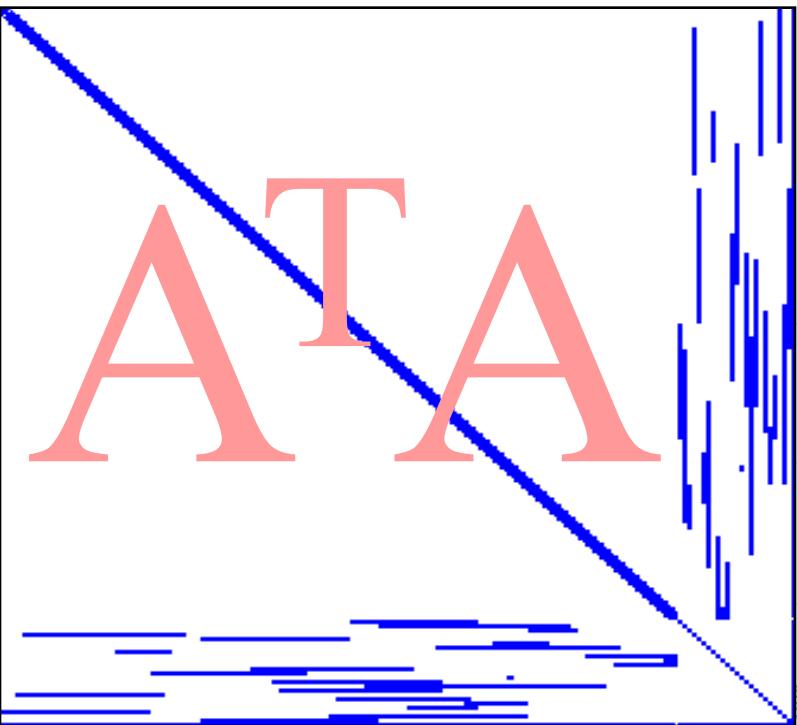


$$P(X, M) = k^* P(x_0) \prod_{i=1}^M P(x_i | x_{i-1}, u_i) \times \prod_{k=1}^K P(z_k | x_{i_k}, l_{j_k})$$

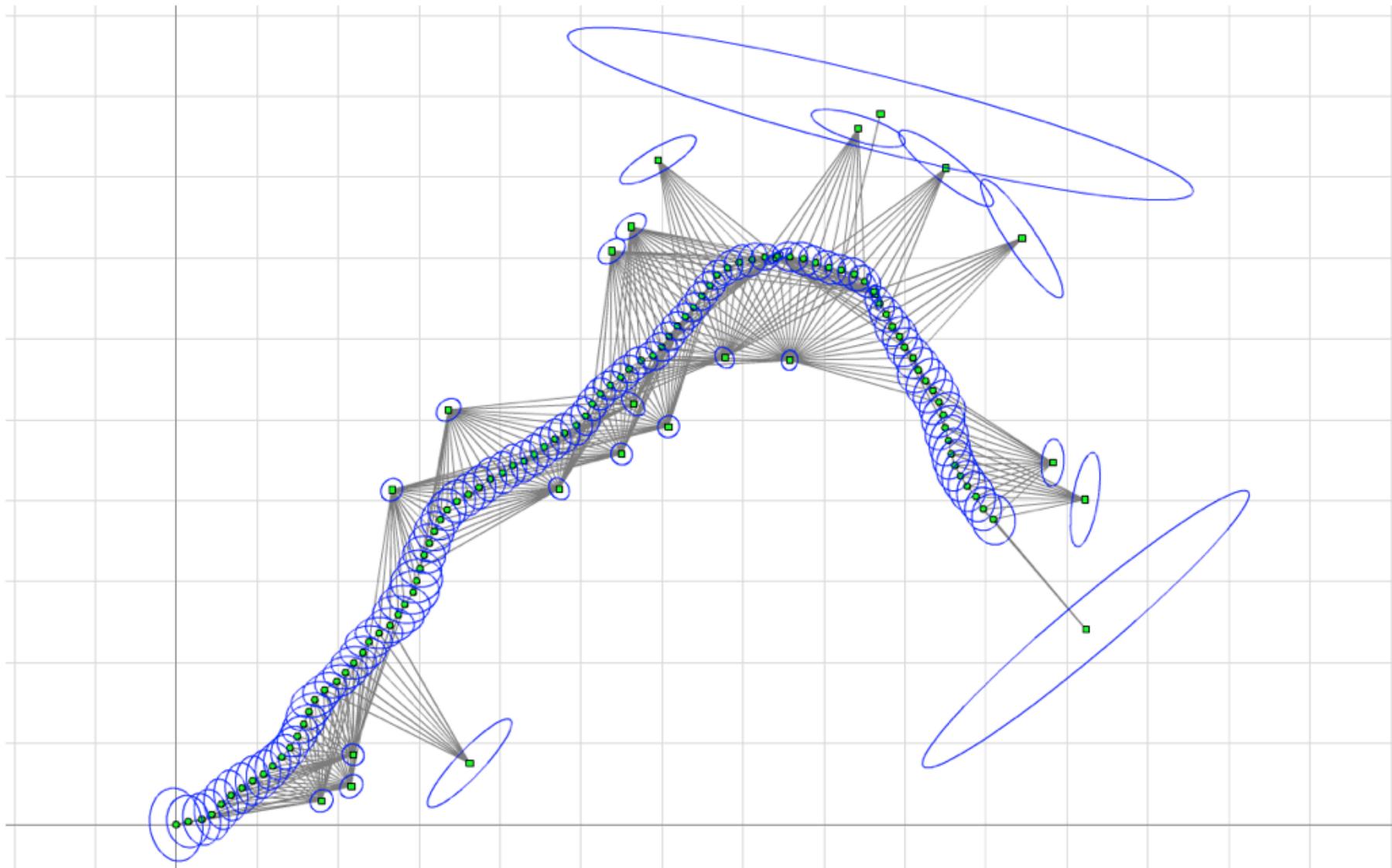
SLAM Factor Graph



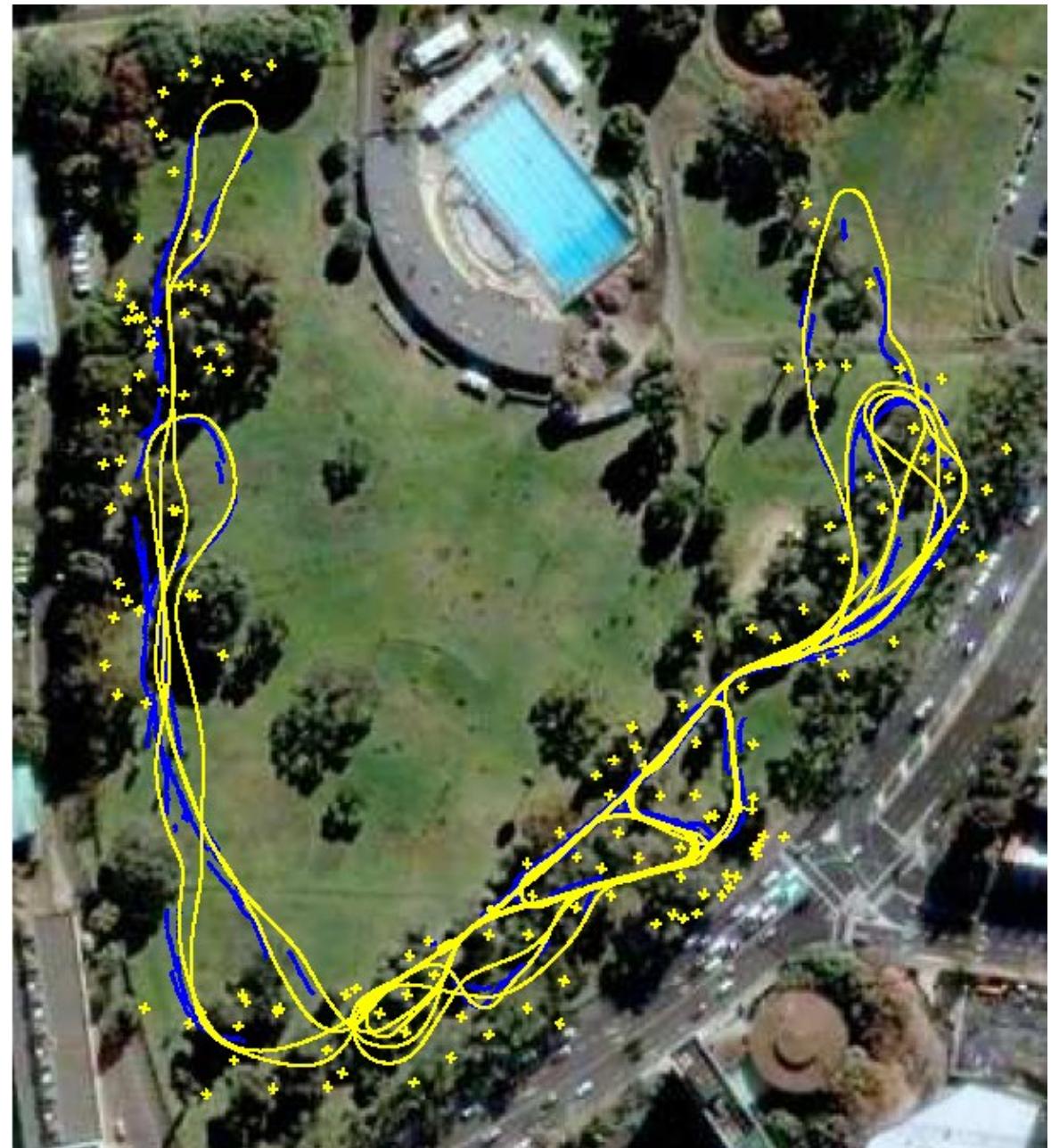
Hessian



End result: Solution + Sigma

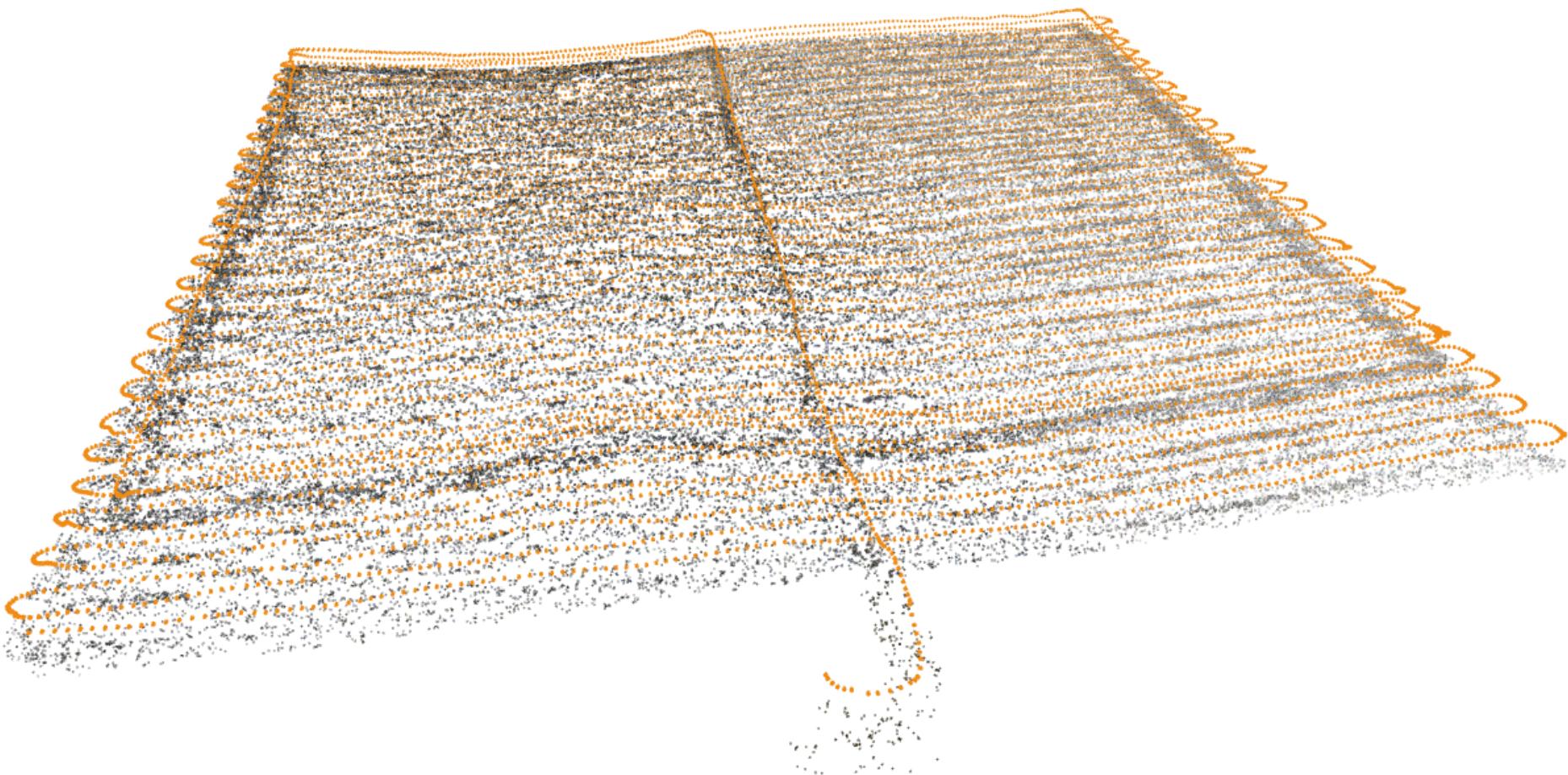


Example:
Victoria Park,
Sidney



Example: Underwater SLAM

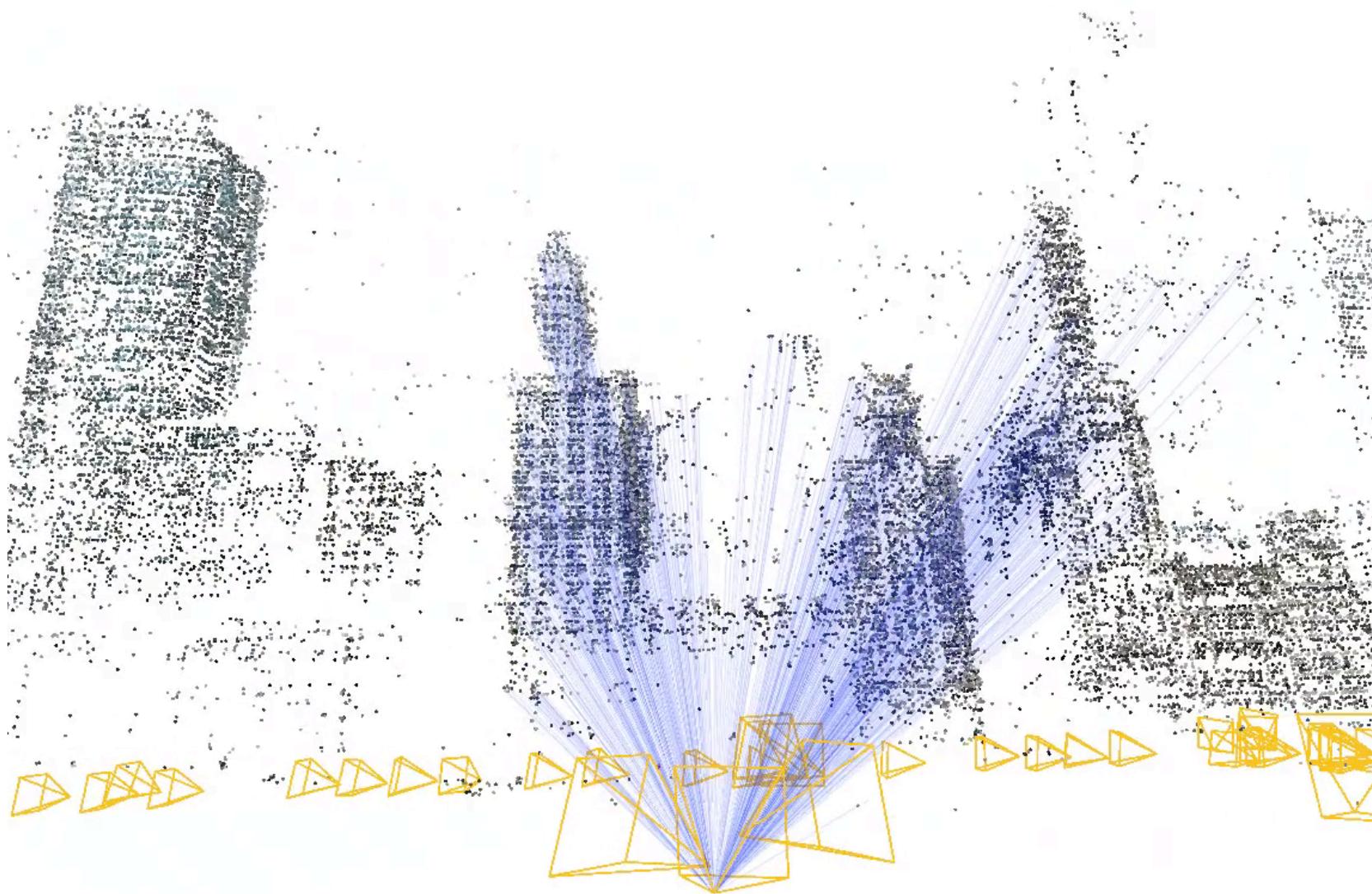
9831 camera poses, 185261 landmarks, and 350988 factors



Structure from Motion (Chicago, movie by Yong Dian Jian)

180 cameras, 88723 points
458642 projections
active camera: 4

Original graph



3D Models from Community Databases

- E.g., Google image search on “Dubrovnik”

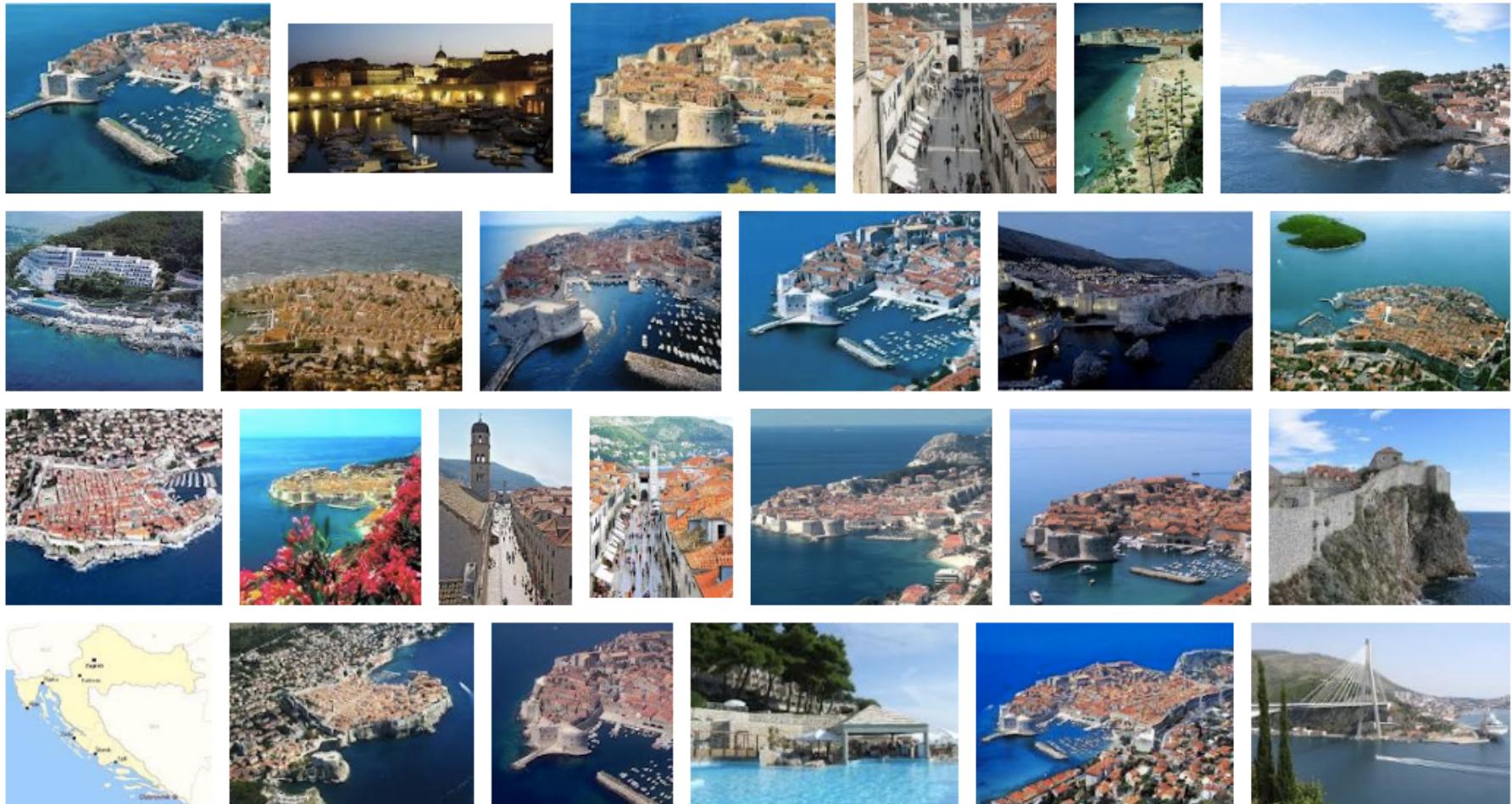


Figure by Aggarwal et al.

3D Models from Community Databases

Agarwal, Snavely, Seitz et al. at UW <http://grail.cs.washington.edu/rome/>



5K images, 3.5M points, >10M factors

Movie by Aggarwal et al.

Frank Dellaert Fall 2019 ³⁶

Hyper-SFM: Efficient Multi-core

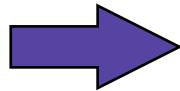


Kai now leads an
autonomous driving
startup in China

[Kai Ni](#), and [Frank Dellaert](#), [HyperSfM](#), [IEEE International Conference on 3D Imaging, Modeling, Processing, Visualization and Transmission \(3DIMPVT\)](#), 2012.

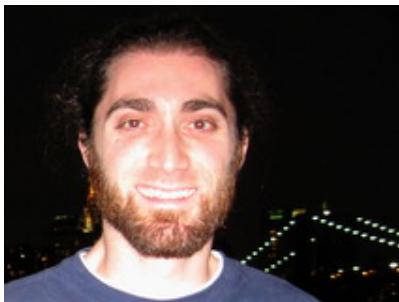
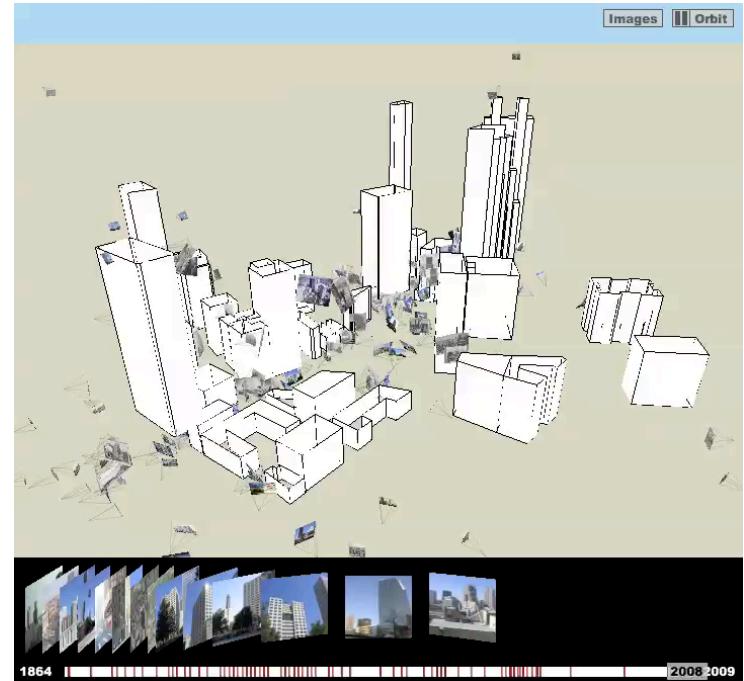
4D Reconstruction

Spatiotemporal Reconstruction



Historical Image Collection

4D Cities: 3D + Time



Grant Schindler

Supported by NSF CAREER, Microsoft
Recent revival: NSF NRI award on 4D
crops for precision agriculture...

4D Reconstruction of Lower Manhattan



[Probabilistic Temporal Inference on Reconstructed 3D Scenes](#), G. Schindler and F. Dellaert,
IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), 2010.

4D Structure over Time



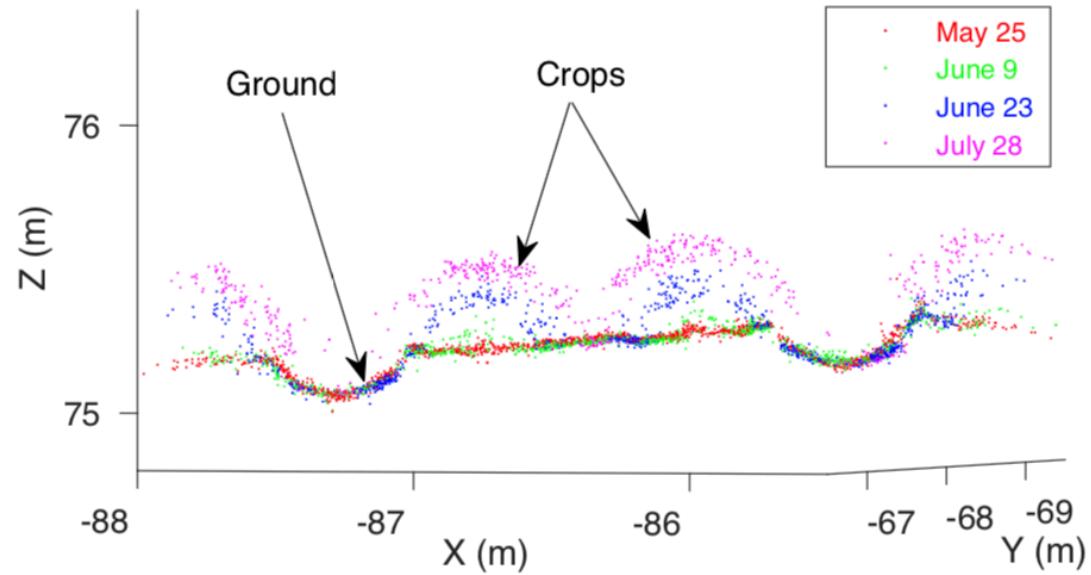
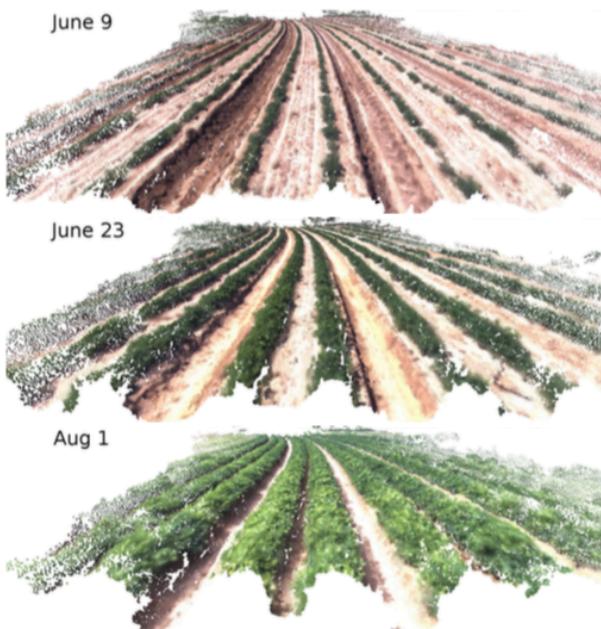
1928

2007

2010

Ilaert Fall 2019 41

4D crop monitoring (Jing Dong)



Results: video (by Jing Dong)

May 25, 2016



May 25, 2016



4D reconstruction results (by PMVS)
and its cross section