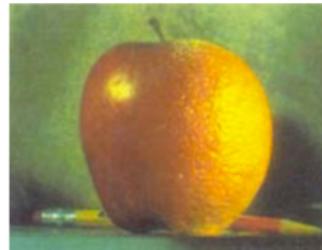


2. Image Formation



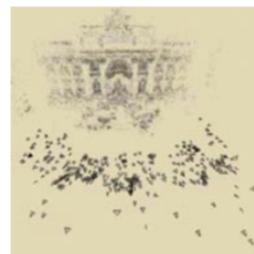
3. Image Processing



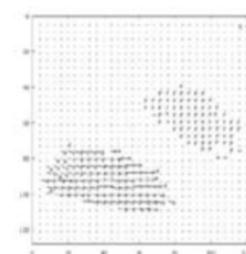
4. Features



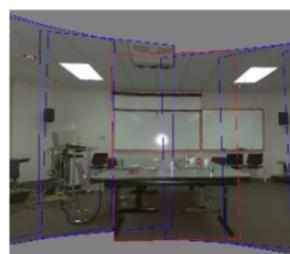
5. Segmentation



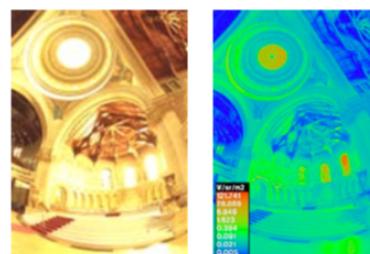
6-7. Structure from Motion



8. Motion



9. Stitching



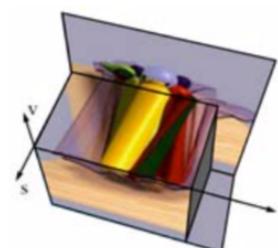
10. Computational Photography



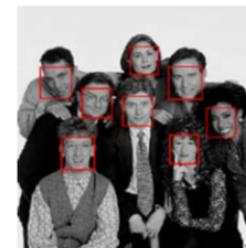
11. Stereo



12. 3D Shape

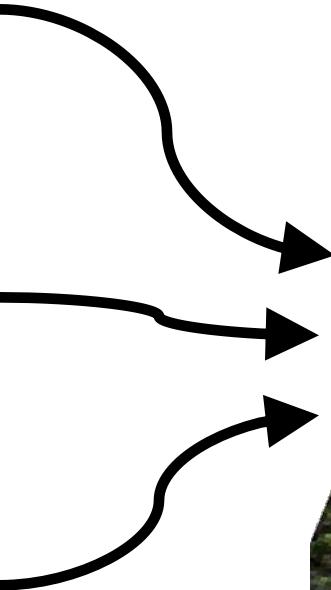


13. Image-based Rendering



14. Recognition

Multiple View Geometry



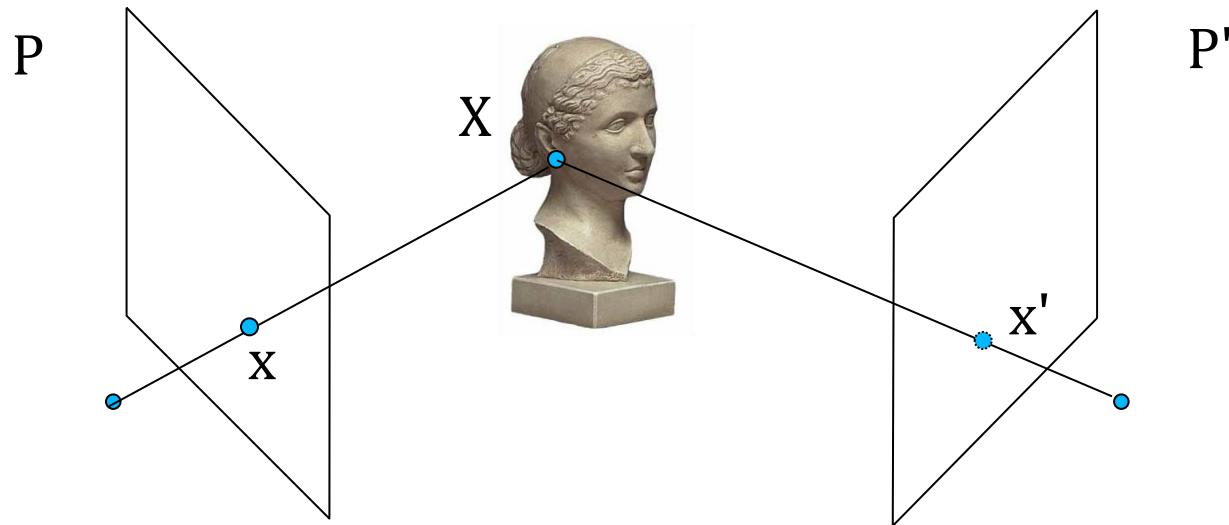
Frank Dellaert

Frank Dellaert Fall 2019

Outline

- Intro
- Camera Review
- Stereo triangulation
- Geometry of 2 views
 - Essential Matrix
 - Fundamental Matrix
- Estimating E/F from point-matches

Why Consider Multiple Views?



Answer: To extract 3D structure via triangulation.

Stereo Rig

Top View

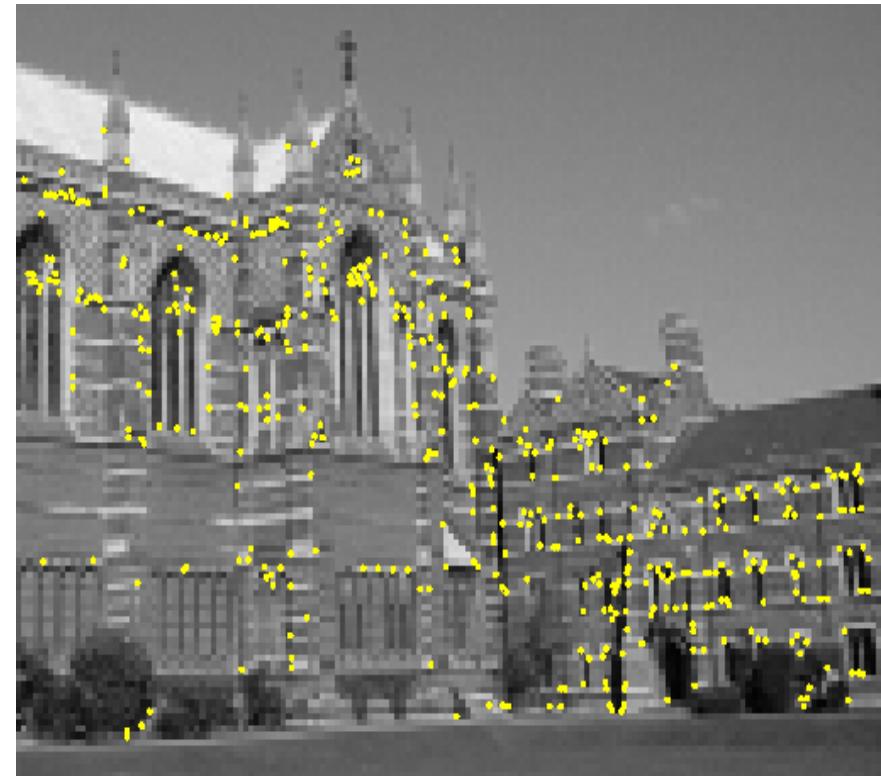
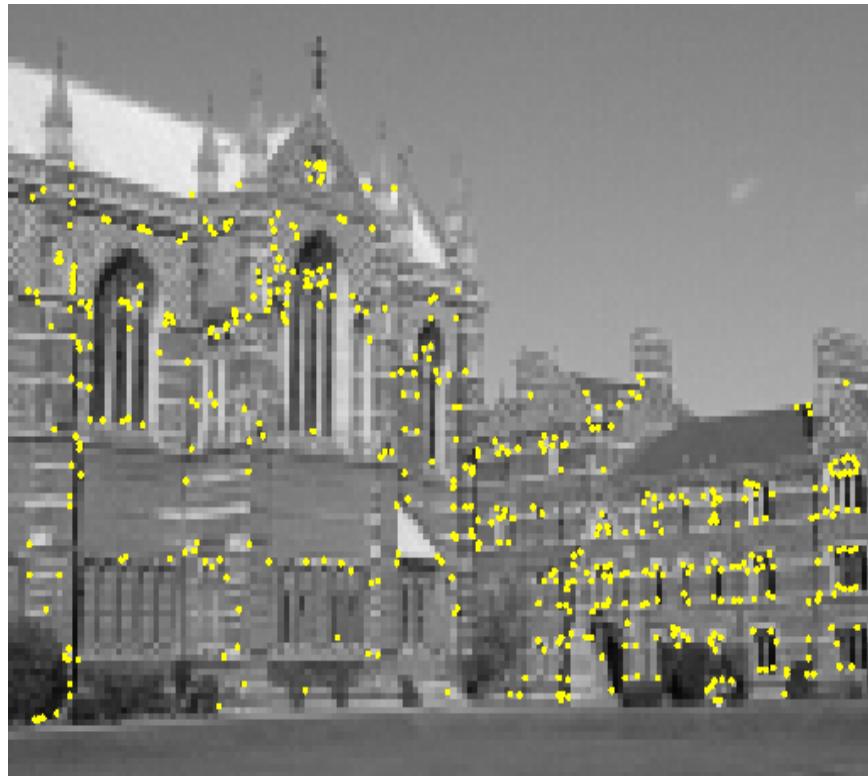


Matches on Scanlines



• Convenient when searching for correspondences.

Feature Matching !



Real World Challenges

Bad News: Good correspondences are hard to find

- Good news: Geometry constrains possible correspondences.
 - 4 DOF between x and x' ; only 3 DOF in X .
 - Constraint is manifest in the **Fundamental matrix**

$$x'^T F x = 0.$$

- F can be calculated either from camera matrices or a set of good correspondences.

Geometry of 2 views ?

- What if we do not know R, t ?
- Caveat:
 - My exposition uses different R, t
 - but more intuitive (IMHO)
 - I use $[R^T | -R^T t] = R^T [I | -t]$ camera matrices
 - Szeliski uses $[R | t]$

Epipolar Geometry

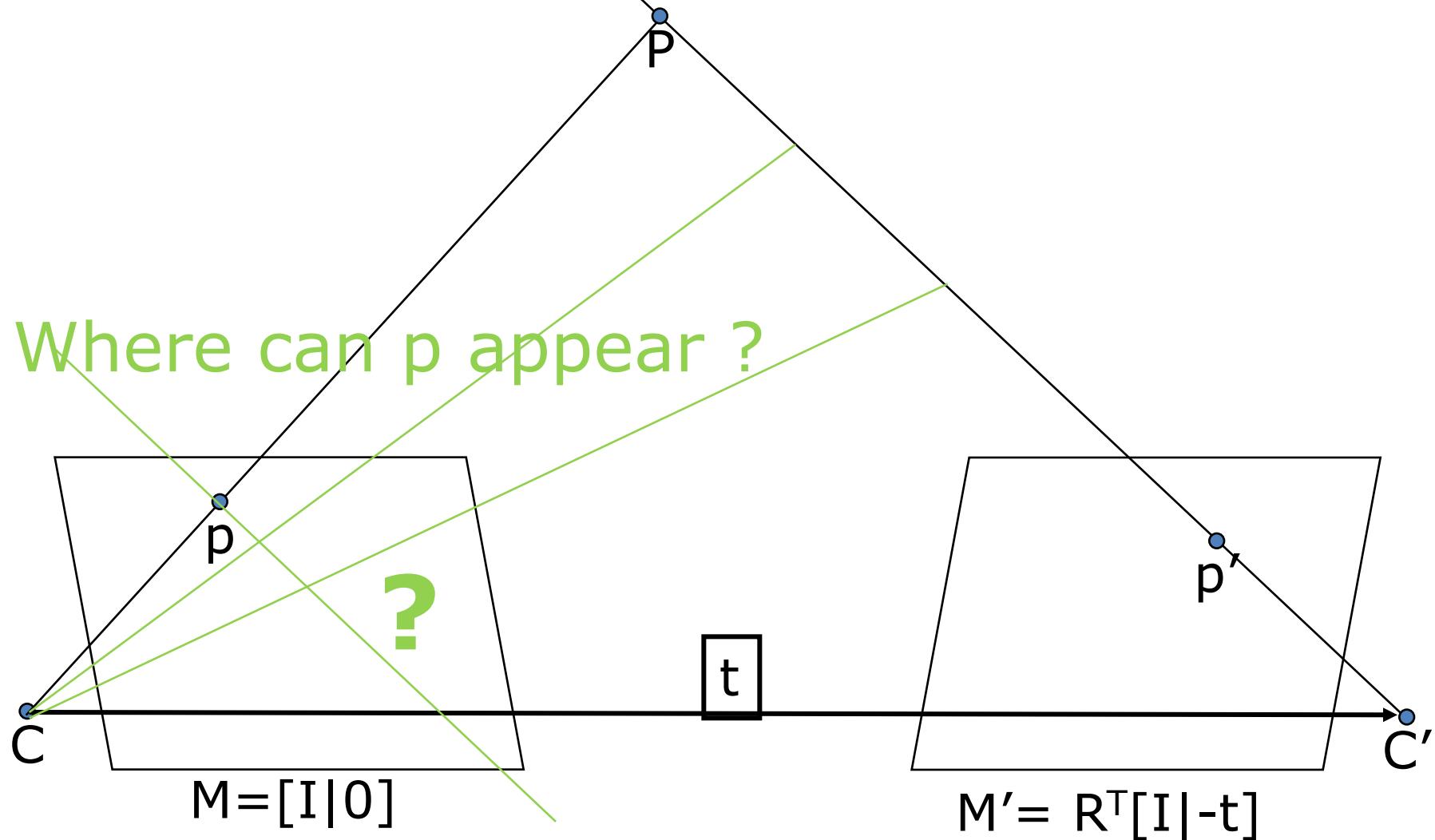


Image of Camera Center



$$M = [I | 0]$$

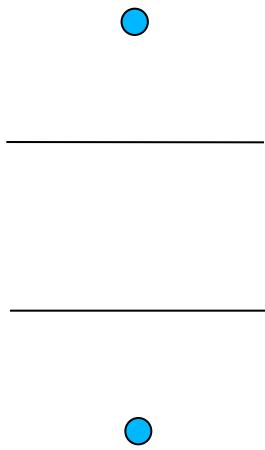


$$M' = R^T[I | -t]$$

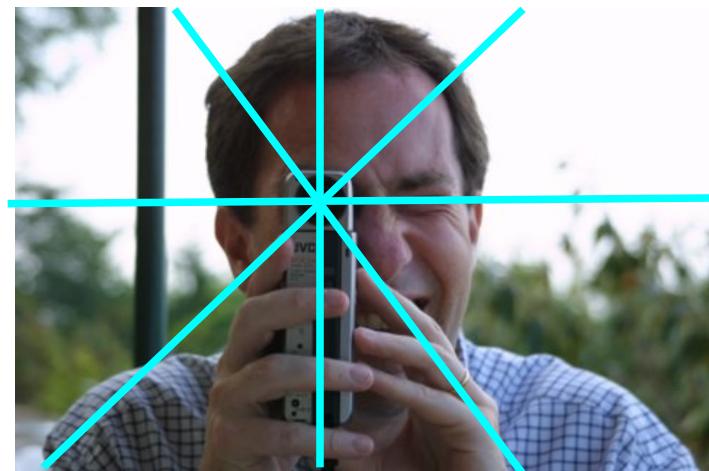
Example:

Cameras Point at Each Other

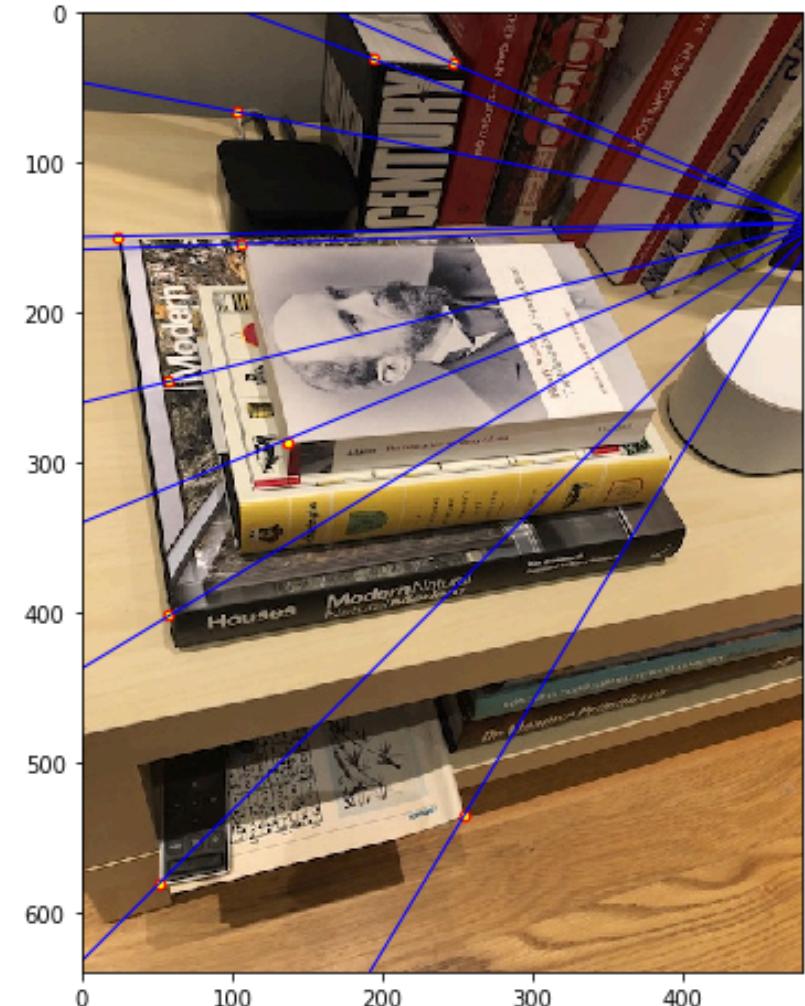
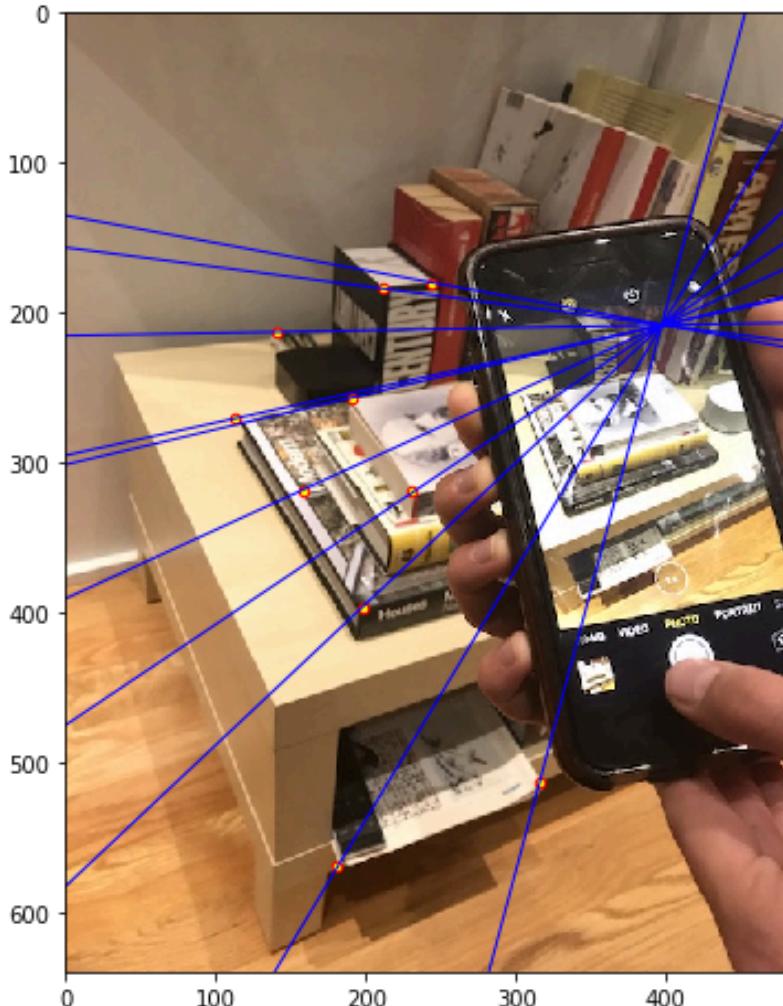
Top View



Epipolar Lines

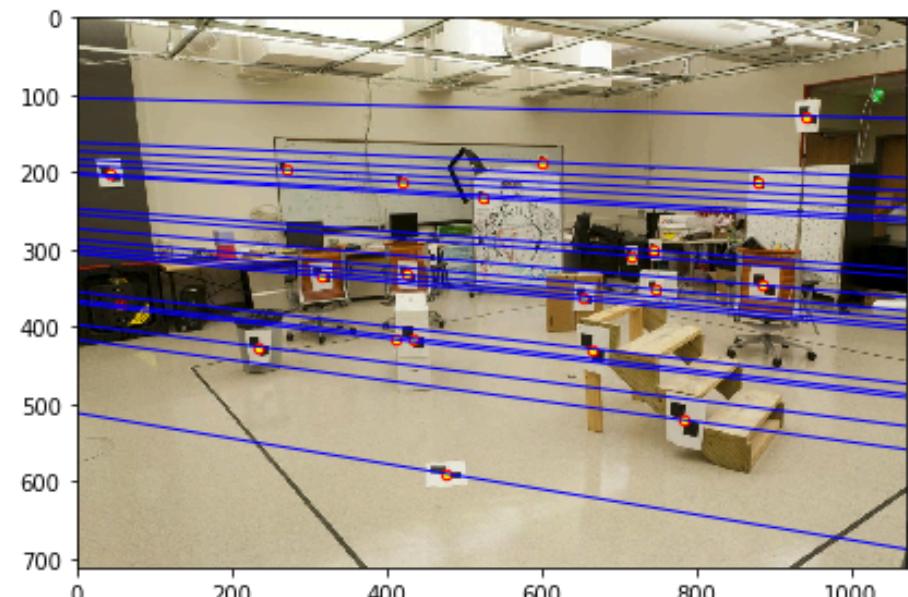
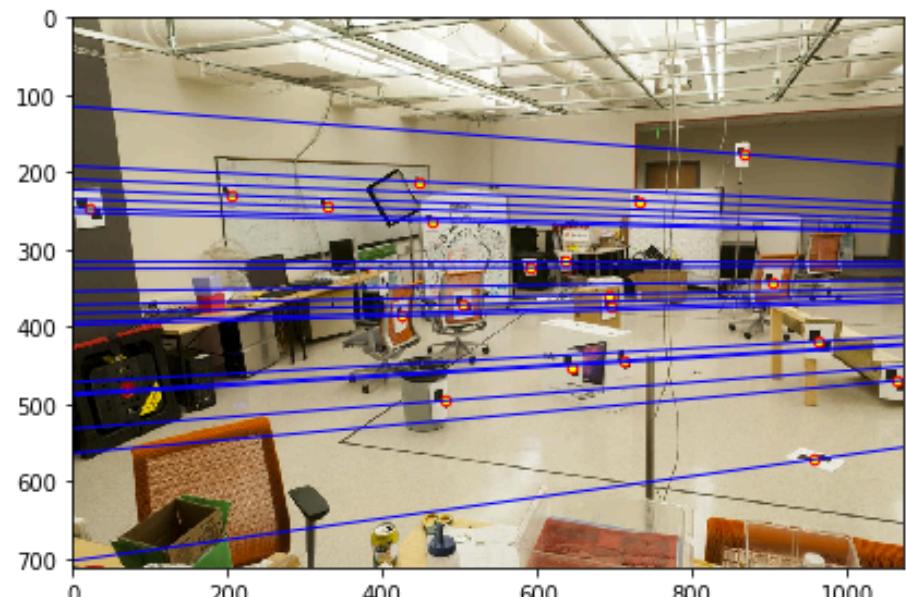


Epipoles



- Epipoles inside the image: zoom-like setup.

Epipoles



- Epipoles in near-stereo config.

Epipoles

- Camera Center C' in first view:

$$e = [I \quad 0] \begin{bmatrix} t \\ 1 \end{bmatrix} = t$$

- Origin C in second view:

$$e' = [R^T \quad -R^T t] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -R^T t$$

Image of Camera Ray ?



$$M = [I | 0]$$



$$M' = R^T[I | -t]$$

Point at infinity

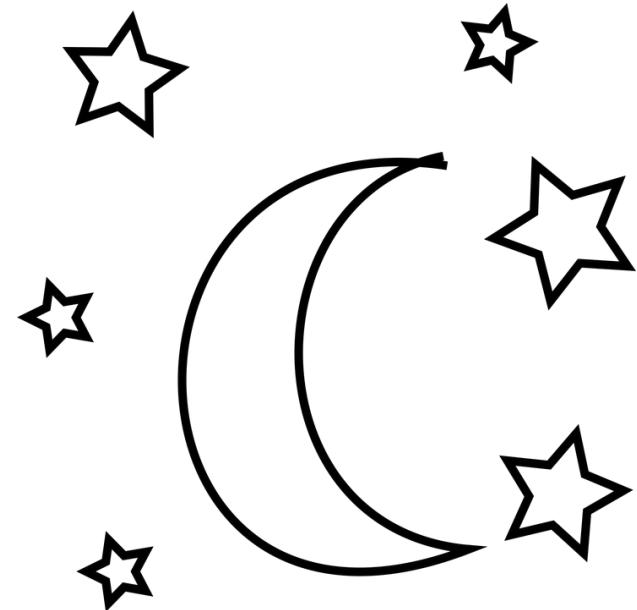
- Given p' , what is corresponding point at infinity $[x \ 0]$?
- Answer for any camera $M'=[A|a]$:

$$p' = [A \ a] * \begin{bmatrix} x \\ 0 \end{bmatrix} = Ax \Rightarrow x = A^{-1}p'$$

- A^{-1} = Infinite homography
- In our case $M'=[R^T|-R^Tt]$: $x = Rp'$

Sidebar: Infinite Homographies

- Homography between
 - image plane
 - plane at infinity
- Navigation by the stars:
 - Image of stars =
function of rotation R only !
 - Traveling on a sphere rotates viewer



Epipolar Line Calculation

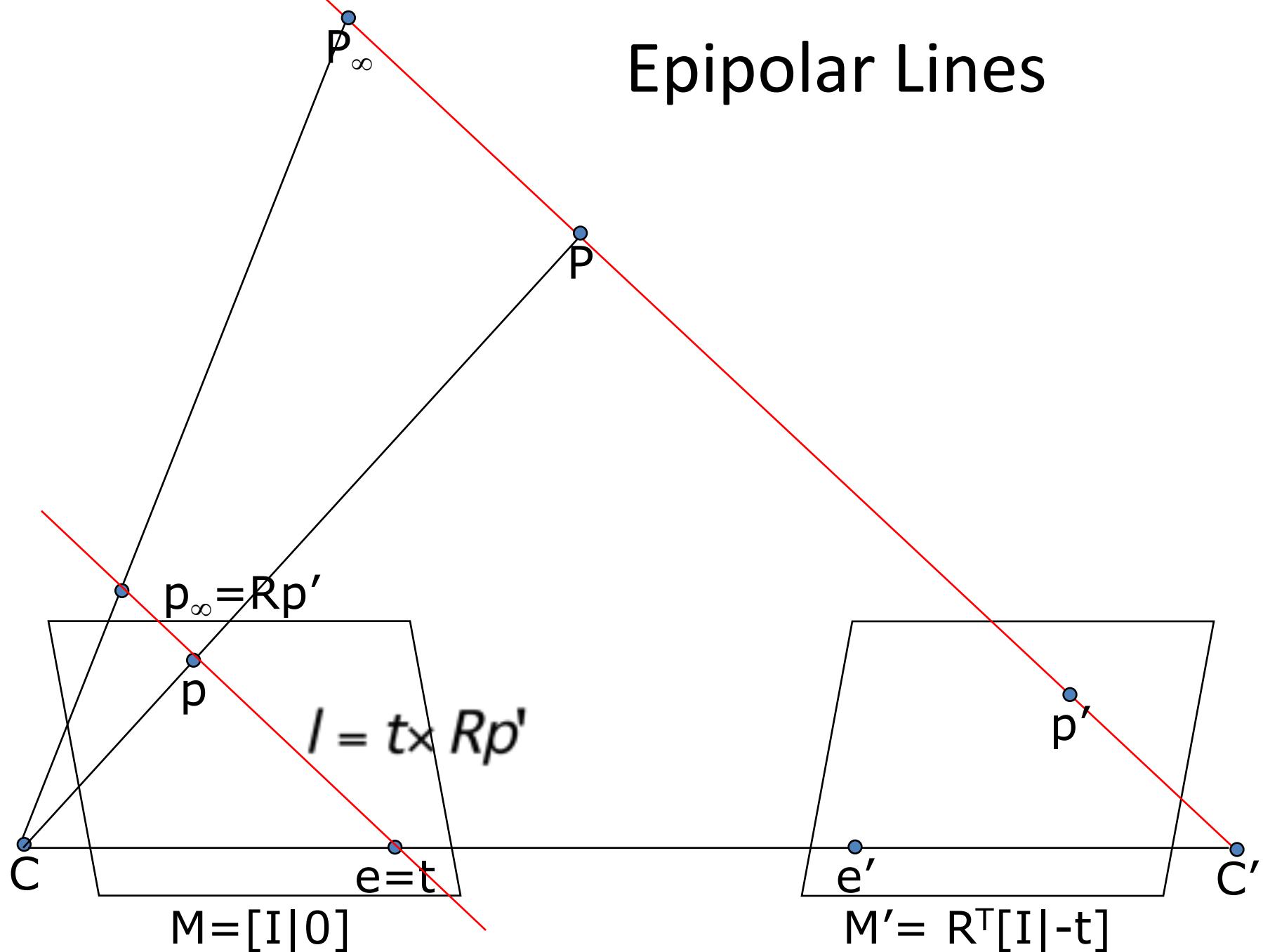
- 1) Point 1 = epipole $e=t$
- 2) Point 2 = point at infinity

$$p_\infty = \begin{bmatrix} I & 0 \end{bmatrix} * \begin{bmatrix} Rp' \\ 0 \end{bmatrix} = Rp'$$

- 3) Epipolar line = join of points 1 and 2

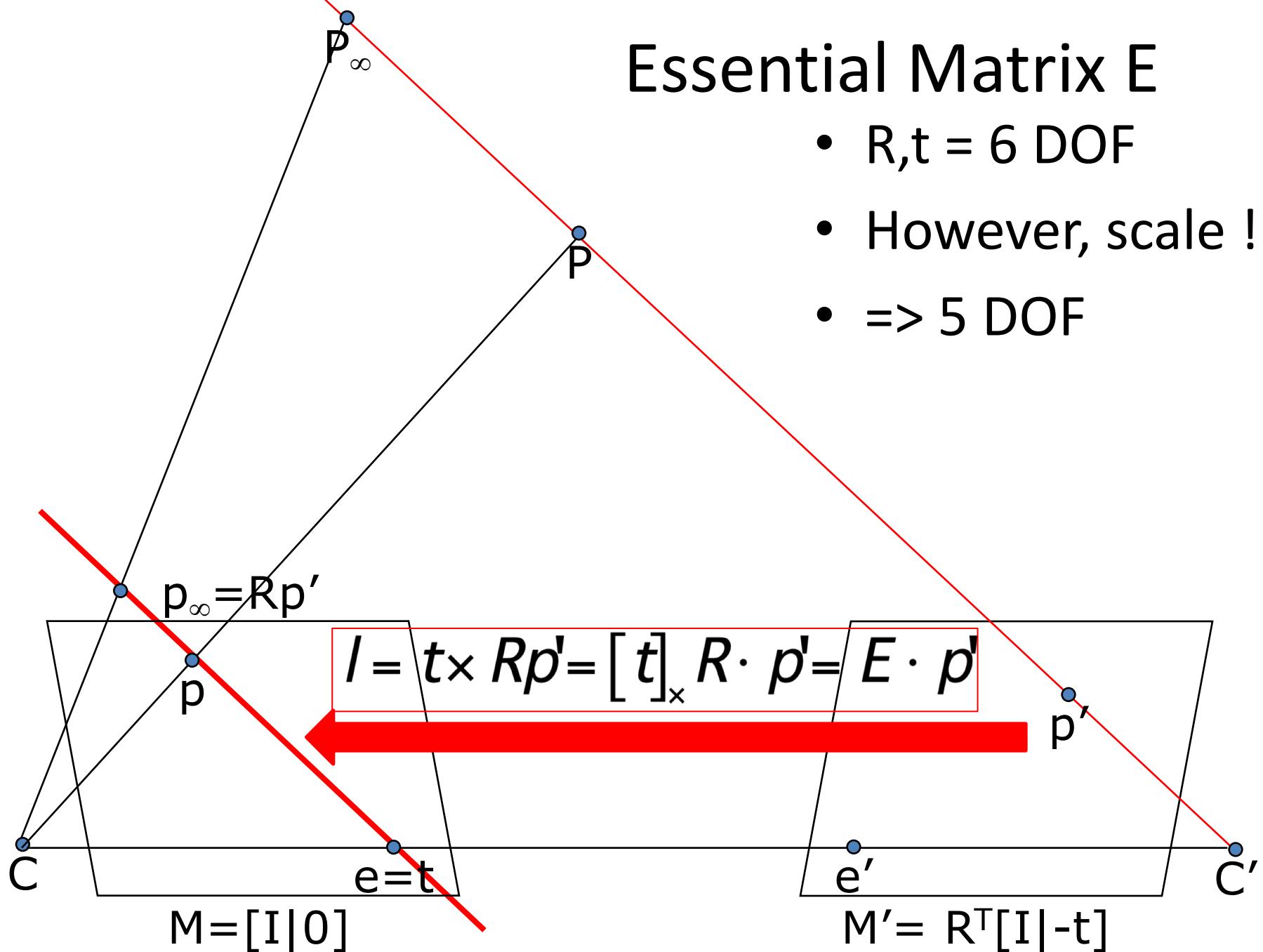
$$l = t \times Rp'$$

Epipolar Lines



Essential Matrix E

- $R, t = 6 \text{ DOF}$
- However, scale !
- $\Rightarrow 5 \text{ DOF}$



Essential Matrix

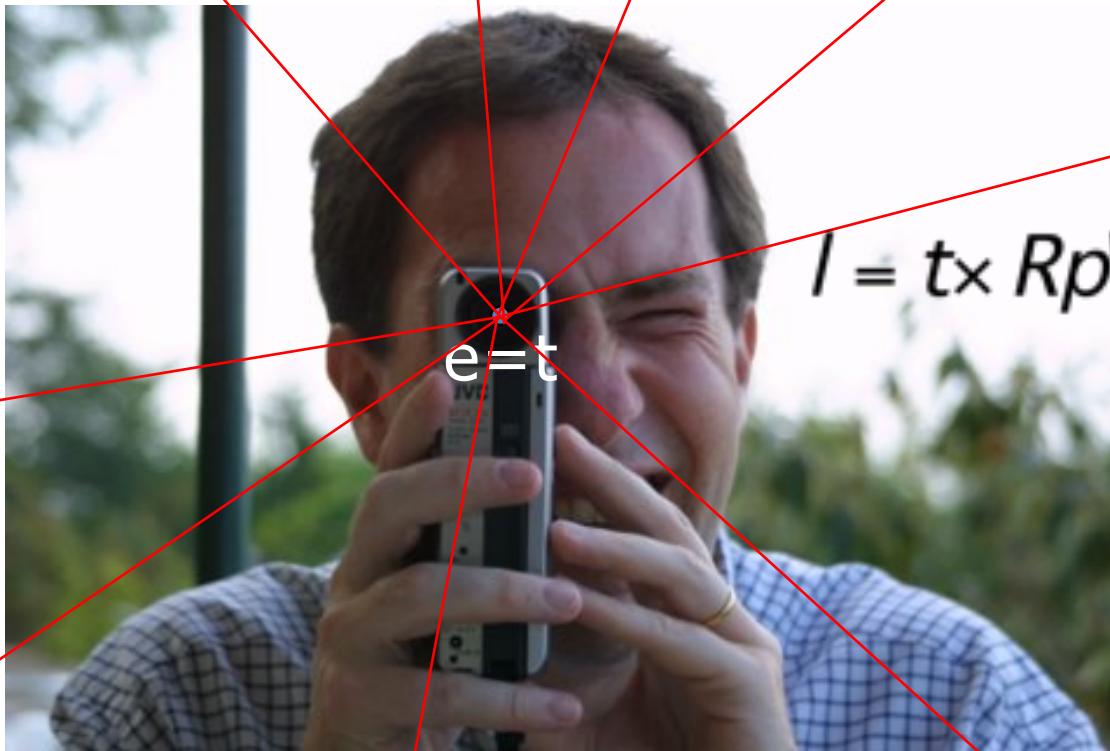
- mapping from p' to l

$$l = t \times Rp' = [t]_x R \cdot p' = E \cdot p'$$

- $E = 3 \times 3$ matrix
- Because p is on l , we have

$$p^T E p' = 0$$

Epipolar lines

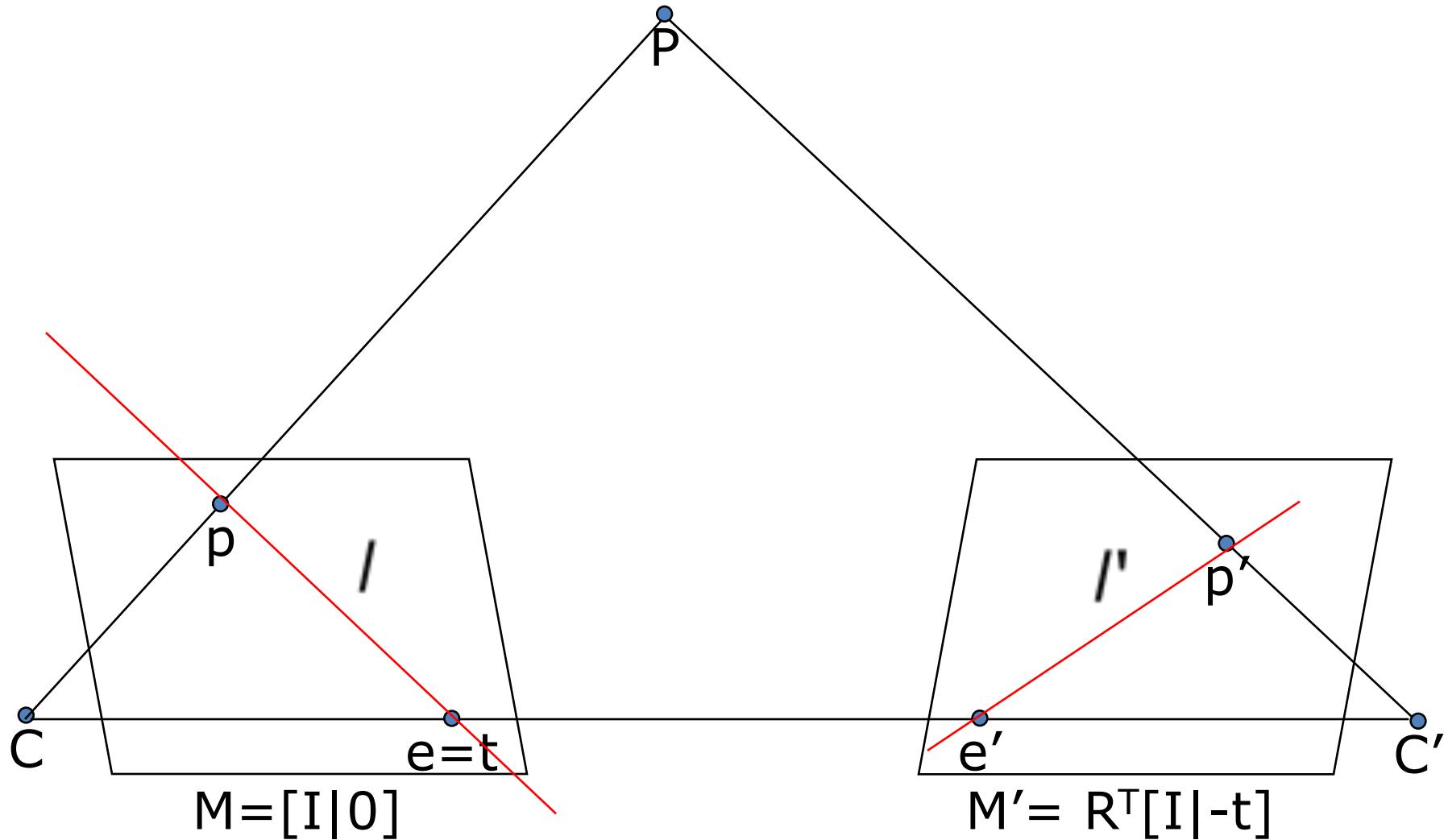


$$l = t \times Rp'$$

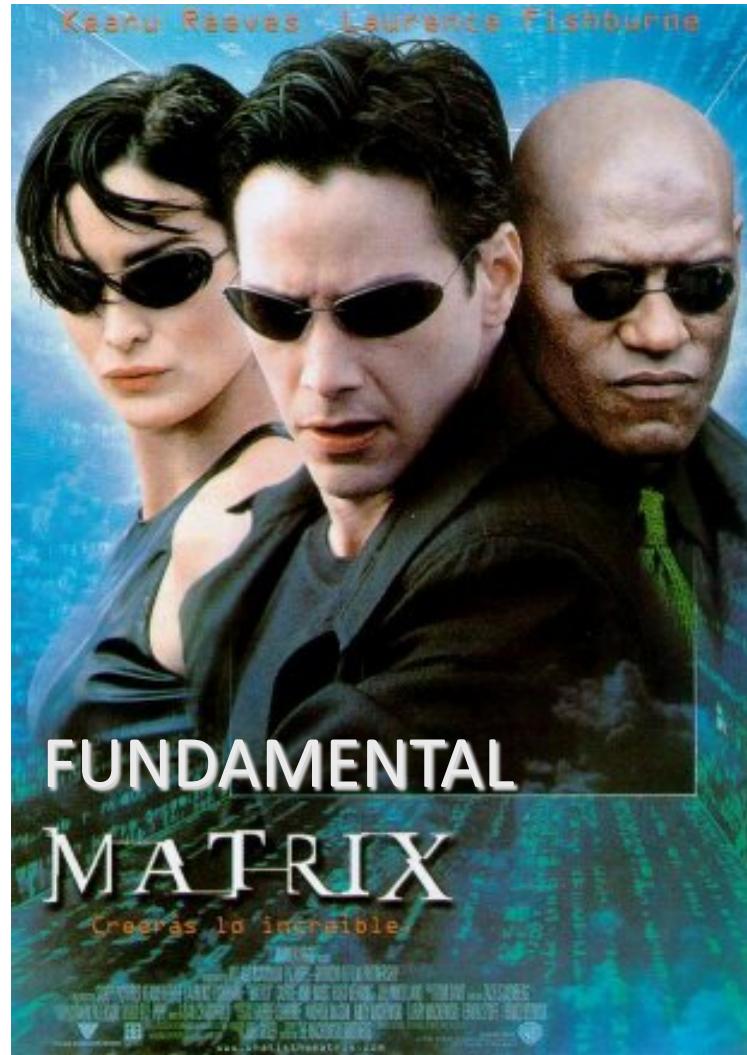
$$e=t$$

$$p_\infty = Rp'$$

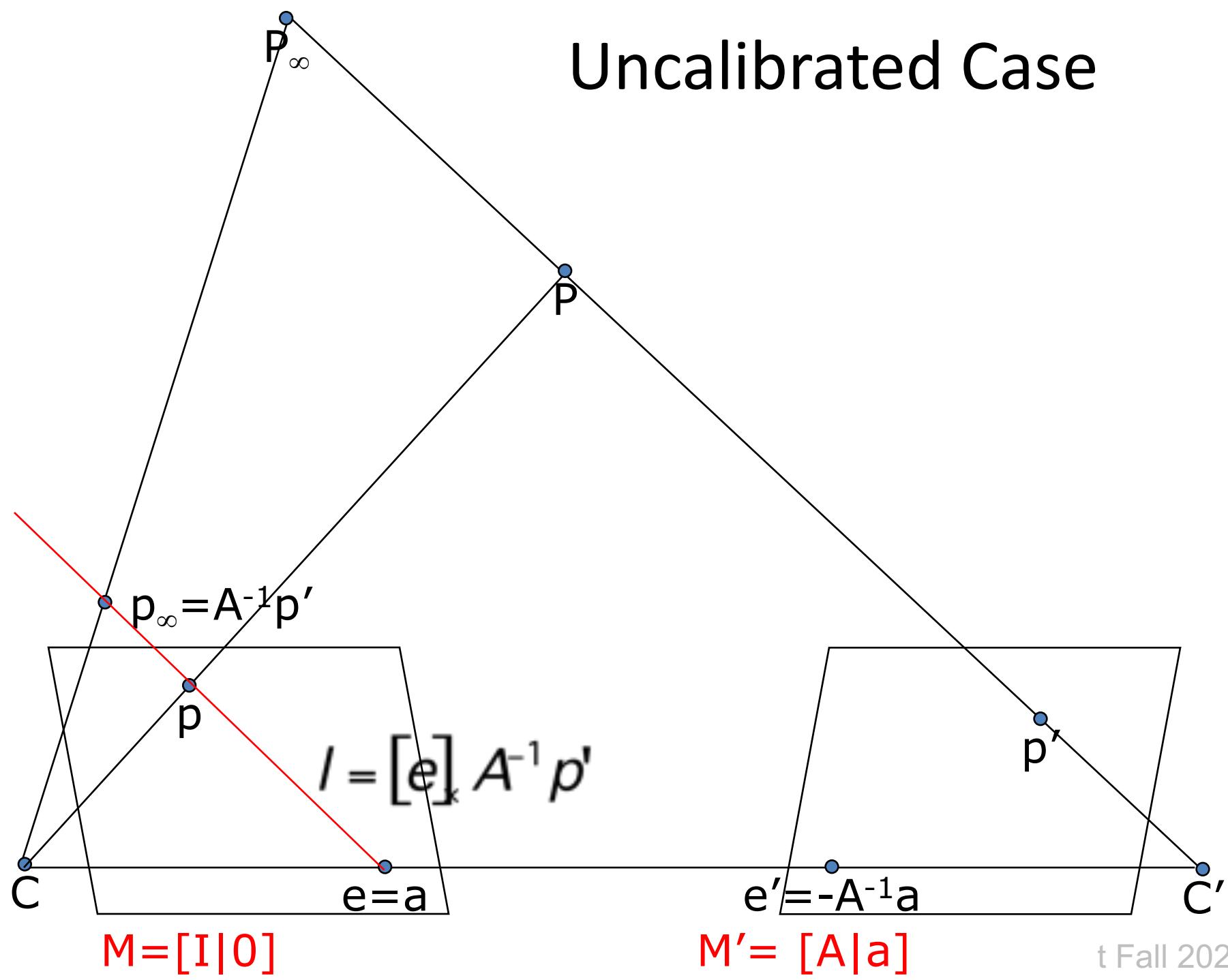
Epipolar Plane



Fundamental Matrix

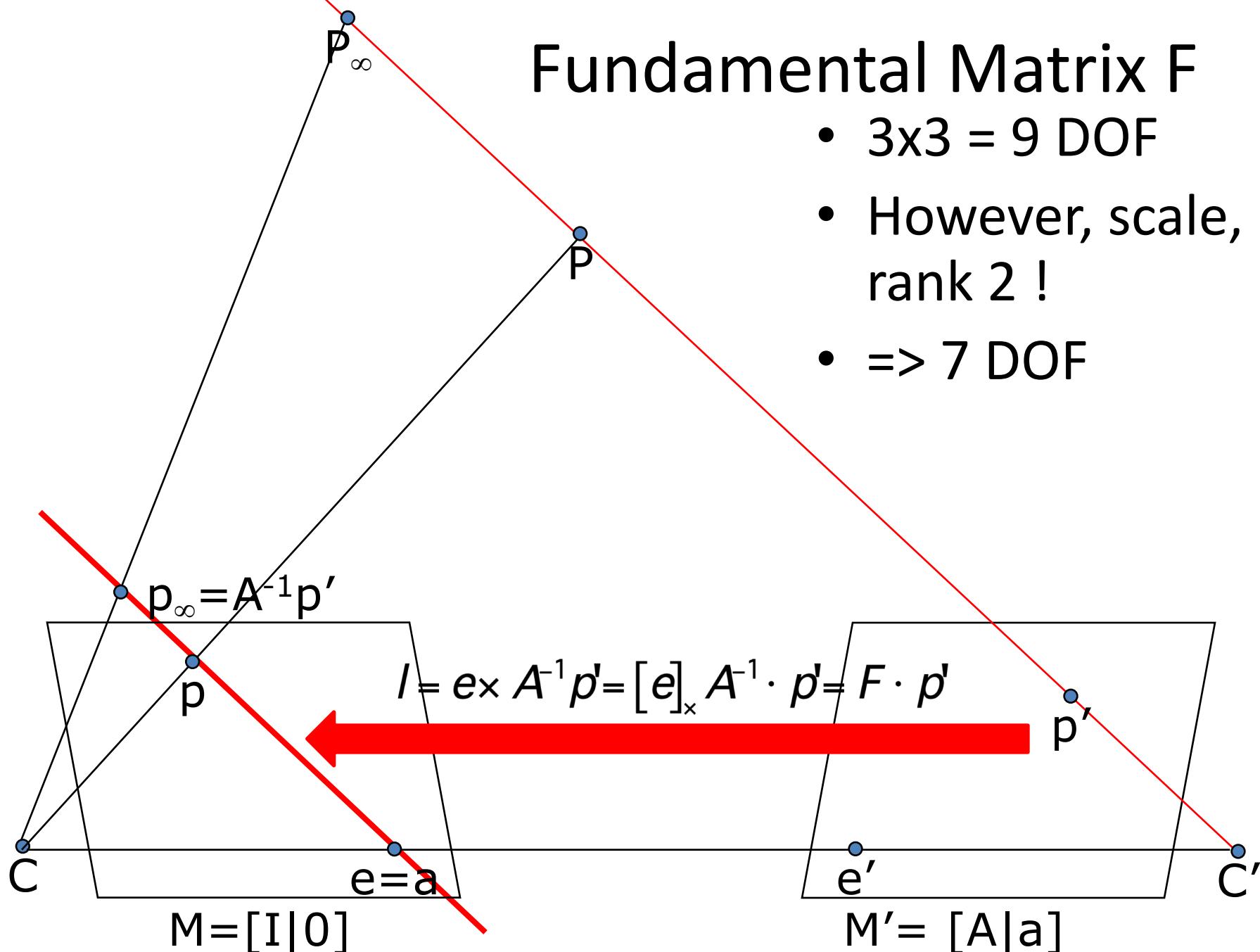


Uncalibrated Case



Fundamental Matrix F

- $3 \times 3 = 9$ DOF
- However, scale, rank 2 !
- $\Rightarrow 7$ DOF



Fundamental Matrix

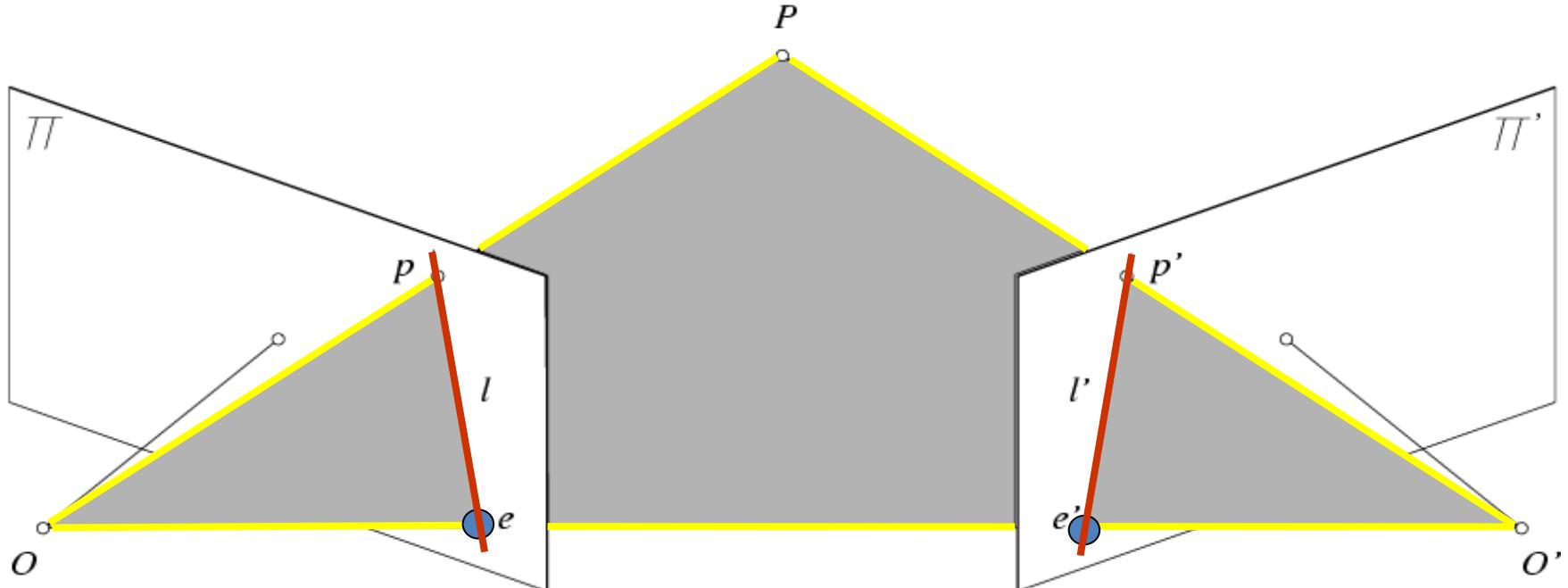
- mapping from p' to l

$$l = ex \ A^{-1} p' = [e]_x A^{-1} \cdot p' = F \cdot p'$$

- $F = 3 \times 3$ matrix
- Because p is on l , we have

$$p^T F p' = 0$$

Uncalibrated Case, Forsyth & Ponce Version



$$\hat{\mathbf{p}}^T \mathcal{E} \hat{\mathbf{p}}' = 0$$

$$\mathbf{p} = \mathcal{K}\hat{\mathbf{p}} \quad \xrightarrow{\hspace{1cm}} \quad \mathbf{p}^T \mathcal{F} \mathbf{p}' = 0 \quad \text{with} \quad \mathcal{F} = \mathcal{K}^{-T} \mathcal{E} \mathcal{K}'^{-1}$$

$$\mathbf{p}' = \mathcal{K}'\hat{\mathbf{p}}'$$

Fundamental Matrix
(Faugeras and Luong, 1992)

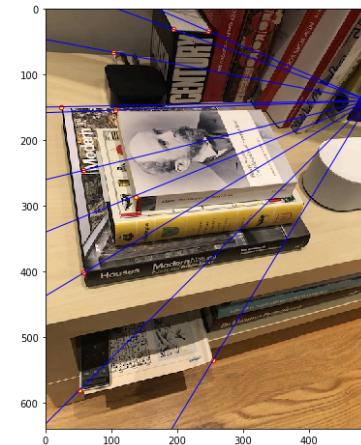
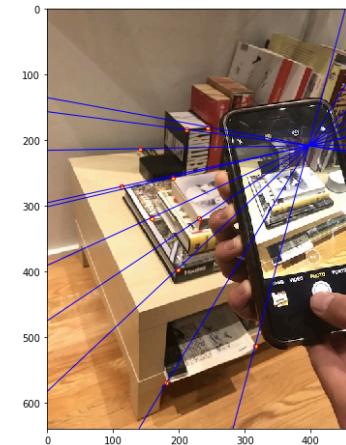
Properties of the Fundamental Matrix

- $\mathcal{F}p'$ is the epipolar line associated with p' .
- $\mathcal{F}^T p$ is the epipolar line associated with p .
- $\mathcal{F}^T e = 0$ and $\mathcal{F}e' = 0$.
- \mathcal{F} is singular.

Non-Linear Least-Squares Approach (Luong et al., 1993)

Minimize

$$\sum_{i=1}^n [d^2(\mathbf{p}_i, \mathcal{F}\mathbf{p}'_i) + d^2(\mathbf{p}'_i, \mathcal{F}^T\mathbf{p}_i)]$$



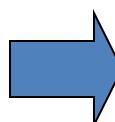
with respect to the coefficients of \mathcal{F} , using an appropriate rank-2 parameterization.

$$\begin{aligned} d(p, l) &= \text{point to line distance} \\ &= (ax + by + cw)/\sqrt{a^2 + b^2} \end{aligned}$$

$$d^2(p, l) = |ax + by + cw|^2/(a^2 + b^2)$$

The Eight-Point Algorithm (Longuet-Higgins, 1981)

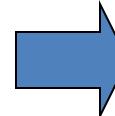
$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$



$$(uu', uv', u, vu', vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$



$$\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$



Minimize:

$$\sum_{i=1}^n (\mathbf{p}_i^T \mathcal{F} \mathbf{p}'_i)^2$$

under the constraint

$$|\mathcal{F}|^2 = 1.$$

The Normalized Eight-Point Algorithm (Hartley, 1995)

- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels:

$$q_i = T p_i \quad q'_i = T' p'_i.$$

- Use the eight-point algorithm to compute \mathcal{F} from the points q_i and q'_i .
- Enforce the rank-2 constraint.
- Output $T^{-1} \mathcal{F} T'$.