

## *Lecture16: Segmentation*

*So Many Stolen PPT slides from friends!!*



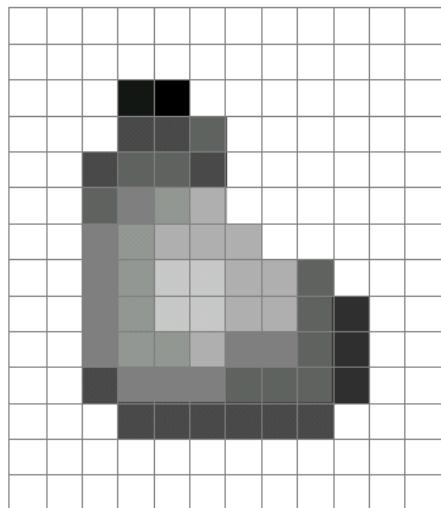
**CS 3630!**



# Image Processing: A Quick Review

# Grayscale Images

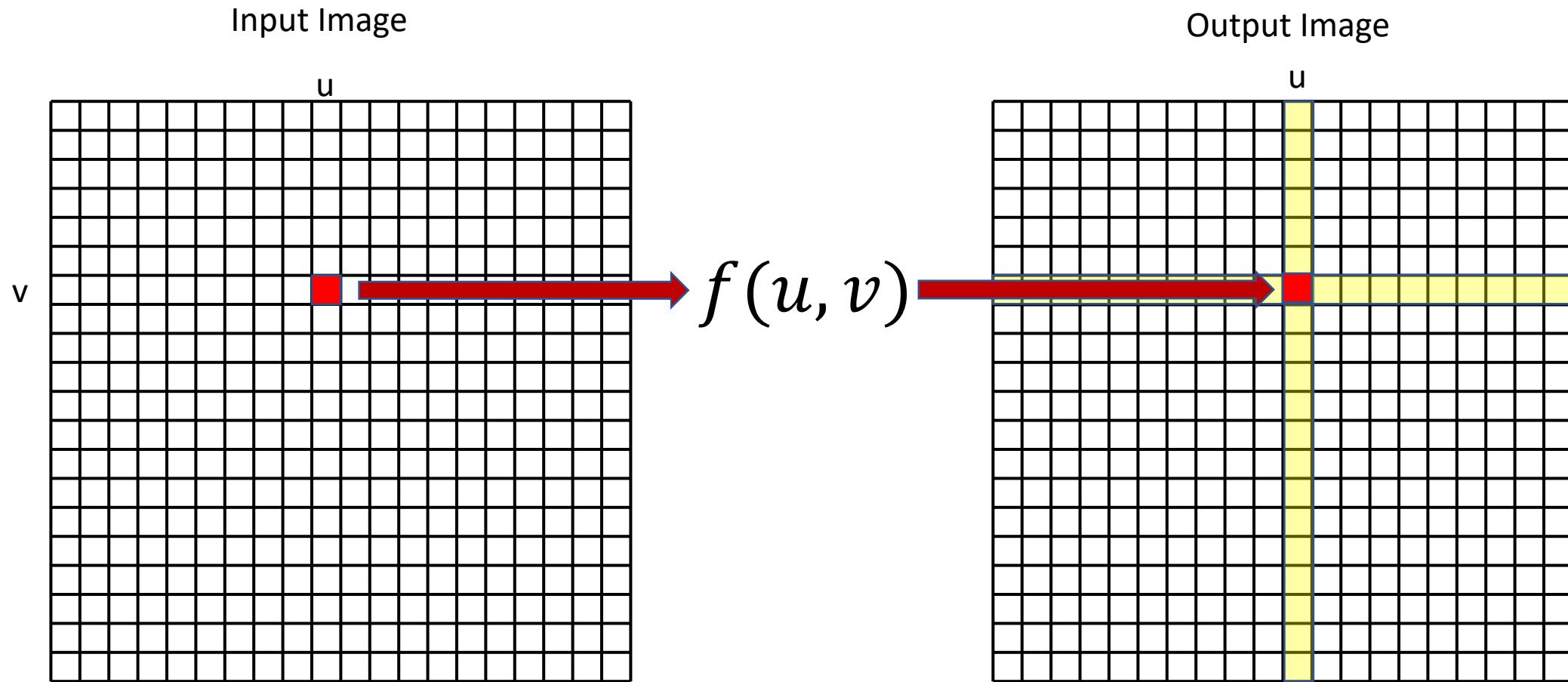
A grid (matrix) of intensity values



255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255	255	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255	255	255	255	255
255	255	127	145	145	175	127	127	95	47	255	255	255	255	255	255
255	255	74	127	127	127	95	95	95	47	255	255	255	255	255	255
255	255	255	74	74	74	74	74	74	74	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255

(common to use one byte per value: 0 = black, 255 = white)

# Monadic operators for filtering



Monadic operations take a single pixel as input and give a single pixel output, and do not consider neighboring pixel values

# Example of some monadic image operations

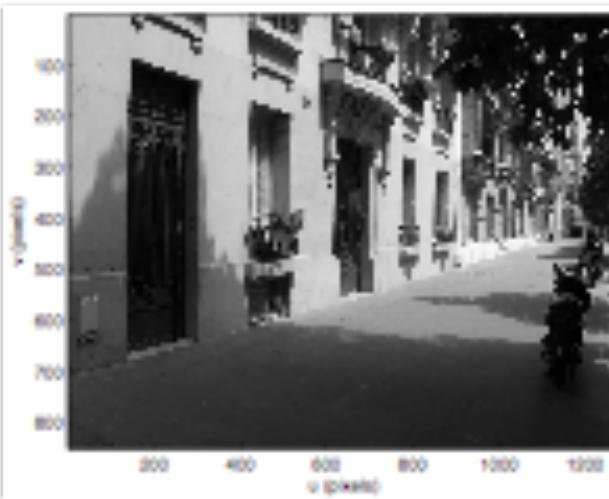
a original

b thresholding (all values greater than some threshold  $\tau$  are mapped to white, and values lower than the threshold are mapped to black)

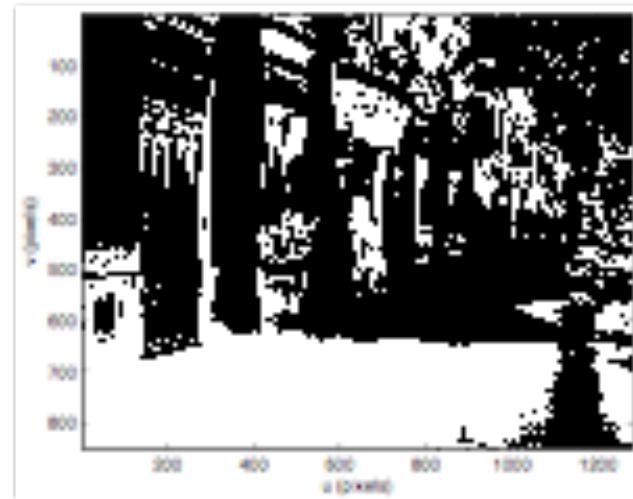
lets look at this one more closely

c histogram normalized

d posterization (conversion of a continuous gradation of tone to several regions of fewer tones, with abrupt changes from one tone to another)



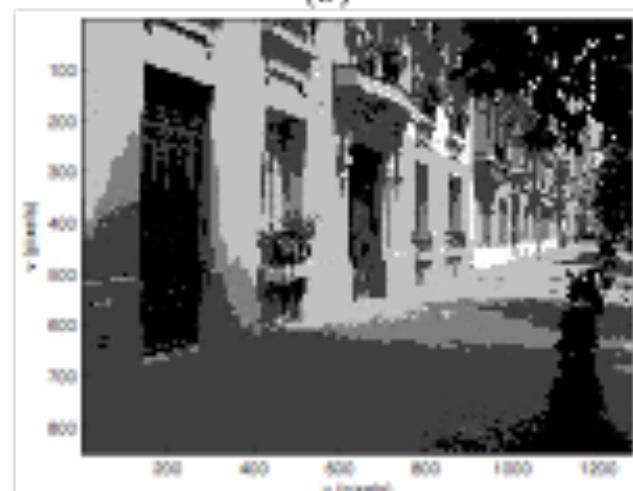
(a)



(b)



(c)



(d)

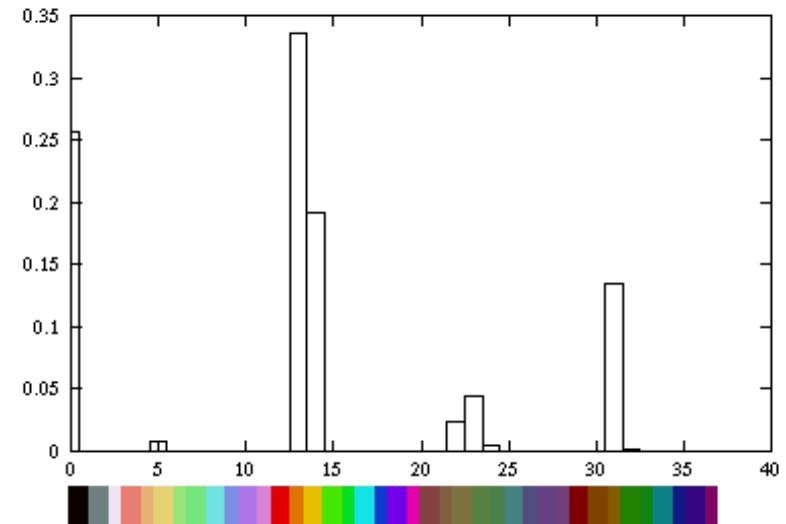
# Color Histogram

- A histogram array,  $\mathcal{H}$ , represents the distribution of colors (or *intensities* if grayscale) in the image  $\mathcal{I}$ .
- For the  $i^{th}$  color  $C_i$ , the value  $\mathcal{H}[i]$  gives the number of pixels of color  $C_i$  in image  $\mathcal{I}$ .

A simple algorithm

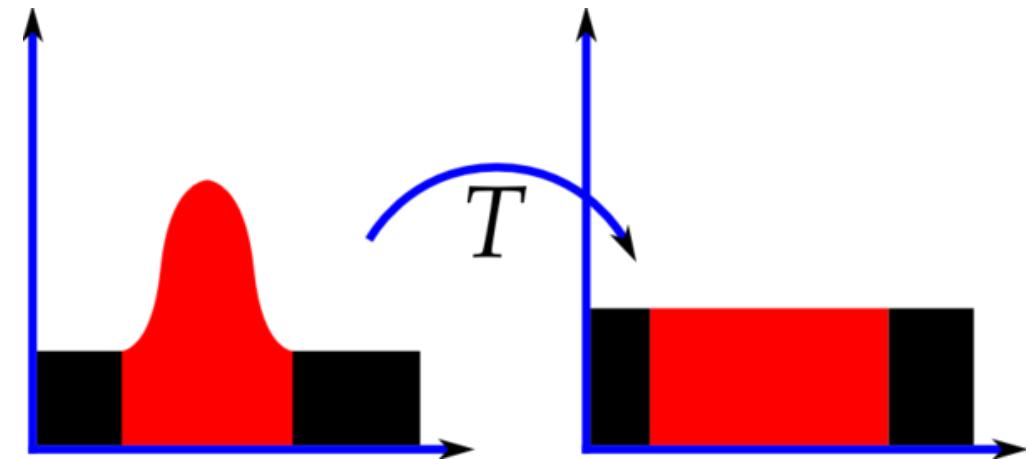
Let  $C(p)$  denote the color label associated to pixel  $p$

1. Initialize:  $\mathcal{H}[i] \leftarrow 0$
2. For pixel  $p \in \mathcal{I}$
3.      $\mathcal{H}[C(p)] += 1$



# Histogram filter

- A histogram can be used to adjust (redistribute) the intensity or color distribution of an image
- Usually increases the contrast of an image by effectively spreading out the most frequent intensity values
- Useful in images with backgrounds and foregrounds that are both bright or both dark



Histograms of an image before and after equalization.

# Image Contrast

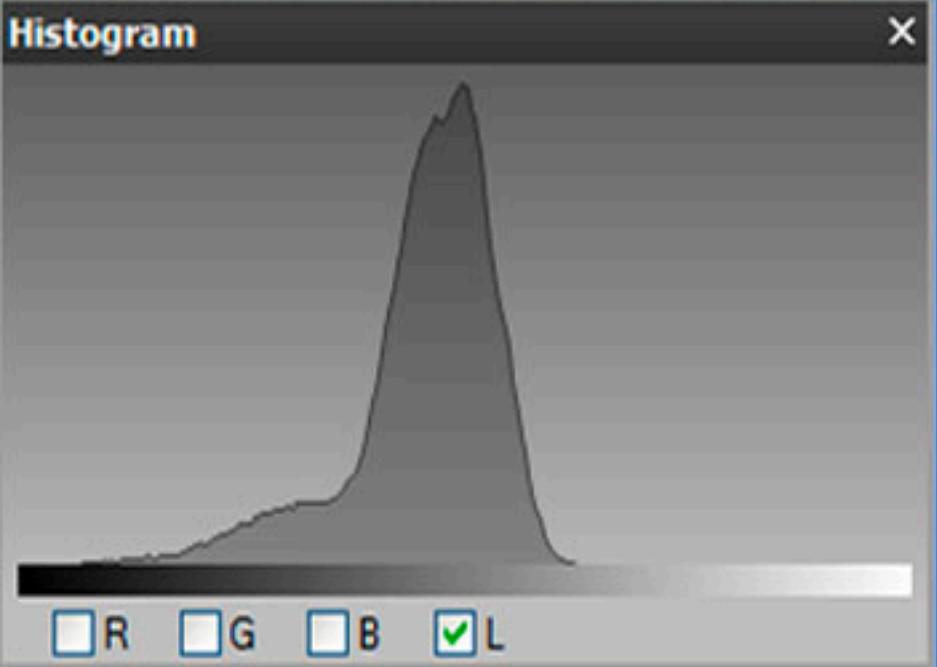
- Contrast is the difference between maximum and minimum pixel intensity in an image (or in a region of an image).



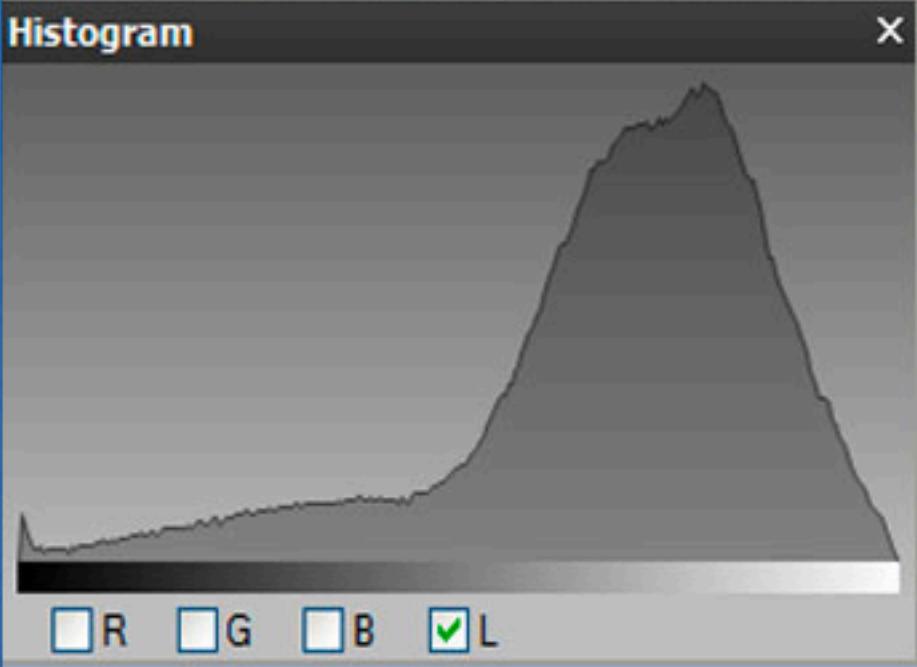
Low contrast



High contrast



Low contrast



High contrast

Capability  
provided by  
[skimage.exposure](#)

# Local Operators

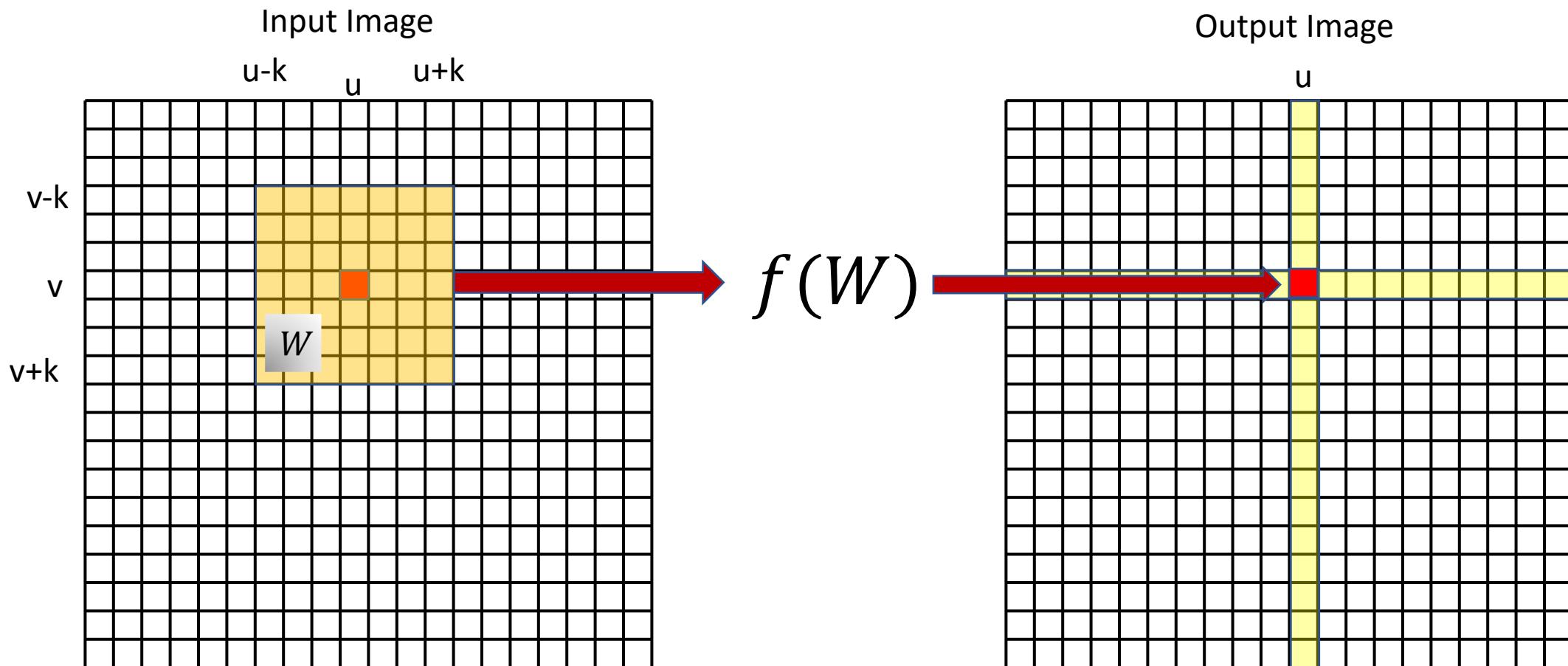
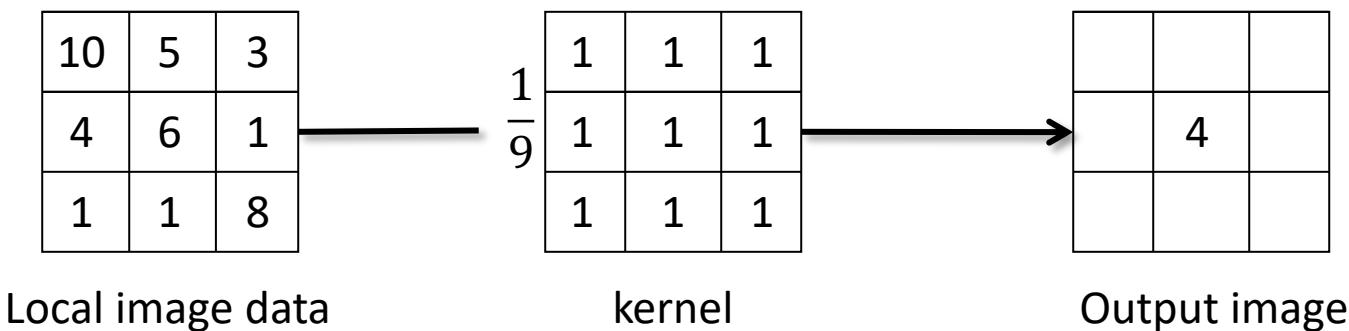


Image processing operations that use a region of pixels in the input image to determine the value of a single pixel in the output image

# Example: Averaging using a linear filtering

- Replace each pixel by a linear combination of its neighbors
- The matrix of the linear combination is called the “kernel,” “mask”, or “filter”

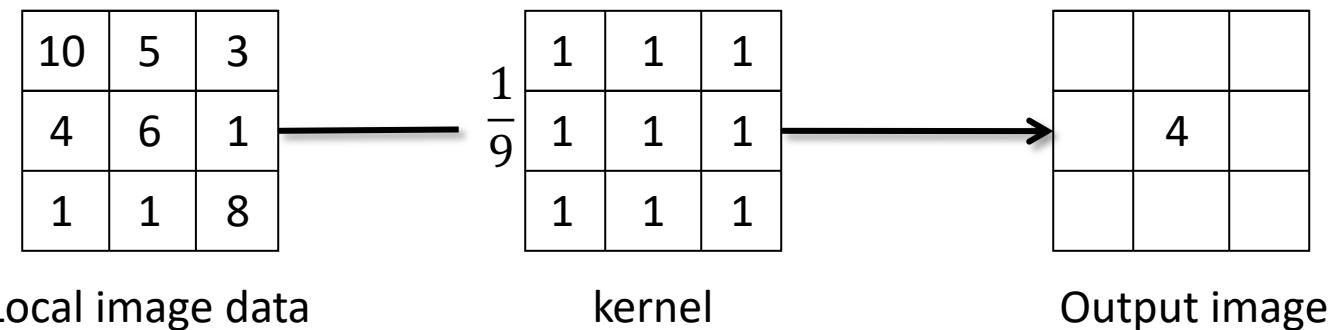


The effect is a blurring of the image.

# Example: Averaging using a linear filtering

Let  $F$  be the image,  $H$  be the kernel (of size  $2k + 1$  by  $2k + 1$ ), and  $G$  be the output image

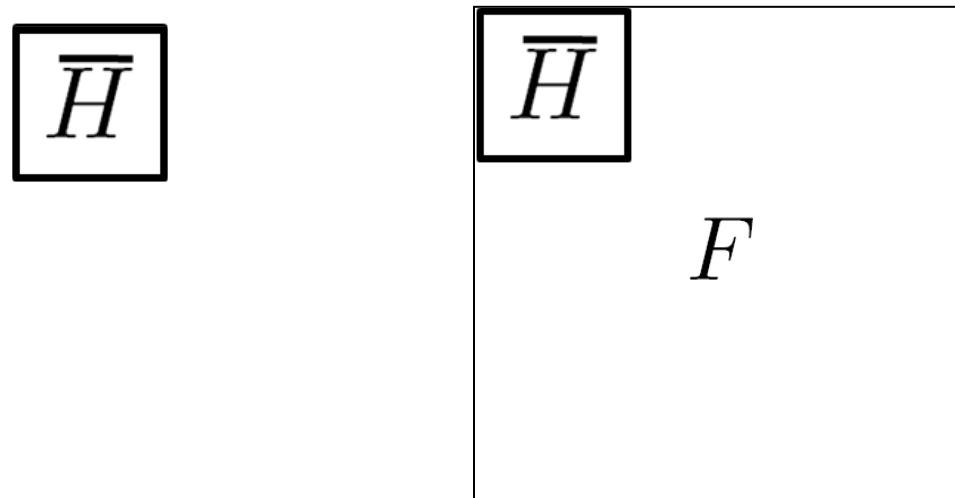
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i + u, j + v]$$



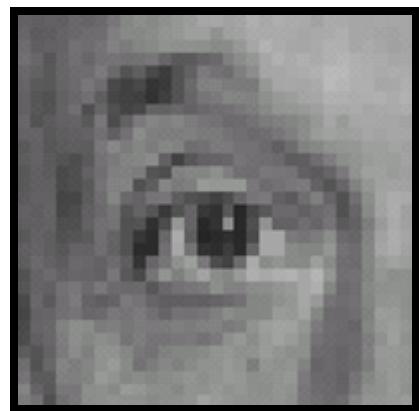
The effect is a blurring of the image.

# Convolution

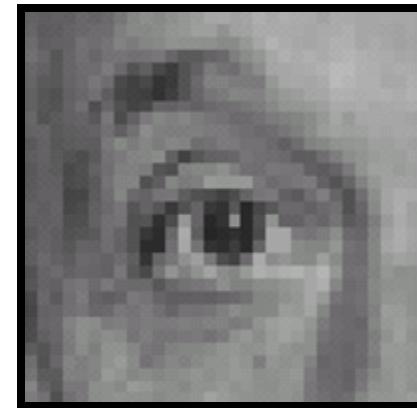
- Applying a kernel to an image in this way is called convolution



# Linear filters: examples



0	0	0
0	1	0
0	0	0

 $=$ 

Original

Identical image

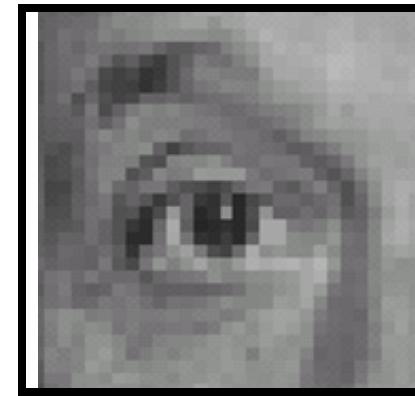
# Linear filters: examples



Original



0	0	0
1	0	0
0	0	0



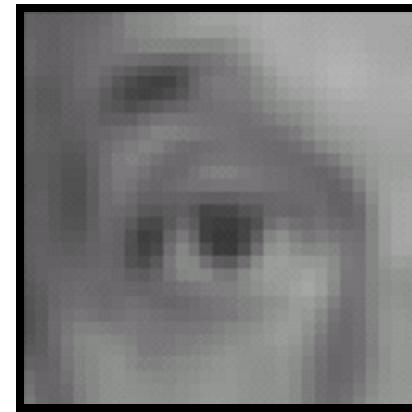
Shifted right  
By 1 pixel

# Linear filters: examples



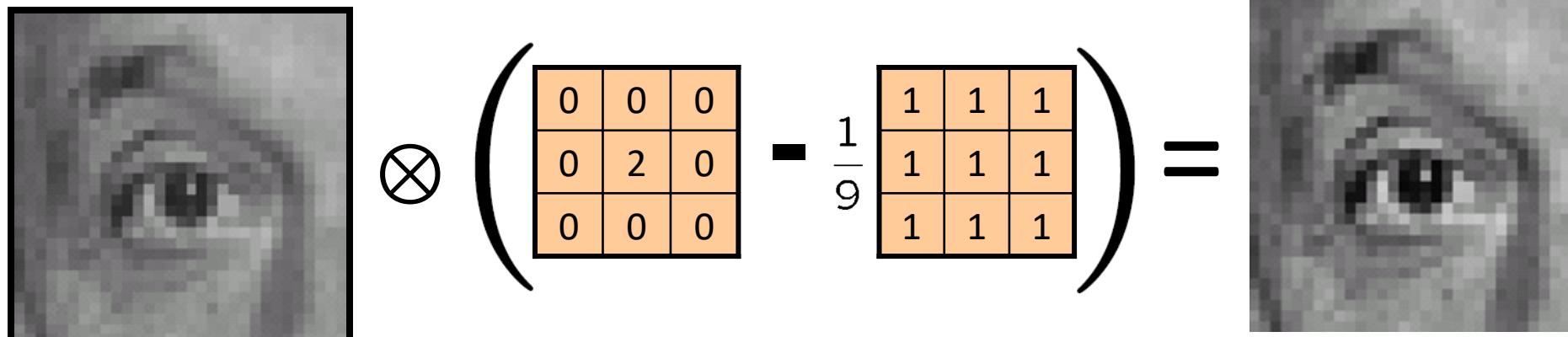
Original

$$\otimes \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} =$$



Mean filter (blurring)

# Linear filters: examples



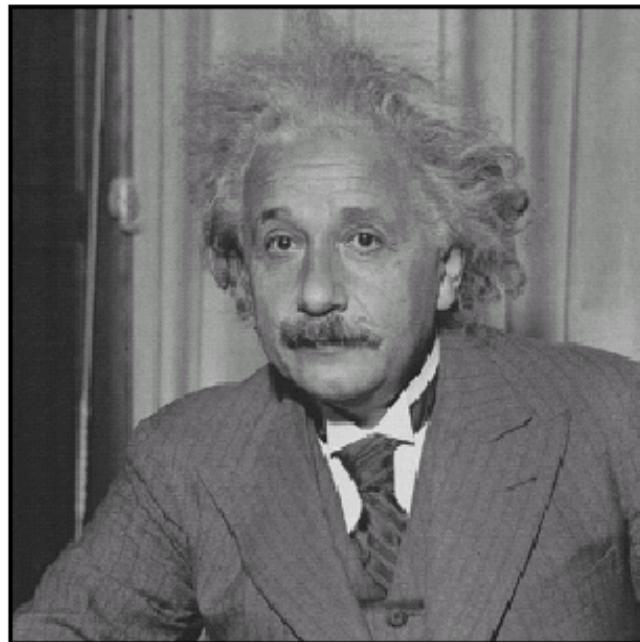
The diagram illustrates the application of a linear filter to a grayscale image of an eye. On the left is the **Original** image. In the center is the filter operation, represented by a tensor product symbol ( $\otimes$ ) followed by a 3x3 kernel matrix. The kernel matrix contains values: the top row is 0, 0, 0; the middle row is 0, 2, 0; and the bottom row is 0, 0, 0. To the right of the matrix is a subtraction operator ( $-$ ) followed by another 3x3 matrix. This second matrix has a value of  $\frac{1}{9}$  in the top-left corner and 1's in all other positions. The result of this operation is an equals sign (=) followed by the **Sharpening filter** output, which shows a more defined and sharper eye image.

Original

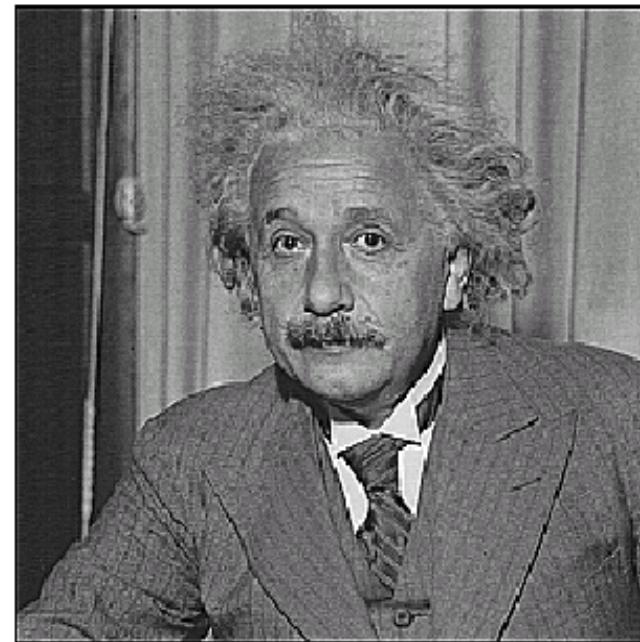
$\otimes \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{array} \right) - \frac{1}{9} \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right) =$

**Sharpening filter**  
(accentuates edges)

# Sharpening

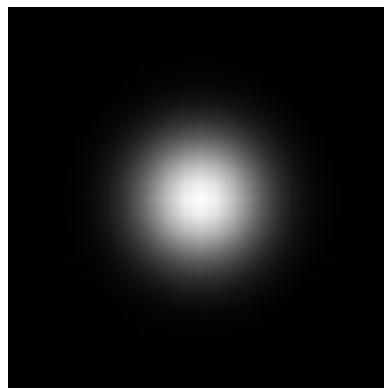
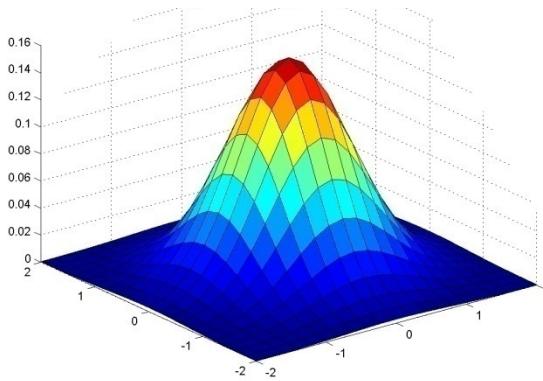


**before**



**after**

# Gaussian Kernel



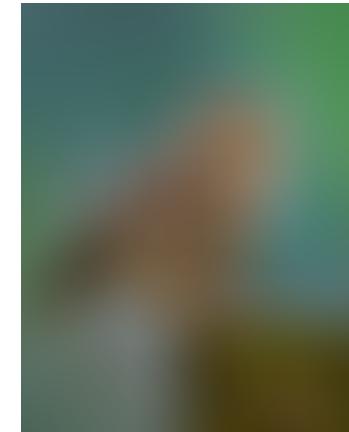
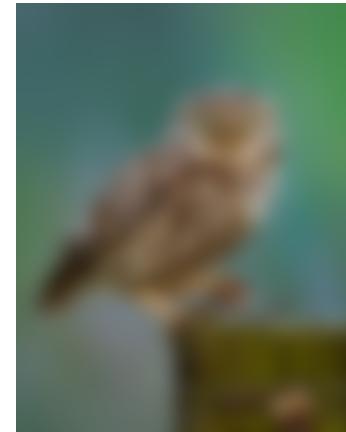
Approximated by:

$$\frac{1}{16}$$

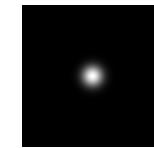
1	2	1
2	4	2
1	2	1

$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

# Gaussian filter



$\sigma = 1$  pixel



$\sigma = 5$  pixels



$\sigma = 10$  pixels



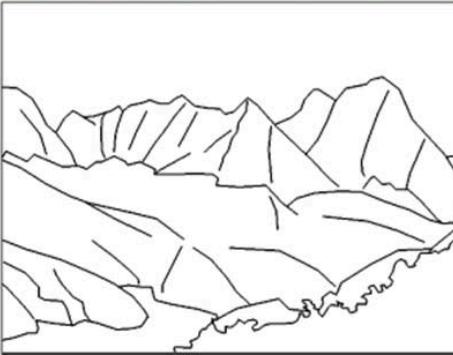
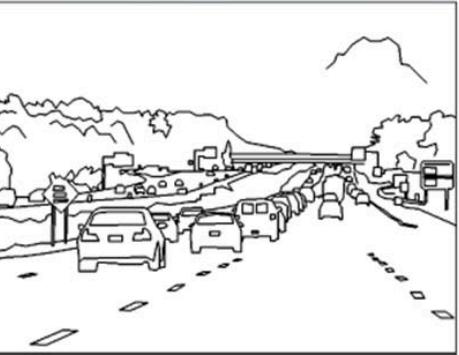
$\sigma = 30$  pixels

# Edge detection



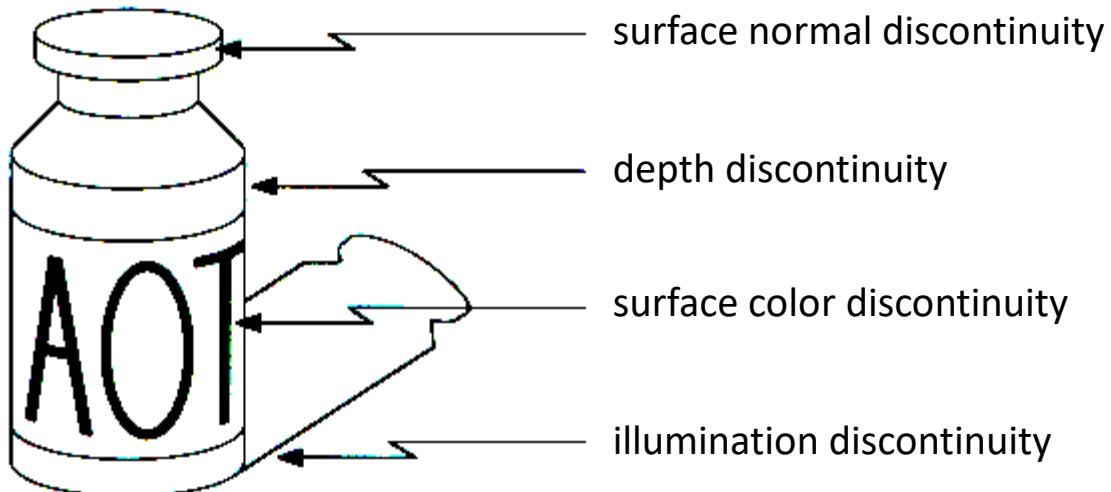
[Winter in Kraków photographed by Marcin Ryczek](#)

# Edge Detection



# Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
- Intuitively, edges carry most of the semantic and shape information from the image



# Edge detection

- **Ideal:** artist's line drawing

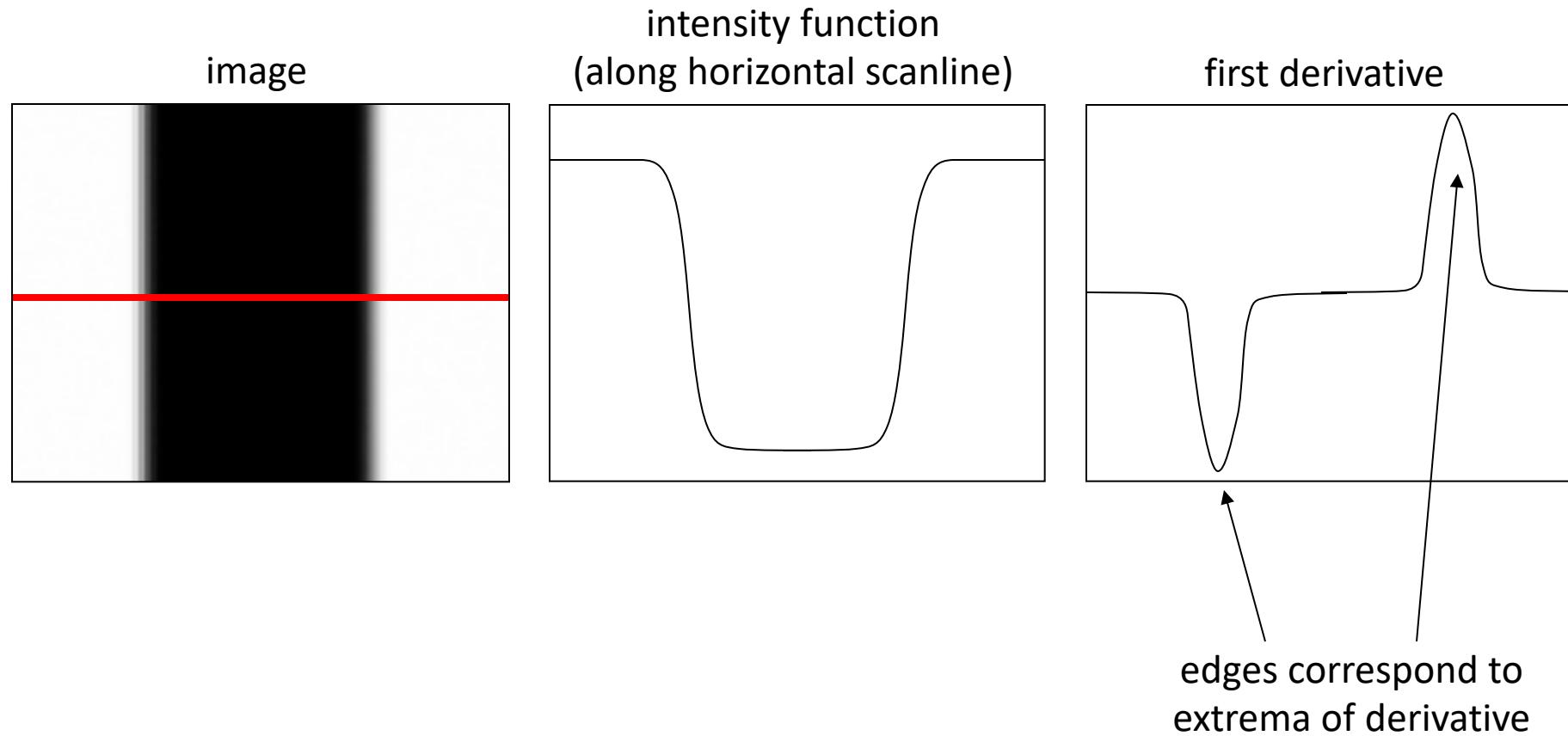


- **Reality:**



# Edge detection

- An edge is a place of rapid change in the image intensity function



# Derivatives with convolution

For 2D function  $f(x,y)$ , the partial derivative is:

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x + 1, y) - f(x, y)}{1}$$

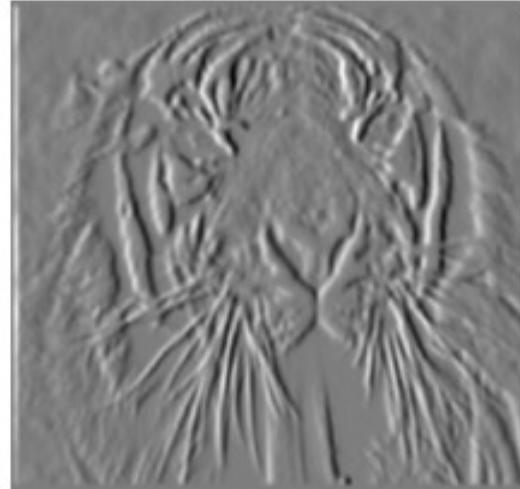
To implement the above as convolution, what would be the associated filter?

# Partial derivatives of an image



$$\frac{\partial f(x, y)}{\partial x}$$

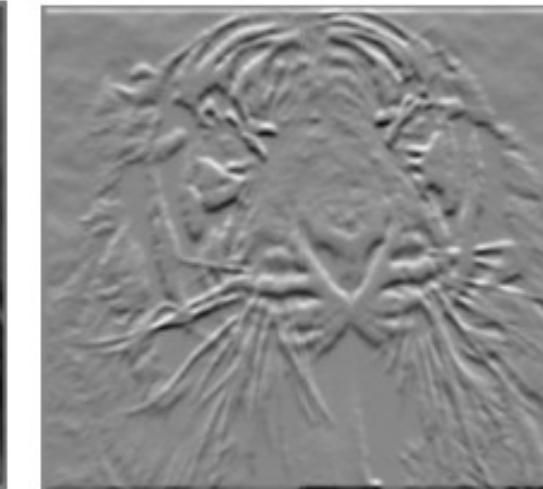
-1	1
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$$\frac{\partial f(x, y)}{\partial y}$$

-1      1  
or  
1      -1

-1	1
1	-1



Which shows changes with respect to x?

# Finite difference filters

- Other approximations of derivative filters exist:

**Prewitt:**  $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

**Sobel:**  $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

**Roberts:**  $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

# Edge Detection

## Through Convolution

**Sobel:**

-1	0	1
-2	0	2
-1	0	1

**Prewitt:**

-1	0	1
-1	0	1
-1	0	1

**Roberts:**

0	1
-1	0

**Canny:**

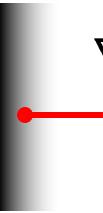
more complex multi-stage algorithm that uses a Gaussian filter and the intensity gradient in an image. One of the most widely used techniques.

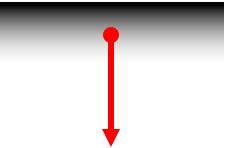


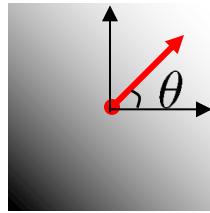
# Image gradient

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The gradient of an image:  $\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$


$$\nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right]$$


$$\nabla f = \left[ 0, \frac{\partial f}{\partial y} \right]$$


$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

The gradient direction is given by  $\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$

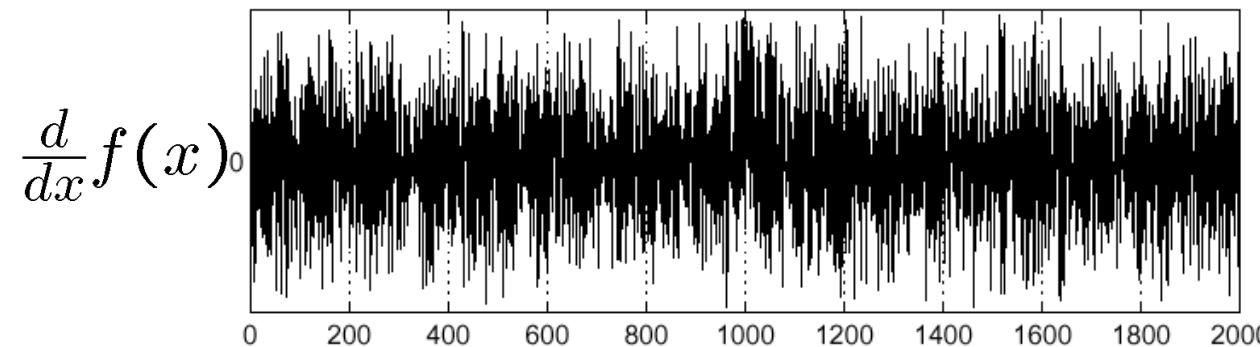
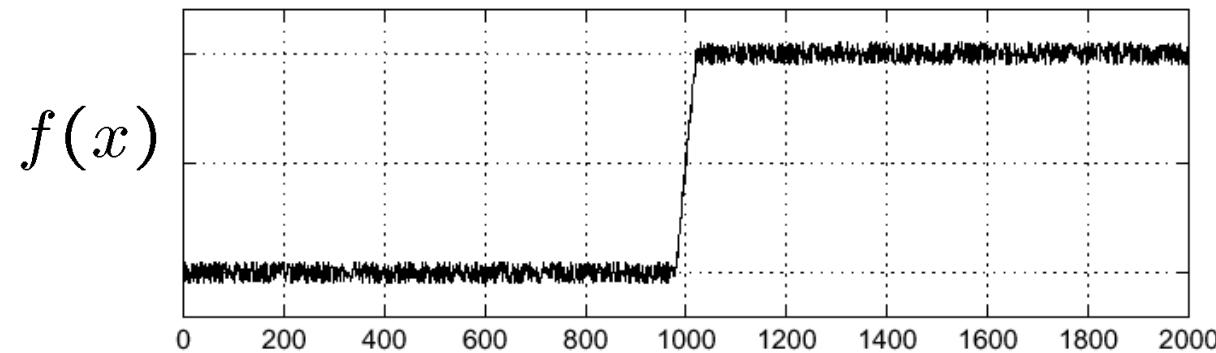
The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}$$

# Effects of noise

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Consider a single row or column of the image



Where is the edge?

Source: S. Seitz

# Gaussian Blurring and Edge Detection

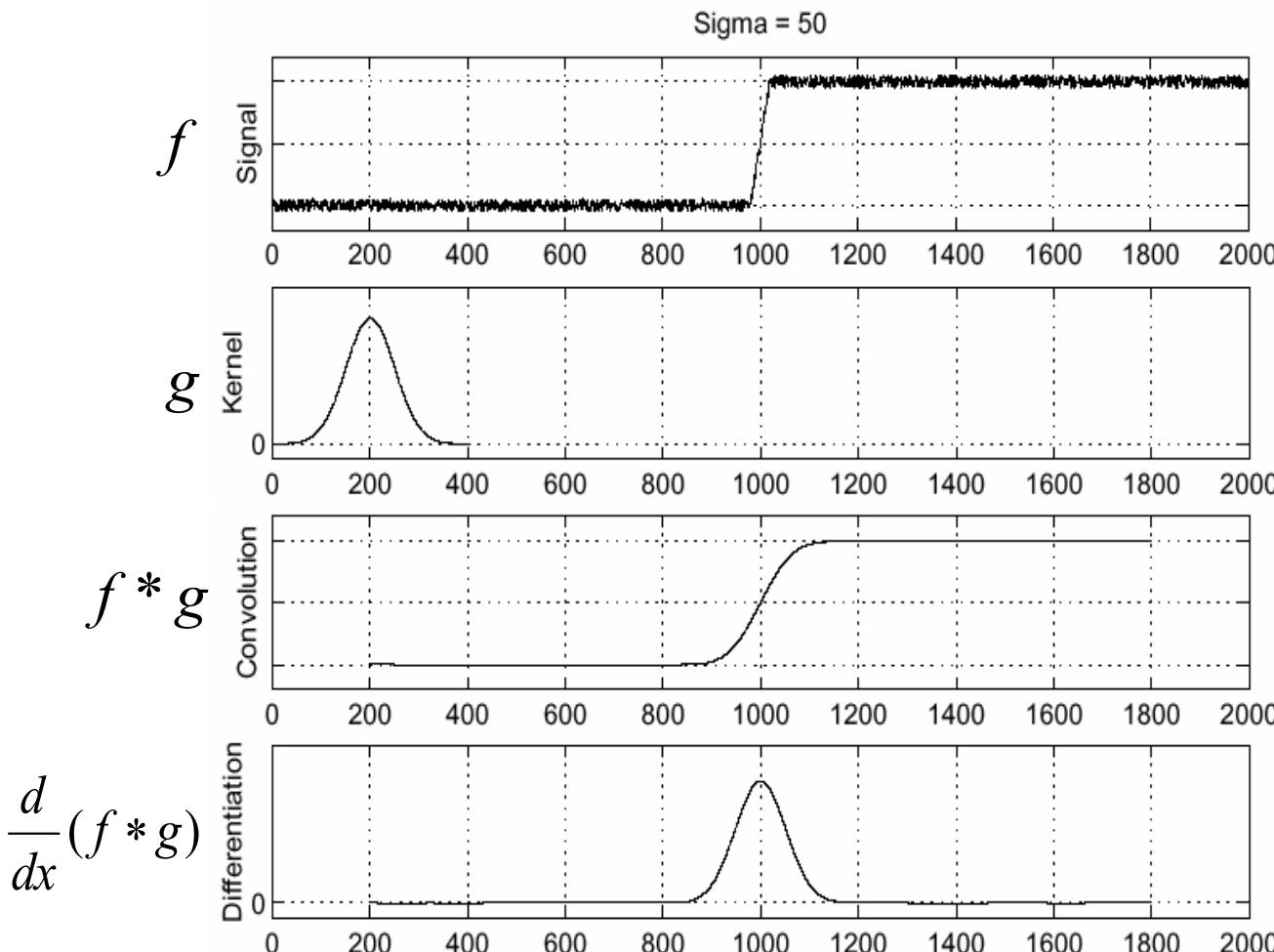
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Blurring the image slightly (Gaussian filter) can improve classification performance by smoothing away random noise in the image

Edge detection can help with some approaches to image classification, but it's not always necessary

# Solution: smooth first

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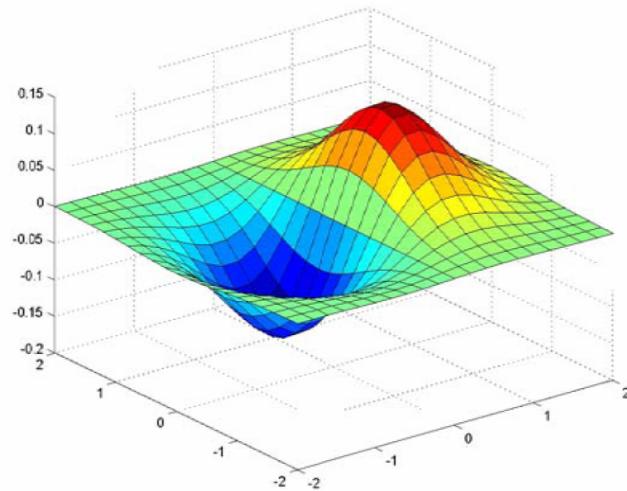


- To find edges, look for peaks in  $\frac{d}{dx}(f * g)$

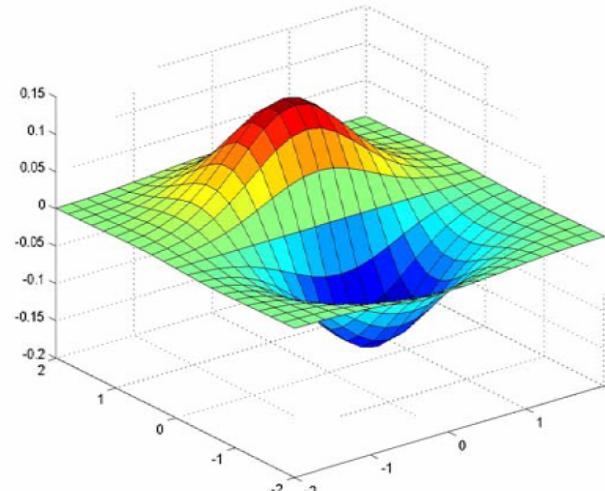
Source: S. Seitz

# Derivative of Gaussian filters

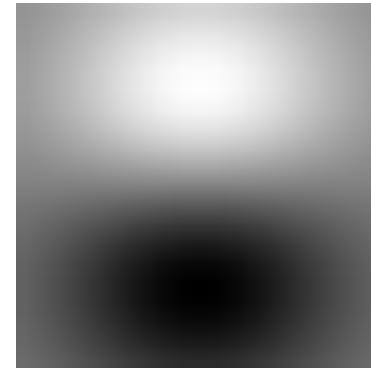
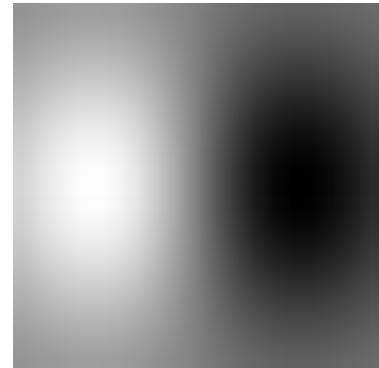
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x-direction



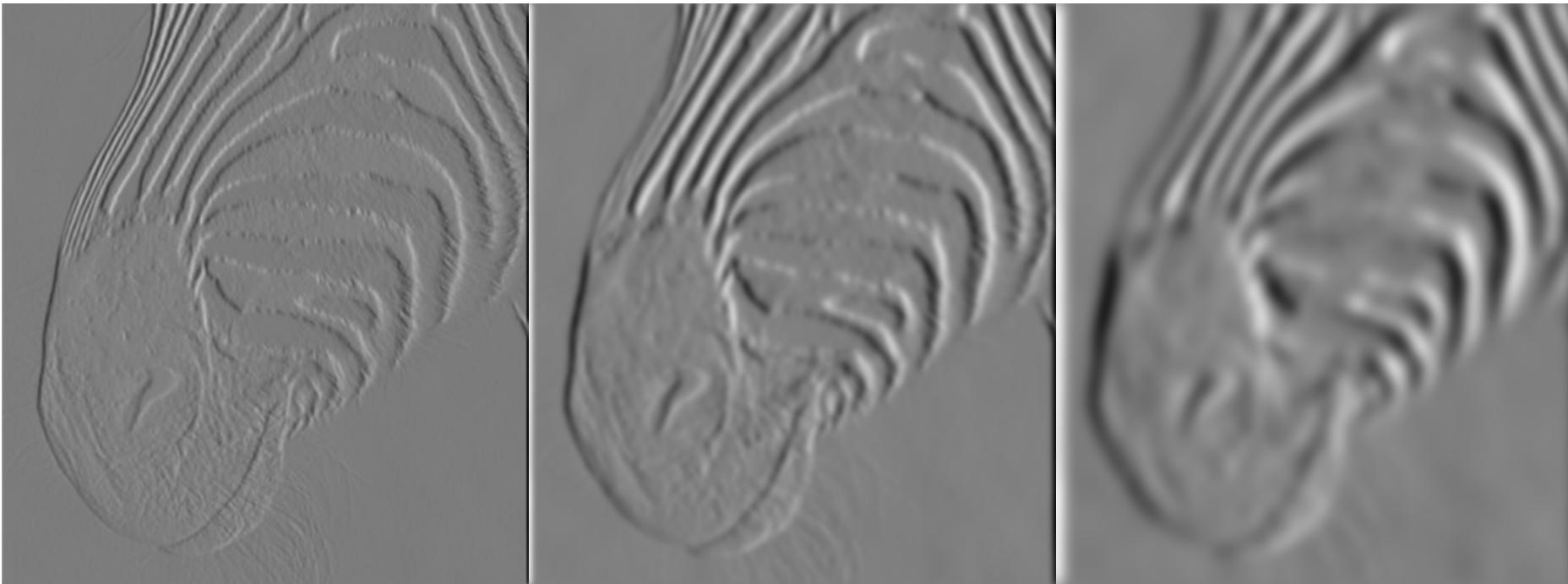
y-direction



Which one finds horizontal/vertical edges?

# Scale of Gaussian derivative filter

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1 pixel

3 pixels

7 pixels

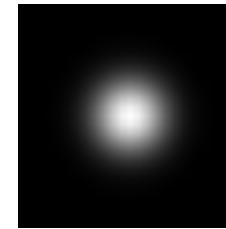
Smoothed derivative removes noise, but blurs edge. Also finds edges at different “scales”

# Review: Smoothing vs. derivative filters

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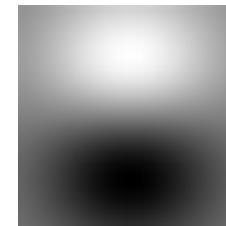
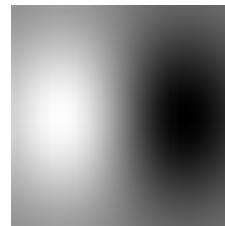
## Smoothing filters

- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter



## Derivative filters

- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions



# Building an edge detector

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original image



final output

# Building an edge detector

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norm of the gradient

# Building an edge detector

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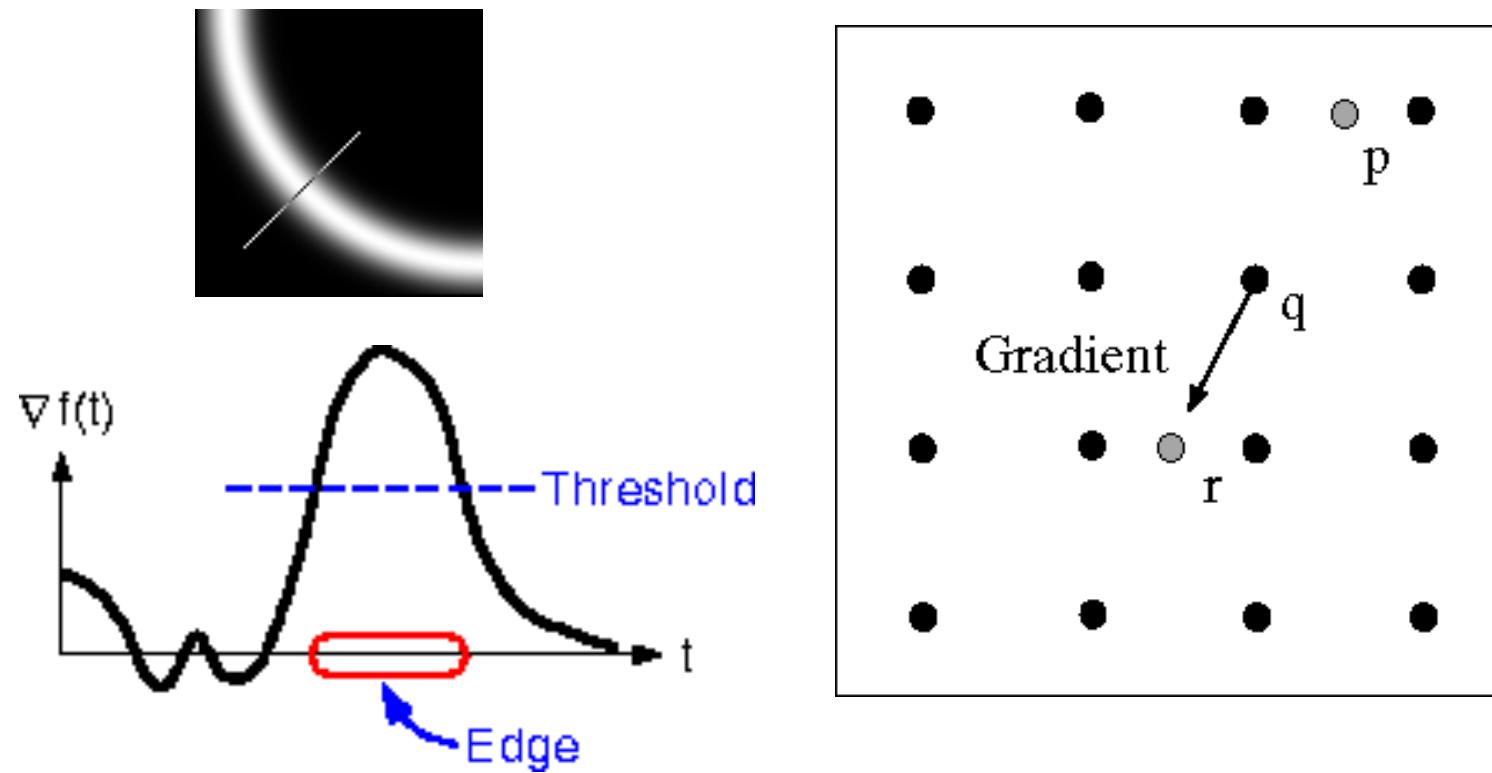


Thresholded norm of the gradient

How to turn  
these thick  
regions of  
the gradient  
into  
curves?

# Non-maximum suppression

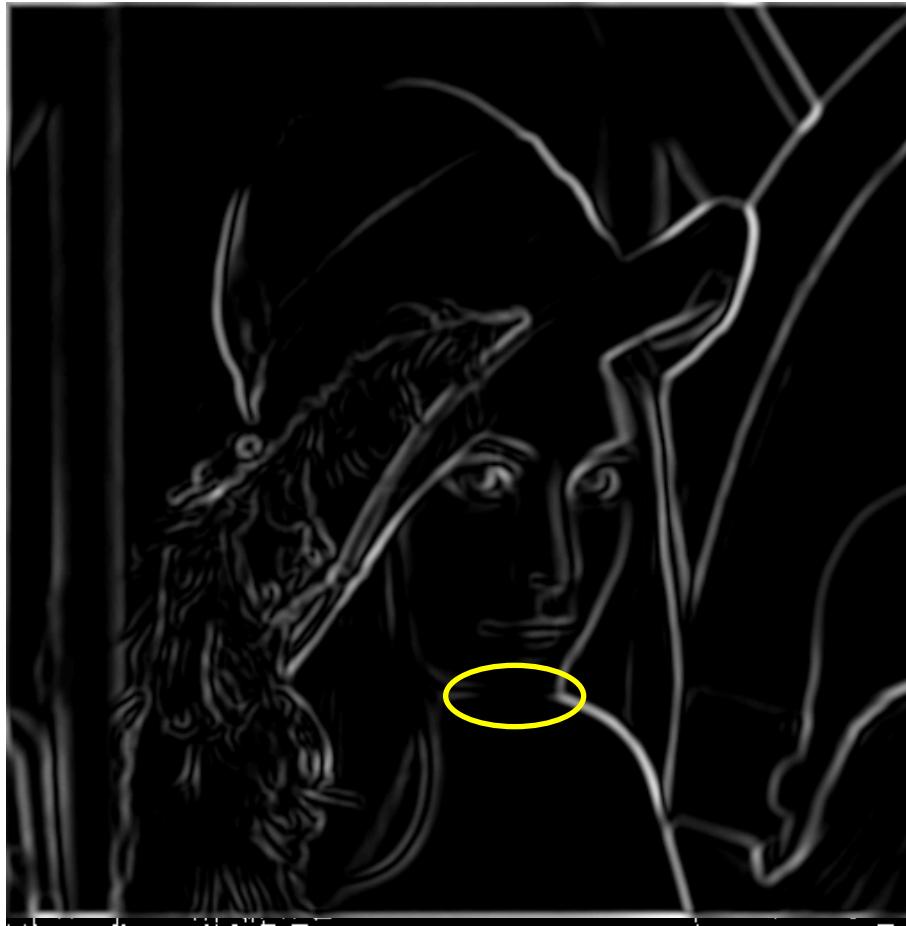
---



- For each location  $q$  above threshold, check that the gradient magnitude is higher than at neighbors  $p$  and  $r$  along the direction of the gradient
  - May need to interpolate to get the magnitudes at  $p$  and  $r$

# Non-maximum suppression

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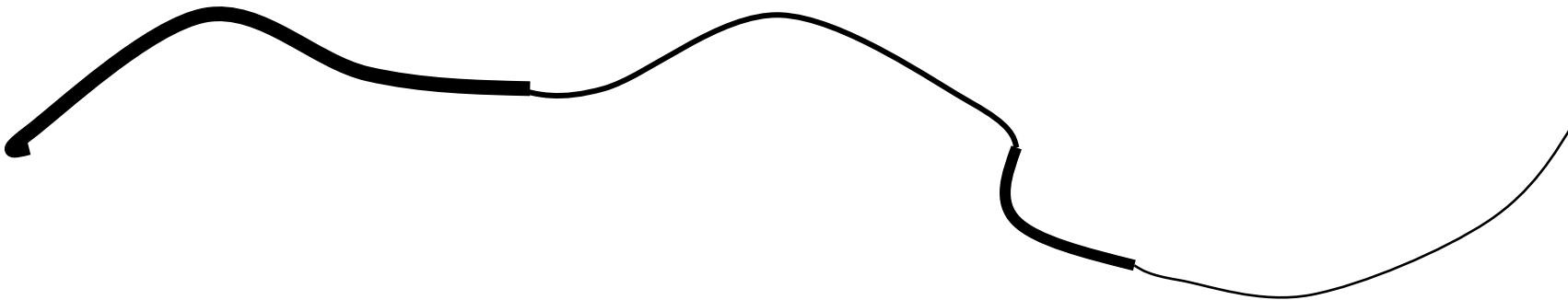


Another problem:  
pixels along  
this edge  
didn't  
survive the  
thresholding

# Hysteresis thresholding

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Use a high threshold to start edge curves, and a low threshold to continue them.



# Hysteresis thresholding

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original image



high threshold  
(strong edges)



low threshold  
(weak edges)



hysteresis threshold

# Recap: Canny edge detector

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1. Compute x and y gradient images
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression:**
  - Thin wide “ridges” down to single pixel width
4. **Linking and thresholding (hysteresis):**
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them

MATLAB: `edge(image, 'canny');`

J. Canny, [A Computational Approach To Edge Detection](#), IEEE Trans.  
Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

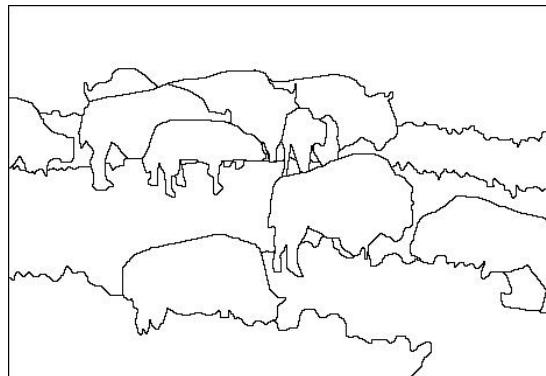
# Image gradients vs. meaningful contours

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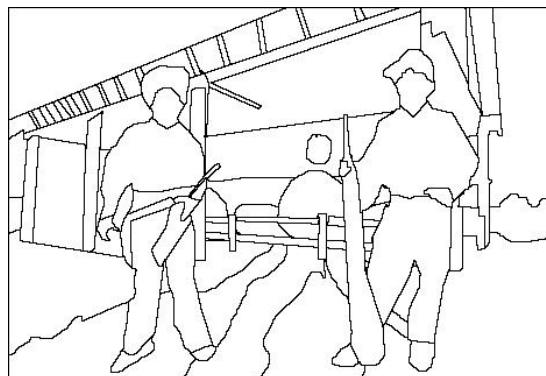
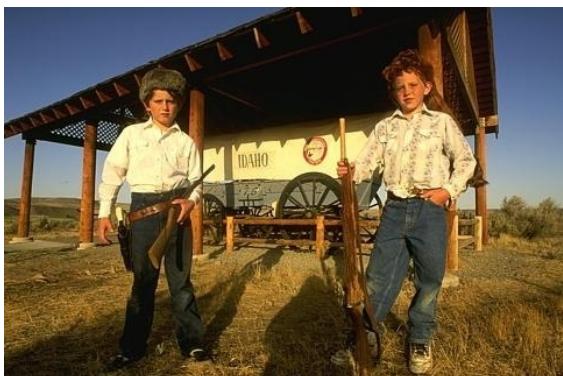
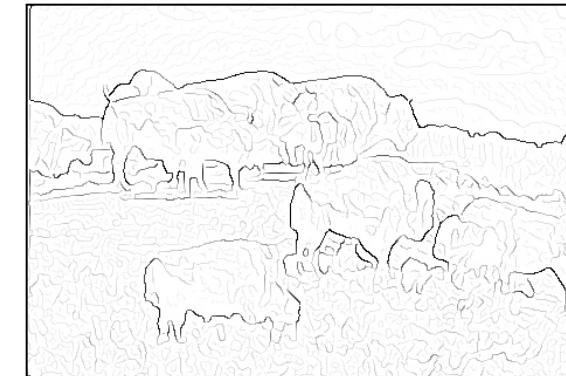
image



human segmentation

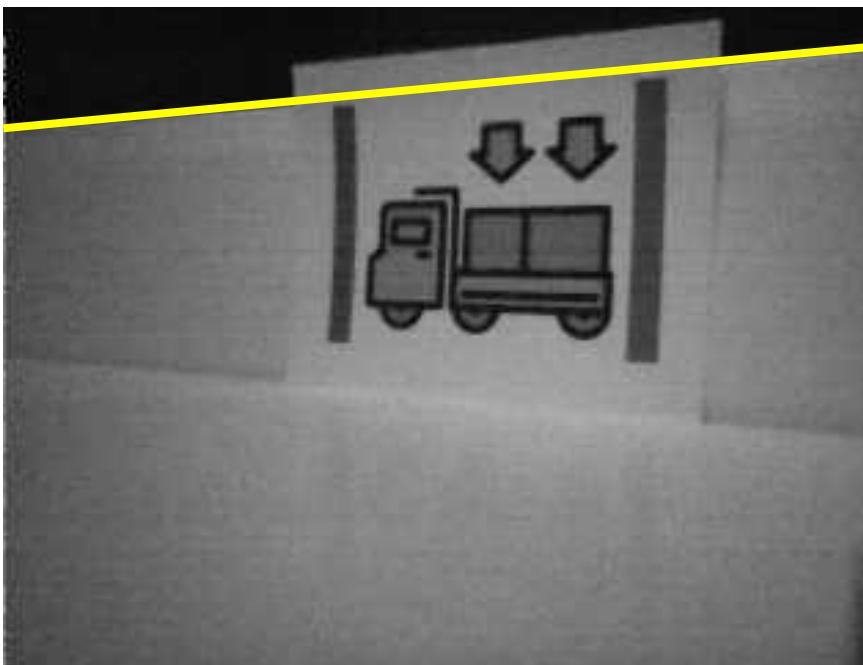
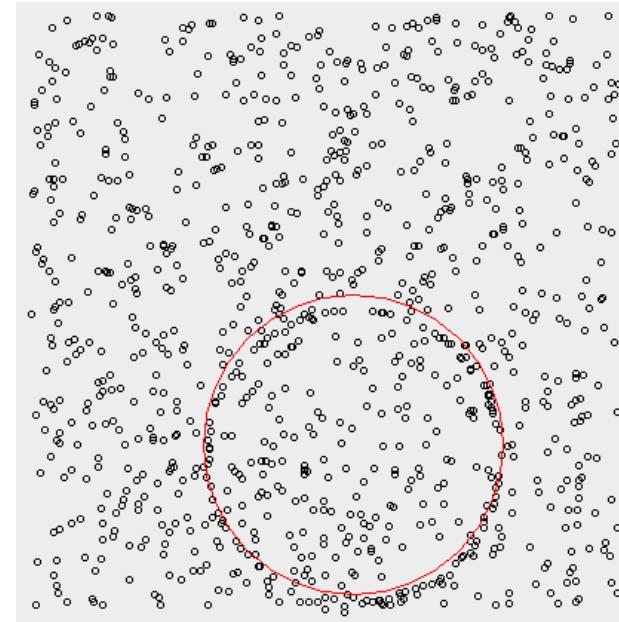
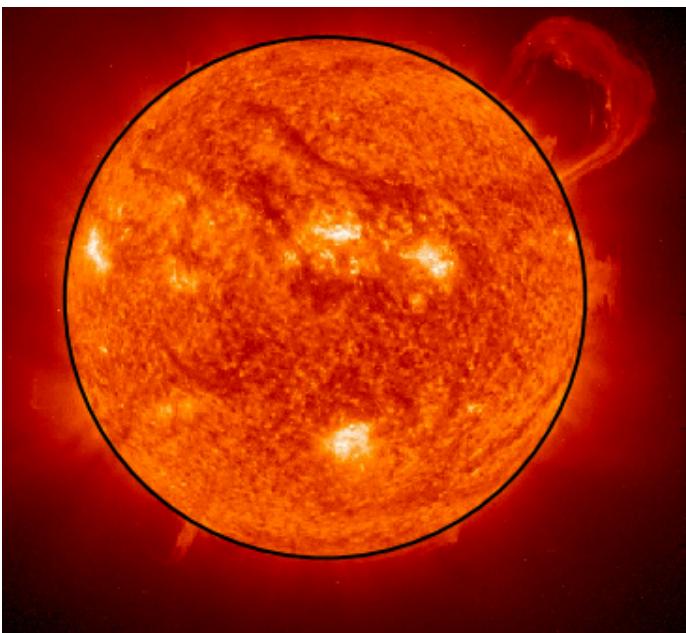


gradient magnitude



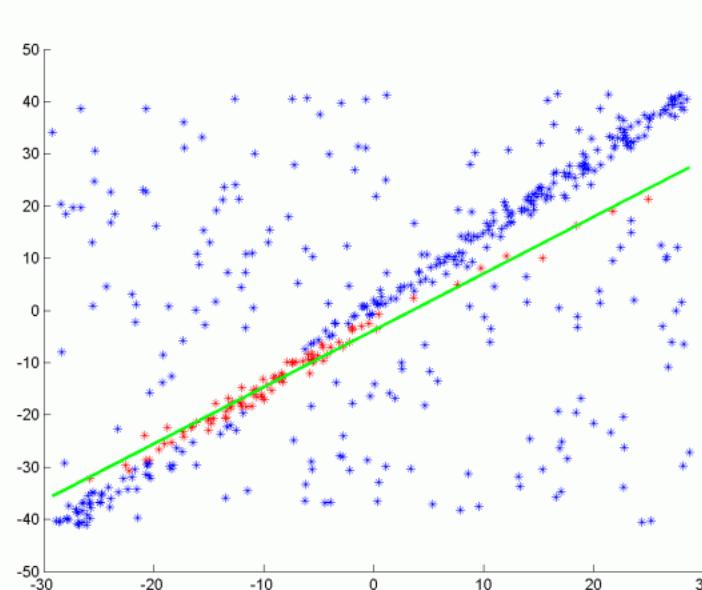
[Berkeley segmentation database](#)

# RANSAC



# RANSAC

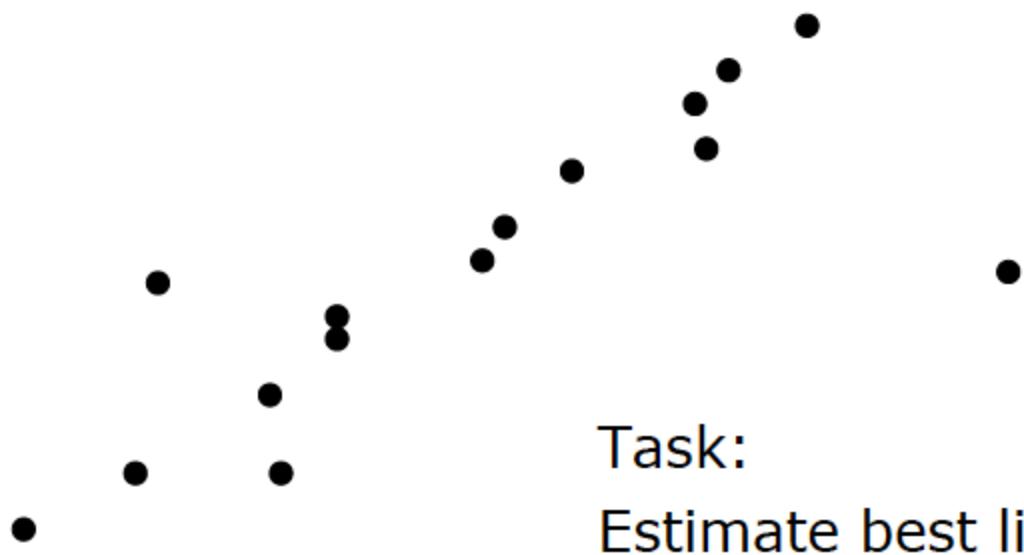
“an iterative method to estimate parameters of a mathematical model from a set of observed data that contains outliers, when outliers are to be accorded no influence on the values of the estimates”



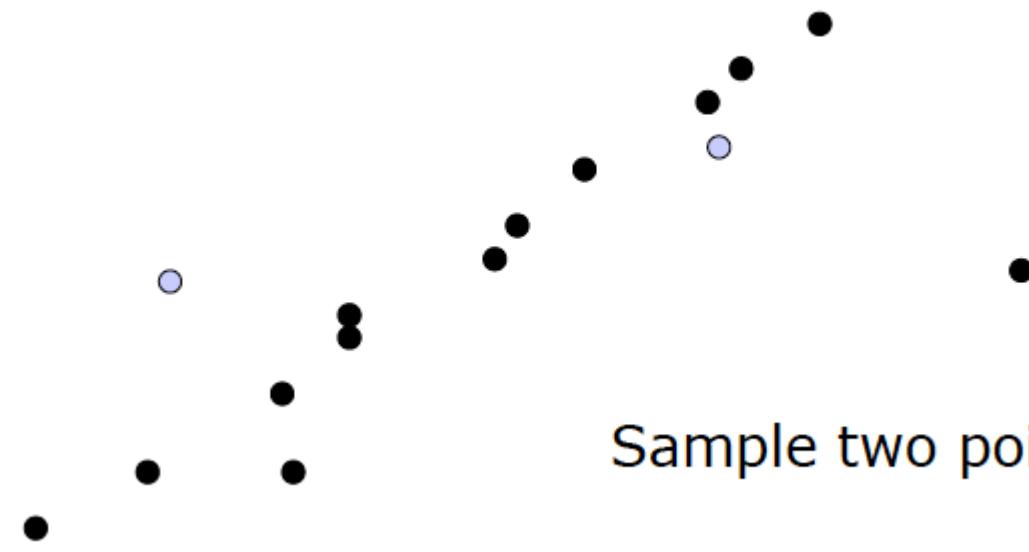
# RANDom SAMple Concensus (RANSAC)

- Randomly choose  $s$  samples/points
  - Typically  $s = \text{minimum sample size that lets you fit a model}$
- Fit a model (e.g., line) to those samples
- Count the number of inliers that approximately fit the model
- Repeat  $N$  times
- Choose the model that has the largest set of inliers

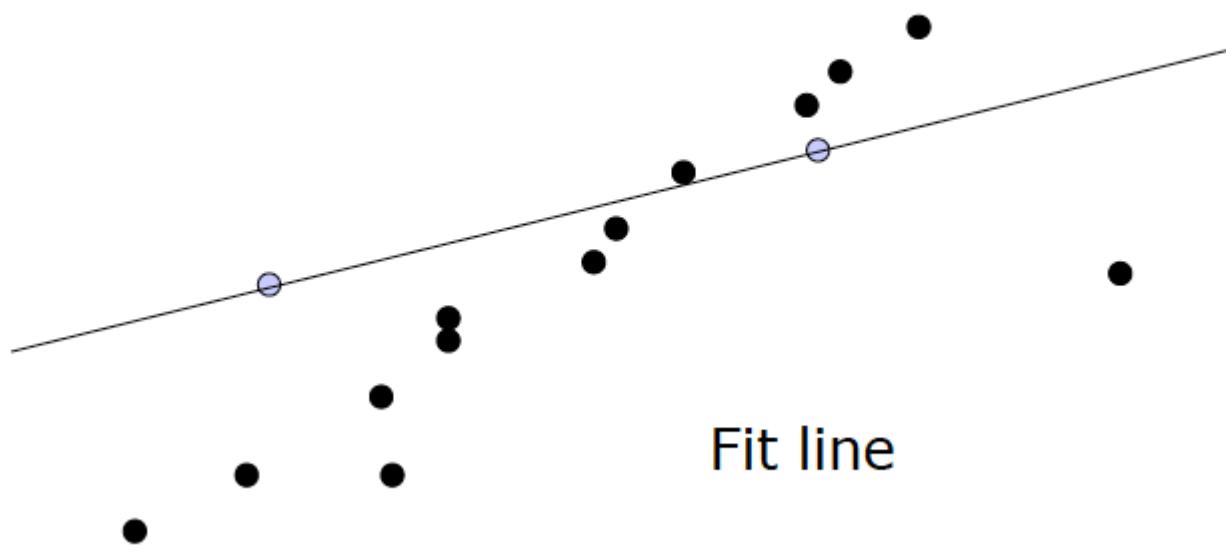
\*better results are often achieved by using all the inliers from this step to create an updated model.



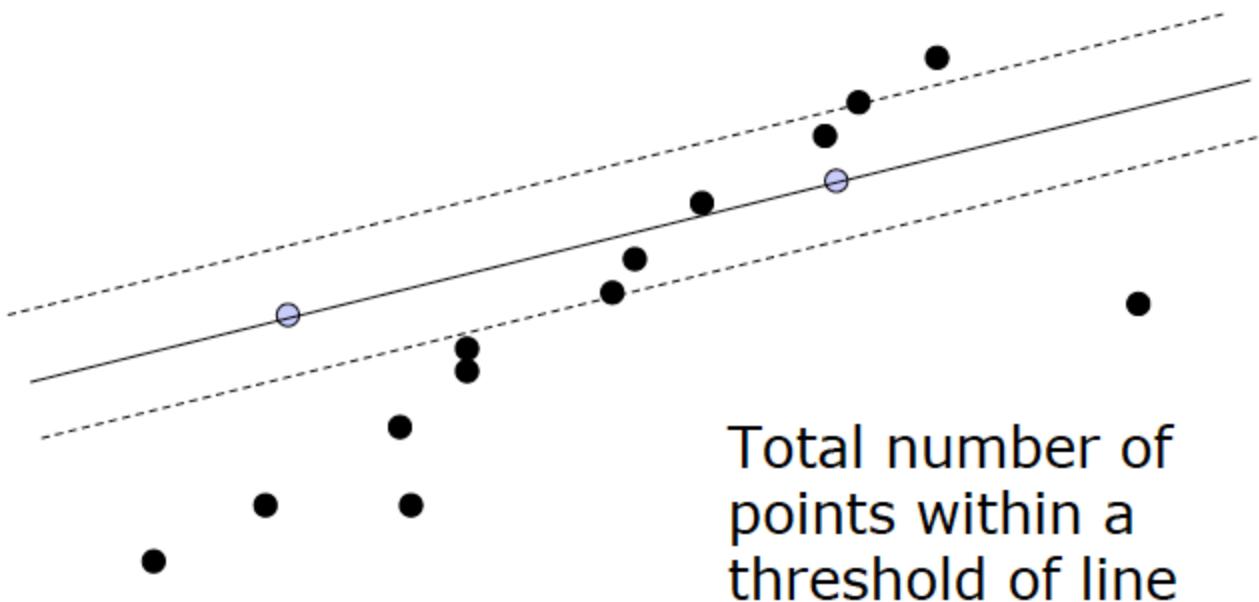
Task:  
Estimate best line



Sample two points



Fit line

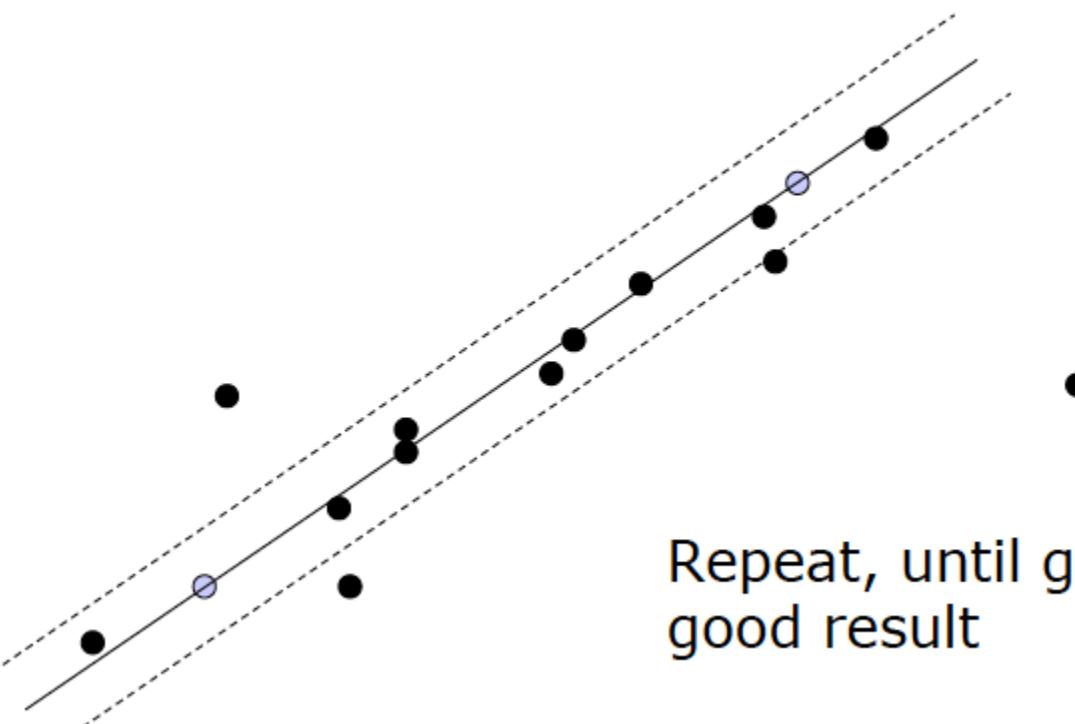




Repeat, until get a  
good result



Repeat, until get a  
good result



Repeat, until get a  
good result

# RANSAC parameters

- **Inlier threshold** related to the amount of noise we expect in inliers
- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we'd like to guarantee
  - Suppose 20% of the data points are outliers, and we want to fit the correct line with 99% probability
  - How many rounds do we need?

Suppose 20% of the points are outliers, and we want to fit the correct line with 99% probability. How many rounds do we need?

- Let  $w$  be the probability of selecting an inlier (.8 in this example)
- We need  $n = 2$  points to fit a model (line)
- What is the probability of selecting  $n = 2$  inliers to create a good model?  $w^2 = .8^2 = 0.64$
- Then the probability of selecting two points that result in a bad model is  $1 - w^2 = .36$
- The probability that all of the  $N$  tries will be bad is  $(1 - w^2)^N$
- If we run  $N$  iterations of RANSAC and want the correct answer with some probability  $p$  (99% above):

$$(1 - w^n)^N \leq 1 - p$$

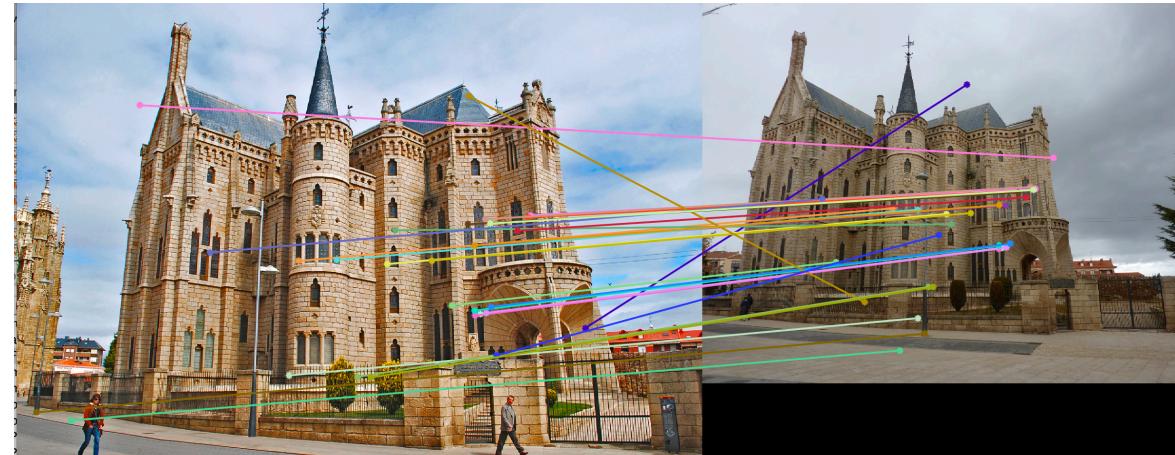
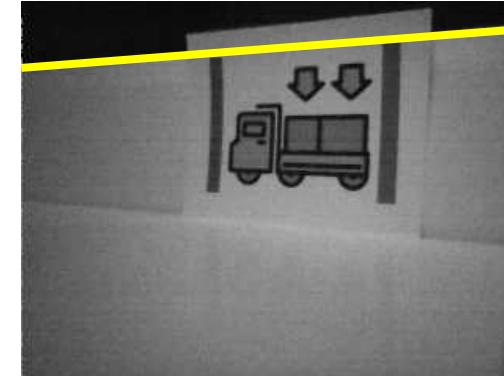
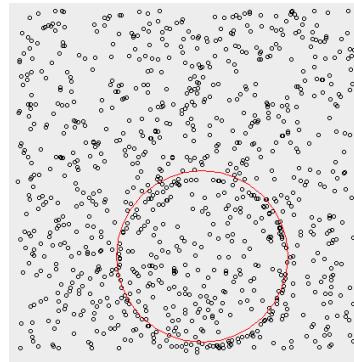
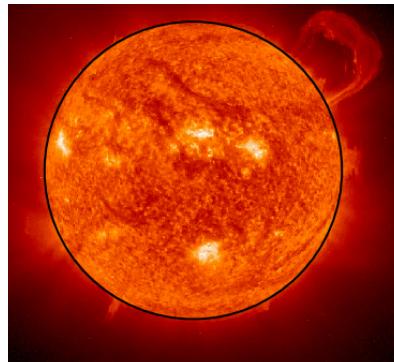
$$N \geq \frac{\log(1 - p)}{\log(1 - w^n)}$$

$$N \geq \frac{\log(1 - .99)}{\log(1 - .8^2)} = 4.5$$

Note:  $\log(1 - w^n) < 0$  !

# RANSAC

RANSAC works extremely well for both low and high-dimensional problems



# References

- RANSAC
  - Autonomous Mobile Robots 4.7.2.4
  - Robotics, Vision and Control 14.2.3