CS 3630

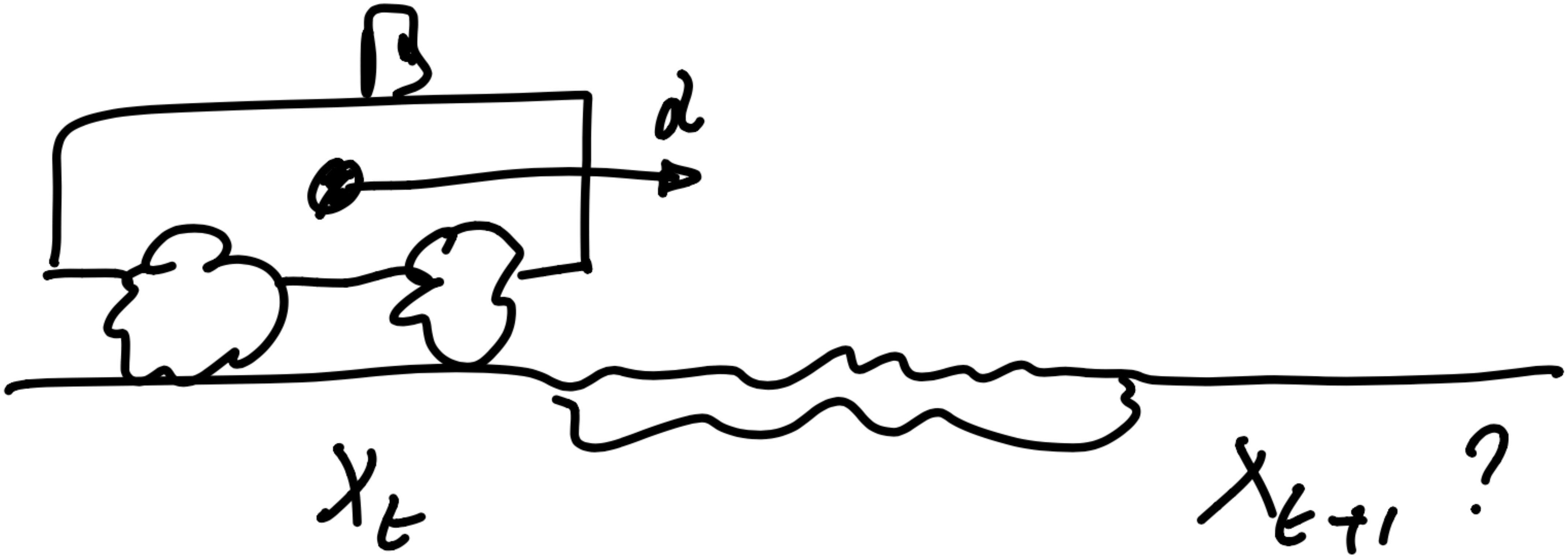
Lecture 3: Probabilistic Actions

Topics

1. **Real Robots**
2. **Atomic State via Discrete Variables**
3. **Probabilistic Outcomes of Actions**
4. **Bayesian vs. Frequentist**
5. **Conditional Probability Distributions**
6. **A Simple Graphical Model**
7. **Sampling**
8. **Forward Simulation**
9. **Factored State Representations**

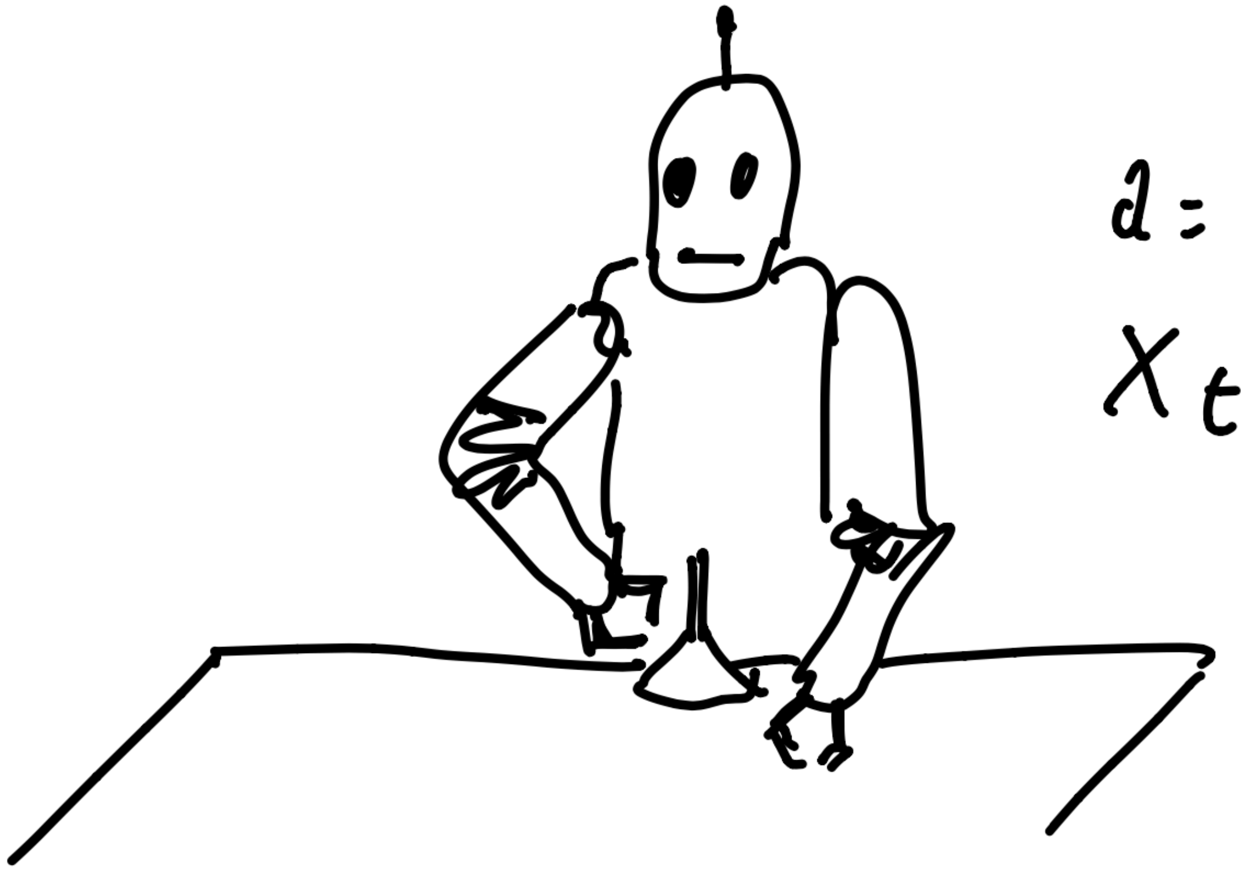
Real Robots

Mobile robot driving in mud



Real Robots

A humanoid attempting to pick up an object

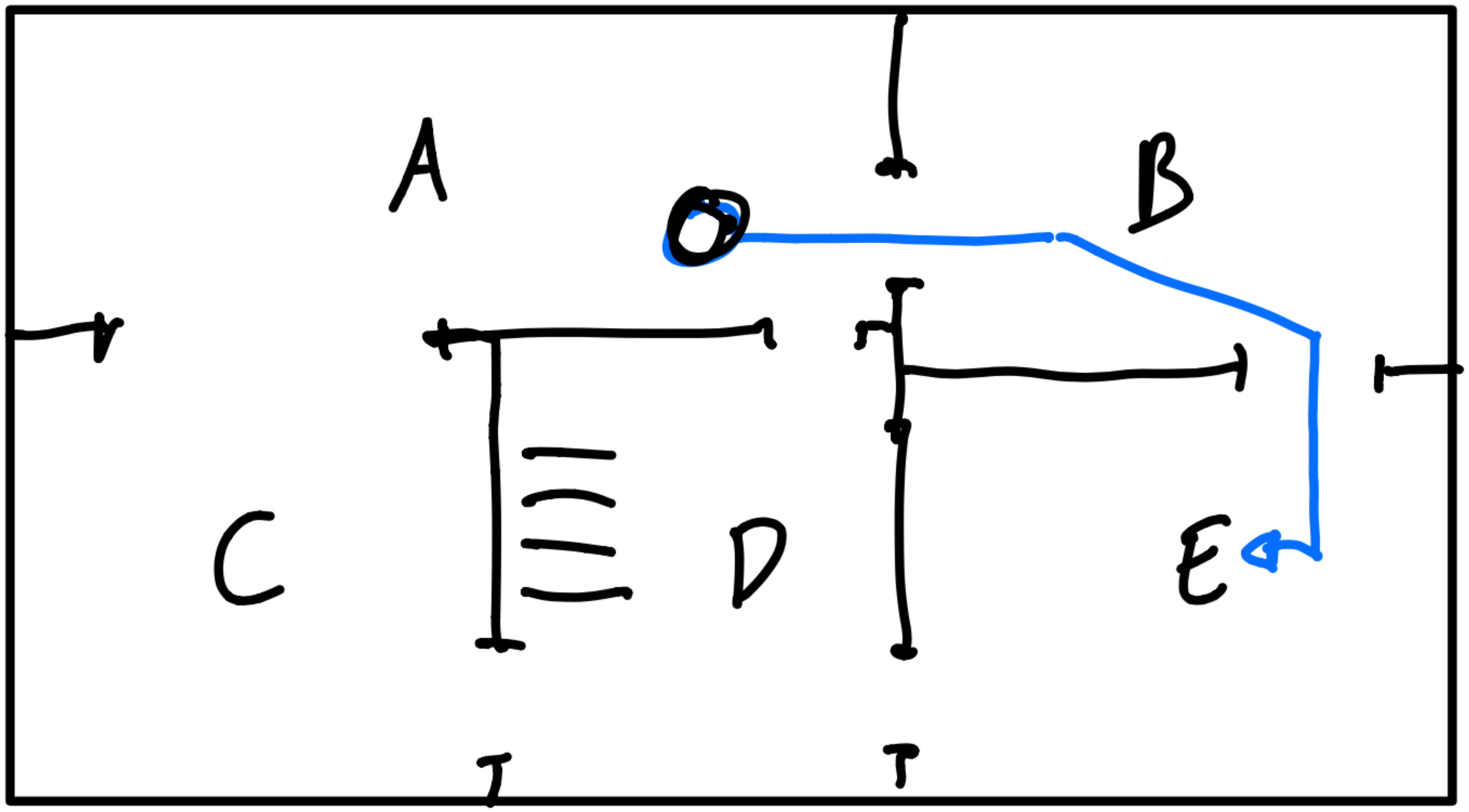


a = pick-up?

x_{t+1} ?

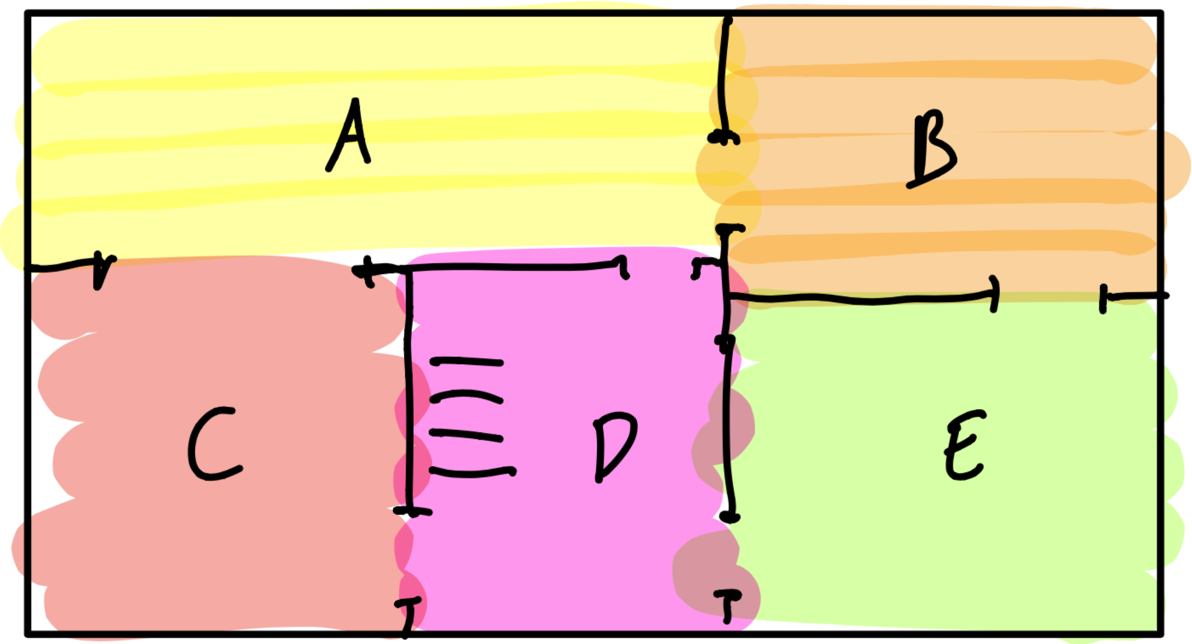
Real Robots

A robot vacuum cleaner facing an uncertain future



Atomic State via Discrete Variables

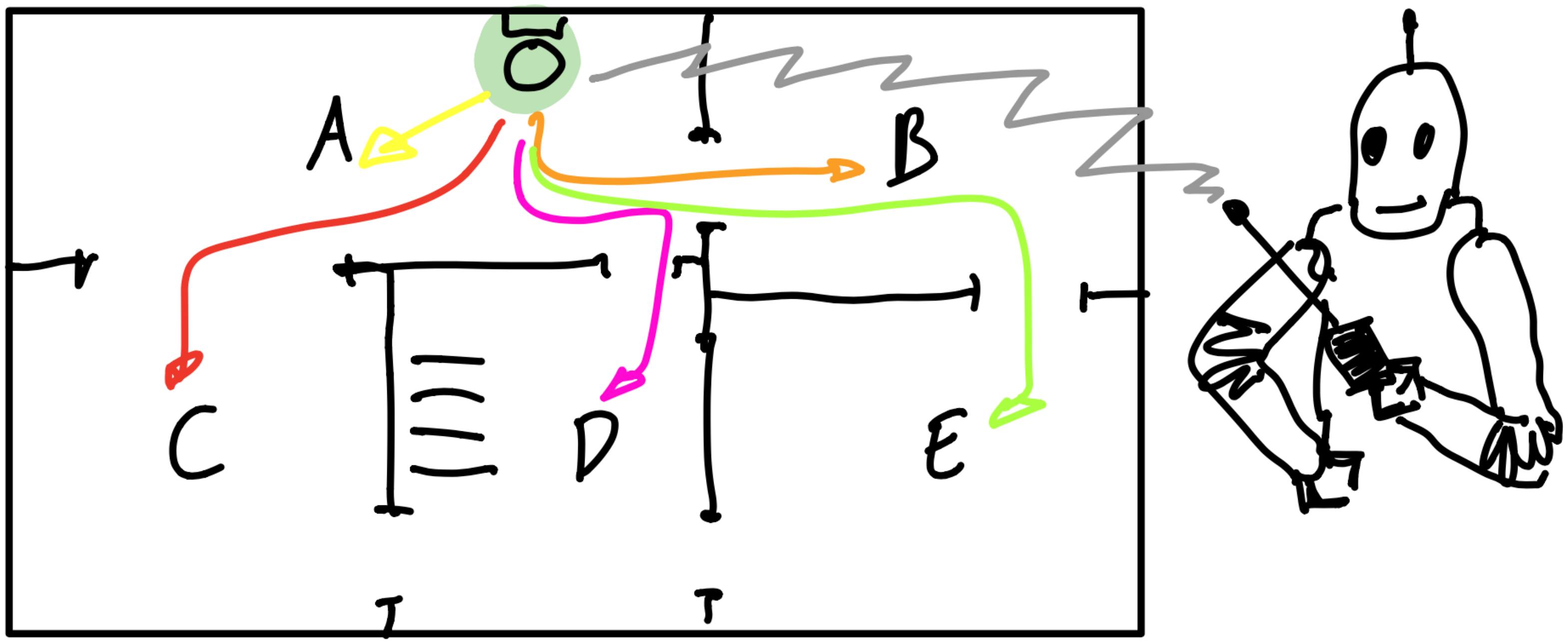
A discrete state representation corresponding to different rooms in the house.



State $x \in \{A, B, C, D, E\}$

Atomic State via Discrete Variables

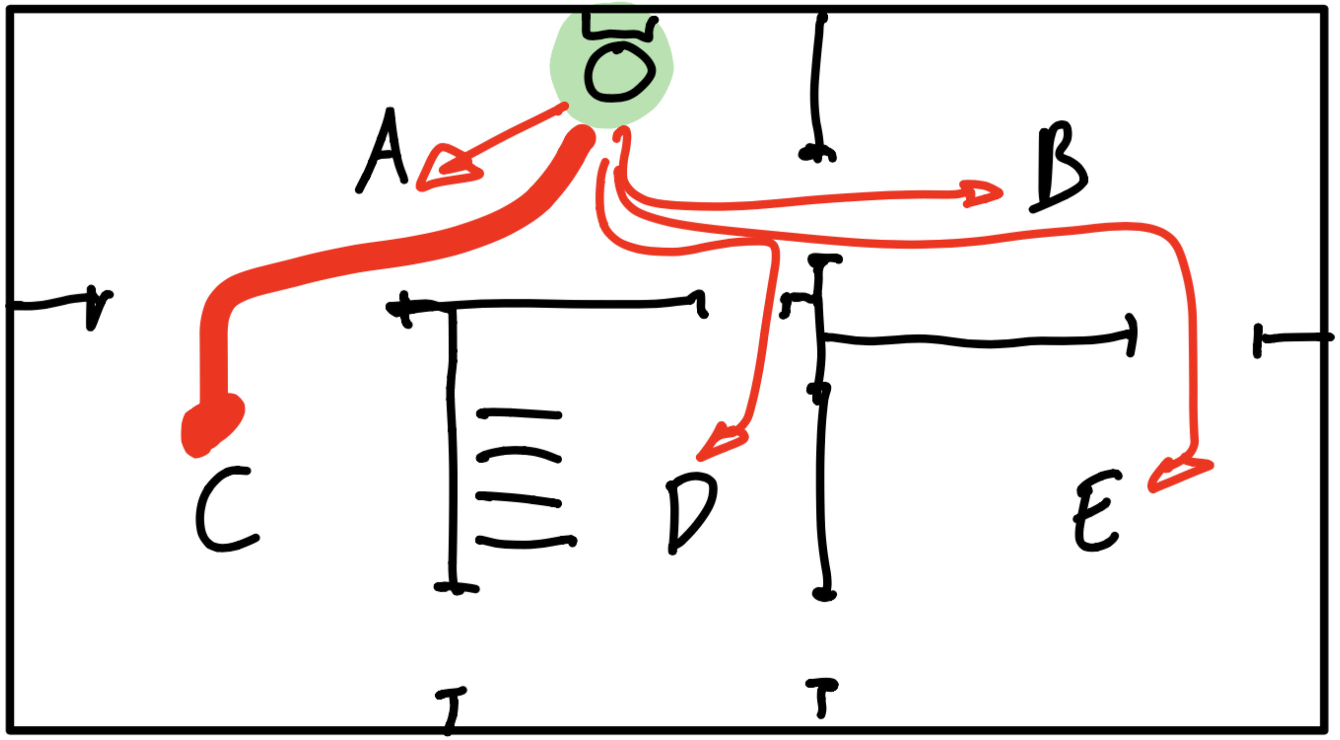
In addition to a set of possible states, we also have a set of possible actions.



Action $a \in \{G_A, G_B, G_C, G_D, G_E\}$

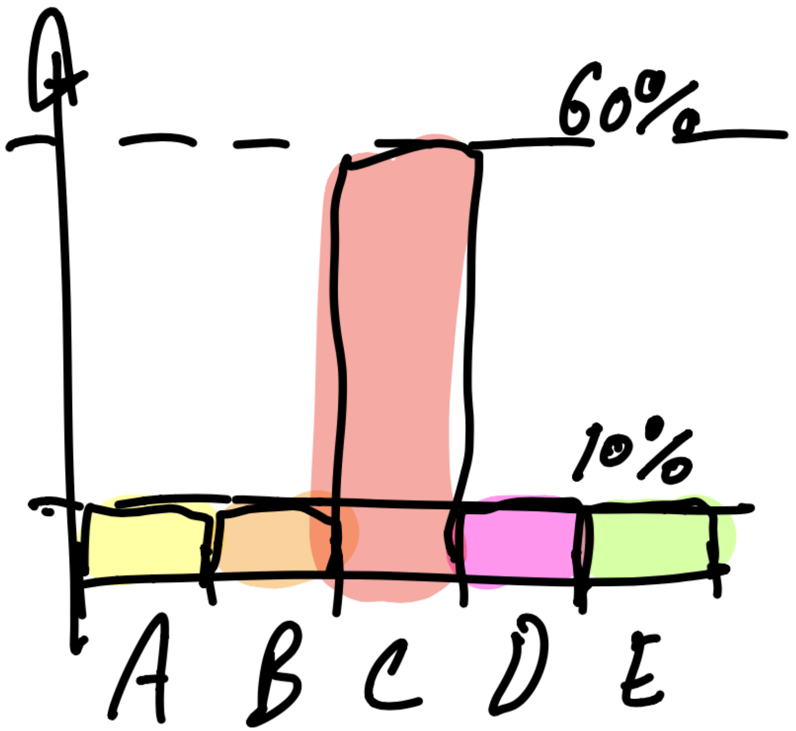
Probabilistic Outcomes of Actions

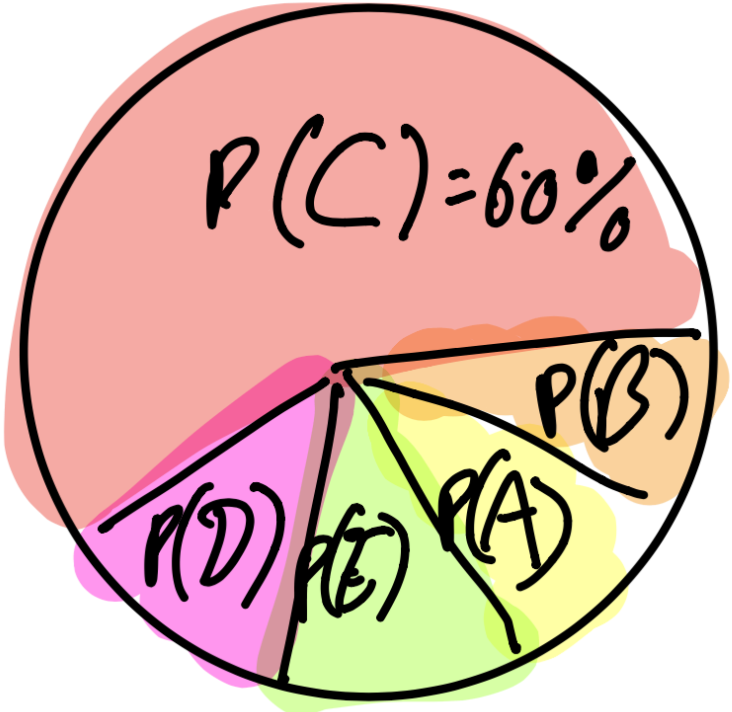
An action, in this case "go to room C", might succeed most of the time but fail some of the time. Below, the thickness of the arrows encodes how likely the robot ends in each room, starting from room A.



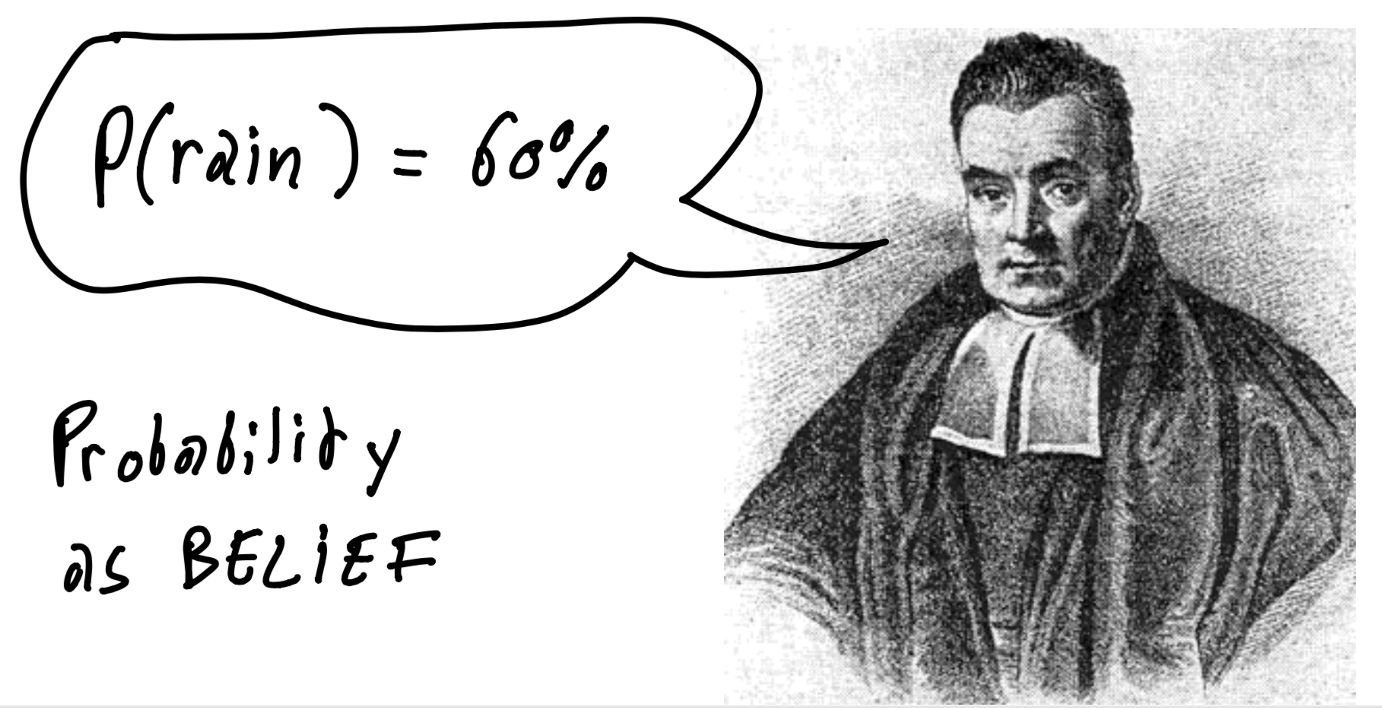
Probabilistic Outcomes of Actions

We can represent the probability $P(x|a=G_C)$ of being in a particular state after taking action G_C as a vector of numbers between 0 and 1, adding up to 1.





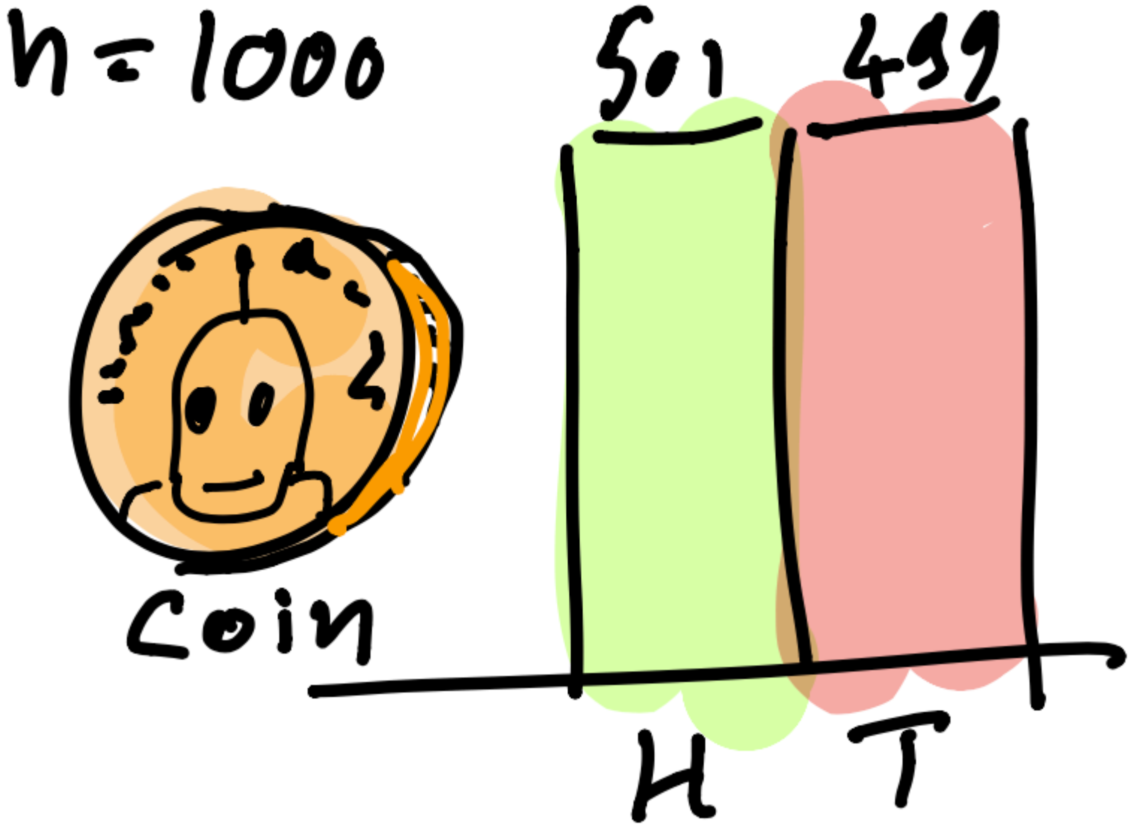
Bayesian vs. Frequentist

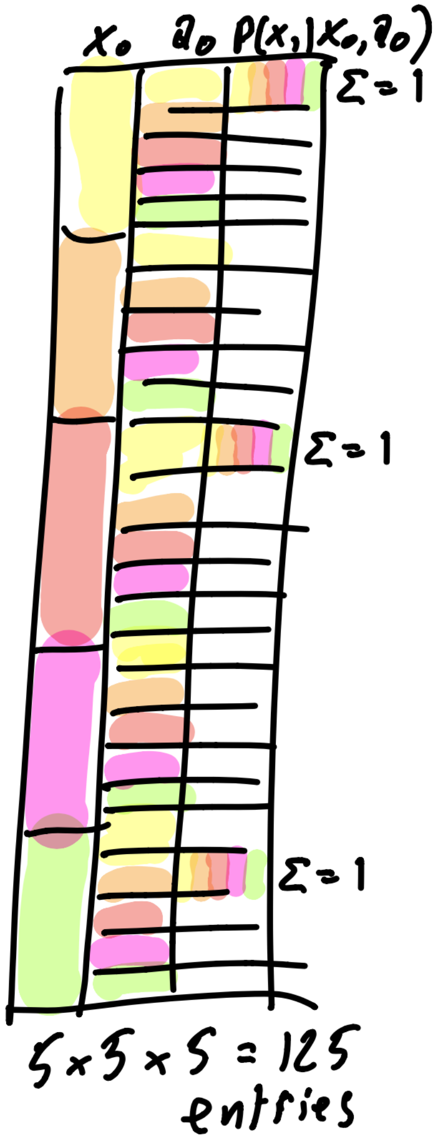


The Reverend Thomas Bayes gave his name to associating probabilities with the strength of beliefs rather than a frequency of events, even though this seems to have been first introduced by Laplace

Bayesian vs. Frequentist

The caricature of the frequentist view involves counting many heads and tails

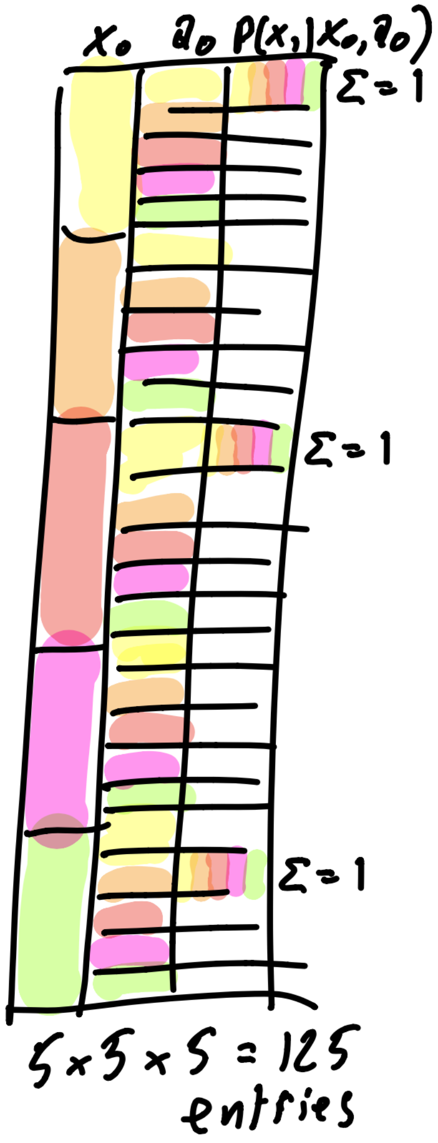




Conditional Probability Distributions

- Conditional Probability, written as $P(x_1|x_0, a_0)$
- Behind the “bar”: known parameters, in this case the starting state and the chosen action.
- To specify: conditional probability table (CPT), specifying a separate PMF on state x_1 for each of the 5 possible starting states x_0 and each of the 5 possible actions a_0.

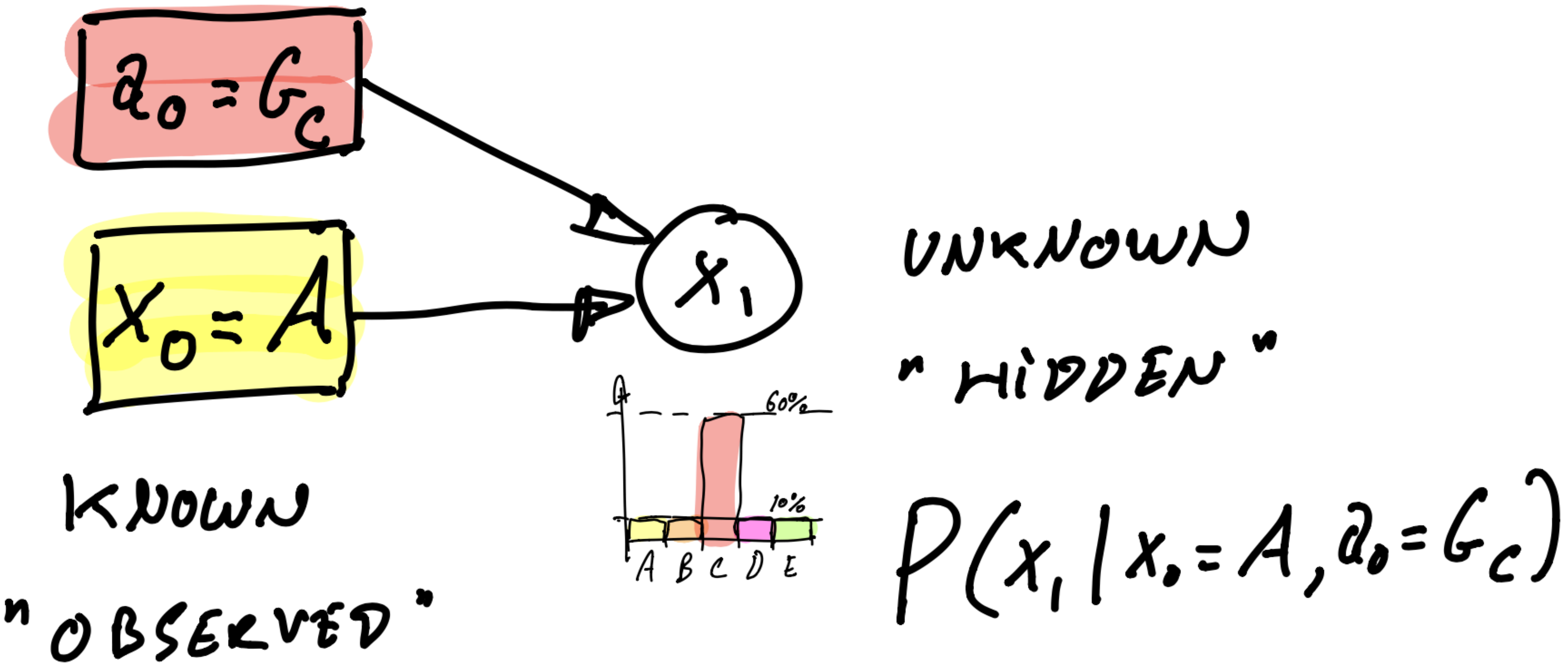
Conditional Probability Distributions



- Q: Even though the CPT on the right has 125 numbers in it, how many independent degrees of freedom do we actually have when specifying this CPT?
- Conditional probability tables do not *have* to be specified as giant tables.
- Q: Come up with a parametric conditional density for the action model of the vacuum robot that is somewhat realistic, yet not completely deterministic.

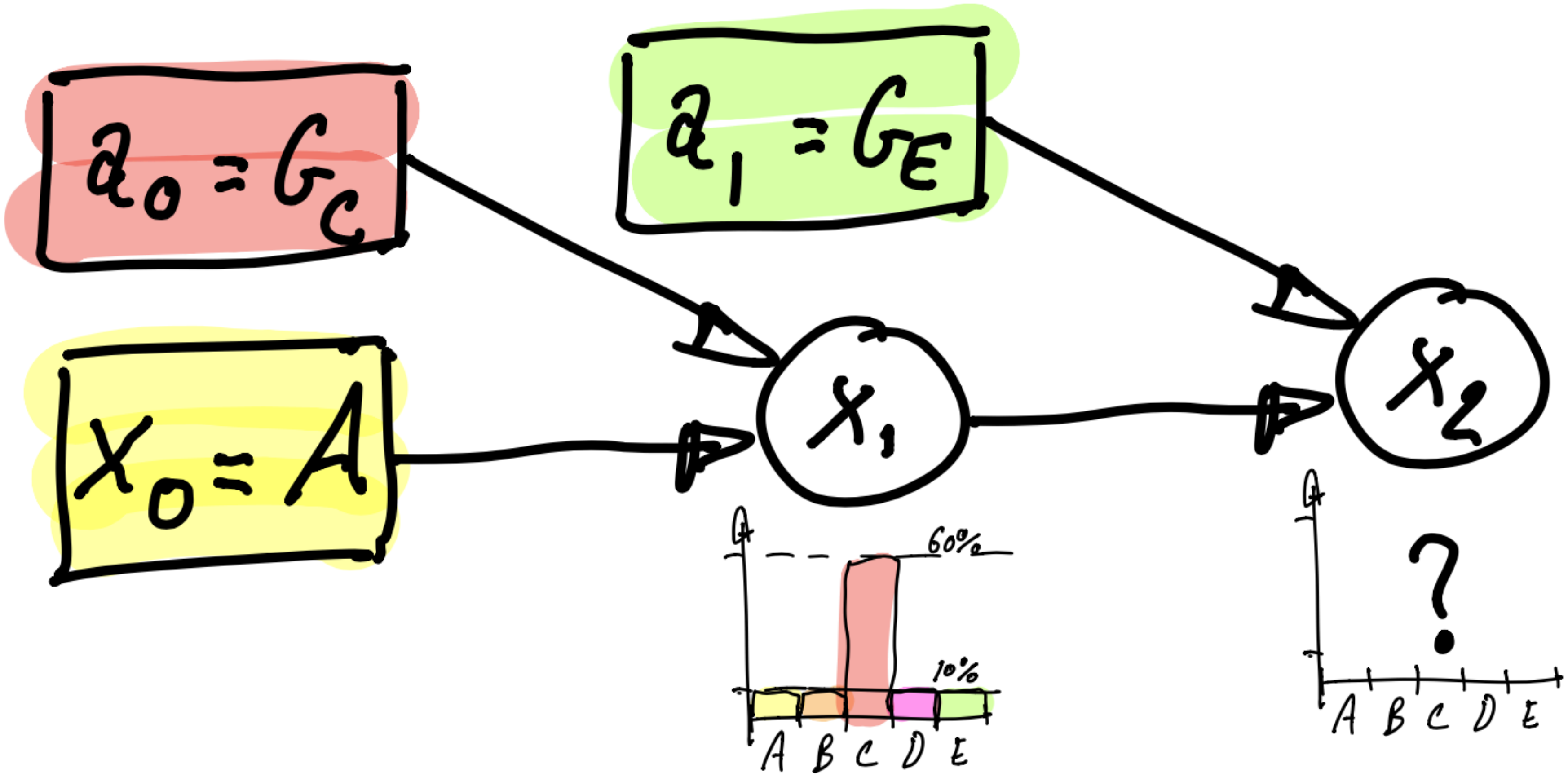
A Simple Graphical Model

Graphical representation of the effect of action a_0 = G_C starting from state x_0 =A



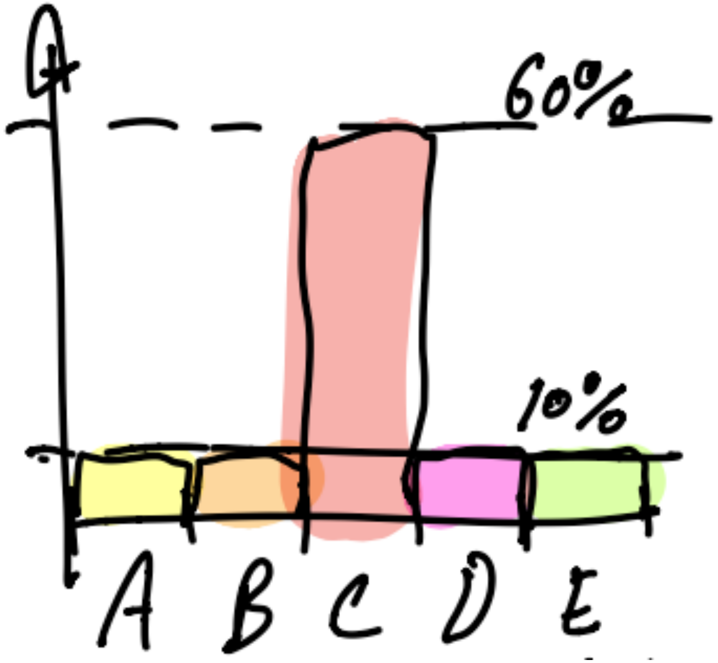
Markov Chains

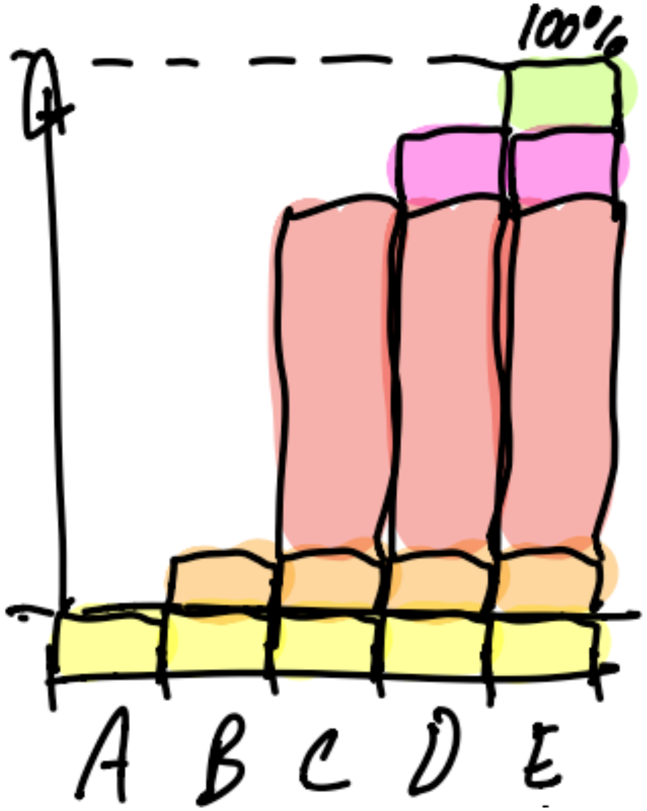
Graphical model of two consecutive actions $a_0 = G_C$ and $a_1 = G_E$



Inverse transform sampling

- Order outcomes
- Form cumulative distribution function (CDF)

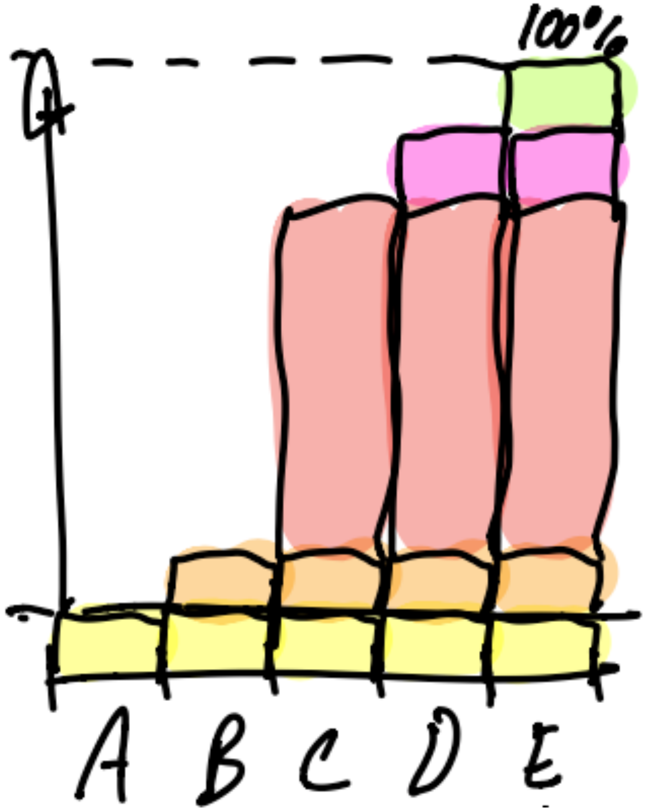




Inverse transform sampling

- Order outcomes
- Form cumulative distribution function (CDF)

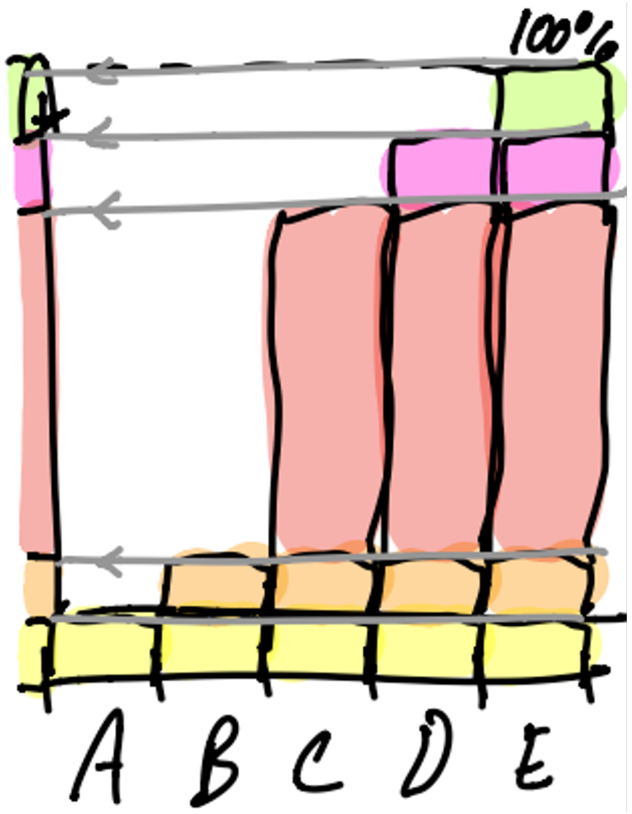
x_i	$P(X = x_i)$	$F(x_i)$
A	0.1	0.1
B	0.1	0.2
C	0.6	0.8
D	0.1	0.9
E	0.1	1.0



Inverse transform sampling

- Order outcomes
- Form cumulative distribution function (CDF)

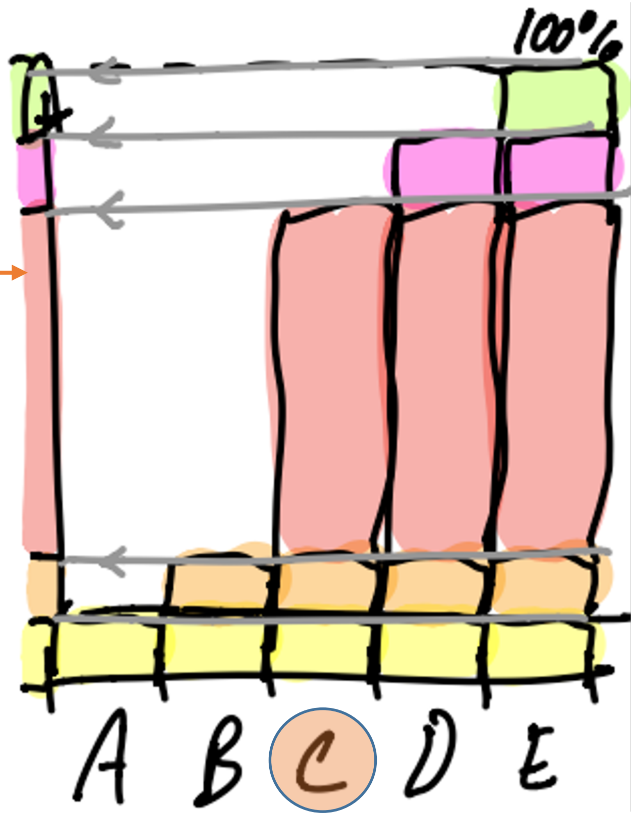
x_i	$P(X = x_i)$	$F(x_i)$
A	0.1	0.1
B	0.1	0.2
C	0.6	0.8
D	0.1	0.9
E	0.1	1.0



Inverse transform sampling

- Order outcomes
- Form cumulative distribution function (CDF)
- Sample random 0.0 <= u <= 1.0

u = 0.70



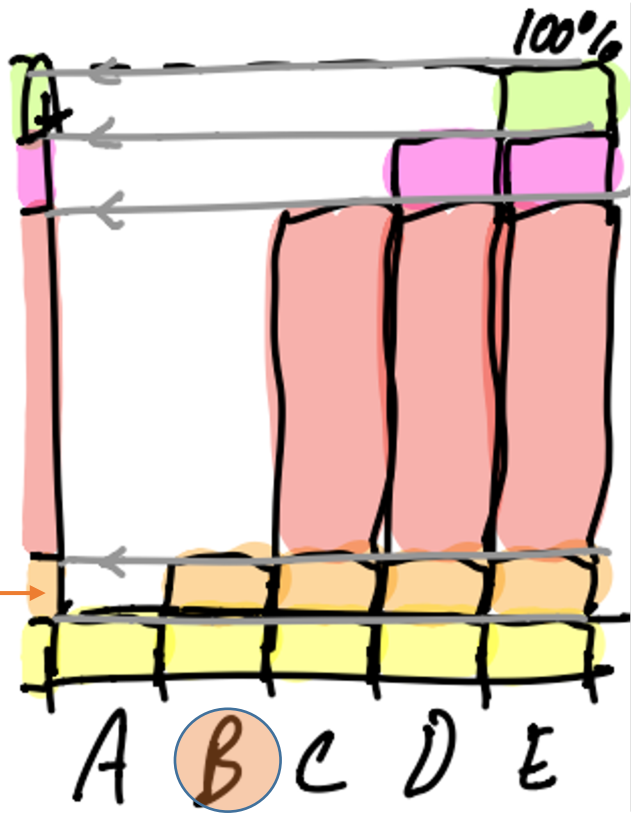
x_i	$P(X = x_i)$	$F(x_i)$
A	0.1	0.1
B	0.1	0.2
C	0.6	0.8
D	0.1	0.9
E	0.1	1.0

Inverse transform sampling

- Order outcomes
- Form cumulative distribution function (CDF)
- Sample random 0.0 <= u <= 1.0

x_i	$P(X = x_i)$	$F(x_i)$
A	0.1	0.1
B	0.1	0.2
C	0.6	0.8
D	0.1	0.9
E	0.1	1.0

u = 0.15

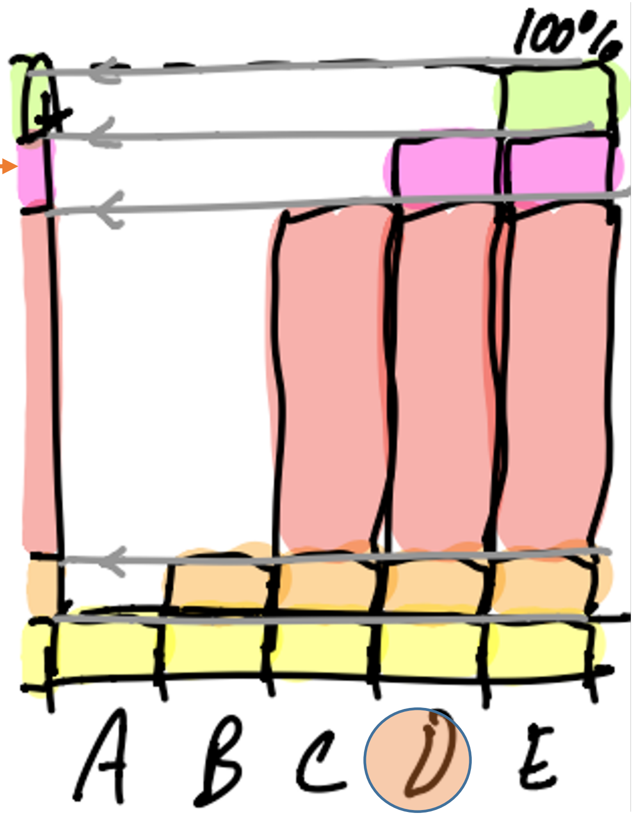


Inverse transform sampling

- Order outcomes
- Form cumulative distribution function (CDF)
- Sample random 0.0 <= u <= 1.0

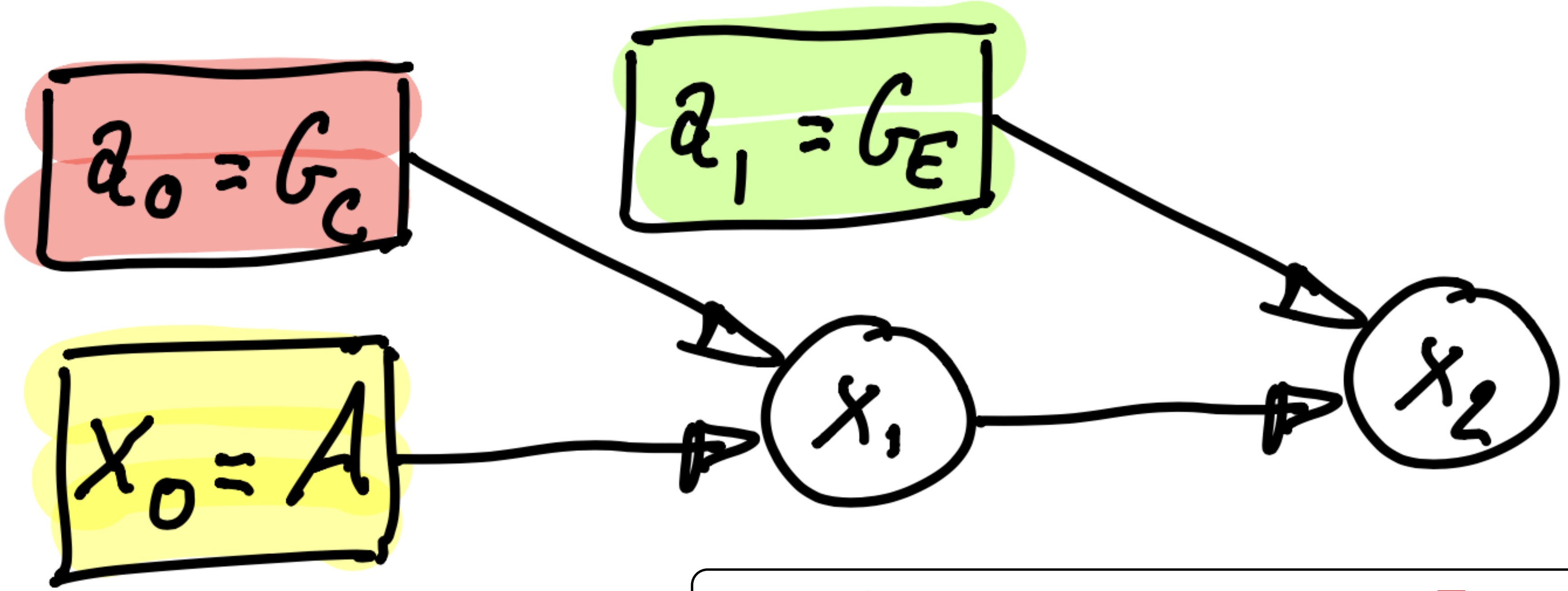
u = 0.86

x_i	$P(X = x_i)$	$F(x_i)$
A	0.1	0.1
B	0.1	0.2
C	0.6	0.8
D	0.1	0.9
E	0.1	1.0



Multiple Rollouts

Repeat multiple times: sample *X1*, retrieve PMF from CPT, then sample X_2

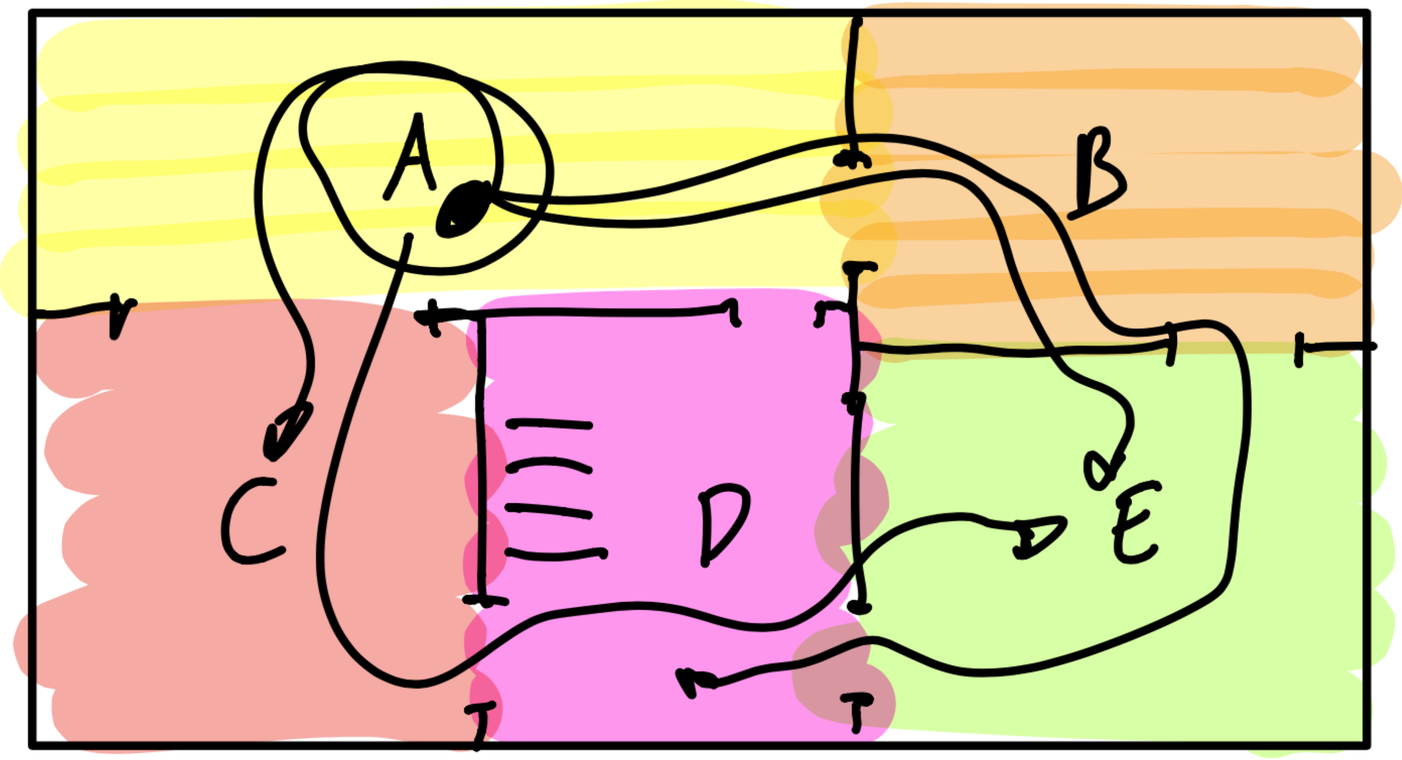


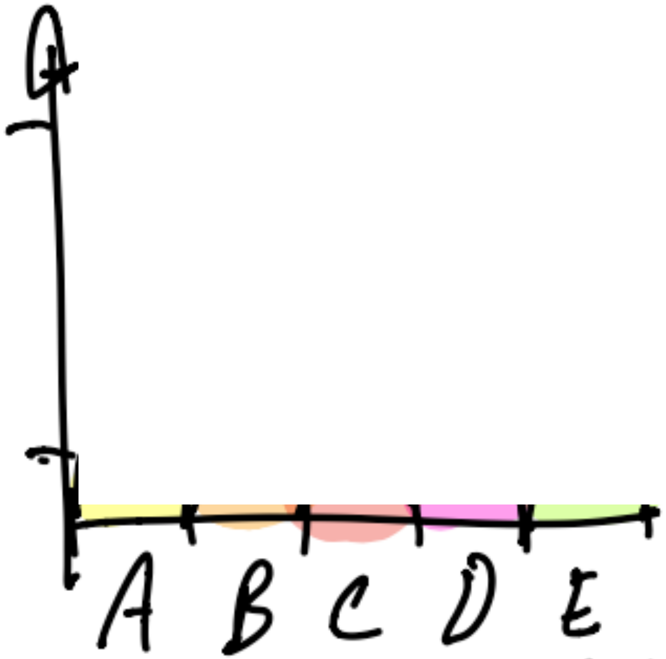
C	Rollout 1	E
A	Rollout 2	C
B	Rollout 3	E
E	Rollout 4	D

Multiple Rollouts

After simulating multiple "rollouts" by simulating in this way, we can approximate the PMF of the final state by construction a **histogram** over the possible values of the state.

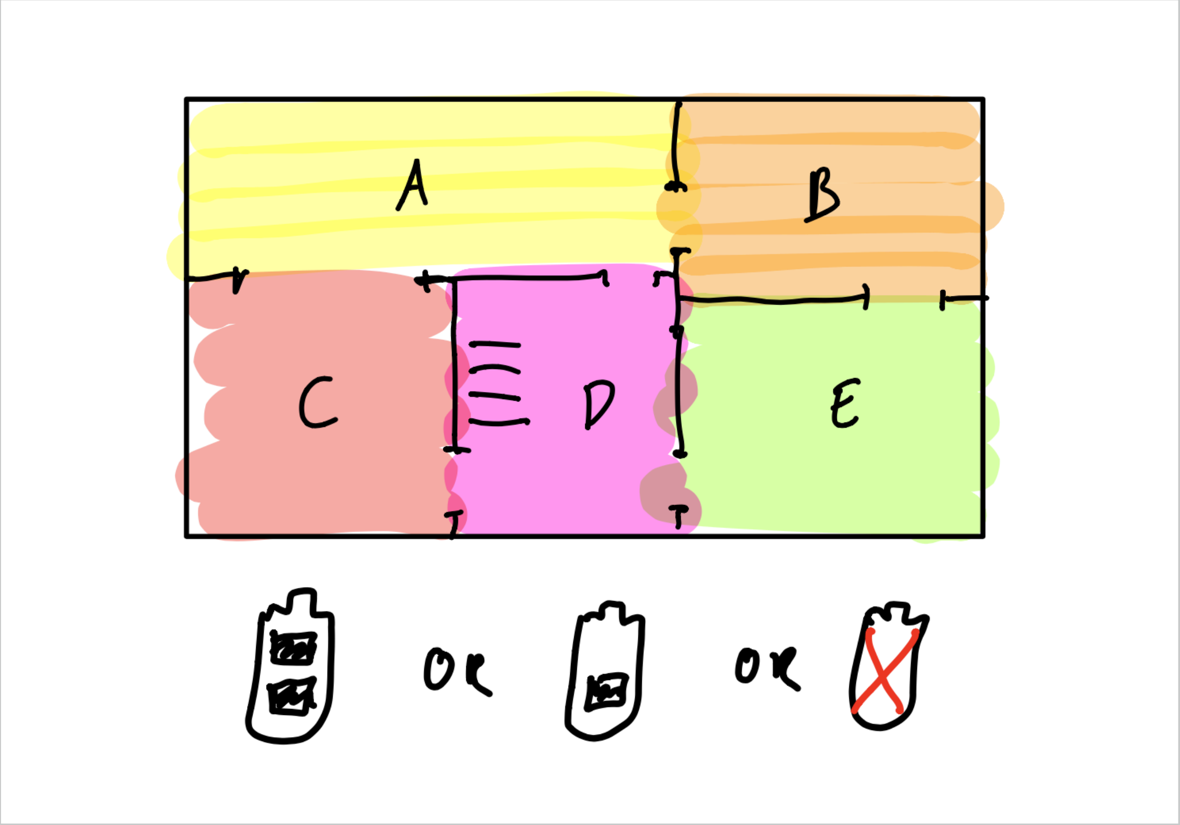
C	Rollout 1	E
A	Rollout 2	C
B	Rollout 3	E
E	Rollout 4	D

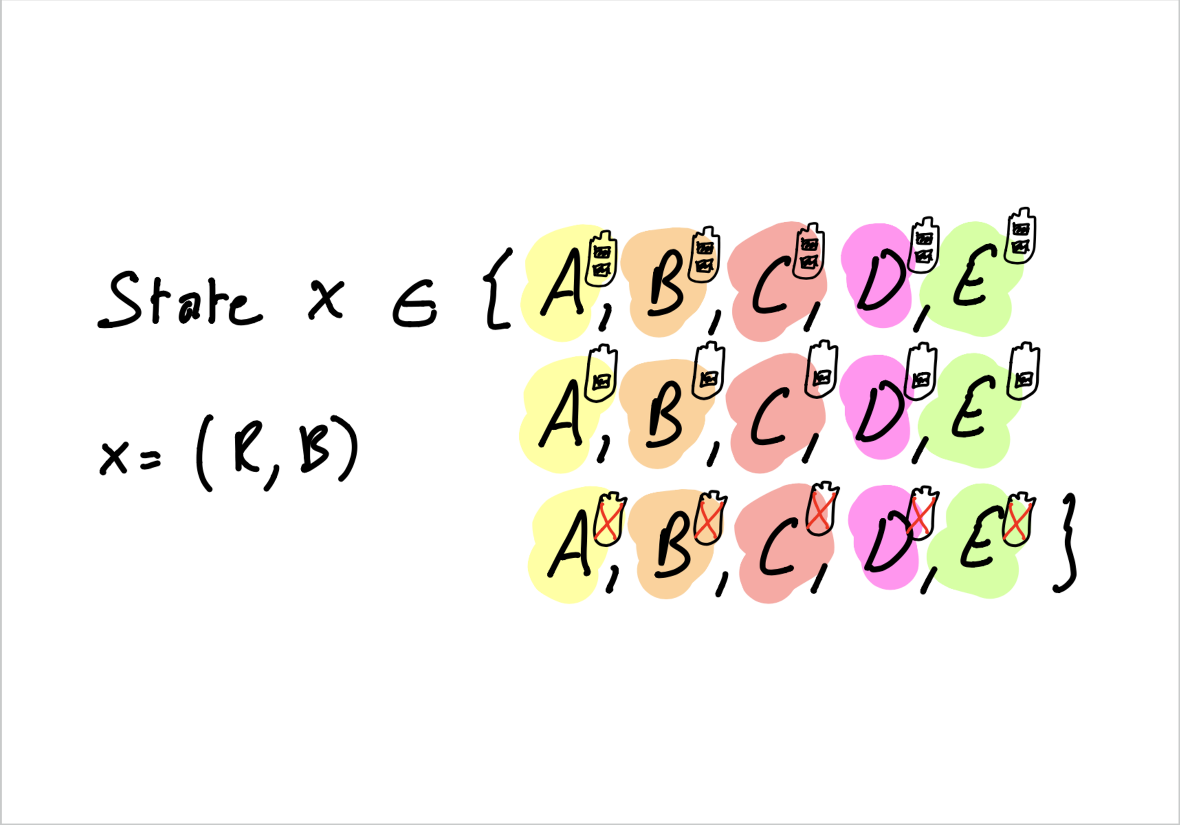




Factored State Representations

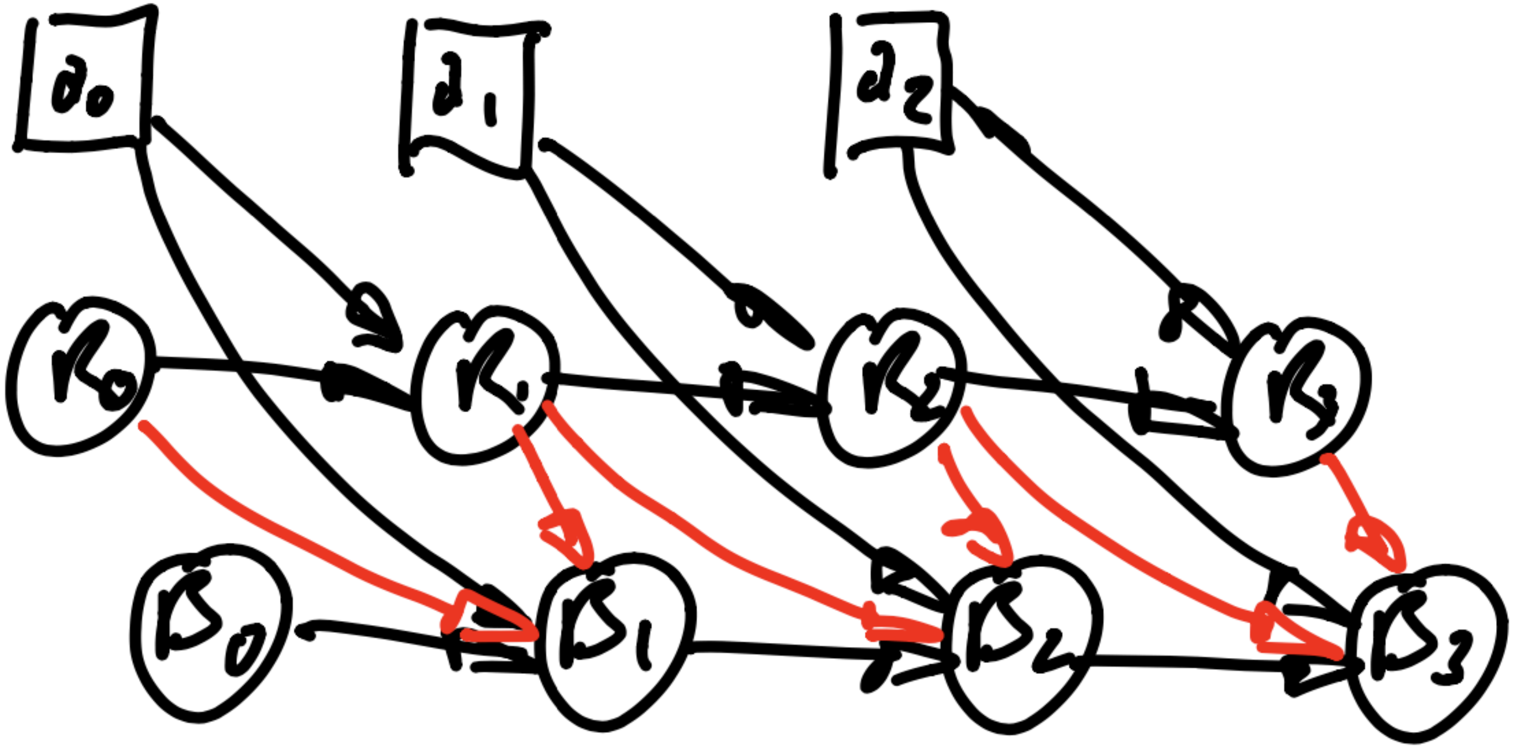
A factored state representation for the vacuum robot might have one variable for the room the robot is in, and another variable describing its battery status.





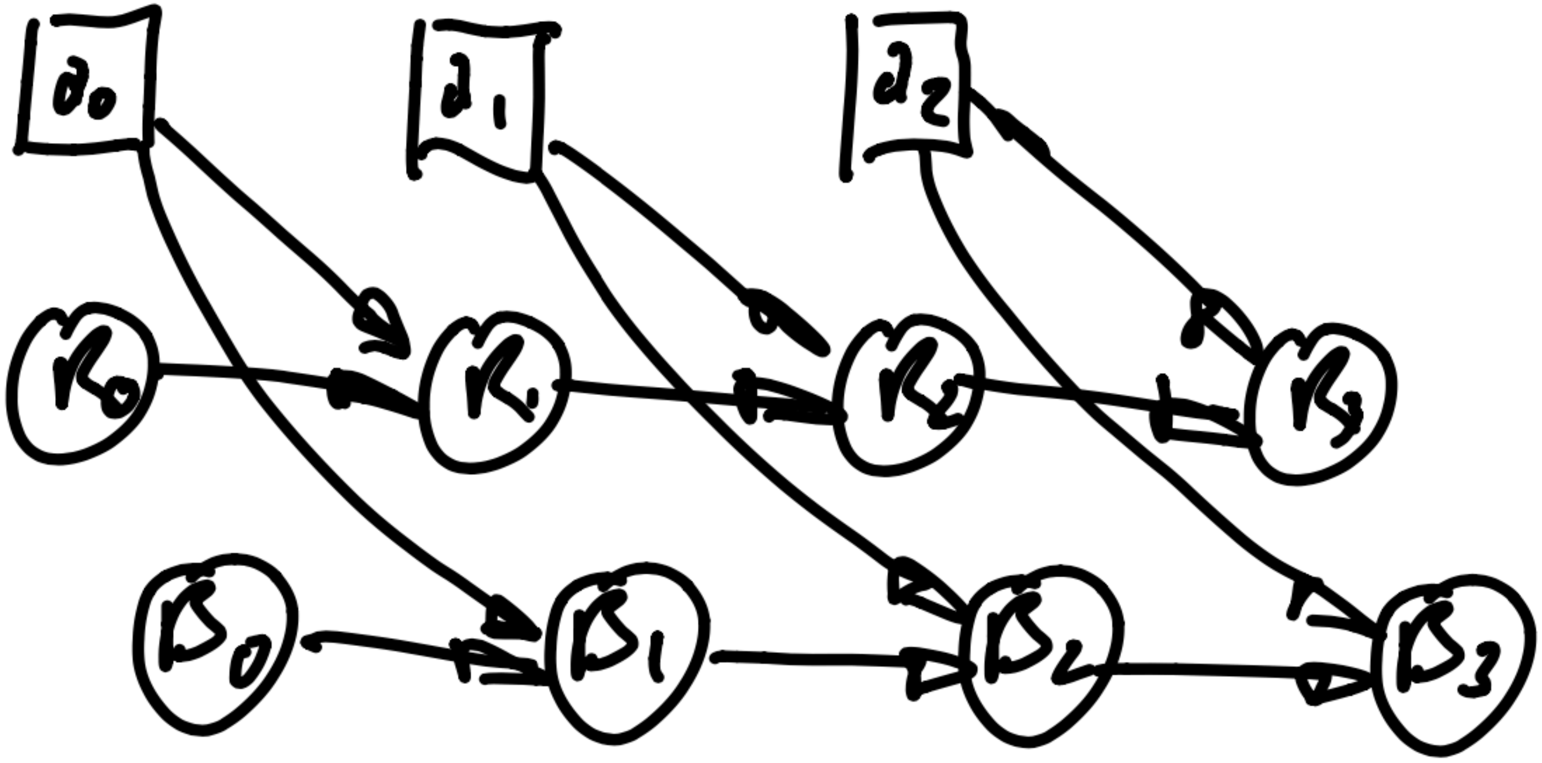
Factored State Representations

A Markov chain for a factored state representation. The red arrows above show that the change in batter status can depend on the room transition



Factored State Representations

If the battery status transition does not depend on the room transition, but only on the action, then we have two independent Markov chains given the state sequence



Summary

- In the **real world, robots** do not always execute actions perfectly, for a variety of reasons.
- We can use **discrete variables** to model the state a robot is in, and actions that connect these states.
- To model the uncertainty with executing an action, we will have to introduce the language of **probability**.
- We take a **Bayesian** view of probability, rather than a frequentist one.
- **Conditional probability distributions** are a way to model how we can affect the state of the robot by actions.
- We can model this **graphically**, using directed edges to specify conditional probabilities on variables.
- We can use **simulation** in a graphical model to explore what a sequence of actions will yield as outcome.
- The graphical model approach allows us to easily extend probabilistic actions to **factored state** representations.