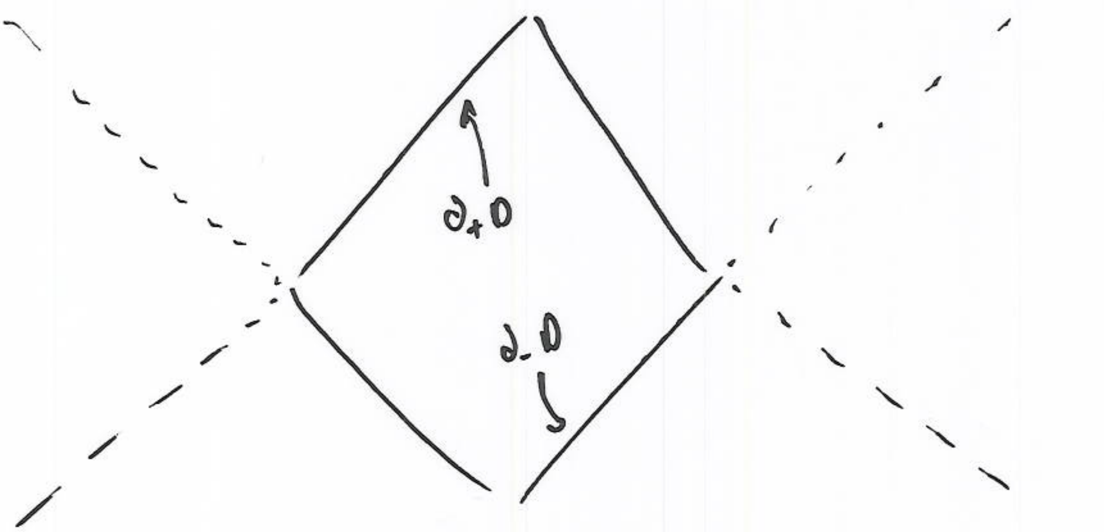
GHY - BT for flat diamond in $\mathbb{M}^2$ (CAUSET) ①

$J^+(\Diamond) \equiv J^+(\partial_+ D)$



$J^-(\Diamond) = J^-(\partial_- D)$

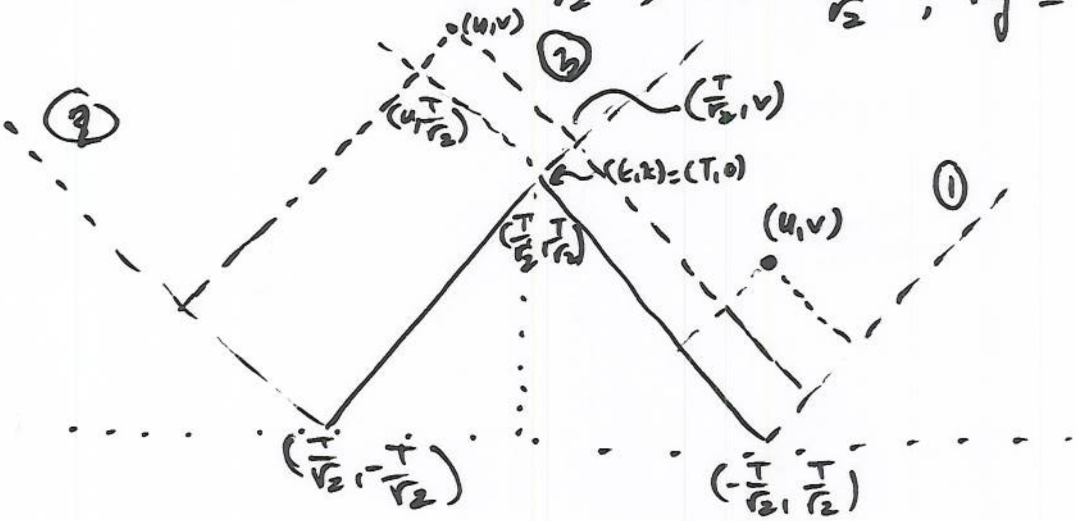
- Call the diamond D.
- There will be two contributions to the GHYBT from the top & bottom $\partial_+ D$ & $\partial_- D$.
- by symmetry they should be the same. So look at $\partial_+ D$.

$$\Delta = N_{min}(J^+(\partial_+ D)) - N_{max}(D) \quad \text{~~Conjecture~~}$$

Calculate $N_{min}(J^+(\partial_+ D))$:

$$N_{min}(J^+(\partial_+ D)) = \int_{J^+(\partial_+ D)} dt\, dx\; e^{-\rho V_\Delta(t,x)} \equiv N_{min}$$

Find V_Δ. lightcone coordinates will be most convenient: $u = \frac{t-x}{\sqrt{2}}$, $v = \frac{t+x}{\sqrt{2}}$, $\ell_g = 1$.



$V_\Diamond = \Delta u \Delta v$

① $V_\Delta = (u + \frac{T}{\sqrt{2}})(v - \frac{T}{\sqrt{2}})$

② $V_\Delta = (u - \frac{T}{\sqrt{2}})(v + \frac{T}{\sqrt{2}})$

③
$$\begin{aligned} V_\Delta &= (u - \tfrac{T}{\sqrt{2}})(v - \tfrac{T}{\sqrt{2}}) + (u - \tfrac{T}{\sqrt{2}})(2\tfrac{T}{\sqrt{2}}) + (2\tfrac{T}{\sqrt{2}})(v - \tfrac{T}{\sqrt{2}}) \\ &= (u - \tfrac{T}{\sqrt{2}})(v + \tfrac{T}{\sqrt{2}}) + 2\tfrac{T}{\sqrt{2}}(v + \tfrac{T}{\sqrt{2}}) - 2\tfrac{T}{\sqrt{2}}(2\tfrac{T}{\sqrt{2}}) \\ &= \underline{(u + \tfrac{T}{\sqrt{2}})(v + \tfrac{T}{\sqrt{2}}) - 2T^2} \end{aligned}$$

$$N_{min} = \int_{-\frac{T}{\sqrt{2}}}^{\frac{T}{\sqrt{2}}} du \int_{\frac{T}{\sqrt{2}}}^{\infty} dv \; \exp[-\rho (u + T/\sqrt{2})(v - T/\sqrt{2})]$$

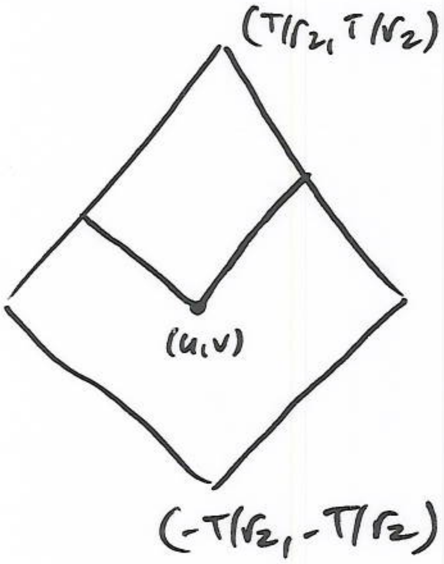
$$+ \int_{\frac{T}{\sqrt{2}}}^{\infty} du \int_{-\frac{T}{\sqrt{2}}}^{\frac{T}{\sqrt{2}}} dv \; \exp[-\rho (u - T/\sqrt{2})(v + T/\sqrt{2})]$$

$$+ e^{2\rho T^2} \int_{\frac{T}{\sqrt{2}}}^{\infty} du \int_{\frac{T}{\sqrt{2}}}^{\infty} dv \; \exp[-\rho (u + \tfrac{T}{\sqrt{2}})(v + T/\sqrt{2})]$$

$$= 2 \int_{0}^{\sqrt{2}T} du \int_{0}^{\infty} dv \; \exp[-\rho u v]$$

$$+ e^{2\rho T^2} \int_{\sqrt{2}T}^{\infty} du \int_{\sqrt{2}T}^{\infty} dv \; \exp[-\rho u v]$$

Calculate $N_{max}(0)$:



$$V_\nabla = (T/\sqrt{2} - u)(T/\sqrt{2} - v)$$

$$N_{max} = \int_{-T/\sqrt{2}}^{T/\sqrt{2}} du \int_{-T/\sqrt{2}}^{T/\sqrt{2}} dv \quad \exp[-\rho(T/\sqrt{2} - u)(T/\sqrt{2} - v)]$$

$$= \int_{-T/\sqrt{2}}^{T/\sqrt{2}} du \int_{-T/\sqrt{2}}^{T/\sqrt{2}} dv \ \exp[-\rho(T/\sqrt{2} + u)(T/\sqrt{2} + v)]$$

$$= \int_{0}^{\sqrt{2}T} du \int_{0}^{\sqrt{2}T} dv \ \exp[-\rho u v]$$