FRW $\rightarrow$ RNC at $(0,0)$

$\cdot$ $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2)$

$\cdot$ Non-zero $\Gamma^{\mu}_{\rho\sigma}$:

$$\Gamma^0_{ij} = -\frac{1}{2}(-g_{ij,0}) = \dot{a}a\,\delta_{ij}$$

$$\Gamma^i_{0j} = \Gamma^i_{j0} = \frac{1}{2}g^{ik}(g_{kj,0}) = \frac{1}{2}a^{-2}\delta^{ik}\delta_{kj}\,2\dot{a}a$$

$$= \delta^i_j\frac{\dot{a}}{a} = \delta^i_j H.$$

$\cdot$ New coords $\tilde{x}^{\bar{\mu}} = A^{\bar{\mu}}{}_{\nu}x^{\nu} + \frac{1}{2}B^{\bar{\mu}}{}_{\rho\sigma}x^{\rho}x^{\sigma}$

s.t. $\cancel{\eta_{\mu\nu} =}\ g_{\mu\nu}\big|_p = \frac{\partial x^{\bar{\mu}}}{\partial x^{\mu}}\frac{\partial x^{\bar{\nu}}}{\partial x^{\nu}}\eta_{\bar{\mu}\bar{\nu}}\big|_p$ at $t = x^{\mu}(p) = (0,0)$

$$\Rightarrow A^{\bar{\mu}}{}_{\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a(0) & 0 \\ 0 & 0 & a(0) \end{pmatrix}$$

$\cdot$ $B^{\bar{\mu}}{}_{\rho\sigma} = A^{\bar{\mu}}{}_{\kappa}\Gamma^{\kappa}_{\rho\sigma}\big|_p$

- Non-zero components of $\Gamma^{\mu}_{\rho\sigma}$ give

$\not{P}$

②

$$B^{0}_{ij} = \Gamma^{0}_{ij}\big|_p = \dot{a} a \delta_{ij}$$

$$B^{\bar{i}}_{0j} = A^{\bar{i}}_{i}\, \Gamma^{i}_{0j}\big|_p = a(0)\delta^{\bar{i}}_{i}\, \Gamma^{i}_{0j}$$

$$= a(0)\delta^{\bar{i}}_{i}\,\delta^{i}_{j} H = \delta^{\bar{i}}_{j}\,\dot{a}(0).$$

$$\Rightarrow \tilde{t} = t + \frac{1}{2}\dot{a}(0)a(0)(x^2+y^2)$$

$$\hat{x} = a(0)x + \cancel{\tfrac{1}{2}}\,\dot{a}(0)\,t\,x$$

$$\hat{y} = a(0)y + \cancel{\tfrac{1}{2}}\,\dot{a}(0)\,t\,y$$

$(+)$

$$g^{\bar{\mu}\bar{\nu}} = \frac{\partial x^{\hat{\mu}}}{\partial x^{\mu}}\frac{\partial x^{\hat{\nu}}}{\partial x^{\nu}}\, g^{\mu\nu}$$

$$g^{\bar{0}\bar{0}} = -1 + \frac{\dot{a}(0)^2 a(0)^2}{a(t)^2}(x^2+y^2)$$

$$g^{\hat{1}\hat{1}} = -\cancel{\tfrac{1}{4}}\,\dot{a}(0)^2 x^2 + \frac{1}{a(t)^2}\left(a(0) + \cancel{\tfrac{1}{2}}\,\dot{a}(0)t\right)^2$$

$$= \frac{a(0)^2}{a(t)^2}\left(1 + \frac{H(0)}{\cancel{4}}t\right)^2 - \frac{\dot{a}(0)^2}{\cancel{4}}x^2$$

$$g^{\hat{1}\hat{1}} = \frac{a_0^2}{a_t^2}(1+H_0 t)^2 - \dot{a}_0^2 x^2$$ where we've defined $a_0 \equiv a(0)$, $a_t \equiv a(t)$ etc.

Check that this has zero first-order variations in t, x, y:

$$g^{\hat{1}\hat{1}} = \frac{a_0^2}{a_0^2(1+H_0 t)^2}(1+H_0 t)^2 - \dot{a}_0^2 x^2 + \text{h.o.}$$

$$= \frac{a_0^2}{a_0^2(1+H_0 t + \frac{1}{2}\frac{\ddot{a}_0}{a_0}t^2)^2}(1+H_0 t)^2 - \dot{a}_0^2 x^2 + \text{h.o.}$$

$$= \left(1 - 2H_0 t - \frac{\ddot{a}_0}{a_0}t^2\right)(1 + 2H_0 t + H_0^2 t^2) - \dot{a}_0^2 x^2 + \text{h.o.}$$

$$= \left(1 - \frac{\ddot{a}_0}{a_0}t^2 + H_0^2 t^2\right) - \dot{a}_0^2 x^2 + \text{h.o.}$$

$$= 1 + \left(H_0^2 - \frac{\ddot{a}_0}{a_0}\right)t^2 - \dot{a}_0^2 x^2 \quad \checkmark$$

$$g^{\hat{2}\hat{2}} = \frac{a_0^2}{a_t^2}(1+H_0 t)^2 - \dot{a}_0^2 y^2$$

$$g^{\hat{0}\hat{1}} = -\frac{\partial \hat{t}}{\partial t}\frac{\partial \hat{x}}{\partial t} + \frac{1}{a_t^2}\frac{\partial \hat{t}}{\partial x}\frac{\partial \hat{x}}{\partial x} + \frac{1}{a_t^2}\frac{\partial \hat{t}}{\partial y}\frac{\partial \hat{x}}{\partial y}$$

$$= -\dot{a}_0 x + \frac{1}{a_t^2}\dot{a}_0 a_0 x a_0(1+H_0 t) + \frac{1}{a_t^2}\dot{a}_0 a_0 y \times 0$$

$$= -\dot{a}_0 x + \frac{a_0^2}{a_t^2}(1+H_0 t)\dot{a}_0 x$$

$$= \dot{a}_0\left(\frac{a_0^2}{a_t^2} - 1\right)x + \frac{\dot{a}_0 H_0 a_0^2}{a_t^2}t x$$

$$g^{\hat{0}\hat{2}} = \dot{a}_0\left(\frac{a_0^2}{a_t^2} - 1\right)y + \frac{a_0^2\dot{a}_0 H_0}{a_t^2}\,ty$$

$$g^{\hat{1}\hat{2}} = -\frac{\partial\hat{x}}{\partial t}\frac{\partial\hat{y}}{\partial t} + \cancel{\frac{\partial\hat{x}}{\partial t}}\,0 + 0$$

$$= \cancel{-(1+H_0t)(1+H_0t)} - \dot{a}_0 x\,\dot{a}_0 y$$

$$= -\dot{a}_0^2 xy$$

Let's compute the Riemann tensor in (t, x, y) coordinates.

$$R^{\mu}{}_{\nu\rho\sigma} = \Gamma^{\mu}_{\nu\sigma,\rho} - \Gamma^{\mu}_{\nu\rho,\sigma} + \Gamma^{k}_{\nu\sigma}\Gamma^{\mu}_{k\rho} - \Gamma^{k}_{\nu\rho}\Gamma^{\mu}_{k\sigma}$$

Non-zero derivatives of Γ:

$$\Gamma^{0}_{ij,0} = \delta_{ij}(\ddot{a}a + \dot{a}^2)$$

$$\Gamma^{i}_{0j,0} = \delta^i_j\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) = \delta^i_j\left(\frac{\ddot{a}}{a} - H^2\right)$$

Then

$$\cancel{R^{0}{}_{\nu\rho\sigma} = \Gamma^{0}_{\nu\sigma,\rho} - \Gamma^{0}_{\nu\rho,\sigma} + \Gamma^{k}_{\nu\sigma}\Gamma^{0}_{k\rho} - \Gamma^{k}_{\nu\rho}\Gamma^{0}_{k\sigma}}$$

$$= \cancel{\delta^i_\nu\delta^j_\sigma\delta^0_\rho\,\delta_{ij}(\ddot{a}a + \dot{a}^2)}$$

Check the expansion:

$\cancel{\hat{t} = \hat{x}^0 =}$ $g^{\hat{0}\hat{0}} = 1^{00} + \alpha R^{0}{}_{\nu}{}^{0}{}_{\sigma} x^{\nu} x^{\sigma}$

$$R^{0}{}_{\nu}{}^{0}{}_{\sigma} = g^{0\rho} R^{0}{}_{\nu\rho\sigma}$$

$$= - R^{0}{}_{\nu 0 \sigma}$$

$$= - \left[\Gamma^{0}_{\nu\sigma,0} - \Gamma^{0}_{\nu 0,\sigma} + \cancel{\Gamma^{k}_{\nu\sigma} \Gamma^{0}_{k0}} - \Gamma^{k}_{\nu 0} \Gamma^{0}_{k\sigma} \right]$$

$$\Rightarrow R^{0}{}_{i}{}^{0}{}_{j} = - R^{0}{}_{i0j} = - \left[\Gamma^{0}_{ij,0} + \cancel{\Gamma^{0}_{ij} \Gamma^{0}_{00}} - \Gamma^{k}_{i0} \Gamma^{0}_{kj} \right]$$

$$= - \left[\delta_{ij} (\ddot{a} a + \dot{a}^2) - \delta^{k}_{i} H \delta_{kj} \dot{a} a \right]$$

$$= - \left(\delta_{ij} (\ddot{a} a + \dot{a}^2) - \delta_{ij} \dot{a}^2 \right)$$

$$\underline{= - \delta_{ij} \ddot{a} a}.$$

$R^{0}{}_{0}{}^{0}{}_{0} = 0$, $R^{0}{}_{i}{}^{0}{}_{0} = 0$, $R^{0}{}_{0}{}^{0}{}_{i} = 0$.

$R'{}_{\nu}{}'{}_{\sigma}$:

$$R'{}_{0}{}'{}_{\rho} = \frac{1}{a_t^2} R^{1}{}_{010}$$

$$= \frac{1}{a_t^2}\left[0 - \Gamma^{1}_{01,0} + 0 - \Gamma^{k}_{01}\Gamma^{1}_{0k} \right]$$

$$= \frac{1}{a_t^2}\left[-\left(\frac{\ddot{a}}{a} - H^2\right) - \delta^{k}_{1} H \delta^{1}_{k} H \right]$$

$$= \frac{1}{a_t^2}\left[-\frac{\ddot{a}_t}{a_t} \right] = -\frac{\ddot{a}_t}{a_t^3}$$

$$R'{}_{0}{}'{}_{i} = \frac{1}{a_t^2} R^{1}{}_{0 1 i}$$

$$= \frac{1}{a_t^2}\left[\cancel{\Gamma^{1}_{0i,1}} \to 0 - 0 + \Gamma^{k}_{0i}\cancel{\Gamma^{1}_{k1}} \to 0 - \Gamma^{k}_{01}\cancel{\Gamma^{1}_{ki}} \to 0 \right]$$

$$= 0$$

$$R'{}_{i}{}'{}_{j} = \frac{1}{a_t^2} R^{1}{}_{i1j} = \frac{1}{a_t^2}\left[\Gamma^{k}_{ij}\Gamma^{1}_{k1} - \Gamma^{k}_{i1}\Gamma^{1}_{kj} \right]$$

$$= \frac{1}{a_t^2}\left[\Gamma^{0}_{ij}\Gamma^{1}_{01} - \Gamma^{0}_{i1}\Gamma^{1}_{0j} \right]$$

$$= \frac{1}{a_t^2}\left[\dot{a}_t a_t \delta_{ij} H_t - \dot{a}_t a_t \delta_{1i}\delta^{1}_{j} \cancel{a} H \right]$$

$$R'{}_{1}{}'{}_{1} = \frac{1}{a_t^2}\left[\dot{a}_t^2 - \dot{a}_t^2\right] = 0$$

$$R'{}_{2}{}'{}_{2} = \frac{1}{a_t^2}\left[\dot{a}_t^2\right] = \frac{\dot{a}_t^2}{a_t^2} = H_t^2 \qquad R'{}_{1}{}'{}_{2} = R'{}_{2}{}'{}_{1} = 0.$$

Now

$$g^{\tilde{1}\tilde{1}} = 1 + \left(H_0^2 - \frac{\ddot{a}_0}{a_0}\right)t^2 - \dot{a}_0^2 x^2 + h.o.$$

whereas

$$\eta^{11} + \alpha R^1{}_{\mu}{}^1{}_{\nu} x^\mu x^\nu = 1 + \alpha\left(H_t^2 a^2 x - \frac{\ddot{a}_t}{a_t^3} t^2\right)$$

$$= 1 + \alpha\ ($$

Don't agree!

To second order, let's invert (+) 8

from page 2.

$$t = \tilde{t} - \frac{1}{2} H(0) \frac{\tilde{x}^2 + \tilde{y}^2}{(1+H(0)t)^2}$$

$$\cancel{\hat{t} = \left(t + \frac{1}{2} H(0) (\tilde{x}^2 + \tilde{y}^2)\right)\left(1 - 2H(0)t + 3H(0)^2 t^2\right) + O(\ldots}$$

$$\cancel{\Rightarrow \frac{3}{2} H_0^3 (\tilde{x}^2 + \tilde{y}^2) t^2 + \left(1 - H(0)^2 (\tilde{x}^2 + \tilde{y}^2)\right) t} + \frac{1}{2} H_0 (\tilde{x}^2 + \tilde{y}^2) - \hat{t} = 0$$

$$t(1+H(0)t)^2 - \tilde{t}(1+H(0)t)^2 = -\frac{1}{2} H(0)(\tilde{x}^2 + \tilde{y}^2)$$

· ~~Figure out~~

GNC: $d\tilde{s}^2 = -d\tilde{t}^2 + h_{ij}(t, \underline{x})\, d\tilde{x}^i d\tilde{x}^j$

$\Downarrow$

RNC: ~~need $g_{\mu\nu}$~~ at $\cancel{(0,0)}$ $(\tilde{t}_0, \tilde{\underline{x}}_0)$.

$ds^2 = \cancel{-dt}$

Do inversion systematically.

We have $x^{\bar{\mu}} = A^{\bar{\mu}}{}_{\nu} x^{\nu} - \frac{1}{2} A^{\bar{\mu}}{}_{\rho} \Gamma^{\rho}_{\alpha\beta} x^{\alpha} x^{\beta}$.

$\cancel{x^\mu = A^\mu{}_\nu x^\nu +}$ RNC inverse relations: 9

$y^\mu = A^\mu{}_\nu x^\nu + B^\mu{}_{\rho\sigma} x^\rho x^\sigma$. Expand to 2nd order.

$x^\mu = (A^{-1})^\mu{}_\nu y^\nu + C^\mu{}_{\rho\sigma} y^\rho y^\sigma$

$$y^\mu = A^\mu{}_\nu (A^{-1})^\nu{}_\rho y^\rho + A^\mu{}_\nu C^\nu{}_{\rho\sigma} y^\rho y^\sigma$$
$$+ B^\mu{}_{\rho\sigma}\left((A^{-1})^\rho{}_\alpha y^\alpha + C^\rho{}_{\alpha\beta} y^\alpha y^\beta\right)$$
$$\left((A^{-1})^\sigma{}_\alpha y^\alpha + O(y^2)\right)$$

$$y^\mu = y^\mu + A^\mu{}_\nu C^\nu{}_{\rho\sigma} y^\rho y^\sigma$$
$$+ B^\mu{}_{\alpha\beta}(A^{-1})^\alpha{}_\rho (A^{-1})^\beta{}_\sigma y^\rho y^\sigma$$

$$\Rightarrow A^\mu{}_\nu C^\nu{}_{\rho\sigma} + B^\mu{}_{\alpha\beta}(A^{-1})^\alpha{}_\rho (A^{-1})^\beta{}_\sigma = 0$$

$$\Rightarrow C^\mu{}_{\rho\sigma} = (A^{-1})^\mu{}_\nu B^\nu{}_{\alpha\beta}(A^{-1})^\alpha{}_\rho (A^{-1})^\beta{}_\sigma$$

Now $B^{\mu}_{\rho\sigma} = -\frac{1}{2} A^{\mu}{}_{\nu} \Gamma^{\nu}_{\rho\sigma}$ so

$$C^{\mu}_{\rho\sigma} = -\tfrac{1}{2}(A^{-1})^{\mu}{}_{\nu} A^{\nu}{}_{\gamma} \Gamma^{\gamma}_{\alpha\beta}(A^{-1})^{\alpha}{}_{\rho}(A^{-1})^{\beta}{}_{\sigma}$$

$$= -\frac{1}{2} \Gamma^{\mu}_{\alpha\beta}(A^{-1})^{\alpha}{}_{\rho}(A^{-1})^{\beta}{}_{\sigma}$$

So to second order we have

$$x^{\mu} = (A^{-1})^{\mu}{}_{\nu} y^{\nu} - \frac{1}{2}\Gamma^{\mu}_{\alpha\beta}(A^{-1})^{\alpha}{}_{\rho}(A^{-1})^{\beta}{}_{\sigma} y^{\rho} y^{\sigma}$$

If we denote y^{μ} by $x^{\bar{\mu}}$ and "$(A^{-1})^{\mu}{}_{\nu} = A^{\mu}{}_{\bar{\nu}}$", i.e. $A^{\mu}{}_{\bar{\nu}} A^{\bar{\nu}}{}_{\rho} = \delta^{\mu}_{\rho}$ we have

$$\cancel{x^{\mu} = A^{\mu}{}_{\bar{\nu}} y^{\bar{\nu}} - \frac{1}{2}\Gamma^{\mu}_{\alpha\beta} A^{\alpha}{}_{\bar{\rho}} A^{\beta}{}_{\bar{\sigma}} y\, x^{\bar{\rho}} x^{\bar{\sigma}}}$$

$$x^{\mu} = A^{\mu}{}_{\bar{\nu}} x^{\bar{\nu}} - \frac{1}{2}\Gamma^{\mu}_{\alpha\beta} A^{\alpha}{}_{\bar{\rho}} A^{\beta}{}_{\bar{\sigma}} x^{\bar{\rho}} x^{\bar{\sigma}}.$$

Where $A^{\mu}{}_{\bar{\nu}}$ and $\Gamma^{\mu}_{\alpha\beta}$ denote the constant matrices that satisfy:

$$\eta_{\bar{\mu}\bar{\nu}} = A^{\mu}{}_{\bar{\mu}} A^{\nu}{}_{\bar{\nu}} g_{\mu\nu}(0, \overbrace{\underline{x}_0}^{\text{GNCs on } \Sigma})$$

$$\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta}(0, \underline{x}_0).$$

What does the $\bar{t}=0$ surface look like in RNCs? ~~Also~~ Since $g_{\mu\nu} = \begin{pmatrix} -1 & \cdots \\ \vdots & h \end{pmatrix}$

$\cancel{X^\mu = 2A^\mu_{\bar\nu}}$ we have $\eta_{\bar\mu\bar\nu} = A^\mu_{\bar\mu} A^\nu_{\bar\nu} g_{\mu\nu}$

$$\underline{\underline{A}} = \begin{pmatrix} 1 & -0- \\ | & \\ 0 & \underline{\underline{a}} \\ | & \end{pmatrix}$$ and a satisfies

$$\cancel{1 = 2aga} \quad \cancel{\text{where } g_{\bar\mu\bar\nu}}$$

$$\underline{\underline{1}} = \underline{\underline{a}}^{-1} \underline{\underline{h}}\, \underline{\underline{a}}^{-1} \quad \text{where} \quad h_{ij} \equiv h_{ij}(0, \underline{x}_0).$$

$$\underline{\underline{a}}^{+1} = \underline{\underline{h}}\, \underline{\underline{a}}^{-1}$$

$$\underline{\underline{1}} = \underline{\underline{h}}\, \underline{\underline{a}}^{-2} \Rightarrow \underline{\underline{a}}^2 = \underline{\underline{h}} \Rightarrow \underline{\underline{a}} = \sqrt{\underline{\underline{h}}}$$

- Since $\underline{\underline{h}}$ is positive-definite it has a unique positive-definite square root.

- ~~Then for $\bar{t}=0$~~

$\cancel{X^\mu =}$

- Recall that $\underline{\underline{A}}$ represents the matrix $A^{\bar\mu}_{\mu}$ in

$$X^{\bar\mu} = A^{\bar\mu}_{\nu} X^\nu - \frac{1}{2} A^{\bar\mu}_{\rho} \Gamma^\rho_{\mu\nu} X^\mu X^\nu$$

where $X^{\bar\mu} = \text{RNC}$, $X^\mu = \text{GNC}$.

[*Clarify notation*: as at the beginning $\tilde{x}^r$ denotes RNCs, x^r denotes GNCs.]

- We want to find the equation for the $t=0$ surface in RNCs. For that we need the inverse relations derived on the last pages. We have:

$$0 = t = x^0 = A^0{}_{\bar{\nu}}\, x^{\bar{\nu}} - \frac{1}{2} \Gamma^0_{\alpha\beta} A^\alpha_{\bar{\mu}} A^\beta_{\bar{\nu}} x^{\bar{\mu}} x^{\bar{\nu}} + \mathcal{O}(\bar{x}^3)$$

where $A^\alpha_{\bar{\beta}}$ are the components of A^{-1}.

Now $A^0_{\bar{i}} = 0$ and

$$\Gamma^0_{\alpha\beta} = \frac{1}{2} g^{00} \left(\cancel{g_{0\alpha,\beta}}^{\,0} + \cancel{g_{0\beta,\alpha}}^{\,0} - g_{\alpha\beta,0} \right)$$

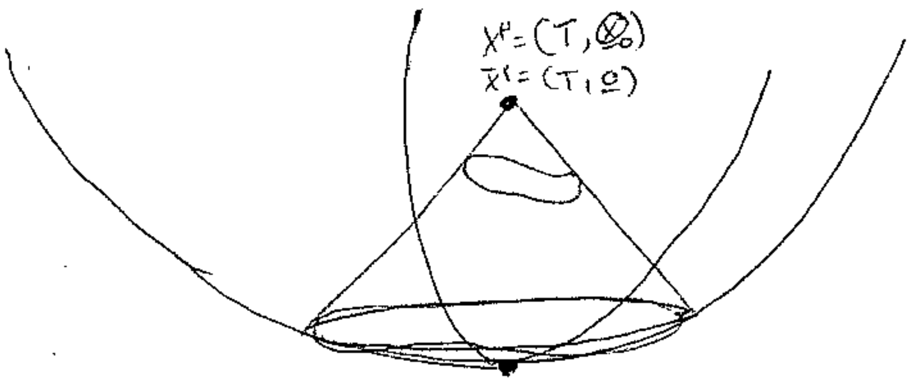
$$= + \frac{1}{2} g_{\alpha\beta,0} \quad (= 0 \text{ unless } \alpha = i, \beta = j).$$

$\Rightarrow$ only non-zero components in $\Gamma^0_{\alpha\beta}$ are Γ^0_{ij}.

So $$0 = \bar{t} - \frac{1}{2} \Gamma^0_{ij} A^i_{\bar{i}} A^j_{\bar{j}} x^{\bar{i}} x^{\bar{j}} + \mathcal{O}(\bar{x}^3)$$

$$\Rightarrow \boxed{\bar{t} = \frac{1}{2} \Gamma^0_{ij} A^i_{\bar{i}} A^j_{\bar{j}} x^{\bar{i}} x^{\bar{j}}}$$

· So the $t=0$ surface is a geodesic surface in RNCs.



· Note that we've rescaled the GNCS ~~to~~ to the particular point $(0, \underline{x}_0)$ by defining $\underline{x} \to \underline{x}' = \underline{x} - \underline{x}_0$ so that $(0, \underline{x}_0) \to (0, \underline{0})$.

~~Note~~ Then $ds^2 = -dt^2 + h_{ij}(t, \underline{x}' + \underline{x}_0)\, dx'^i dx'^j$

so the h_{ij} that enters in A & Γ is that evaluated at $(0, \underline{x}_0)$ in the original GNC.

· Also note that $X^\mu = (T, \underline{0})$ corresponds to $\bar{X}^\mu(T, \underline{0})$

hence
$$\bar{X}^{\bar{\mu}} = A^{\bar{\mu}}{}_{\nu} X^\nu - \frac{1}{2} A^{\bar{\mu}}{}_{\nu} \Gamma^{\nu}_{\rho\sigma} X^\rho X^\sigma$$

$$X^{\bar{0}} = X^0 - \frac{1}{2} \Gamma^0_{\rho\sigma} X^\rho X^\sigma$$

$$= X^0 - \frac{1}{2} \Gamma^0_{ij} X^i X^j = T.$$

$$X^{\bar{i}} = 0 - A^{\bar{i}}_{i} \Gamma^{i}_{\rho\sigma} X^\rho X^\sigma \quad \text{but } \Gamma^i_{00} = 0 \text{ so}$$

$$= 0 \;\forall\, \bar{i}.$$

- We need to integrate from the bottom geodesic to the backward lightcone emanating at $(T, \underline{0})$. What's the equation for this lightcone?

- Suvrat's analysis $\Rightarrow$ only need to find lightcone in tangent space at $(T, \underline{0})$; then treat light-rays as straight as any corrections due to geodesic acceleration occur at ~~sth~~ orders higher than T^2.

- To find the light-cone note that at $(T, \underline{0})$

$$g_{\bar\mu\bar\nu}(T, \underline{0}) = \eta_{\bar\mu\bar\nu} - \frac{1}{3} T^2 R_{\bar\mu 0 \bar\nu 0} + O(T^3).$$

so the null geodesic vector $\zeta^{\bar\mu}$ satisfies

$$-(\zeta^0)^2 + \underline{\zeta}^2 = \frac{1}{3} R_{\bar\mu 0 \bar\nu 0} \zeta^{\bar\mu} \zeta^{\bar\nu}.$$

- Recall $R^{\bar\mu}{}_{\bar\rho\bar\nu\bar\sigma} = \Gamma^{\bar\mu}_{\bar\rho\bar\sigma,\bar\nu} - \Gamma^{\bar\mu}_{\bar\rho\bar\nu,\bar\sigma} + \Gamma^{\bar\mu}_{\bar\nu\bar\kappa}\Gamma^{\bar\kappa}_{\bar\rho\bar\sigma} - \Gamma^{\bar\mu}_{\bar\sigma\bar\kappa}\Gamma^{\bar\kappa}_{\bar\rho\bar\nu}$ so

$$R^{\bar\mu}{}_{0\bar\nu 0} = \Gamma^{\bar\mu}_{00,\bar\nu} - \Gamma^{\bar\mu}_{0\bar\nu,0} + \Gamma^{\bar\mu}_{\bar\nu\bar\kappa}\Gamma^{\bar\kappa}_{00} - \Gamma^{\bar\mu}_{0\bar\kappa}\Gamma^{\bar\kappa}_{0\bar\nu}$$