

We calculate the GHY boundary term for the flat causal diamond in  $D$  dimensions via two limiting procedures. Take standard polar coordinates  $(t, r, \boldsymbol{\theta})$ . The tips of the diamond are at  $(\pm T, \mathbf{0})$ . Consider two  $\epsilon$ -families of surfaces:

- Timelike lozenges (TL):

$$t^2 - (T - r)^2 + \epsilon^2 = 0$$

- Spacelike lozenges (SL):

$$r^2 - (T - |t|)^2 + \epsilon^2 = 0$$

both approaching the diamond as  $\epsilon \rightarrow 0$ . If we calculate the boundary action associated with these surfaces for non-zero  $\epsilon$  and take the limit of that expression as  $\epsilon \rightarrow 0$  we obtain the following results. For the TL family we get

$$S_{GHY} \sim \begin{cases} 4 \log \left( \frac{2T}{\epsilon} \right) & \text{for } D = 2 \\ 2V_{D-2}T^{D-2} \left[ -\log \left( \frac{2T}{\epsilon} \right) + \frac{D-1}{D-2} + H_{D-3} \right] & \text{for } D > 2 \end{cases}$$

where  $H_n$  is the  $n^{th}$  harmonic number and for the SL family we obtain

$$S_{GHY} \sim \begin{cases} 4 \log \left( \frac{2T}{\epsilon} \right) & \text{for } D = 2 \\ 2V_{D-2}T^{D-2} \frac{D-1}{D-2} & \text{for } D > 2 \end{cases}$$

where  $V_n$  is the volume of the  $n$ -sphere. So the limits agree for  $D = 2$  but disagree for  $D > 2$ . The SL limit gives a finite constant boundary term for  $D > 2$  whereas the TL limit gives a logarithmic divergence in all  $D$ .