

We calculate the GHY boundary term for the flat causal diamond in D dimensions via two limiting procedures. Take standard polar coordinates $(t, r, \boldsymbol{\theta})$. The tips of the diamond are at $(\pm T, \mathbf{0})$. Consider two ϵ -families of surfaces:

- Timelike lozenges (TL):

$$t^2 - (T - r)^2 + \epsilon^2 = 0$$

- Spacelike lozenges (SL):

$$r^2 - (T - |t|)^2 + \epsilon^2 = 0$$

both approaching the diamond as $\epsilon \rightarrow 0$. If we calculate the boundary action associated with these surfaces for non-zero ϵ and take the limit of that expression as $\epsilon \rightarrow 0$ we obtain the following results. For the TL family we get

$$S_{GHY} \sim \begin{cases} 4 \log \left(\frac{2T}{\epsilon} \right) & \text{for } D = 2 \\ 2V_{D-2}T^{D-2} \left[-\log \left(\frac{2T}{\epsilon} \right) + \frac{D-1}{D-2} + H_{D-3} \right] & \text{for } D > 2 \end{cases}$$

where H_n is the n^{th} harmonic number and for the SL family we obtain

$$S_{GHY} \sim \begin{cases} 4 \log \left(\frac{2T}{\epsilon} \right) & \text{for } D = 2 \\ 2V_{D-2}T^{D-2} \frac{D-1}{D-2} & \text{for } D > 2 \end{cases}$$

where V_n is the volume of the n -sphere. So the limits agree for $D = 2$ but disagree for $D > 2$. The SL limit gives a finite constant boundary term for $D > 2$ whereas the TL limit gives a logarithmic divergence in all D .