Information Processing and Computer Vision (TIVA)
TP4 - Graph-Cut Based Image Segmentation.

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1 The MRF problem

The binary segmentation problem consists in labelling each pixel of a grayscale image as belonging to the foreground or the backgroung, thanks to pre-labeled image regions (one inside the background, and another one inside the foreground).

For this purpose, we will find the *min-cut* of the graph constituted by $N = width * height$ nodes, one node for each pixel of the image, plus 2 additional nodes the source S and the sink T which designate respectively "foreground" and "background".
On this graph, we add an edge between each pixel node p and T (resp. S), with a weight $U_p = -\log(p_r + 10^{-5})$ where p_r equals the probability that the pixel p belongs to the foreground (resp. background), estimated here as the frequence of the intensity color of the pixel p in the histogram of the pixels intensity of the foreground (resp. background) pre-labeled region. Moreover, we add edges between every pair of neigbours of pixels p, q (q at the left, right, top, or down of p) and take for their weights 10 if these have the same labels, and 0 if not, in order to penalize irregularities.

This *min-cut* problem can be synthetized by the research of the labels' assignement Y (X intensity of pixels, Y labels for each pixel) that minimizes the following energy, sum of the unary potentials and the pairwise potentials :

$$\sum_p U_p(y_p|x_p) + 10 \sum_{||p-q||_1=1} \delta_{y_p!=y_q}$$

2 The results : obtained segmentations and accuracy

2.1 Obtained segmentation maps

On figure 1, we show the given image without and with the pre-labeled regions as foreground (white) and background (black). Figure 2 shows the histogram of the pixels intensity of the pre-labeled regions. Figure 3 gives the result of the segmentation when minimizing only the unary potentials, and figure 4 gives the result for the whole MRF energy minimization (with pairwise potential or "regularization" term). Figure 5 is the ground truth of the segmentation.

We can observe easily that the add of the regularization term with the pairwise potentials really helps the segmentation.

Indeed we see on figure 3 that when we minimize only the unary potentials term, some pixels inside the pieces, that are darker, are labelled as background because their color matches more with the distribution of colors of the background pre-labeled region.

On figure 4, when we add to the unary term the pairwise potentials term in the MRF energy to minimize, these same pixels, that were not well identified in the unary potentials minimization, are in this case well labelled as foreground : indeed the neighbours of these pixels, lighter, are always more inclined to be considered as foreground because of the unary term, and since the pairwise potentials penalizes different labels for neigbouring pixels, the pixels that are darker inside the pieces will tend to be considered themselves as foreground.

Thus with a high regularization coefficient (10 here), we get with the pairwise potentials term a segmentation map that is far more closer to the ground truth than with the unary potentials only.

2.2 Accuracy assessments

We denote by y the segmentation map computed with the MRF minimization, and by z the ground truth segmentation.

Accuracy can first be evaluated by computing the fraction of pixels well predicted :

$$A_1 = \frac{1}{N} \sum_p \delta_{y_p == z_p}$$

If we want to focus more on the quality of the segmentation of the pieces (foreground), we can also compute the "Intersection over Union" quotient :

$$A_2 = \sum_p \frac{\delta_{y_p == FG, z_p == FG}}{\delta_{\{y_p == FG\} \mathcal{U} \{z_p == FG\}}}$$

To compare the two measurements,

$$A_1 = \frac{\delta_{y_p = FG, z_p = FG} + \delta_{y_p = BG, z_p = BG}}{N}, \; A_2 = \frac{\delta_{y_p = FG, z_p = FG}}{N - \delta_{y_p = BG, z_p = BG}}$$

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Accuracy measure - Minimized Energy	Only with unary potentials	With pairwise potentials
A_1 ("Simple matching")	0.978	0.997
A_2 ("Intersection over Union")	0.931	0.989

The computed accuracies show both the improvement of the segmentation's quality with the add of the pairwise potentials term in the MRF energy.

3 Ideas of further improvements

— First, we could have assigned the costs $U_p = -\log(p + 10^{-10})$ only for the edges linking S/T to the pixel nodes that are not in pre-labeled regions : to guide the segmentation in the best way, we could indeed choose directly cost of 0 (resp. $+\infty$) on the edge $p - S$ and $+\infty$ (resp. 0) on the edge $p - T$ for each pixel p in the background (resp. foreground) pre-labeled region.

— Secondly, we have estimated the probability p_r that the pixel p belongs to the foreground by computing the frequence of the intensity of p in the foreground pre-labeled region. But since the pre-labeled region is just a piece of the foreground, it could have been more accurate if we had first modelized the histogram of the intensity of the pre-labeled region pixels as a gaussian distribution, or as a sum of gaussians (using for instance gaussian mixture models and EM algorithm) : indeed, if the intensity of a pixel belonging to a piece and located outside the foreground pre-labeled region was 200 and if the pixel intensity of the foreground pre-labeled region was in average 190 but only went up toward 199 only, thus the cost on the edge $p - T$ would be $-\log(0+10^{-5}) = 5$ without such modelization as a gaussian, while, in reality, it would be near $-\log(1+10^{-5}) = 0$ if we had modelized the distribution as a gaussian centered in 190 for instance.

On the example, even without these two improvements, we obtain satisfying results. Indeed, pieces and background have there very different intensity distributions (really different averages, and low variances) as we see it with the histograms (thus, improvement 2 is not necessary), and, as a consequence of low variances, the intensity distribution of the pre-labeled region is really near from the intensity distribution of the whole back/foreground region (which limits the importance of improvement 1).

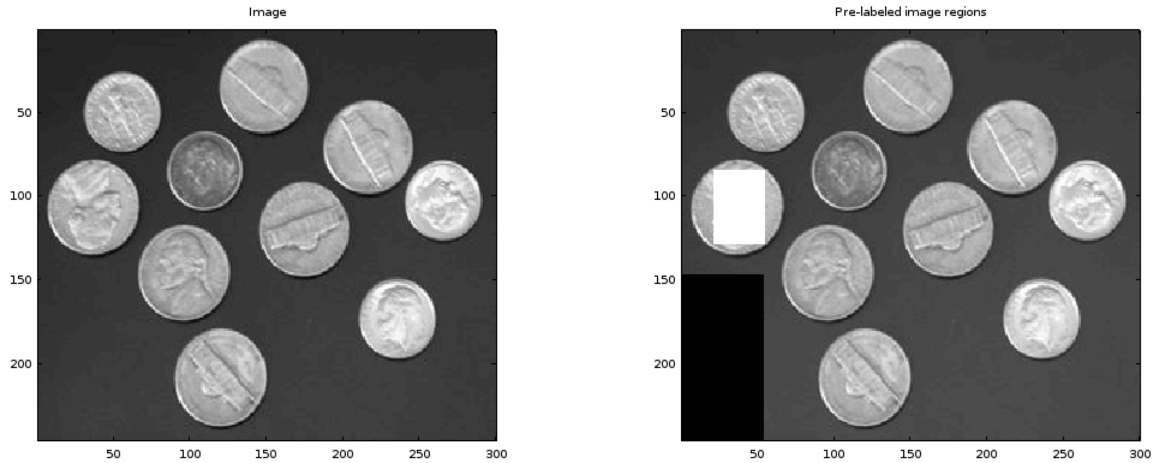

FIGURE 1 – The given image we want to segment, and with the pre-labeled regions as foreground (white) and background (black)

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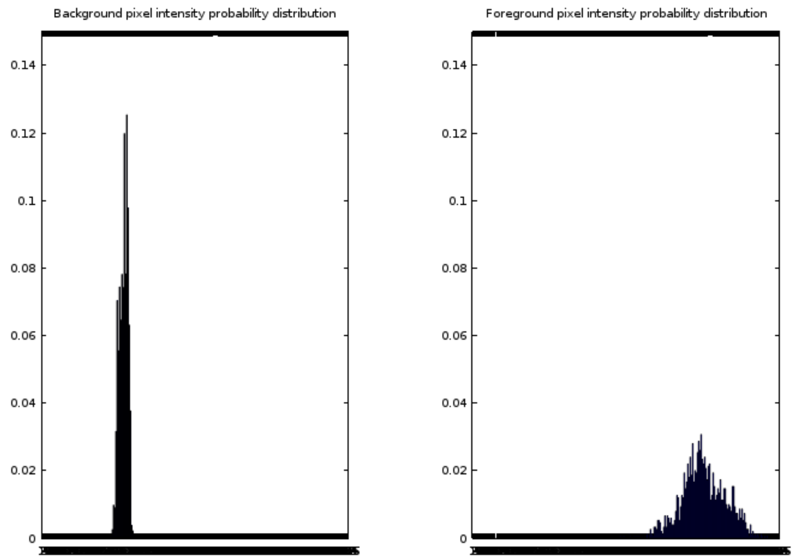

FIGURE 2 – Histograms of the intensity of the pre-labeled regions

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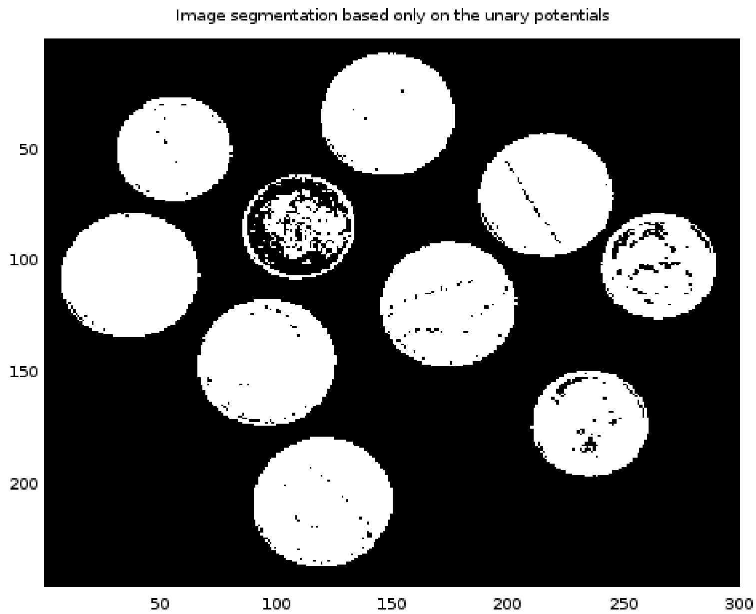

FIGURE 3 – Result of the segmentation when minimizing only the unary potentials

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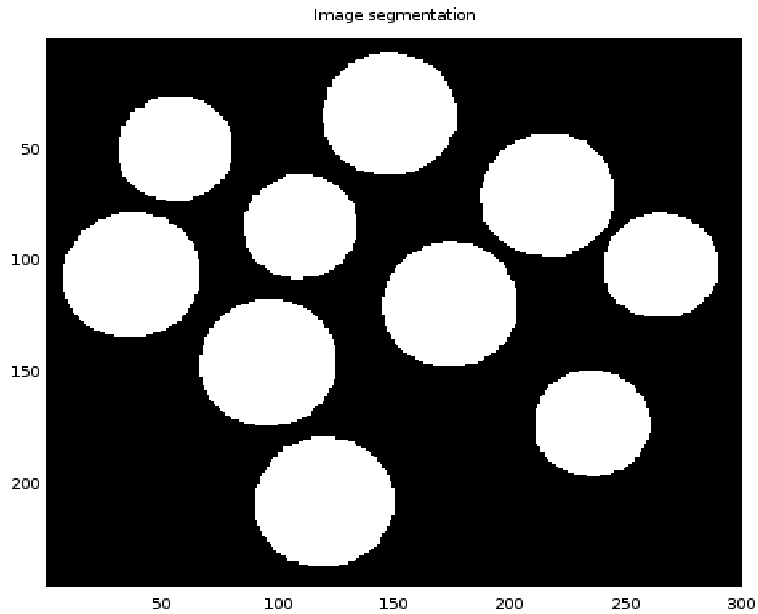

FIGURE 4 – Result of the segmentation for the whole MRF energy minimization (with pairwise potential term)

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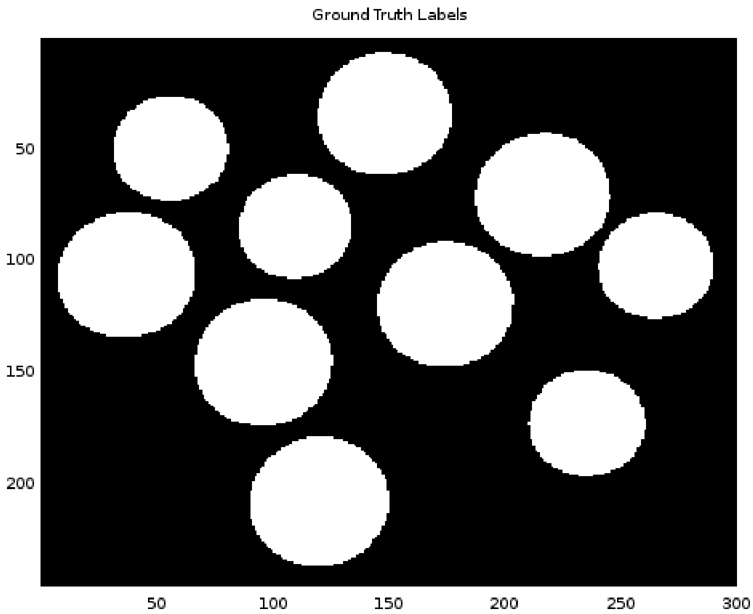

FIGURE 5 – Ground truth of the segmentation
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