force

$$V = fx \quad \Rightarrow \quad H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + fx$$

Solve $$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - fx\right)K(x,t|x',t') = i\hbar\,\delta(x-x')\,\delta(t-t')$$

Fourier transform: $\int dx\,dt\; e^{-ikx}\, e^{i\omega t}$

$$\Rightarrow \left(\hbar\omega - \frac{\hbar^2k^2}{2m} - if\frac{d}{dk}\right) G(k,\omega|x',t') = i\hbar\, e^{-ikx'+i\omega t'}$$

where $G(k,\omega|x',t') \equiv \int dx\,dt\; e^{-ikx}\, e^{i\omega t}\, K(x,t|x',t')$

and $K(x,t|x',t') = \int \frac{dk\,d\omega}{(2\pi)^2} e^{ikx}\, e^{-i\omega t}\, G(k,\omega|x',t')$

Now have ordinary differential equation for G:

$$\frac{dG}{dk} + \left(\frac{i\hbar\omega}{f} - \frac{i\hbar^2k^2}{2mf}\right)G = -\frac{\hbar}{f}\, e^{-ikx'+i\omega t'}$$

multiply by integrating factor $e^{\int^k dk'\left(\frac{i\hbar\omega}{f} - \frac{i\hbar^2k'^2}{2mf}\right)} = e^{\frac{i\hbar\omega}{f}k - \frac{i\hbar^2}{6mf}k^3}$

$$\Rightarrow \frac{d}{dk}\left(G\, e^{\frac{i\hbar\omega}{f}k - \frac{i\hbar^2}{6mf}k^3}\right) = -\frac{\hbar}{f}\, e^{\frac{i\hbar\omega}{f}k - \frac{i\hbar^2}{6mf}k^3 - ikx' + i\omega t'}$$

$$G = -\frac{\hbar}{f}\, e^{-\frac{i\hbar\omega}{f}k + \frac{i\hbar^2}{6mf}k^3} \int^k e^{-\frac{i\hbar^2}{6mf}k'^3 + \frac{i\hbar\omega}{f}k' - ik'x'}\, dk'\; e^{i\omega t'}$$

$$K = \int \frac{dk\,d\omega}{(2\pi)^2}\, e^{ikx - i\omega t}\, G$$

$$K = -\frac{\hbar}{f}\int\frac{dk}{2\pi}\int_{\infty}^{k} dk'\; e^{\frac{i\hbar^2}{6mf}(k^3 - k'^3) + ikx - ik'x'} \overbrace{\int\frac{d\omega}{2\pi}\, e^{-i\omega\left(t-t'+\frac{\hbar}{f}(k-k')\right)}}^{\delta\left(t-t'+\frac{\hbar}{f}(k-k')\right) = \frac{f}{\hbar}\delta\left(k-k'+\frac{f}{\hbar}(t-t')\right)}$$

↑ pick for causality: integral enforces $k < k' \Rightarrow t > t'$ from δ

δ also enforces $k' = k + \frac{f}{\hbar}(t-t') \Rightarrow k'^3 - k^3 = 3\frac{f}{\hbar}(t-t')k^2 + 3\frac{f^2}{\hbar^2}(t-t')^2k + \frac{f^3}{\hbar^3}(t-t')^3$

also get $-$ sign from flipping limits of k' integral. Now

$$K = \int\frac{dk}{2\pi}\, e^{-\frac{i\hbar^2}{6mf}\left(3\frac{f}{\hbar}(t-t')k^2 + 3\frac{f^2}{\hbar^2}(t-t')^2k + \frac{f^3}{\hbar^3}(t-t')^3\right) + ikx - i\left(k+\frac{f}{\hbar}(t-t')\right)x'}\;\theta(t-t')$$

Now carry out integral by completing square and treating as Gaussian integral.