Computational Complexity and Linguistic Theory

Guy Emerson

What I'll Cover

- Turing Completeness of Unification
- Complexity in Phonology
- Judicious Incoherence

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  - Powerful formalism for developing theory
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  - Constraints on human behaviour

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- Computation types and wrapper types (including append-lists) do not change the formalism

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- 2021 More careful implementation
- 2023 Dan: this looks fun

Turing Completeness of Unification

- Separating formalism from theory: new applications can be surprising
- Comments welcome on draft paper!
  `https://www.cl.cam.ac.uk/~gete2/wrapper.pdf`

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- Lamont (2023): Optimality Theory is Turing-complete
- Hao (2024): Single-tape Optimality Theory is PSPACE-complete (at least as hard as NP)
- Emerson and Lamont (in prep.): Single-tape Harmonic Grammar is finite-state

Optimality Theory

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Harmonic Grammar

- Underlying phonological representation is distinct from surface realisation
  - e.g. English plural *-s*: [s], [z], [əz]
- A grammar is a set of constraints (which may conflict), and a **weighting** of those constraints
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  - At each step, sufficient to consider a finite set of candidates
- Relies on comparing candidates as a single score
  - In contrast, Optimality Theory requires comparison for each constraint

Harmonic Grammar vs. Optimality Theory

- Theoretical commitments very similar
  - Explain complex phenomena as interactions between simple constraints
- Computational behaviour very different
  - Finite-state vs. PSPACE-complete

10 Years of My Semantics Research…

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- Truth is painful

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- Truth is useful
  - A truth-conditional model can learn and generalise meanings in a more human-like way
- Truth is painful
  - A truth-conditional model is intractable with respect to dimensionality of feature space

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  - Expressive
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Judicious Incoherence

- A computational system cannot be all three:
  - Expressive
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  - Coherent
- Human cognition is expressive and tractable, so cannot be coherent

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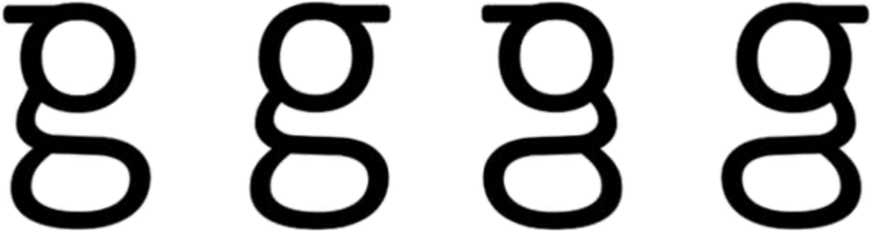
- Bayesian inference is #P-complete (at least as hard as NP-complete), even in restricted settings and even when approximated (Roth, 1996)

Example: Looptail g



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  - $p(shape|class)$ skewed to handwritten form

Amortised Variational Inference Revisited

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Amortised Variational Inference Revisited

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- Zhao et al. (2019) alternative view:
  - VAE objective minimises KL-divergence between
  - generative model $p_\theta(z)p_\theta(x|z)$
  - inference model $p_{\mathcal{D}}(x)p_\phi(z|x)$

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- Can be solved by iterating over all (x, y) pairs (Arnold & Press, 1989)

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- This includes all neural network models

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- Theorem:
  - If $p(x|y), p(y|x) > 0$, this is co-NP-complete.
  - In the general case, this is Σ_2^P-complete.

Succinct Compatibility is in co-NP

- If $p(x|y)$ and $p(y|x)$ are incompatible, we can find a certificate (x_1, x_2, y_1, y_2)

- Verify by checking:
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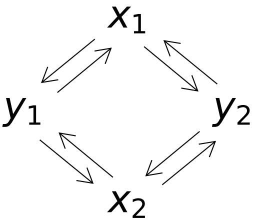

Coherence as Regulariser

- Given observed (x_1, y_1), sample alternatives x_2, y_2.
- Regularise $\dfrac{p(x_1|y_1)p(y_1|x_2)p(x_2|y_2)p(y_2|x_1)}{p(y_1|x_1)p(x_1|y_2)p(y_2|x_2)p(x_2|y_1)} \approx 1$

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- Model can appear coherent in a subspace (polynomially bounded in size)
- First steps: experiments planned on BabyLM

Judicious Incoherence

- Cognitive models must be expressive and tractable
- Expressive tractable models are incoherent
- Humans and models can be judiciously incoherent