



Computational Complexity and Linguistic Theory

Guy Emerson

What I'll Cover

- Turing Completeness of Unification
- Complexity in Phonology
- Judicious Incoherence

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 - Powerful formalism for developing theory
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 - Constraints on human behaviour

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- Computation types and wrapper types (including append-lists) do not change the formalism

Timeline



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- 2021 More careful implementation
- 2023 Dan: this looks fun

Turing Completeness of Unification

- Separating formalism from theory: new applications can be surprising
- Comments welcome on draft paper!

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- Lamont (2023): Optimality Theory is Turing-complete
- Hao (2024): Single-tape Optimality Theory is PSPACE-complete (at least as hard as NP)
- Emerson and Lamont (in prep.): Single-tape Harmonic Grammar is finite-state

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Harmonic Grammar

- Underlying phonological representation is distinct from surface realisation
 - e.g. English plural -s: [s], [z], [əz]
- A grammar is a set of constraints (which may conflict), and a **weighting** of those constraints
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 - At each step, sufficient to consider a finite set of candidates
- Relies on comparing candidates as a single score
 - In contrast, Optimality Theory requires comparison for each constraint

Harmonic Grammar vs. Optimality Theory

- Theoretical commitments very similar
 - Explain complex phenomena as interactions between simple constraints
- Computational behaviour very different
 - Finite-state vs. PSPACE-complete

10 Years of My Semantics Research...

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- Truth is useful
 - A truth-conditional model can learn and generalise meanings in a more human-like way
- Truth is painful
 - A truth-conditional model is intractable with respect to dimensionality of feature space

Judicious Incoherence

- A computational system cannot be all three:
 - Expressive
 - Tractable
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Judicious Incoherence

- A computational system cannot be all three:
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 - Coherent
- Human cognition is expressive and tractable, so cannot be coherent

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- Bayesian inference is #P-complete (at least as hard as NP-complete), even in restricted settings and even when approximated (Roth, 1996)

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- Can be solved by iterating over all (x, y) pairs (Arnold & Press, 1989).

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- This includes all neural network models.

Succinct Compatibility Problem

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Succinct Compatibility Problem

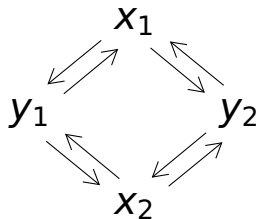
- Given succinctly encoded $p(x|y)$ and $p(y|x)$, are they compatible with some $p(x, y)$?
- Theorem:
 - If $p(x|y), p(y|x) > 0$, this is co-NP-complete.
 - In the general case, this is Σ_2^P -complete.

Succinct Compatibility is in co-NP

- If $p(x|y)$ and $p(y|x)$ are incompatible, we can find a certificate (x_1, x_2, y_1, y_2)
- Verify by checking:
 - $$\begin{aligned} & p(x_1|y_1)p(y_1|x_2)p(x_2|y_2)p(y_2|x_1) \\ & \neq \\ & p(y_1|x_1)p(x_1|y_2)p(y_2|x_2)p(x_2|y_1) \end{aligned}$$

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Coherence as Regulariser

- Given observed (x_1, y_1) , sample alternatives x_2, y_2 .
- Regularise $\frac{p(x_1|y_1)p(y_1|x_2)p(x_2|y_2)p(y_2|x_1)}{p(y_1|x_1)p(x_1|y_2)p(y_2|x_2)p(x_2|y_1)} \approx 1$

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- Model can appear coherent in a subspace (polynomially bounded in size)
- First steps: experiments planned on BabyLM

Judicious Incoherence

- Cognitive models must be expressive and tractable
- Expressive tractable models are incoherent
- Humans and models can be judiciously incoherent