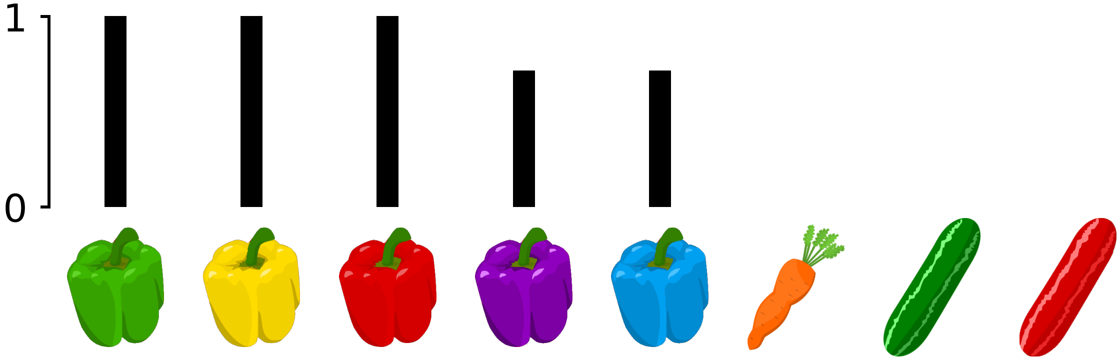
# Bernoulli Fields

## Probability distributions for vague semantics and pragmatics

Guy Emerson

# Truth-Conditional Functions



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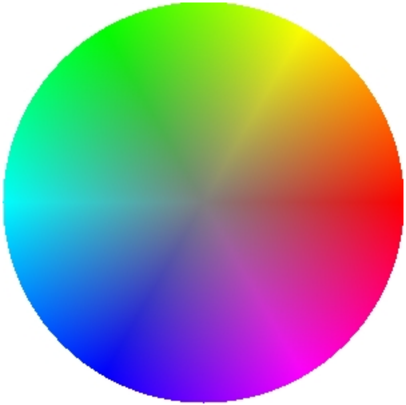
  - $\mathbb{P}\left(T_y{=}\top\right) = f(y)$, $\qquad \mathbb{P}\left(T_y{=}\bot\right) = 1 - f(y)$
- What about $\mathbb{P}\left(T_x{=}\top, T_y{=}\top\right)$?

# Example: Colour Space



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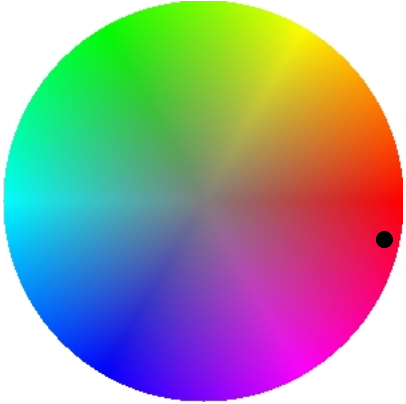

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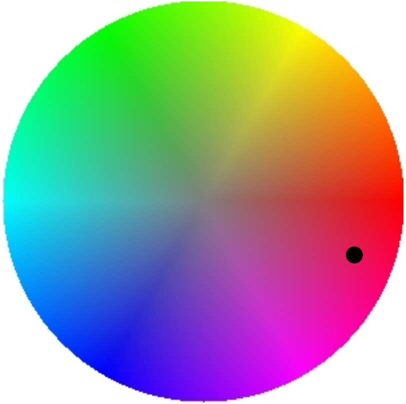

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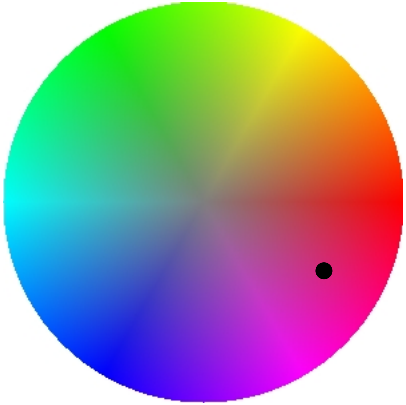

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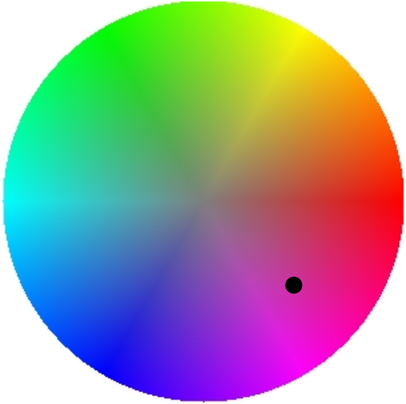

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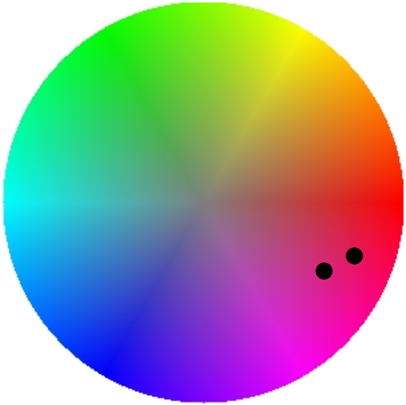

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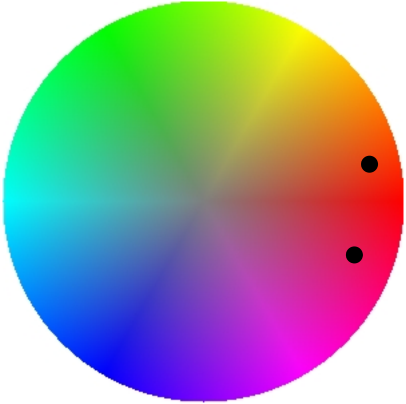

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- Language use is coherent: the extension of “red” is self-consistent within a discourse

# Jointly Distributed Truth Values

| | $T_y = \perp$ | $T_y = \top$ |
|---|---|---|
| $T_x = \perp$ | $p_{\perp\perp}$ | $p_{\perp\top}$ |
| $T_x = \top$ | $p_{\top\perp}$ | $p_{\top\top}$ |

# Jointly Distributed Truth Values

| | $T_y = \bot$ | $T_y = \top$ |
|---|---|---|
| $T_x = \bot$ | $p_{\bot\bot}$ | $p_{\bot\top}$ |
| $T_x = \top$ | $p_{\top\bot}$ | $p_{\top\top}$ |

- 4 outcomes; probabilities sum to 1, so 3 degrees of freedom

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- $f(x)$ and $f(y)$ are 2 degrees of freedom

# Jointly Distributed Truth Values

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  - No dependence: $p_{\top\top} = f(x)f(y)$
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- Dependence as: covariance? correlation coefficient?
  - not useful as parameters:
    range depends on $f(x)$ and $f(y)$

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- Dependence as **odds ratio**: $\omega = \dfrac{p_{\top\top}p_{\perp\perp}}{p_{\top\perp}p_{\perp\top}}$
- 3 degrees of freedom: $f(x)$, $f(y)$, $\omega(x, y)$

# Local Dependence

- $f : x \mapsto$ marginal probability of truth
- $\omega : x, y \mapsto$ odds ratio (degree of dependence)
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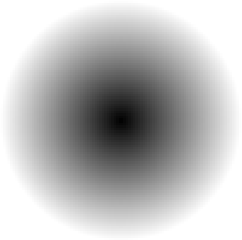
- For details, see forthcoming book chapter "Probabilistic Lexical Semantics: From Gaussian Embeddings to Bernoulli Fields"

# Distributions for Regions

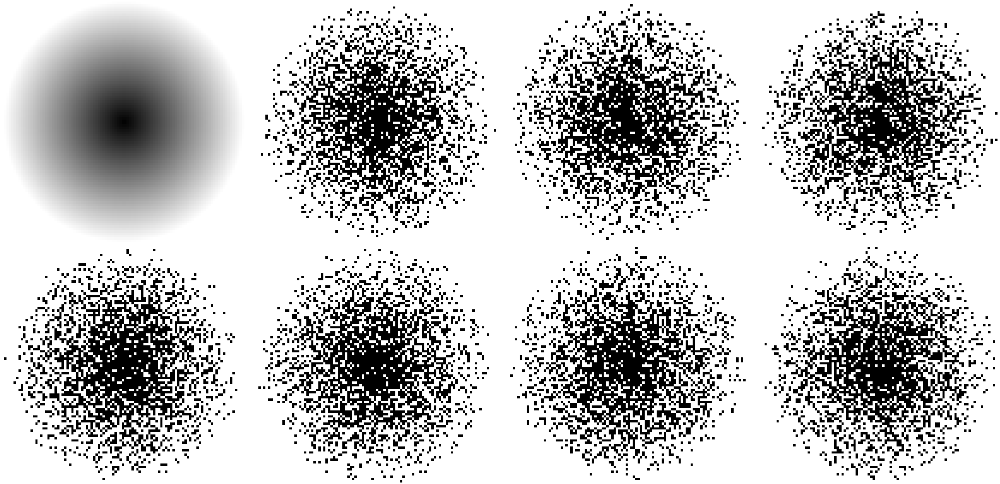
Joint distribution for truth values
$\Leftrightarrow$ Distribution for regions

# Distributions for Regions

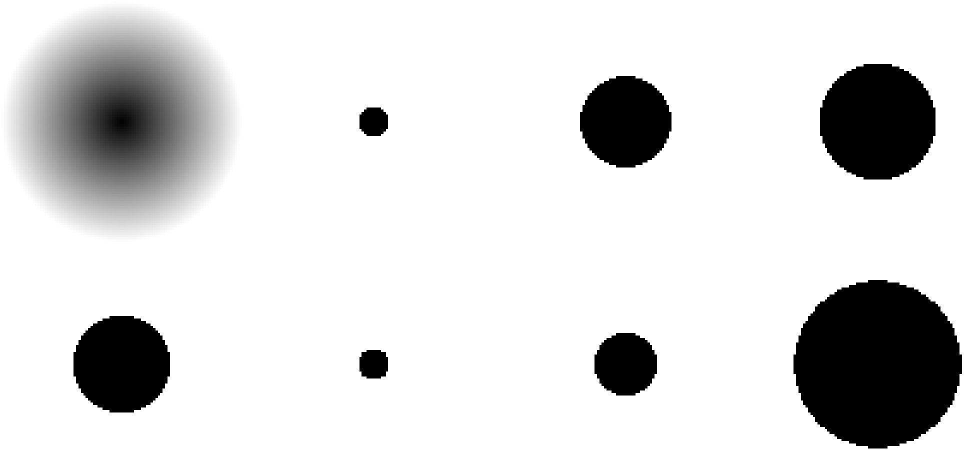


$f$, darker is higher probability of truth

# Independence



# Maximum Dependence



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# Summary

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- Experiments on real data underway...