# EE211A Digital Image Processing Winter 2018 Homework 1 (Deadline: 01/29/2018 6 pm)

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Problem 1 Implement your own function that performs a convolution operation between a grayscale image and a filter. a) Using your convolution function, filter the test image with Sobel filter. Explain your observations. b) Using your convolution function, filter the test image with Gaussian filter for three different  $\sigma$  ( $\sigma$  = 1,  $\sigma$ =5,  $\sigma$ =15). Explain your observations.

# Results:

(a):

This part is to apply Sobel filter to process the image. There're several methods to deal with the boarders. In this task, I flipped the image firstly in both right and left directions and then flipped the image we obtained upside and downside, as shown in Fig. 1. So the image is actually 9 times as large as the original one.

Then I convolve the image with x-sobel filter and y-sobel filter, respectively. It can be known that in the process of x-sobel-filter convolution, the outlines of the image in x direction are stressed (Fig 3.) while in the y-process, the outlines in y direction are stressed, too (Fig 4.). Finally I add them together and after normalization we can know that the process of Sobel-filter convolution can extract the outlines of the image. Thus Sobel filter is mostly used in edge detection.

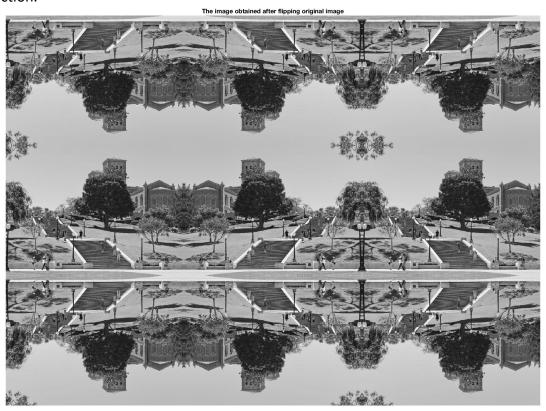


Fig. 1. The image obtained after flipping the original image

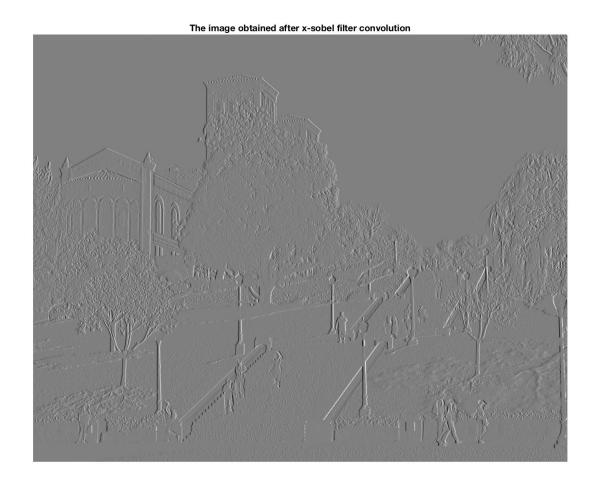


Fig. 2. The image obtained after x-sobel filter convolution

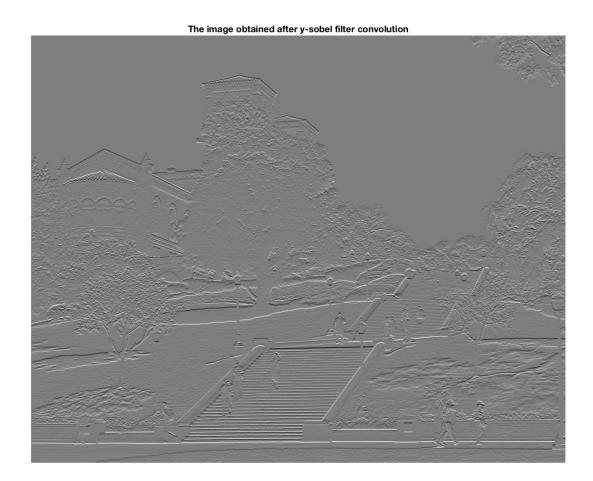


Fig. 3. The image obtained after y-sobel filter convolution

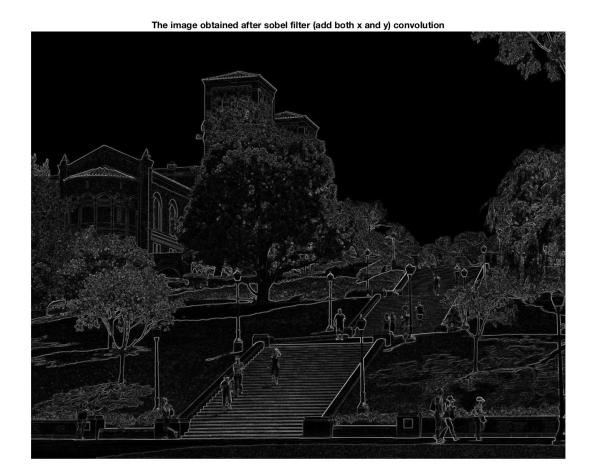


Fig. 4. The image obtained after sobel filter (add both x and y) convolution

```
Code:
pgm_name = 'Test_image.pbm';
data = imread(pgm_name);
data = im2double(data);
figure(1)
imshow(data);
[m n] = size(data);
sobel x = [-1 \ 0 \ 1; -2 \ 0 \ 2; -1 \ 0 \ 1];
sobel y = [-1 -2 -1; 0 \ 0 \ 0; 1 \ 2 \ 1];
data_1 = [data(:,n:-1:1) \ data(:,:) \ data(:,n:-1:1)];
newdata = [data 1(m:-1:1,:); data 1(:,:); data 1(m:-1:1,:)];
imshow(newdata);
title('The image obtained after flipping original image')
%sobel_x
data\_sobel\_x = [];
for i = 1:m
                 for j = 1:n
                                                         data sobel x(i,j) = (-1)*newdata(m+i-1,n+j-1)+1*newdata(m+i-1)
1, n+j+1+(-2)*newdata(m+i,n+j-1)+2*newdata(m+i,n+j+1)+(-1)*newdata(m+i+1,n+j-1)*newdata(m+i+1,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newdata(m+i,n+j-1)*newd
1)+1*newdata(m+i+1,n+j+1);
```

```
end
end
figure(2)
imshow(data_sobel_x,[]);
title('The image obtained after x-sobel filter convolution')
%sobel_y
data sobel_y = [];
for i = 1:m
                  for j = 1:n
                                                           data\_sobel\_y(i,j) = (-1)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1,n+j-1)+(-2)*newdata(m+i-1,n+j-1
1, n+j+(-1)*newdata(m+i-1, n+j+1)+1*newdata(m+i+1, n+j-1)
1)+2*newdata(m+i+1, n+j)+1*newdata(m+i+1, n+j+1);
                   end
end
figure(3)
imshow(data_sobel_y,[]);
title('The image obtained after y-sobel filter convolution')
figure(4)
imshow(abs(data sobel x)+abs(data sobel y),[]);
title('The image obtained after sobel filter (add both x and y) convolution')
```

(b):

In this part we filter the image with Gaussian filter and the 3 values of standard deviations are tried ( $\sigma = 1$ ,  $\sigma = 5$ ,  $\sigma = 15$ ). In order to deal with the boarders, I did the same flipping steps as part (a), shown in Fig 5.

From the images obtained after the convolution, it is obvious that the Gaussian filter process can blur the images. In addition, the value of  $\sigma$  (standard deviation) can also effect the results. The larger the  $\sigma$ , the more the Gaussian filter blurs the image. Moreover, the size of the Gaussian filter I apply in this process varies according to the value of  $\sigma$ . The size of the filter is usually  $5\sigma$  or  $6\sigma$ . In this problem I choose the sizes as 5\*5, 29\*29 and 225\*225, respectively, which correspond to the  $\sigma$  value of 1, 5 and 15.

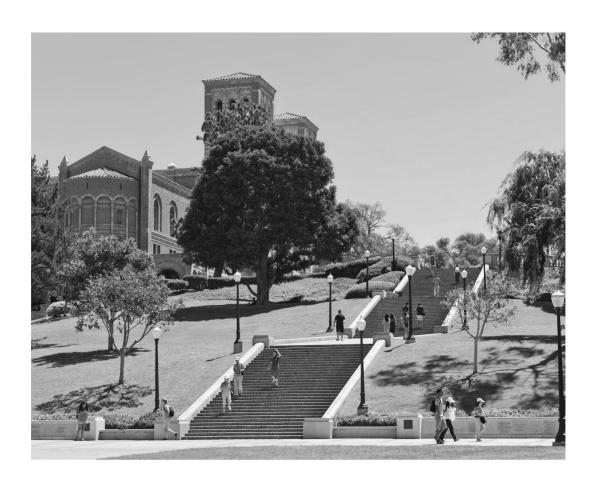


Fig. 4. The original image

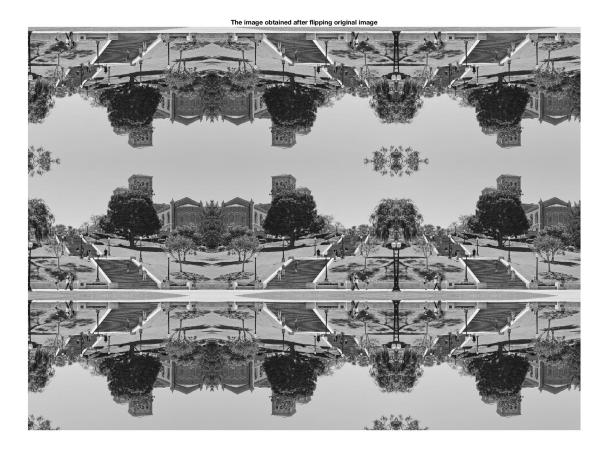


Fig. 5. The image obtained after flipping the original image

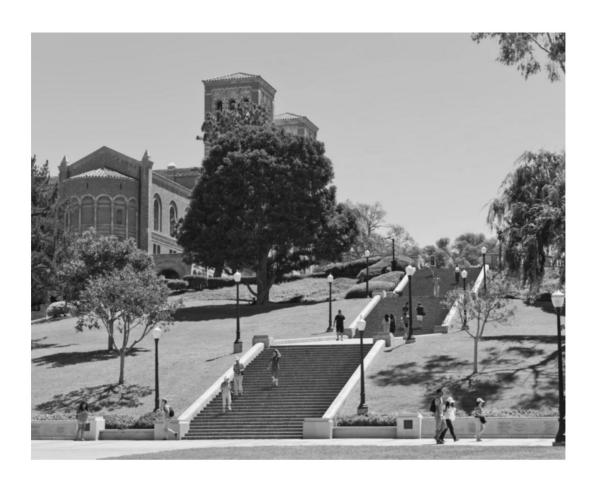


Fig. 6. The image obtained after convolution with gaussian filter when signma=1



Fig. 7. The image obtained after convolution with gaussian filter when signma=5  $\,$ 



Fig. 8. The image obtained after convolution with gaussian filter when signma=15

```
Code:
pgm_name = 'Test_image.pbm';
data = imread(pgm name);
data = im2double(data);
figure(1)
imshow(data);
title('The original image')
[m n] = size(data);
%flip the image right and left
data_1 = [data(:,n:-1:1) data(:,:) data(:,n:-1:1)];
%flip the image up and down
newdata = [data_1(m:-1:1,:); data_1(:,:); data_1(m:-1:1,:)];
%generate gaussian filter with sigma=1,size=5*5
gaussian 1=[];
for i=1:5
        gaussian_1(i,j) = 1/(2*pi)*exp(-((i-3)^2+(j-3)^2)/2);
    end
end
%generate gaussian filter with sigma=5,size=29*29
gaussian_5=[];
```

```
for i=1:29
    for j=1:29
        gaussian 5(i,j) = 1/(50*pi)*exp(-((i-15)^2+(j-15)^2)/50);
    end
end
%generate gaussian filter with sigma=15,size=225*225
gaussian 15=[];
for i=1:225
    for j=1:225
        gaussian 15(i,j) = 1/(450*pi)*exp(-((i-113)^2+(j-113)^2)/450);
    end
end
%gaussian sigma=1
data 1=[];
for i = 1:m
    for j = 1:n
        conmatrix 1 = \text{newdata}(m+i-2:m+i+2,n+j-2:n+j+2).*gaussian 1;
        data_1(i,j) = sum(conmatrix_1(:));
    end
end
figure(2)
imshow(data_1);
title('The image obtained after convolution with gaussian filter, sigma=1')
%gaussian sigma=5
data_5=[];
for i = 1:m
    for j = 1:n
        conmatrix 5 = \text{newdata}(m+i-14:m+i+14,n+j-14:n+j+14).*gaussian 5;
        data_5(i,j) = sum(conmatrix_5(:));
    end
end
figure(3)
imshow(data_5);
title('The image obtained after convolution with gaussian filter, sigma=5')
%gaussian sigma=15
data 15=[];
for i = 1:m
    for j = 1:n
        conmatrix 15 = \text{newdata}(m+i-112:m+i+112,n+j-112:n+j+112).*gaussian 15;
        data_15(i,j) = sum(conmatrix_15(:));
    end
end
figure(4)
imshow(data_15);
title('The image obtained after convolution with gaussian filter, sigma=15')
```

Problem 2 Implement your own function that performs bilateral filtering to a grayscale image. Using your own function, perform bilateral filtering operation on the test image for the following  $\sigma$ s and  $\sigma$ r values. Explain your observations.

$\sigma_s$ =3, $\sigma_r$ =0.1	$\sigma_s$ =3, $\sigma_r$ =0.3	$\sigma_s = 3, \sigma_r = 10$
$\sigma_s = 10,  \sigma_r = 0.1$	$\sigma_s = 10, \sigma_r = 0.3$	$\sigma_{s}$ =10, $\sigma_{r}$ =10
$\sigma_s = 25, \sigma_r = 0.1$	$\sigma_s = 25, \sigma_r = 0.3$	$\sigma_{\rm s}$ =25, $\sigma_{\rm r}$ =10

## Result:

In this part I tried 9 combinations of different  $\sigma_s$  and  $\sigma_r$ . In this task I choose the filter sizes as 6  $\sigma_s$ . Thus, when  $\sigma_s$  equals to 3, the filter is 17\*17 (not 18 in order to make the calculation of mid point more convenient). When  $\sigma_s$  equals to 10, the filter is 59\*59 and when  $\sigma_s$  equals to 25, the filter is 149\*149.

The results after convolution with such filters are shown below:

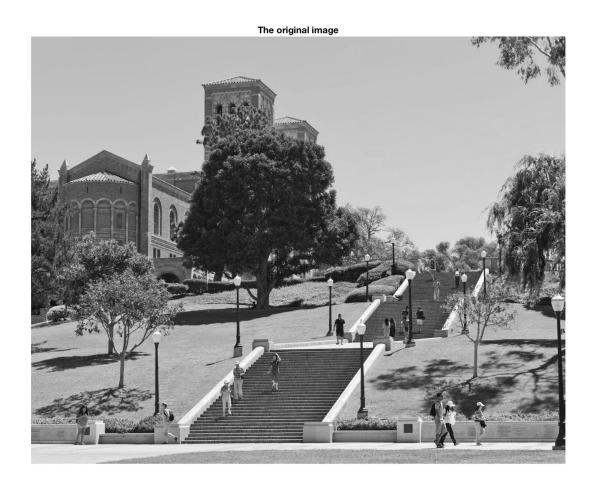


Fig 9. The original image

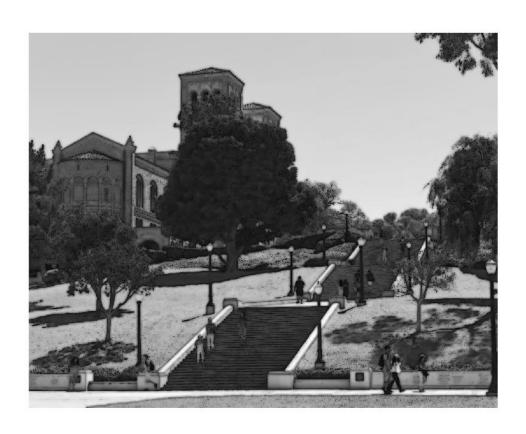


Fig 10. Bilateral filter when  $\sigma_s$  = 3,  $\sigma_r$  = 0.1



Fig 11. Bilateral filter when  $\sigma_s$  = 3,  $\sigma_r$  = 0.3



Fig 12. Bilateral filter when  $\sigma_s$  = 3,  $\sigma_r$  = 10



Fig 13. Bilateral filter when  $\sigma_s$  = 10,  $\,\sigma_r$  = 0.1



Fig 14. Bilateral filter when  $\sigma_s$  = 10,  $\,\sigma_r$  = 0.3



Fig 15. Bilateral filter when  $\sigma_s$  = 10,  $\,\sigma_r$  = 10



Fig 16. Bilateral filter when  $\sigma_s$  = 25,  $\,\sigma_r$  = 0.1



Fig 17. Bilateral filter when  $\sigma_s$  = 25,  $\,\sigma_r$  = 0.3



Fig 18. Bilateral filter when  $\sigma_s$  = 25,  $\,\sigma_r$  = 10

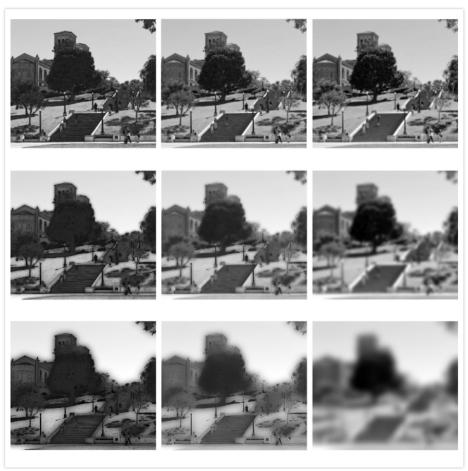


Fig 19. All images corresponding to the order shown in the table

# Observation:

From the images we obtained, it can be known that the images after convolution are different according to the values of  $\sigma s$  and  $\sigma r$ . When is  $\sigma r$  and  $\sigma s$  are getting larger, the image will be more obscure. If  $\sigma r$  is fixed, the image will be darker and more obscure while  $\sigma s$  increases. If is  $\sigma s$  fixed, the image will be lighter and more obscure while  $\sigma r$  increases.

From the equation, we can know that  $\sigma$ s influences the Gaussian convolution in geometric aspect, the further a pixel is to the chosen point, the less that filter effects it. Thus we can say that the special Gaussian filter only has obvious impact on pixels near the chosen pixel. Besides,  $\sigma$ r influences the Gaussian convolution in pixel-value difference aspect, the larger the difference between one pixel value to a chosen point's pixel value, the less that filter effects it. Thus we can say that the r-Gaussian filter only has obvious impact on pixels who has similar pixel value as the chosen point's pixel value.

## Code:

```
pgm name = 'Test_image.pbm';
data = imread(pgm_name);
data = im2double(data);
figure(1)
imshow(data);
title('The original image')
[m n] = size(data);
%flip the image right and left
data_1 = [data(:,n:-1:1) data(:,:) data(:,n:-1:1)];
%flip the image up and down
newdata = [data_1(m:-1:1,:);data_1(:,:);data_1(m:-1:1,:)];
sigma_s = [3\ 10\ 25]; \ s=3, \ size=17*17, \ s=10, \ size=59*59, \ s=25, \ size=149*149
%sigma r
sigma r = [0.1 \ 0.3 \ 10];
%generate gaussian filter with sigma_s=3,size=17*17
gaussian s1=[];
for i=1:17
   for j=1:17
       gaussian s1(i,j) = 1/(2*pi*9)*exp(-((i-9)^2+(j-9)^2)/(2*9));
   end
end
gaussian s1=(gaussian s1)./gaussian s1(9,9);
%generate gaussian filter with sigma s=10,size=59*59%gaussian s2=[]
for i=1:59
   for j=1:59
       gaussian_s2(i,j) = 1/(2*pi*100)*exp(-((i-30)^2+(j-30)^2)/200);
   end
end
gaussian s2=(gaussian s2)./gaussian s2(30,30);
%generate gaussian filter with sigma s=25, size=17*17
gaussian s3=[];
for i=1:149
   for j=1:149
       gaussian s3(i,j) = 1/(2*pi*625)*exp(-((i-75)^2+(j-75)^2)/(2*625));
   end
end
gaussian s3=(gaussian s3)./gaussian s3(75,75);
%%gaussian sigma r with filter s1, size=17*17
for a = 1:3
    data 1 = [];
    r = sigma_r(a);
    for i = 1:m
        for j = 1:n
            for p = 1:17
                for q = 1:17
                    gaussian r1(p,q) = 1/(2*pi*r^2)*exp(-(newdata(m+i,n+j)-
newdata(m+i+p-9, n+j+q-9))^2/(2*r^2));
                end
            A = gaussian s1.*gaussian r1.*newdata(m+i-8:m+i+8,n+j-8:n+j+8);
            data_1(i,j)=sum(A(:));
        end
    end
    %data_1 = data_1./max(max(data_1));
```

```
figure
    imshow(data 1,[])
%%gaussian sigma r with filter s2, size=59*59
for a = 1:3
    data_2 = [];
    r = sigma_r(a);
    for i = 1:m
        for j = 1:n
            for p = 1:59
                for q = 1:59
                    gaussian r2(p,q) = 1/(2*pi*r^2)*exp(-(newdata(m+i,n+j)-
newdata(m+i+p-30, n+j+q-30))^2/(2*r^2));
                end
            end
            A = gaussian s2.*gaussian r2.*newdata(m+i-29:m+i+29,n+j-
29:n+j+29);
            data 2(i,j)=sum(A(:));
        end
    end
    %data_2 = data_2./max(max(data_2));
    figure
    imshow(data_2,[])
%%gaussian sigma r with filter s3,size=149*149
for a = 1:3
    data_3 = [];
    r = sigma_r(a);
    for i = 1:m
        for j = 1:n
            for p = 1:149
                for q = 1:149
                    gaussian_r3(p,q) = 1/(2*pi*r^2)*exp(-(newdata(m+i,n+j)-
newdata(m+i+p-75, n+j+q-75))^2/(2*r^2));
            end
            A = gaussian s3.*gaussian r3.*newdata(m+i-74:m+i+74,n+j-
74:n+j+74);
            data_3(i,j)=sum(A(:));
        end
    end
    data 3 = data 3./max(max(data 3));
    figure
    imshow(data 3,[])
end
```