# CS136 — Project proposal
# Game theory for supply chain: incentives and equilibria

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In this particular supply chain setting, a supplier ships an inventory to a retailer for storage in the retailer's warehouses before the inventory is sold. The retailer sets the retail price and compensates the supplier, at the wholesale price, after the inventory is sold. The supplier is also charged for storage in the warehouse. The supplier can adjust the wholesale price and the quantity of inventory. The retailer can adjust the retail price and the warehouse storage price to send different incentives to the supplier. The retail price set by the retailer will directly impact demand, and combined with the warehouse storage price, it will also impact the amount of inventory the supplier is willing to ship to the retailer.

This scenario reveals a trade-off for the retailer in setting both the retail price and the warehouse storage price. If the retailer selfishly maximizes their profit by setting the retail price to the selfish price, demand will be lower. The supplier will have to pay high warehouse storage fees. A supplier that is constantly recomputing their optimal strategy will "retaliate" by sending less inventory in the future in order to avoid paying excessive warehouse cost. The retailer might then fall short of supply.

This loss of economic efficiency due to the disjoint decision-making of two independent profit-maximizing agents is referred to as double-marginalization. In this situation, the supplier, the retailer, and consumers are worse-off than with vertical integration.

In the context of a long-term supplier-retailer relationship, modeled as a repeated 2-player game, Game Theory predicts that different strategic behaviours can emerge. This project will explore conditions for Folks' theorems to hold in this supply chain context. In particular under what conditions on the parameters (discount factors, elasticities) can a cooperative equilibrium be observed?

To approach the problem, we will start with strong assumptions that will then be partially relaxed:

- The retailers' warehouse has unlimited capacity
- We start with a simple model with one supplier and one retailer. We will then generalize it to scenarios with multiple suppliers
- Demands in the products from different suppliers are not substitutable.
- When considering the many-supplier model, we might relax the unlimited capacity assumption.

To build a first intuition, consider a simplistic scenario. If the retailer and the supplier "cooperate" – the retailer by setting a retail price below the selfish price, the supplier by shipping a sufficient amount of inventory – the respective pay-offs of the retailer and the supplier are $(3, 3)$. Given the same amount of supply, if the retailer selfishly optimizes the retail price, his pay-off is 5. The supplier earns less because his payment only depends on the retail price through demand, and is charged additional warehouse storage fees. Their pay-off falls to 1. On the other hand, if the retailer cooperatively sets the price whereas the supplier does not provide enough inventory, the pay-off for the retailer falls to 1 (low retail price), and the pay-off for the supplier is 5 (high demand and low warehouse storage fees). If both the supplier and the retailer defect, then their pay-offs are (2, 2): the retailer has high prices but not enough supply, the supplier has low warehouse storage fees but does not sell high volumes. The pay-offs described here are report

| | | Retailer | |
|---|---|---|---|
| | | cooperative pricing | selfish pricing |
| Supplier | cooperative inventory | (3,3) | (1, 5) |
| | under-supply | (5,1) | (2,2) |

Table 1: Pay-offs matrix – Illustration for Prisoners' Dilemma in supply-chain context.

We use the following notations:
$\Pi^r$ profit function of the retailer.
$\Pi_j^s$ profit function of the $j$-th supplier.
$Q_i$ sales of the product produced by supplier $i$.
$S_i$ shelf-space allocated to manufacturer $i$.
$\alpha$ constant.
$\mu_i$ price elasticity of demand for manufacturer $i$'s product.
$\gamma$ shelf-space elasticity at retailer.
$P_j$ price for the product of manufacturer $i$.
$W_i$ manufacturer $i$'s price to retailer.
$C_i$ unit constant production cost of making the product for manufacturer $i$.

The demand satisfies:

$$Q_i = \alpha\ S_i^{\gamma}\ P_i^{\mu_i}$$

which can be simplified to $Q_i = \alpha\ P_i^{\mu_i}$ when ignoring the shelf-space allocation problem.

The retailer maximizes their profit:

$$\Pi^r = \sum_{j=1}^{n}(P_j - W_j)Q_j = \alpha\ \sum_{j=1}^{n} S_j^{\gamma}\ P_j^{\mu_j}(P_j - W_j)$$

In the simplified case of one retailer/one supplier and no consideration of shelf space, the retailer seeks to maximize. $\alpha P_j^{\mu_j}(P_j - W_j)$ In the classical setting, the suppliers maximize:

$$\Pi_j^s = (W_j - C_j)Q_j = \alpha(W_j - C_j)\ S_j^{\gamma}\ P_j^{\mu_j}$$

We will adjust these equations with warehouse storage cost and solve the maximization problems under different set of constraints to fill in a pay-off matrix with realistic predictions. In practice the pay-offs in table 1 depend on any number of the variables described earlier. Our approach is to explore the set of equilibria that arise depending on the parameters and variables, including $P_j$, $\mu_i$, $W_i$, $\delta$ (discount factor), $\gamma$.

# References

[1] Li, X., Nukala, S., Mohebbi, *Game theory methodology for optimizing retailers' pricing and shelf-space allocation decisions on competing substitutable products.* S. Int J Adv Manuf Technol (2013) 68: 375. https://doi.org/10.1007/s00170-013-4735-1