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ORIGINAL ARTICLE

Game theory methodology for optimizing retailers' pricing and shelf-space allocation decisions on competing substitutable products

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Abstract This paper examines the influence of competition among supply chain partners on product demand. A power law demand function that depends on product pricing and shelf-space allocation (SSA) is used. The exponents in the power law are given by the elasticities of demand. In order to achieve the optimal pricing and SSA strategies in the presence of competition, game-theory-based methodologies—Cournot and Stackelberg games—are employed. For each type of game, a Nash equilibrium is achieved by optimizing the profit as a function of demand and price. A case study is presented to demonstrate the potential of this methodology. The results of this study indicate (1) how to achieve optimal pricing and SSA strategies, (2) how manufacturers can influence demand for a product, (3) that both prices and profits decrease using the Stackelberg game as compared with the Cournot game, and (4) that coordination beyond simple knowledge of price would be beneficial for improving overall profits.

Keywords Game theory · Supply chain management · Cournot games · Stackelberg games · Shelf-space allocation (SSA) · Pricing · Direct and cross elasticities

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1 Introduction

In the presence of pressing competition, retailers often need to make decisions about which assortment of products to display, what price to set on them, and how much shelf space to allocate for each product. Considering the prime issues of today's consumer markets, we aim to develop an integrated demand model that takes into account pricing and shelf-space allocation (SSA), along with respective cross and direct elasticities, in the presence of competition between substitutable goods and among retailers. Although extensive research has been done in the area of demand modeling, the current model is the first, to the best of our knowledge, to integrate all three of the most important variables—price, SSA, and competition—to drive strategic retail decisions.

Ample research has been performed in the area of demand models addressing the issues pertaining to optimal SSA. Models developed in Refs. [1, 2] considered SSA as the main variable. These models were later criticized as they ignored substitution or complementary effects [3]. Subsequent research [3, 4] suggested models that took into account main and cross-space elasticities, different product profit margins, and inventory-management costs. Their results indicated that higher shares went to products with higher profit margins but failed to provide integer solutions. An integer programming model was built in Ref. [5] but failed to offer allocation rules based on the solution. Studies like these indicate that most retailers try to allocate shelf space based on three factors: (1) profitability, (2) customer satisfaction, and (3) competition. Since these elements often conflict, the retailers cannot make adequate decisions at a faster pace. Therefore, in order to assist these retailers with decision-making, a number of heuristics-based software solutions, such as APOLLO, COSMOS, SPACEMAN, etc.

have been developed. For example, the work of Buzzell et al. [6] developed the rules for COSMOS, which removes the least profitable item from the product display list and allocates its space to the most profitable one. However, these software solutions are not effective and cannot be used regularly because they cannot address issues of competition and substitution.

Competition and substitution play a crucial role in the effort to estimate demand for a certain product. Studies such as those done by Inman and Winner [7] state that about half to two thirds of consumers make purchase decisions about which brand to buy when they are actually at the point of purchase. Subsequently, the research in Ref. [8] examined consumer response to stock-outs across eight categories at retailers worldwide and pointed to the importance of demand estimation for retailers: their study reported that during a stock-out, 45 % of customers will substitute, i.e., buy one of the competing, but available, items from that category; 15 % will delay purchase; 31 % will go to another store; and 9 % will not buy any item at all. That is, this study suggests that when faced with stock-outs, most consumers will either buy substitute items at the same retailer or switch to another store. These numbers show the importance of competition among substitutable products to retailers in terms of their pricing strategies while making decisions on future sales. Typically, competition among equally viable products (from a product pricing and substitutability standpoint) is achieved at a retailer level by the amount of shelf space allocated to each of the products. Research in Ref. [9] showed that the amount of shelf space allocated to a product has a significant impact on the sales volume of that product; in response, retailers regularly make decisions about the types of products to display (product assortment) and the amount of shelf space to allocate for each of the products, thereby influencing customers' purchasing decisions as noted by Wang and Gerchak [10].

Furthermore, the research in Ref. [11] reported that the simple act of a customer's looking at a brand increases its consideration probability by between 30 and 120 %. Such results clearly explain why manufacturers are continually battling for higher shares of the shelf. Studies like these suggest the importance of product availability that influences the demand and profit thereby emphasizing the importance of competition and shelf space while trying to predict future demand for a product.

Another critical aspect of a product's demand curve is how much of that product is demanded when the price changes, i.e., the price elasticity of the product [12] states that the price elasticity of demand underlines the importance of price in estimating demand for a product. Typically, the price elasticity of demand is negative—consumers buy more of a product at a lower price and less when the product is at a higher price, all other things being equal, as noted by Karl and Ray [13]. Warehouse/retail outlets such as Costco and Sam's Club are examples of this. On the other hand, the price elasticity of demand may occasionally become positive—the higher the price, the higher the demand. Such an anomaly can be traced back to the customers' perception that a higher-priced product either offers higher quality or more operational features, in which case competing products are not likely to be easily substitutable. This study addresses the problem of demand specification at competing retailers. The demand model developed and analyzed considers the effect of optimal SSA and pricing strategies when manufacturers and retailers are competing to spur demand for their products. In particular, this demand specification model considers the fact that manufacturers as well as competing retailers can actually intervene in the process of decision-making regarding optimal SSA and pricing of a product through their pricing policies.

Accurate demand measurement is very important in an integrated supply chain, mostly in order to avoid the "bullwhip effect" or excess ordering of scarce goods, and to reduce excess holding and stock-outs. At the retail level of a supply chain, demand specification is crucial for a variety of management decisions, such as determining the ideal product assortment to carry, how much of each item should be stocked, and how often the stock should be replenished. Abundant research in the area of demand specification and inventory models has provided valuable tools and interesting demand models.

By using available tools and demand models from the literature, we have formulated and evaluated a refined demand model that takes care of most of the limitations. The current demand model accounts for, first, the impact of manufacturers: The shelf-space-optimization models (as well as user-friendly commercial tools) use margins on items as given and recommend allocation rules based on relative profitability. These margins depend on the wholesale prices, which are within manufacturers' control, thus indicating that a manufacturer is actually able to influence the allocation process through manipulation of wholesale prices. This role must be recognized formally.

Second, the current demand model accounts for the fact that profits of competing manufacturers are also interdependent, as SSA signifies the importance of a given manufacturer's prospects [14]. As stated by Gruen et al. [8] regarding the competition that is inherent among retailers, 31 % of customers go to another store to get a certain product if they do not find it in the store where they are shopping. Aside from the unavailability of a product at a certain store, other factors such as price differences are also possible reasons for competition among retailers. Retailers often charge different prices for a product and its substitutes [12], a phenomenon known as price discrimination. There is a vast literature in economics on price discrimination theory (see, for example, Ref. [15] for a review of this literature). In addition, a number of important papers in the marketing field (e.g., Refs. [16, 17]) have discussed different forms of price discrimination and how it can be implemented in practice. Price discrimination at retailers as well as among manufacturers produces competition between different retailers selling the same products (e.g., Nike at Foot Locker vs. Nike at Big 5) and substitutes at different prices (e.g., Nike and Reebok running shoes) and also between manufacturers selling substitutable products (e.g., Nike and Reebok shoes). SSA of a product vs. its substitutes also plays a micro-level part in manufacturers' competition.

The above-cited literature and reasoning suggest that the demand specification problem should include manufacturer as well as retail competition among manufacturers, adding a new dimension to existing demand models. To better understand the interactive nature of competition among multiple retailers and manufacturers, it is necessary to understand the economics underlying markets and pricing. When retailers or manufacturers sell a product, they face demand/supply curve under which customers are willing to purchase a quantity dependent upon the price and marketing mix variables, if any, that influence the demand. Depending on the market, suppliers either produce a specific quantity or set a specific price. The supply chain members make these choices while anticipating their competitors' possible outcomes, as has been shown by Shah [18] and Sela and Vleugels [19]. In strategic decision-making situations, a supply chain member chooses strategies that will maximize her returns, given the strategic choices of her competitors. The main idea of this paper is to incorporate the options and interactions of the supply chain members into a formal demand measurement model. Game theory provides the best tool for studying strategic decision-making situations of this type.

Game theory studies decisions that are made in environments in which supply chain members interact as stated by Webb [20]. The situations in which game theory has been applied in the real world—primarily in the fields of economics and war—reveal its selective usefulness for solving problems of competition. The two significant areas where this tool has been applied have been economics and war.

In these times of economic uncertainty and different consumer mindset, we use this theoretical game approach to define and develop our understanding of real oligopolies (in which a market is dominated by only a few sellers) in order to understand better the integration of the supply chain. For this purpose, we choose to define a demand model similar to that of Herr'an's [14], to integrate competition, pricing, and SSA when there are two competing retailers with two competing manufacturers. As in Herr'an's study [14], the manufacturers compete with one another through their substitutable products, which influence the SSA at each of the retailers; this influences the demand for that product while the retailers compete with one another through their pricing strategies for each of the substitutable products.

To ensure clear understanding of the workings of such demand models, a pilot study was conducted using the demand model developed by Herr'an et al. [14] but relaxing their assumptions. The model contains two competing manufacturers battling for shelf space at a single retailer. The assumptions that have been relaxed in this study include the following: (1) choice of direct- and cross-price elasticities and (2) a cost-based pricing strategy that applies the same markup to both brands by retailers. The analytical model thus developed was evaluated using numerical simulations. The initial starting values for this simulation were obtained from Ref. [14]. Our plotted results in the pilot were precisely similar to those of Herr'an et al.'s [14] and provided useful insights into game-theory concepts in the presence of competition. Utilizing the knowledge obtained from the pilot study, a new demand model that incorporated competition between two retailers and two manufacturers was developed.

The remainder of the work is organized as follows: In Section 2, the literature on analyzing competition among supply chain members is reviewed and contributions of this study are highlighted. Section 3 presents a detailed formulation to derive game theoretic models addressing pricing and SSA for substitutable products in the presence of competition amongst retailers. In Sections 4 and 5, optimal policies are obtained for retailers and manufacturers, respectively. Section 6 gives results on implementing the proposed models for a case study. Finally, the conclusions are given in Section 7.

2 Literature review

Supply chain scholars have taken various perspectives to analyze competition within and between upstream (i.e., manufacturers) and downstream members (i.e., retailers) of supply chains. One important stream has focused on developing price-dependent demand functions such that the challenge is to make joint pricing decisions and use price to compete for market demand. The majority of the existing works in the literature have utilized game theory tools to analyze the problem and obtain optimal solutions. For instance, Chen [21] and Doyle [22] studied the pricing game in the Bertrand model. Chen et al. [23] applied a game theory to study the optimal joint pricing and inventory decisions in the newsboy problem. Choi and Jagpal [24] studied a duopoly pricing game when firms are risk averse and face parameter uncertainty. They showed that the optimal policy is for a firm to act as a Stackelberg price follower and suggested that it is advantageous for a firm which has accurate estimates of demand parameters to share these estimates with the other firms. Luo et al. [25] considered a dynamic pricing strategy for a duopoly game in the airline industry. The pure strategy Nash equilibrium is found, and two optimal dynamic pricing strategies are proposed. Chiu et al. [26] applied the game theory to study some strategic actions for retailers to fight a price war. Esmaeili [27] introduced a new approach to include the marketing effort in joint pricing and lot sizing models leading to profit increase for the manufacturer.

The second stream of research is dedicated to the SSA problem and is well argued that it is a potential source of conflict between retailers and manufacturers. To deal with this issue, Martín-Herrán and Taboubi [28] proposed a differential game formalism for determining the SSA by a retailer such that two competing brands exist. To be able to characterize Stackelberg equilibrium, they assumed that the two brands' manufacturers behave myopically, i.e., each player observes only the evolution of her brand goodwill stock. Martín-Herrána et al. [29] extended their model and proposed a differential game whereby each manufacturer can influence the allocation decision by her advertising spending to improve her brand's goodwill which in turn affects the demand for her product. In the proposed game, Stackelberg open-loop equilibrium is characterized and shown to be time consistent where manufacturers are leaders and the retailer is the follower. Hansen et al. [30] presented a retail shelf-space decision model that incorporates a nonlinear profit function, vertical and horizontal location effects, and product cross elasticity. They discussed the impact of the number of item facings, vertical location, and horizontal location and found the vertical location effect is approximately double the size of the horizontal location effect on profit performance.

The third and recent stream of research, closer to this paper, investigates pricing and SSA simultaneously for strategic decision making. An assumption in this category of studies is that the manufacturer can influence, by her wholesale price or advertising policy, the retailer's SSA. Among the most recent works in the literature, Amrouche and Zaccour [31] proposed a comprehensive game-theoretic model in which one national-brand manufacturer, acting as a leader, maximizes her own profit and one retailer, selling the national brand and her private label and acting as a follower, maximizes her category profit. They characterized the resulting Stackelberg equilibrium in terms of the amount of shelf space allocated to these brands as well as their prices. Murray et al. [32] developed a model that jointly optimizes a retailer's decisions for product prices, display facing areas, display orientations and shelf-space locations in a product category. Their model considers both the width and height of each shelf, allowing products to be stacked.

The review of the literature reveals that simultaneous SSA and pricing decisions for substitutable products have not deeply been addressed. Hence, our model contributes to the existing literature in several key ways. In the first place, we develop game theoretic models to address pricing and SSA, along with respective cross and direct elasticities for substitutable products in the presence of competition amongst retailers. Optimal policies are derived for both balanced and unbalanced supply and demand. In the second place, unlike the most existing models in the literature considering one retailer or one manufacturer, we investigate two manufacturers/two retailers case where unbalanced supply and demand is also considered. In the third place, the validity of those models is investigated using numerical simulations under four different scenarios that reflect various vertical and horizontal integrations for the problem.

3 Model formulation

3.1 Indices

i	Number of manufacturers/products
j	Number of retailers

3.2 Notations

$\prod_j^r$	Profit function of the jth retailer
$\prod_i^m$	Profit function of the ith manufacturers
Q_{ij}	Sales/demand/supply of the product produced by manufacturer i to the jth retailers
S_{ij}	Shelf space allocated to the product of manufacturer i at retailer j
α	Constant
μ_{ij}	Price elasticity of demand for manufacturer i's product at retailer j
γ_j	Shelf-space elasticity at retailer j
P_{ij}	Price for the product of manufacturer i at retailer j
W_{ij}	Manufacturer i's price to retailer j
C_{ij}	Unit constant production of making the product at manufacturer i for retailer j
ε_{ij}	Cross-price elasticity of the manufacturer i's product for the jth retailer

3.3 Formulation

For the purpose of this study, it is assumed that the prices of substitutable products are different; the shelf space is normalized to 1. It means that for the case of two manufacturers and two retailers $S_{21} = 1 - S_{11}$ and $S_{22} = 1 - S_{12.}$

It is also assumed that there are no trade promotions, so the cost for the shelf space is constant. According to personal communication with the customer insight and strategy team at Wal-Mart, Inc, trade promotions do not exist in this retail chain. This is clearly a simplifying and realistic assumption for product categories and brands where the manufacturers have to pay retailers to have access to their shelf space. Following Corstjens and Doyle [3, 4], Desmet and Renaudin [33], and Zufreyden [5], we adopt the following sales/demand/supply function for the two retailers/two manufacturers problem:

$$Q_{ij} = \alpha \cdot S_{ij}^{\gamma_j} \cdot P_{kj}^{\varepsilon_{kj}} \cdot P_{il}^{\varepsilon_{il}} \cdot P_{ij}^{\mu_{ij}} \quad i,j,k,l = 1,2;\ i \neq k;\ j \neq l \tag{1}$$

Equation (1) represents the retailer *j*'s demand for the product of the *i*th manufacturer as a function of their shelf space, direct price, and competitor price (cross price) along with elasticities. It should be noted that index *l* for retailers and index *k* for manufacturers are also used to show that values for *i* and *k* as well as *j* and *l* are not equal. For example, if *i* is equal to 1 and *j* is equal to 1, *k* and *l* will be 2 since it is a two retailers/two manufacturers problem and we have i≠k and j≠l. The choice of $\alpha>0;\mu_{ij}\geq 0;\ \varepsilon_{ij}\geq 0;0<\gamma_j<1$ is adapted from Ref. [14]. Derivation for Eq. (1) is provided in Appendix 1.

Unlike Refs. [5] and [14], the demand/supply function proposed in Equation (1) specifically includes the cross-shelf-space elasticities between products within the same product category. The inequalities $\mu_1\geq\varepsilon_{11}$, $\mu_1\geq\varepsilon_{12}$; $\mu_2\geq\varepsilon_{21}$, $\mu_2\geq\varepsilon_{22}$ reflect the accepted assumption in economics, which states that own price elasticity is higher, in absolute value, than cross-price elasticity.

The demand specification model described in this paper accounts for substitution effects and is of interest to mass distribution industries where substitutable products/brands are sold in competing stores. However, this is not the most general model, as a symmetry assumption has been made regarding substitution effects similar to those of Bergen and John [34] and Trivedi [35]. Indeed it is readily seen that:

Retailer1 manufacturer/product1 :

$$Q_{11} \geq 0; \frac{\partial Q_{11}}{\partial S_{11}} \geq 0; \frac{\partial Q_{11}}{\partial P_{11}} \leq 0; \frac{\partial Q_{11}}{\partial P_{21}} \geq 0; \frac{\partial Q_{11}}{\partial P_{12}} \geq 0 \tag{2}$$

Retailer1 manufacturer/product2 :

$$Q_{21} \geq 0; \frac{\partial Q_{21}}{\partial (1-S_{11})} \geq 0; \frac{\partial Q_{21}}{\partial P_{21}} \leq 0; \frac{\partial Q_{21}}{\partial P_{22}} \geq 0; \frac{\partial Q_{21}}{\partial P_{11}} \geq 0 \tag{3}$$

Retailer2 manufacturer/product 1 :

$$Q_{12} \geq 0; \frac{\partial Q_{12}}{\partial S_{12}} \geq 0; \frac{\partial Q_{12}}{\partial P_{12}} \leq 0; \frac{\partial Q_{12}}{\partial P_{22}} \geq 0; \frac{\partial Q_{12}}{\partial P_{11}} \geq 0 \tag{4}$$

Retailer2 manufacturer/product 2 :

$$Q_{22} \geq 0; \frac{\partial Q_{22}}{\partial (1-S_{12})} \geq 0; \frac{\partial Q_{22}}{\partial P_{22}} \leq 0; \frac{\partial Q_{22}}{\partial P_{12}} \geq 0; \frac{\partial Q_{22}}{\partial P_{21}} \geq 0 \tag{5}$$

The proofs of the above inequalities are provided in Appendix 1. The above inequalities indicate that sales of brands 1 and 2 are nonnegative, increasing in the shelf space allocated to that brand and in the competing brand's retail price, with each decreasing in its own retail price and the competing brand's shelf space. This is applicable to both the retailers and manufacturers. In economics, it is believed that these assumptions are acceptable in the context of consumer products belonging to the same category and where the stores carrying them are of the same type. But one should note that the above assumptions do not imply symmetric elasticities.

Considering a two retailers/two manufacturers distribution channel (in which, in game theory methodology, the participants are also known as players), we propose a model that accounts for the brand and store substitution effects generated by the shelf space and pricing decisions while playing non-cooperative and cooperative games. This is the most parsimonious structure that would enable us to account for vertical and horizontal integration (manufacturer–retailer; manufacturer–manufacturers/retailer–retailer, respectively).

It is clear that while playing any game, the aim of each player is to maximize her own profit. Therefore maximizing profit will be the objective function for all the players. Below is the objective function for retailers and manufacturers, respectively:

$$\text{Ratailers}: \quad \underset{P_{ij}}{\text{Max}} \prod\nolimits_j^r = \sum\nolimits_{i=1}^{2} \left(P_{ij} - W_{ij}\right) Q_{ij} \quad \forall j = 1,2 \tag{6}$$

$$\text{Manufacturers}: \underset{W_{ij}}{\text{Max}} \prod\nolimits_i^m = \sum\nolimits_{j=1}^{2} \left(W_{ij} - C_{ij}\right) Q_{ij} \quad \forall i = 1,2 \tag{7}$$

Substitute Q_{ij} from Eq. (1) and utilize the procedure outlined below to achieve optimal pricing and strategies that drive the demand and profits for each of the supply-chain partners. The Stackelberg equilibrium can be obtained by optimizing the follower (retailers or manufacturers depending on the scenario being scrutinized) objective function to get their reaction functions (i.e., SSA and sale price) to leaders' strategies. Then, plugging the reaction function in the leader's objective function (profit) leads to the Nash equilibrium. In the next section, we present the procedure for obtaining retailers' response functions.

4 Retailers' Response Functions

The following propositions characterize both retailers' reaction functions for SSA and retail prices at the Nash equilibrium. The optimal prices and allocation strategies are obtained by solving the first-order optimality conditions for the retailers' profit maximization. After optimization of profit function, the following propositions are attained.

Proposition 1 This proposition provides the retailers with optimal SSA and pricing values in the presence of competition when the manufacturer's prices are exogenous.

1. Assuming there is an interior solution, both of the retailers' reaction functions for the shelf space allocated to the two products are given as follows:

$$\text{Retailer 1}: \mathrm{S}_{11}^{*} = \frac{[\mathrm{P}_{11} - \mathrm{W}_{11}]^{\gamma_1 - 1}}{(\mathrm{P}_{11} - \mathrm{W}_{11})^{\gamma_1 - 1} + \left[(\mathrm{P}_{21} - \mathrm{W}_{21}) \cdot \mathrm{P}_{11}^{(\varepsilon_{11} - \mu_{11})} \cdot \mathrm{P}_{21}^{(\mu_{21} - \varepsilon_{21})} \cdot \mathrm{P}_{22}^{\varepsilon_{22}} \cdot \mathrm{P}_{12}^{-\varepsilon_{12}}\right]^{\gamma_1 - 1}} \tag{8}$$

$$\text{Retailer 2}: \mathrm{S}_{12}^{*} = \frac{[\mathrm{P}_{12} - \mathrm{W}_{12}]^{\gamma_2 - 1}}{(\mathrm{P}_{12} - \mathrm{W}_{12})^{\gamma_2 - 1} + \left[(\mathrm{P}_{22} - \mathrm{W}_{22}) \cdot \mathrm{P}_{12}^{(\varepsilon_{12} - \mu_{12})} \cdot \mathrm{P}_{22}^{(\mu_{12} - \varepsilon_{22})} \cdot \mathrm{P}_{21}^{\varepsilon_{21}} \cdot \mathrm{P}_{11}^{-\varepsilon_{11}}\right]^{\gamma_2 - 1}} \tag{9}$$

2. The retailers' sales prices for both the products at the equilibrium are given as follows:

   Retailer 1: sales price of product 1 at retailer 1 is P_{11}:

$$P_{11}^{\mu_{11} - \varepsilon_{11}}[\mu_{11} W_{11} - (\mu_{11} + 1) P_{11}] = \varepsilon_{11} \cdot \left(\frac{1 - S_{11}}{S_{11}}\right)^{\gamma_1} (P_{21} - W_{21}) P_{12}^{-\varepsilon_{12}} P_{21}^{\mu_{21} - \varepsilon_{21}} P_{22}^{\varepsilon_{22}} \tag{10}$$

   Sales price of product 2 at retailer 1 is given as P_{21}:

$$P_{21}^{\mu_{21} - \varepsilon_{21}}[\mu_{21} W_{21} - (\mu_{21} + 1) P_{21}] = \varepsilon_{21} \cdot \left(\frac{S_{11}}{1 - S_{11}}\right)^{\gamma_1} (P_{11} - W_{11}) P_{22}^{-\varepsilon_{22}} P_{11}^{\mu_{11} - \varepsilon_{11}} P_{12}^{\varepsilon_{12}} \tag{11}$$

   Retailer 2: sales price of product 1 at retailer 2 is given as P_{12}

$$P_{12}^{\mu_{12} - \varepsilon_{12}}[\mu_{12} W_{12} - (\mu_{12} + 1) P_{12}] = \varepsilon_{12} \cdot \left(\frac{1 - S_{12}}{S_{12}}\right)^{\gamma_2} (P_{22} - W_{22}) P_{11}^{-\varepsilon_{11}} P_{22}^{\mu_{22} - \varepsilon_{22}} P_{21}^{\varepsilon_{21}} \tag{12}$$

   Sales price of product 2 at retailer 2 is given as P_{22}

$$P_{22}^{\mu_{22} - \varepsilon_{22}}[\mu_{22} W_{22} - (\mu_{22} + 1) P_{22}] = \varepsilon_{22} \cdot \left(\frac{S_{12}}{1 - S_{12}}\right)^{\gamma_2} (P_{12} - W_{12}) P_{21}^{-\varepsilon_{21}} P_{12}^{\mu_{12} - \varepsilon_{12}} P_{11}^{\varepsilon_{11}} \tag{13}$$

Proof Assuming an interior solution at the first-order optimality condition for the retailer's maximization problem with regard to SSA as well as to price,

Retailer 1:

$$\frac{\partial \Pi_1^r}{\partial S_{11}} = \gamma_1 S_{11}^{\gamma_1 - 1}\left[(P_{11} - W_{11}) \cdot \alpha \cdot P_{11}^{\mu_{11}} \cdot P_{12}^{\varepsilon_{12}} \cdot P_{21}^{\varepsilon_{21}}\right] - \gamma_1 (1 - S_{11})^{\gamma_1 - 1}\left[(P_{21} - W_{21}) \cdot \alpha \cdot P_{21}^{\mu_{21}} \cdot P_{22}^{\varepsilon_{22}} \cdot P_{11}^{\varepsilon_{11}}\right] = 0 \tag{14}$$

$$\frac{\partial \Pi_1^r}{\partial P_{11}} = P_{11}^{\mu_{11}}\left[\mu_{11} + 1 - \frac{\mu_{11}}{P_{11}} \cdot W_{11}\right] \cdot \left[\alpha \cdot P_{12}^{\varepsilon_{12}} \cdot P_{21}^{\varepsilon_{21}} \cdot S_{11}^{\gamma_1}\right] + P_{11}^{\varepsilon_{11} - 1}\left[\varepsilon_{11} \cdot \alpha \cdot (1 - S_{11})^{\gamma_1} \cdot (P_{21} - W_{21}) \cdot P_{21}^{\mu_{21}} \cdot P_{22}^{\varepsilon_{22}}\right] = 0 \tag{15}$$

$$\frac{\partial \Pi_1^r}{\partial P_{21}} = \varepsilon_{21} P_{21}^{\varepsilon_{21} - 1}\left[\alpha S_{11}^{\gamma_1} \cdot P_{12}^{\varepsilon_{12}} \cdot P_{11}^{\mu_{11}} (P_{11} - W_{11})\right] + \left[(\mu_{21} + 1) P_{21}^{\mu_{21}} - \mu_{21} P_{21}^{\mu_{21} - 1} \cdot W_{21}\right] \cdot \alpha (1 - S_{11})^{\gamma_1} \cdot P_{22}^{\varepsilon_{22}} \cdot P_{11}^{\varepsilon_{11}} = 0 \tag{16}$$

Retailer 2:

$$\frac{\partial \Pi_2^r}{\partial S_{12}} = \gamma_2 S_{12}^{\gamma_2 - 1}\left[(P_{12} - W_{12}) \cdot \alpha \cdot P_{11}^{\varepsilon_{11}} \cdot P_{22}^{\varepsilon_{22}} \cdot P_{12}^{\mu_{12}}\right] - \gamma_2 (1 - S_{12})^{\gamma_2 - 1}\left[(P_{22} - W_{22}) \cdot \alpha \cdot P_{21}^{\varepsilon_{21}} \cdot P_{12}^{\varepsilon_{12}} \cdot P_{22}^{\mu_{22}}\right] = 0 \tag{17}$$

$$\frac{\partial \Pi_2^r}{\partial P_{12}} = \left[(\mu_{12}+1)P_{12}^{\mu_{12}} - \mu_{12}W_{12}P_{12}^{\mu_{12}-1}\right]\cdot\left[\alpha S_{12}^{\gamma_2}P_{11}^{\varepsilon_{11}}P_{22}^{\varepsilon_{22}}\right] + \varepsilon_{12}P_{12}^{\varepsilon_{12}-1}\left[\alpha(1-S_{12})^{\gamma_2}P_{22}^{\mu_{22}}P_{21}^{\varepsilon_{21}}(P_{22}-W_{22})\right] = 0 \quad (18)$$

$$\frac{\partial \Pi_2^r}{\partial P_{22}} = \left[(\mu_{22}+1)P_{22}^{\mu_{22}} - \mu_{22}W_{22}P_{22}^{\mu_{22}-1}\right]\cdot\left[\alpha(1-S_{12})^{\gamma_2}P_{21}^{\varepsilon_{21}}P_{12}^{\varepsilon_{12}}\right] + \varepsilon_{22}P_{22}^{\varepsilon_{22}-1}\left[\alpha S_{12}^{\gamma_2}P_{12}^{\mu_{12}}P_{11}^{\varepsilon_{11}}(P_{12}-W_{12})\right] = 0 \quad (19)$$

The analytical solutions presented in this section for retail prices are not closed-form solutions. Hence, a numerical NLP technique is used to obtain optimal values.

Most of the literature on retailer demand optimization models assumes wholesale prices to be exogenous. But in this paper, we consider wholesale price as a tool used by the manufacturer to obtain the desired share of the shelf since there are no trade promotions. Notice also that the retailers reaction functions satisfy 0, for all parameters and wholesale price values. Hence, the solution is indeed interior.

The shelf space allocated to each brand is decreasing in its wholesale price and increasing in the competitive brand's wholesale price. Below are the partial derivatives of the optimal prices and shelf spaces with respect to wholesale prices offered to retailers.

Retailer 1:

$$\frac{\partial S_{11}^*}{\partial W_{11}} = \frac{c\cdot(P_{11}-W_{11})^{2c-1} - c\cdot(P_{11}-W_{11})^{c-1}\left[(P_{11}-W_{11})^c + \left[(P_{21}-W_{21})\cdot P_{11}^{(\varepsilon_{11}-\mu_{11})}\cdot P_{21}^{(\mu_{21}-\varepsilon_{21})}\cdot P_{22}^{\varepsilon_{22}}\cdot P_{12}^{-\varepsilon_{12}}\right]^c\right]}{\left[(P_{11}-W_{11})^c + \left[(P_{21}-W_{21})\cdot P_{11}^{(\varepsilon_{11}-\mu_{11})}\cdot P_{21}^{(\mu_{21}-\varepsilon_{21})}\cdot P_{22}^{\varepsilon_{22}}\cdot P_{12}^{-\varepsilon_{12}}\right]^c\right]^2} \quad (20)$$

$$\frac{\partial S_{11}^*}{\partial W_{21}} = \frac{c\cdot(P_{21}-W_{21})^{c-1}\left(P_{11}^{(\varepsilon_{11}-\mu_{11})}\cdot P_{21}^{(\mu_{21}-\varepsilon_{21})}\cdot P_{22}^{\varepsilon_{22}}\cdot P_{12}^{-\varepsilon_{12}}\right)^c(P_{11}-W_{11})^c}{\left[(P_{11}-W_{11})^c + \left[(P_{21}-W_{21})\cdot P_{11}^{(\varepsilon_{11}-\mu_{11})}\cdot P_{21}^{(\mu_{21}-\varepsilon_{21})}\cdot P_{22}^{\varepsilon_{22}}\cdot P_{12}^{-\varepsilon_{12}}\right]^c\right]^2} \quad (21)$$

$$\frac{\partial P_{11}^*}{\partial W_{11}} = \frac{P_{11}^{\mu_{11}-\varepsilon_{11}}\mu_{11}}{\varepsilon_{11}\left(\frac{1-S_{11}}{S_{11}}\right)^{\gamma_1}(P_{21}-W_{21})P_{12}^{-\varepsilon_{12}}P_{21}^{\mu_{21}-\varepsilon_{21}}P_{22}^{\varepsilon_{22}}} \quad (22)$$

$$\frac{\partial P_{21}^*}{\partial W_{11}} = \frac{-\varepsilon_{21}\left(\frac{S_{11}}{1-S_{11}}\right)^{\gamma_1}P_{22}^{-\varepsilon_{22}}P_{11}^{\mu_{11}-\varepsilon_{11}}P_{12}^{\varepsilon_{12}}}{P_{21}^{\mu_{21}-\varepsilon_{21}}\left[\mu_{21}W_{21}-(\mu_{21}+1)P_{21}\right]} \quad (23)$$

$$\frac{\partial P_{11}^*}{\partial W_{21}} = \frac{-\varepsilon_{11}\left(\frac{1-S_{11}}{S_{11}}\right)^{\gamma_1}P_{12}^{-\varepsilon_{12}}P_{21}^{\mu_{21}-\varepsilon_{21}}P_{22}^{\varepsilon_{22}}}{P_{11}^{\mu_{11}-\varepsilon_{11}}\left[\mu_{11}W_{11}-(\mu_{11}+1)P_{11}\right]} \quad (24)$$

$$\frac{\partial P_{21}^*}{\partial W_{21}} = \frac{P_{21}^{\mu_{21}-\varepsilon_{21}}\mu_{21}}{\varepsilon_{21}\cdot\left(\frac{S_{11}}{1-S_{11}}\right)^{\gamma_1}(P_{11}-W_{11})P_{22}^{-\varepsilon_{22}}P_{11}^{\mu_{11}-\varepsilon_{11}}P_{12}^{\varepsilon_{12}}} \quad (25)$$

Where:

$c = \gamma_1 - 1$

Retailer 2:

$$\frac{\partial S_{12}^*}{\partial W_{12}} = \frac{d\cdot(P_{12}-W_{12})^{2d-1} - d\cdot(P_{12}-W_{12})^{d-1}\left[(P_{12}-W_{12})^d + \left[(P_{22}-W_{22})\cdot P_{12}^{(\varepsilon_{12}-\mu_{12})}\cdot P_{22}^{(\mu_{12}-\varepsilon_{22})}\cdot P_{21}^{\varepsilon_{21}}\cdot P_{11}^{-\varepsilon_{11}}\right]^d\right]}{\left[(P_{12}-W_{12})^d + \left[(P_{22}-W_{22})\cdot P_{12}^{(\varepsilon_{12}-\mu_{12})}\cdot P_{22}^{(\mu_{12}-\varepsilon_{22})}\cdot P_{21}^{\varepsilon_{21}}\cdot P_{11}^{-\varepsilon_{11}}\right]^d\right]^2} \quad (26)$$

$$\frac{\partial S_{12}^{*}}{\partial W_{22}}=\frac{d\cdot(P_{22}-W_{22})^{d-1}\left(P_{12}^{(\varepsilon_{12}-\mu_{12})}\cdot P_{22}^{(\mu_{12}-\varepsilon_{22})}\cdot P_{21}^{\varepsilon_{21}}\cdot P_{11}^{-\varepsilon_{11}}\right)^{d}(P_{12}-W_{12})^{d}}{\left[(P_{12}-W_{12})^{d}+\left[(P_{22}-W_{22})\cdot P_{12}^{(\varepsilon_{12}-\mu_{12})}\cdot P_{22}^{(\mu_{12}-\varepsilon_{22})}\cdot P_{21}^{\varepsilon_{21}}\cdot P_{11}^{-\varepsilon_{11}}\right]^{d}\right]^{2} \tag{27}$$

$$\frac{\partial P_{12}^{*}}{\partial W_{12}}=\frac{\mu_{12}P_{12}^{\mu_{12}-\varepsilon_{12}}}{\varepsilon_{12}\cdot\left(\frac{1-S_{12}}{S_{12}}\right)^{\gamma_2}(P_{22}-W_{22})P_{11}^{-\varepsilon_{11}}P_{22}^{\mu_{22}-\varepsilon_{22}}P_{21}^{\varepsilon_{21}}} \tag{28}$$

$$\frac{\partial P_{22}^{*}}{\partial W_{12}}=\frac{-\varepsilon_{22}\cdot\left(\frac{S_{12}}{1-S_{12}}\right)^{\gamma_2}P_{21}^{-\varepsilon_{21}}P_{12}^{\mu_{12}-\varepsilon_{12}}P_{11}^{\varepsilon_{11}}}{P_{22}^{\mu_{22}-\varepsilon_{22}}[\mu_{22}W_{22}-(\mu_{22}+1)P_{22}]} \tag{29}$$

$$\frac{\partial P_{12}^{*}}{\partial W_{22}}=\frac{-\varepsilon_{12}\cdot\left(\frac{1-S_{12}}{S_{12}}\right)^{\gamma_2}P_{11}^{-\varepsilon_{11}}P_{22}^{\mu_{22}-\varepsilon_{22}}P_{21}^{\varepsilon_{21}}}{P_{12}^{\mu_{12}-\varepsilon_{12}}[\mu_{12}W_{12}-(\mu_{12}+1)P_{12}]} \tag{30}$$

$$\frac{\partial P_{22}^{*}}{\partial W_{22}}=\frac{\mu_{22}P_{22}^{\mu_{22}-\varepsilon_{22}}}{\varepsilon_{22}\cdot\left(\frac{S_{12}}{1-S_{12}}\right)^{\gamma_2}(P_{12}-W_{12})P_{21}^{-\varepsilon_{21}}P_{12}^{\mu_{12}-\varepsilon_{12}}P_{11}^{\varepsilon_{11}}} \tag{31}$$

Where:

$d=\gamma_2-1$

Item (2) in proposition 1 indicates that the sale price set by the retailer depends on the direct- and cross-price elasticities of the sales and shelf space allocated, along with the sale price of the substitutable product and its wholesale price. The sale price of a product increases when the price elasticity for the brand decreases and when its cross-price elasticity increases. This result is intuitive because it confirms that the retailer will increase the sale price or retail margin for those brands whose sales are less sensitive to a variation in their own price, but also when the brand's sales are very sensitive to the price variations of the competing brand. Although the final equilibrium expression of the shelf space allocated by the retailer to each brand will be obtained after computing the retail and wholesale prices, we can nevertheless characterize the ratio of shelf space, assuming $c=\gamma_1-1$.

Proposition 2 This proposition helps the retailers determine the allocation ratios based on revenues generated. The equilibrium ratio of brands' respective shelf space is given as the ratio of revenues generated with regard to substitutable product:

$$\frac{S_{11}^{*}}{1-S_{11}^{*}}=\left(\frac{Q_{11}\cdot(P_{11}-W_{11})}{Q_{21}\cdot(P_{21}-W_{21})}\right)^{\frac{1}{2c+1}} \tag{32}$$

$$\frac{S_{11}^{*}}{1-S_{11}^{*}}=\left(\frac{(P_{11}-W_{11})^{\frac{1}{c}}}{(P_{21}-W_{21})^{\frac{1}{c}}}\right)\cdot P_{11}^{\mu_{11}-\varepsilon_{11}}\cdot P_{21}^{\varepsilon_{21}-\mu_{21}}\cdot P_{12}^{\varepsilon_{12}}\cdot P_{22}^{-\varepsilon_{22}} \tag{33}$$

Ratio of brand sales is given by:

$$\frac{Q_{11}}{Q_{21}}=\left(\frac{S_{11}^{*}}{1-S_{11}^{*}}\right)^{\gamma_1}\cdot P_{11}^{\mu_{11}-\varepsilon_{11}}\cdot P_{21}^{\varepsilon_{21}-\mu_{21}}\cdot P_{12}^{\varepsilon_{12}}\cdot P_{22}^{-\varepsilon_{22}} \tag{34}$$

Therefore, the allocation rule stated above explicitly indicates that the brands' relative shares of the shelf space must be equal to their own profits. In the next section, we present the procedure for obtaining the manufacturers' response functions.

5 Manufacturers' response functions

The manufacturers take into account both retailers' reaction functions and play a Nash game. Therefore, both manufacturers' prices at equilibrium are obtained by incorporating the retailer's reaction functions into their profit functions and then maximizing them. The main assumption while evaluating objective function in this section is that each manufacturer offers a different price to each retailer. There could be several reasons behind this price discrimination. Some of them could be the quantity ordered, the relationships maintained and visibility offered for certain products over others. Here, we investigate the problem for two cases. In the first case, supply is assumed to be equal to demand (balanced case) while in the second case they are not equal (unbalanced case).

5.1 Balanced supply and demand

A manufacturer fixes her wholesale price by maximizing her profit function with regard to her prices subject to retailers' optimal SSA and prices, as seen in Section 4. The generalized profit function for the manufacturer is

$$\underset{W_{ij}}{\text{Max}} \prod\nolimits_i^m = \sum\nolimits_{j=1}^2 \left(W_{ij} - C_{ij}\right).\alpha.P_{kj}^{\varepsilon_{kj}}.P_{ij}^{\varepsilon_{il}} \cdot P_{ij}^{\mu_{ij}} \cdot S_{ij}^{\gamma_j}$$
$$i = 1,2;\ i \neq k;\ j \neq l \tag{35}$$

Since the retail prices are nonlinear, it is not advisable to substitute them in the profit function of the manufacturer. Therefore, our next step is to evaluate the profit function of manufacturers to obtain the optimal wholesale prices at Nash equilibrium.

Proposition 3 below summarizes results for a case in which both manufacturers are active players and offer cooperation to the retailers as leaders. In such a scenario, the game is a two-stage sequential one. Nash equilibrium is determined recursively by first obtaining both the retailers' reaction functions and then determining the manufacturers' optimal participation prices.

Proposition 3 This proposition holds good for competing manufacturers, providing the manufacturers with tools (wholesale prices) with which they can intervene in the retail decision-making process when supply and demand are balanced.

Assuming that there is an interior solution, the manufacturers' equilibrium wholesale prices are given as

$$\left(W_{ij} - C_{ij}\right)\begin{bmatrix} \gamma_j P_{ij}^{\mu_{ij}} P_{kj}^{\varepsilon_{kj}} S_{ij}^{\gamma_j - 1} P_{il}^{\varepsilon_{il}} \frac{\partial S_{ij}^*}{\partial W_{ij}} + \varepsilon_{kj} P_{ij}^{\mu_{ij}} P_{kj}^{\varepsilon_{kj}-1} S_{ij}^{\gamma_j} P_{il}^{\varepsilon_{il}} \frac{\partial P_{kj}^*}{\partial W_{ij}} \\ + \\ \varepsilon_{il} P_{ij}^{\mu_{ij}} P_{kj}^{\varepsilon_{kj}} S_{ij}^{\gamma_j} P_{il}^{\varepsilon_{il}-1} \frac{\partial P_{il}^*}{\partial W_{ij}} + \mu_{ij} P_{ij}^{\mu_{ij}-1} P_{kj}^{\varepsilon_{kj}} S_{ij}^{\gamma_j} P_{il}^{\varepsilon_{il}} \frac{\partial P_{ij}^*}{\partial W_{ij}} \end{bmatrix}$$
$$+P_{ij}^{\mu_{ij}} P_{kj}^{\varepsilon_{kj}} S_{ij}^{\gamma_j} P_{il}^{\varepsilon_{il}} = 0 \tag{36}$$

$0 < \gamma_1 < 1;\ \ 0 < \gamma_2 < 1;\ \ i,k = 1,2$ and $\mathrm{k} \neq \mathrm{i}, \mathrm{l}, \mathrm{j}$
$= 1,2$ and $\mathrm{l} \neq \mathrm{j}$

Proof Assuming an interior solution, the first-order optimality condition for each of the manufacturers' profit maximization problems is given as follows:

1. Partial derivatives of manufacturer 1's profit with regard to wholesale price offered to retailer 1 for product 1:

$$\frac{\partial \Pi_1^m}{\partial W_{11}} = 0 \tag{37}$$

$$\alpha \cdot P_{11}^{\mu_{11}} \cdot P_{21}^{\varepsilon_{21}} \cdot P_{12}^{\varepsilon_{12}} \cdot S_{11}^{\gamma_1}$$
$$\cdot \begin{bmatrix} 1 + (W_{11} - C_{11}) \cdot \\ \left(\frac{\mu_{11}}{P_{11}} \cdot \frac{\partial P_{11}}{\partial W_{11}} + \frac{\varepsilon_{21}}{P_{21}} \cdot \frac{\partial P_{21}}{\partial W_{11}} + \frac{\gamma_1}{S_{11}} \cdot \frac{\partial S_{11}}{\partial W_{11}}\right) \end{bmatrix} = 0 \tag{38}$$

$$W_{11}^* = C_{11} - \frac{1}{\left(\frac{\mu_{11}}{P_{11}} \cdot \frac{\partial P_{11}}{\partial W_{11}} + \frac{\varepsilon_{21}}{P_{21}} \cdot \frac{\partial P_{21}}{\partial W_{11}} + \frac{\gamma_1}{S_{11}} \cdot \frac{\partial S_{11}}{\partial W_{11}}\right)} \tag{39}$$

2. Partial derivatives of profit regarding to wholesale price offered to retailer 2 for product 1:

$$\frac{\partial \Pi_1^m}{\partial W_{12}} = 0 \tag{40}$$

$$\alpha \cdot P_{12}^{\mu_{12}} \cdot P_{22}^{\varepsilon_{22}} \cdot P_{11}^{\varepsilon_{11}} \cdot S_{12}^{\gamma_2} \tag{41}$$

$$\cdot \begin{bmatrix} 1 + (W_{12} - C_{11}) \cdot \\ \left(\frac{\mu_{12}}{P_{12}} \cdot \frac{\partial P_{12}}{\partial W_{12}} + \frac{\varepsilon_{22}}{P_{22}} \cdot \frac{\partial P_{22}}{\partial W_{12}} + \frac{\gamma_2}{S_{12}} \cdot \frac{\partial S_{12}}{\partial W_{12}}\right) \end{bmatrix} = 0$$

$$W_{12}^* = C_{11} - \frac{1}{\left(\frac{\mu_{12}}{P_{12}} \cdot \frac{\partial P_{12}}{\partial W_{12}} + \frac{\varepsilon_{22}}{P_{22}} \cdot \frac{\partial P_{22}}{\partial W_{12}} + \frac{\gamma_2}{S_{12}} \cdot \frac{\partial S_{12}}{\partial W_{12}}\right)} \tag{42}$$

For manufacturer 2, likewise:

1. Partial derivatives of manufacturer's profit with regard to wholesale price offered to retailer 1 for product 2:

$$\frac{\partial \Pi_2^m}{\partial W_{21}} = 0 \tag{43}$$

$$\alpha \cdot P_{21}^{\mu_{21}} \cdot P_{11}^{\varepsilon_{11}} \cdot P_{22}^{\varepsilon_{22}} \cdot (1 - S_{11})^{\gamma_1} \tag{44}$$

$$\cdot \begin{bmatrix} 1 + (W_{21} - C_{21}) \cdot \\ \left(\frac{\mu_{21}}{P_{21}} \cdot \frac{\partial P_{21}}{\partial W_{21}} + \frac{\varepsilon_{11}}{P_{11}} \cdot \frac{\partial P_{11}}{\partial W_{21}} - \frac{\gamma_1}{1 - S_{11}} \cdot \frac{\partial S_{12}}{\partial W_{21}}\right) \end{bmatrix} = 0$$

$$W_{21}^* = C_{21} - \frac{1}{\left(\frac{\mu_{21}}{P_{21}} \cdot \frac{\partial P_{21}}{\partial W_{21}} + \frac{\varepsilon_{11}}{P_{11}} \cdot \frac{\partial P_{11}}{\partial W_{21}} - \frac{\gamma_1}{1 - S_{11}} \cdot \frac{\partial S_{12}}{\partial W_{21}}\right)} \tag{45}$$

2. Partial derivatives of manufacturer's profit with regard to wholesale price offered to retailer 2 for product 2:

$$\frac{\partial \Pi_2^m}{\partial W_{22}} = 0 \tag{46}$$

$$\alpha \cdot P_{22}^{\mu_{22}} \cdot P_{12}^{\varepsilon_{12}} \cdot P_{21}^{\varepsilon_{21}} \cdot (1 - S_{12})^{\gamma_2} \tag{47}$$

$$\cdot \begin{bmatrix} 1 + (W_{22} - C_{21}) \cdot \\ \left(\frac{\mu_{22}}{P_{22}} \cdot \frac{\partial P_{22}}{\partial W_{22}} + \frac{\varepsilon_{12}}{P_{12}} \cdot \frac{\partial P_{12}}{\partial W_{22}} - \frac{\gamma_2}{1 - S_{12}} \cdot \frac{\partial S_{12}}{\partial W_{22}}\right) \end{bmatrix} = 0$$

$$W_{22}^{*} = C_{21} - \frac{1}{\left(\frac{\mu_{22}}{P_{22}} \cdot \frac{\partial P_{22}}{\partial W_{22}} + \frac{\varepsilon_{12}}{P_{12}} \cdot \frac{\partial P_{12}}{\partial W_{22}} - \frac{\gamma_2}{1-S_{12}} \cdot \frac{\partial S_{12}}{\partial W_{22}}\right)} \tag{48}$$

Restriction on substitution due to nonlinear structure of the retail prices as well as partial derivatives leads to numerical analysis. The complexity of retail price substitution and simplification reduces our capability to derive the upper and lower bounds on the wholesale price of the manufacturer as in Ref. [14]. Hence, we are unable to comment on the concavity of the whole sale prices and sensitivity analysis of the lower and upper bounds.

5.2 Unbalanced supply and demand

The generalized profit function for the manufacturer when supply is not equal to demand is as follow:

$$\underset{W_{ij}}{\text{Max}} \prod_i^{'m} = \sum_{j=1}^{2} \left(W_{ij} - C_{ij}\right).\alpha.W_{ij}^{\mu_{ij}} \cdot W_{il}^{\varepsilon_{il}} \; i = 1,2; \; i \neq k; \; j \neq l \tag{49}$$

Similar to the previous section, the next step is to evaluate the profit function of manufacturers to obtain the optimal wholesale prices at Nash equilibrium. Proposition 4 below summarizes results for a case in which both manufacturers are active players and offer cooperation to the retailers as leaders.

Proposition 4 This proposition holds good for competing manufacturers, providing the manufacturers with tools (wholesale prices) with which they can intervene in the retail decision-making process while demand and supplies are not balanced.

Assuming an interior solution, the first-order optimality condition for each of the manufacturers' profit maximization problems is given as follows:

1. Partial derivatives of manufacturer i's profit with regard to wholesale price offered to retailer 1 for product i:

$$\frac{\partial \prod_l^{'\mathrm{m}}}{\partial W_{i1}} = 0 \tag{50}$$

$$\begin{aligned} &\alpha W_{i2}^{\varepsilon_{i2}} \left[(\mu_{i1}+1) W_{i1}^{\mu_{i1}} - \mu_{i1} C_{i1} W_{i1}^{\mu_{i1}-1}\right] \\ &\quad + (W_{i2} - C_{i2}) \alpha W_{i2}^{\mu_{i2}} \varepsilon_{i1} W_{i1}^{\varepsilon_{i1}-1} = 0 \end{aligned} \tag{51}$$

$$\begin{aligned} &W_{i1}^{\mu_{i1}-\varepsilon_{i1}} \left[\mu_{i1} C_{i1} W_{i2}^{\varepsilon_{i2}} - W_{i1}(\mu_{i1}+1) W_{i2}^{\varepsilon_{i2}}\right] \\ &\quad = \varepsilon_{i1}(W_{i2} - C_{i2}) W_{i2}^{\mu_{i2}} \end{aligned} \tag{52}$$

2. Partial derivatives of profit regarding to wholesale price offered to retailer 2 for product i:

$$\frac{\partial \prod_l^{'\mathrm{m}}}{\partial W_{i2}} = 0 \tag{53}$$

$$\begin{aligned} &(W_{i1} - C_{i1}) \alpha W_{i1}^{\mu_{i1}} \varepsilon_{i2} W_{i2}^{\varepsilon_{i2}-1} \\ &\quad + \alpha W_{i1}^{\varepsilon_{i1}} \left[(\mu_{i2}+1) W_{i2}^{\mu_{i1}} - \mu_{i2} C_{i2} W_{i2}^{\mu_{i2}-1}\right] = 0 \end{aligned} \tag{54}$$

$$\begin{aligned} &W_{i2}^{\mu_{i2}-\varepsilon_{i2}} \left[\mu_{i2} C_{i2} W_{i1}^{\varepsilon_{i1}} - W_{i2}(\mu_{i2}+1) W_{i1}^{\varepsilon_{i1}}\right] \\ &\quad = \varepsilon_{i2}(W_{i1} - C_{i1}) W_{i1}^{\mu_{i1}} \end{aligned} \tag{55}$$

6 Case study—results and discussion

To evaluate both vertical and horizontal integrations, this paper considers four game plans that reflect various integration strategies and their workings using game theory. The four game plans are as follows:

- Game plan 1—Cournot game (CG) played at both the retailer and manufacturer level, considering elasticities and wholesale prices to be exogenous.
- Game plan 2—manufacturer 1 plays a Stackelberg game (SG) individually with both the competing retailers as followers. CG is played within the manufacturers' group.
- Game plan 3—manufacturer 2 plays an SG individually with both the competing retailers as followers. The optimal retail prices are obtained from sub-scenario 2. CG is played within the manufacturers group.
- Game plan 4—an SG is played at both retailers and manufacturers to find optimal wholesale prices; the retail prices are obtained from sub-scenario 2.

The game plans discussed above suppose that supply-chain members interact with each other through a combination of options that they choose. The best of the four game plans is evaluated and selected for a case study using numerical analysis. Newton Raphson's method for non-linear optimization is utilized for numerical evaluation since the retail and wholesale prices lack a closed-form analytical solution. R-software is used for this purpose and is set to reach convergence by the 10,000th iteration. It is worth mentioning that for such problems multiple equilibrium points can and do exist. Without loss of generality, this provides the opportunity to carry out further analysis for the underlined real-world problem and examine their significance to derive managerial scenarios and recommendations. The optimal values obtained are reported and discussed below.

Table 1 Fixed parameters used to further our numerical analysis

Parameter	Value
ε_{12}	1.5
ε_{22}	1.7
ε_{11}	1.1
ε_{21}	1.3
W_{12} (\$)	1.2
W_{22} (\$)	1.3
W_{11} (\$)	1
W_{21} (\$)	1.1
C_{12} (\$)	0.89
C_{22} (\$)	0.89
C_{11} (\$)	0.79
C_{21} (\$)	0.79
μ_{12}	4.5
μ_{21}	5
γ_1	0.75
γ_2	0.75

These results are believed to be the best for the equilibrium strategies, and deviation from these optimal values indicates deviation from equilibrium, which would not be beneficial to any of the supply-chain members. Tabulated below are the results for a two retailer/two manufacturers vertically and horizontally integrated supply chain.

Table 1 presents the initial starting values chosen for the numerical simulation. These values are taken from Ref. [14].

Table 2 depicts optimal pricing and SSA values at Cournot equilibrium (game plan 1) when the wholesale prices are exogenous (taken from Table 1). Results of this table correlate to proposition 1 and also indicate that the cost-advantaged manufacturer sells his product for a lower price, which translates into a lower retail price and higher SSA. This finding is similar to that of Herr'an et al.'s [14].

The results in Table 3 are indicative of the leader–follower relationship among retailers and Cournot between the manufacturers (game plans 2 and 3). Consider retailer 1 as a leader who fixes prices for his products first and retailer 2 as a follower who is a new entrant and follows the leader by observing his moves. This scenario would represent a retailer SG, and this table presents us with optimal pricing values for retailer 2. Considering these optimal retail prices from the SG as exogenous, the wholesale prices for the Cournot manufacturers are recomputed and are also presented in Table 3.

Table 2 Optimal values for both retailers in Cournot game; indicative of game plan 1

Prices (\$/unit)								Shelf space (%)			
P^*_{11}	P^*_{21}	P^*_{12}	P^*_{22}	W^*_{11}	W^*_{12}	W^*_{21}	W^*_{22}	S^*_{11}	S^*_{21}	S^*_{12}	S^*_{22}
1.24	1.72	1.83	2.34	1	1.1	1.2	1.3	72	28	69	31

Table 3 Optimal values for mixed game strategies, with retailers playing Stackelberg game; retailer 1 as leader and manufacturers playing a Cournot game, indicative of game plans 2 and 3

Prices (\$/unit)								Shelf space (%)			
P^*_{11}	P^*_{21}	P^*_{12}	P^*_{22}	W^*_{11}	W^*_{12}	W^*_{21}	W^*_{22}	S^*_{11}	S^*_{21}	S^*_{12}	S^*_{22}
1.24	1.72	0.7	1.14	1.51	1.17	1.5	0.5	72	28	76.82	23.18

The results of Table 4 are indicative of the SG at all levels of the supply chain (game plan 4). Utilizing the optimal pricing values for retailers and for manufacturer 1, who is considered the leader from Table 3, optimal wholesale prices for manufacturer 2, who is the follower, are recomputed (the so-called prisoner's dilemma). The results for wholesale prices are connected to proposition 3 and the entire Table 4 results are aligned with the SG rule, where the leaders get better payoffs. These results also provide us enough evidence to recommend that transparency beyond knowledge of prices when one is choosing strategies in an integrated supply chain is required to have a win–win situation and to avoid a prisoner's dilemma situation.

Employing the results stated in Tables 2, 3, and 4, profits/losses are computed for various game plans in order to analyze the outcomes to provide valuable insights. Table 5 clearly indicates greater overall profits and prices across the integrated supply chain for the Cournot equilibrium when each player looks out for his own good and makes the move that is most beneficial for him. These results are consistent with findings in the classical economic literature and indicate that knowledge of the leader's price alone is not beneficial. For example, let us consider Wal-Mart, which offers "everyday low prices" to be the leader, and Kroger to be the follower. Following its strategy, Wal-Mart prices normal bananas at \$0.58/lb and organic bananas at \$0.71/lb. Kroger knows these values and prices its bananas at \$0.55 and \$0.68/lb, but it also has a loyalty card program (Wal-Mart does not offer loyalty programs) which further reduces its price to \$0.50/lb. Considering that both retailers allocate different shelf space, availability becomes another issue for the customers. If we assume that both stores sell the same

Table 4 Optimal values of M1 and M2 in a Stackelberg game at two levels with M1 as the leader purely Stackelberg equilibrium inactive of game plan 4

Prices (\$/unit)								Shelf space (%)			
P^*_{11}	P^*_{21}	P^*_{12}	P^*_{22}	W^*_{11}	W^*_{12}	W^*_{21}	W^*_{22}	S^*_{11}	S^*_{21}	S^*_{12}	S^*_{22}
1.24	1.72	0.7	1.14	1.51	1.17	1.08	0.64	72	28	76.82	23.18

Table 5 Final outcome of various strategies employed

Players	Cournot equilibrium profits	Mixed equilibrium profits	Stackelberg equilibrium profits	Leader/ follower for Stackelberg game
Retailer 1	$102,836.37	$143.30	$641.99	Follower
Retailer 2	$71,102.14	$1,724.92	$1,183.86	Leader
Manufacturer 1	$14,182.57	$1,001.43	$1,001.43	Leader
Manufacturer 2	$71,065.17	−$831.87	−$789.50	Follower
Overall supply chain profit	$259,186.25	$2,037.79	$2,037.79	
Summary	Cournot > mixed/Stackelberg. This profit table clearly indicates that cooperation beyond knowledge of prices is advisable			

amounts of bananas, the retailers would still not make similar profits due to different working models that consider different variables, such as promotions, offers, vendor pricing, distance from each other, customer types present in that demographic area, along with the day of the week and the pricing and availability of other goods. Therefore, transparency beyond pricing and SSA is very important in order to achieve complete coordination among all the members of a supply chain. This type of transparency is not possible to achieve, especially during tense economic times, as retail giants like Wal-Mart do not share any of their information with either their competing retailers or with any of their research vendors. Hence, Cournot equilibriums are best in such situations.

7 Conclusions and recommendations

The current research pursues a systematic approach in analyzing Cournot and Stackelberg oligopolies with increasingly complex and realistic demand and profit functions in order to obtain optimal pricing and SSA values. These optimal values are then translated into specific demand and profit values allied to various members of the supply chain. Interesting insights based on these optimal values, overall profits, and strategies followed are revealed in this process. Analysis of this complex integrated supply chain utilizing game theory allows us to make the following inferences and to provide some recommendations:

- The profits/losses in Table 5 suggest the Cournot equilibrium to be the best strategy. This is a very pragmatic strategy followed by all retailers, but especially by the retail giant Wal-Mart.
- Manufacturers can intervene in the process of SSA through their wholesale prices (proposition 3). Lower wholesale prices translate to lower retail prices, which in turn result in more shelf space allocated for that product (proposition 1).
- The SG played among the members is very close to a real-world scenario, in which the leader represents a tycoon in that area and the follower represents a new entrant into the business.
  - The prices and profits of the leaders were more than those of the follower;
  - The follower's SSA was less than that of the leader.
- Prices and profits are less for sequential simultaneous SG compared with CG for all the members and the entire supply chain. This is consistent with the established results in classical economic literature.
- Overall, the analysis of various strategies applied across the two scenarios suggests the necessity to confirm the level and intensity of information sharing among the members of an integrated supply chain.
- Wal-Mart could use this type of demand model to integrate its pricing, merchandising, and replenishment teams to provide better coordinated optimal pricing and allocation strategies that would increase its overall sales compared with its competitors, such as Target, Kroger, etc.

This study can be extended in several ways. First, cannibalization and penetration in the retail markets by any of the big retailers can be analyzed, especially when retailers are making real-estate decisions on placement of stores in certain trade areas. Second, the existence of multiple equilibrium points and their significance in a case study can also be demonstrated whereby presenting discussion about their implications yields practical/managerial insights. Finally, the presented optimal policies for the unbalanced supply and demand case can be further analyzed for a case study to examine the dynamics of the given supply chain.

Appendix 1

In economics and business studies, the price elasticity of demand (PED) is an "elasticity that measures the nature and degree of the relationship between changes in quantity demanded of a good to changes in its price." By definition, price elasticity (μ_i) of demand is given by:

$$\mu_i = -\frac{\%\,\text{change in quantity demanded}}{\%\,\text{change in price}} = -\frac{\frac{\Delta Q_i}{Q_i}}{\frac{\Delta P_i}{P_i}}$$

Using differential calculus, by integrating the equation

$$\mu_i = -\frac{\mathrm{dQ_i}}{\mathrm{dP_i}} \cdot \frac{P_i}{Q_i}$$

yields the power law form given by:

$$Q \propto P_i^{\mu_i}$$

The cross-price elasticity is defined as "the measure of responsiveness of the quantity demand of a good to a change in the price of a competing substitutable product from another manufacturer." By definition, cross elasticity (ε_k) of demand is given by:

$$\varepsilon_k = \frac{\%\,\text{change in the quantity demanded of product k}}{\%\,\text{change in price of product l}} = \frac{\frac{\Delta Q_i}{Q_i}}{\frac{\Delta P_k}{P_k}}$$

Using differential calculus: $\varepsilon_k = \frac{\mathrm{dQ_i}}{\mathrm{dP_k}} \cdot \frac{P_k}{Q_i}$ and integrating this equation yields the power law form given by:

$$Q_i \propto P_k^{\varepsilon_k}$$

Similarly, the elasticity (ε_k) of shelf space (S_i) is defined as: "elasticity that measures the nature and degree of the relationship between changes in quantity demanded of a good to changes in the amount of shelf space allocated to that product." By definition, direct shelf-space elasticity (γ_j) of demand is given by:

$$\gamma_j = \frac{\%\,\text{change in quantity demanded}}{\%\,\text{change in shelf space allocated to that product}} = -\frac{\frac{\Delta Q_i}{Q_i}}{\frac{\Delta S_i}{S_i}}$$

Using differential calculus: $\gamma_j = \frac{\mathrm{dQ_i}}{\mathrm{dS_i}} \cdot \frac{S_i}{Q_i}$. Integrating this equation yields the power law form given by:

$$Q_i \propto S_i^{\ \gamma_j}$$

The cross-shelf-space elasticity is defined as "the measure of responsiveness of the quantity demand of a good to a change in the shelf space allocated to a competing substitutable product from another manufacturer." By definition, cross elasticity (γ_l) of demand is given by:

$$\gamma_l = \frac{\%\,\text{change in the quantity demanded of product i}}{\%\,\text{change in shelf space allocated to product k}} = \frac{\frac{\Delta Q_i}{Q_i}}{\frac{\Delta S_k}{S_k}}$$

Using differential calculus: $\gamma_l = \frac{\mathrm{dQ_i}}{\mathrm{dS_k}} \cdot \frac{S_k}{Q_i}$. Integrating this equation yields the power law form given by:

$$Q_i \propto S_k^{\gamma_l}$$

Direct PED is an "elasticity that measures the nature and degree of the relationship between changes in quantity demanded of a good to changes in its price." By definition, price elasticity (μ_i) of demand is given by

$$\mu_i = -\frac{\%\,\text{change in quantity demanded}}{\%\,\text{change in price at the competing retailer}} = -\frac{\frac{\Delta Q_i}{Q_i}}{\frac{\Delta P_{ij}}{P_{ij}}}$$

Using differential calculus: $\mu_i = -\frac{\mathrm{dQ_i}}{\mathrm{dP_{ij}}} \cdot \frac{P_{ij}}{Q_i}$. Integrating this equation yields the power law form given by:

$$Q \propto P_{ij}^{\mu_i}$$

Combining the above four equations, the demand Q_i for a product i is given by

$$Q_i = \alpha \cdot S_i^{\gamma_j} \cdot S_k^{\gamma_l} \cdot P_i^{\mu_i} \cdot P_k^{\varepsilon_k} \quad \text{for i, k} = 1, 2$$

$$Q_i = \alpha S_i^{\gamma_1}\left(1 - S_i^{\gamma_1}\right) P_{il}^{\varepsilon_{il}} P_{kj}^{\varepsilon_{kj}} P_{ij}^{\mu_{ij}}$$

$$Q_k = \alpha S_k^{\gamma_j}\left(1 - S_k^{\gamma_j}\right) P_{kl}^{\varepsilon_{kl}} P_{ij}^{\varepsilon_{ij}} P_{kj}^{\mu_{kj}}$$

where $\alpha > 0,\ \mu_k \geq 0, \varepsilon_k \geq 0, 0.$

The demand function of Q_i, also known as Cobb–Douglas, is very well known in the economics literature. The power law formulation above allows us to account for the interaction between the variables and also has the property that elasticities are constant. We also have:

- $Q_i \geq 0$: The proof of this is very simple and intuitive. There cannot be negative amounts of any product supplied or demanded. Therefore the quantity of sales >0, which is strictly positive.
- $\frac{\partial Q_i}{\partial S_i} \geq 0$

Proof Using Eq. (1), we take the first-order derivative with respect to shelf-space allocated to manufacturer K:

$$Q_i = \alpha S_i^{\gamma_j} S_k^{\gamma_l} P_i^{\mu_i} P_k^{\varepsilon_k}$$

$$\frac{\partial Q_i}{\partial S_i} = \alpha \gamma_j S_i^{\gamma_j - 1} S_k^{\gamma_l} P_i^{\mu_i} P_k^{\varepsilon_k} \geq 0$$

The above equation is not negative because all the prices involved are positive, and, in addition, SSA cannot be negative, the shelf-space direct elasticity value is also observed to be between 0 and 1, which restricts its negativity, and the constant alpha and gamma values are strictly positive. Therefore, the above equation cannot be negative.

- $\frac{\partial Q_i}{\partial P_k} \geq 0$

Proof The proof for the above inequality is similar to the previous one. Here, again a first-order derivative is taken, and similar restrictions on shelf space and price are applied. The only difference is that we have cross-price elasticity multiplied to the quantity demanded. The cross-shelf-space elasticity is always positive, and therefore the final inequality is positive and greater than 0.

$$\frac{\partial Q_i}{\partial P_k} = \alpha \varepsilon_k S_i^{\gamma_j} S_k^{\gamma_l} P_i^{\mu_i} P_k^{\varepsilon_k - 1} \geq 0$$

- $\frac{\partial Q_i}{\partial P_i} \leq 0$

Proof This inequality follows a similar pattern to the two before it. Take a first-order derivative with similar restrictions applied on shelf space and price. The difference observed is the direct price elasticity multiplied to the quantity demanded. The direct shelf-space elasticity carries a negative sign, and therefore introduces negativity into the equation. Hence, it is less than zero.

$$\frac{\partial Q_i}{\partial P_i} = \alpha(\mu_i) S_i^{\gamma_j} S_k^{\gamma_l} P_i^{\mu_i - 1} P_k^{\varepsilon_k} \le 0$$

All these above inequalities have been tested in the pilot phase of the study using numerical analysis in order to evaluate our assumptions.

Proofs	Values	Proofs	Values
$\frac{\partial Q_1}{\partial P_1^*} \le 0$	−0.2011	$\frac{\partial Q_2}{\partial P_1^*} \ge 0$	0.0230
$\frac{\partial Q_1}{\partial P_2^*} \ge 0$	0.0388	$\frac{\partial Q_2}{\partial P_2^*} \le 0$	−0.0666
$\frac{\partial Q_1}{\partial S_1^*} \ge 0$	0.0582	$\frac{\partial Q_2}{\partial S_1^*} \ge 0$	0.0099
$\frac{\partial Q_1}{\partial S_2^*} \ge 0$	0.0713	$\frac{\partial Q_2}{\partial S_2^*} \ge 0$	0.0489

Appendix 2

Although the direct- and cross-shelf-space elasticities are defined earlier as two independent quantities, the normalization constraint on the shelf space imposes a relationship between these two elasticities. Consider the demand of two products at a retailer as

$$Q_1 = \alpha \cdot S_1^{\gamma_1} \cdot S_2^{\gamma_2} \cdot P_1^{\mu_1} \cdot P\,_2^{\varepsilon_1}$$
$$Q_2 = \alpha \cdot S_2^{\gamma_1} \cdot S_1^{\gamma_2} \cdot P_2^{\mu_2} \cdot P_1^{\varepsilon_1}$$

Since $S_1 + S_2 = 1$,

$$\frac{\partial S_2}{\partial S_1} = -1;$$

$$\frac{\partial S_1}{\partial S_2} = -1;$$

$$\frac{\partial Q_1}{\partial S_1} = Q_1 \cdot \left[\frac{\gamma_1}{S_1} - \frac{\gamma_2}{S_2}\right]$$

$$\frac{\partial Q_1}{\partial S_1} = \frac{Q_1}{S_1} \cdot \left[\gamma_1 - \gamma_2 \cdot \frac{S_1}{S_2}\right]$$

$$\frac{\frac{\partial Q_1}{Q_1}}{\frac{\partial S_1}{S_2}} = \left[\gamma_1 - \gamma_2 \cdot \frac{S_1}{S_2}\right] \to \text{Direct}-\text{shelf}-\text{space elasticity}$$

Similarly solving for cross-shelf-space elasticity

$$\frac{\partial Q_1}{\partial S_2} = Q_1 \cdot \left[\frac{\gamma_2}{S_2} - \frac{\gamma_1}{S_1}\right]$$
$$\frac{\partial Q_1}{\partial S_2} = \frac{Q_1}{S_2} \cdot \left[\gamma_2 - \gamma_1 \cdot \frac{S_2}{S_1}\right]$$

$$\frac{\frac{\partial Q_1}{Q_1}}{\frac{\partial S_1}{S_2}} = \left[\gamma_1 - \gamma_2 \cdot \frac{S_1}{S_2}\right] \to \text{Cross}-\text{shelf}-\text{space elasticity}$$

Simple algebraic manipulations applied to the above equation and it could be rewritten as:

$$\frac{\frac{\partial Q_1}{Q_1}}{\frac{\partial S_2}{S_2}} = -\frac{S_2}{S_1} \cdot \underbrace{\left[\gamma_1 - \gamma_2 \cdot \frac{S_1}{S_2}\right]}_{\text{Direct}-\text{shelf}-\text{space elasticity}}$$
$$\text{Cross}-\text{shelf}-\text{space elasticity} = -\frac{S_2}{S_1} \cdot \text{Direct}-\text{shelf}-\text{space elasticity}$$

Thus, cross elasticity is proportional to shelf-space ratio, with the proportionality factor being minus the direct elasticity.

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