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Conference Paper · June 2015

DOI: 10.1109/SPS.2015.7168285

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***Abstract*—Due to the stochastic nature of EEG signals, various nonlinear patterns and methods have been applied in order to obtain characteristic understanding of their dynamic behavior [6]. The Fractal Dimension (FD) is an appropriate tool to analyzed EEG signals and can be calculated by means of the Higuchi's algorithm. Nevertheless, this algorithm depends of the k parameter to improve the speed of calculation. The aim of this work is to analyze the sensitivity of the k parameter due to segmentation, overlap, and noise over a signal. After that, with a better k parameter we applied the FD on EEG brain signals recorded while subjects were executing cognitive task. To analyze the statistical differences for each cognitive mental task, the hypothesis Wilcoxon signed-rank test was applied. The results for all tested brain bands used in this study reported a statistical difference ($p < 0.05$) in 9 out of 10 pairs of mental tasks. The proposed approach reported is a good tool for cognitive tasks discrimination. We have also determine better k parameter for different conditions therefore these results can be used for future studies.**

***Index Terms*—Fractal Dimension, EEG signals, Higuchi's method, cognitive task.**

I. Introduction

Brain signals can give information on neuronal activity behavior during the development of various mental tasks such as imaginary motion, hypnosis, cognitive, including drug effects [6], [4],[21], [20]. They allow us to study EEG dynamics or anomalies from diseases, i.e. Alzheimer or Parkinson [5], [12]. These brain signals can be recorded by means of electrodes placed on the human scalp.

EEG signals have high nonlinear characteristics and are highly stochastic [6], for this reason, in order to obtain further information it is important to study and analyze its nonlinear characteristics, using such technics as Detrended Fluctuation Analysis [21] and Approximate Entropy [4]. Due to the nonlinear characteristics of EEG signals, Fractal Dimension (FD) analysis can be used to quantify the irregularity or complexity of brain signals [5]. The FD is commonly used in EEG applications such as epilepsy, cognitive task, sleep [5], [1], [19], [3], [14]. Two approaches exist to calculate the FD, in time domain and in phase space domain. In the case of the first approach, the FD is calculated over the time domain to quantify the self-similarity of an object.Thus, it allows us to analyze brain signals for transient detection but also the main advantage is that computations are fast. Second, the phase space domain of nonlinear system have one or more attractors with Fractal Dimension. The principal feature of attractor dimension is not changed under different initial conditions. Nevertheless, to calculated the FD requires high computational complexity [3]. Scientific literature there are reports of calculations of Fractal Dimension analysis of EEG signals recorded while the subjects were in the state of relaxation , or hypnosis, or executing mental task.

Also, an algorithm in conjunction with artificial neural networks has been used to predict the starting stage of epilepsy meanwhile obtaining a classification accuracy over 80% [19]. The scientific literature reported that different algorithms to calculate the FD but Higuchi's algorithm is more frequently used by several authors [3], [1],[23],[17],[16]. Nevertheless, the Higuchi's algorithm is necessary to tune up a k parameter because the correct election of k parameter has an important role to obtain reliable FD values. Several authors have utilized Fractal Dimension by means of the Higuchi's algorithm but their selection of k parameter is not well specified. In [1], they analyzed the k parameter between 3 and 10 after that they selected the k = 6 value. In [3], they analyzed different segmentation lengths of data to study FD behavior, the authors did not mention the k parameter. In [14], a wide range of k parameter was analyzed, from $2 - 80$, and they selected three $k = 9, 25, 50$ values. In these studies, they did not analyzed the influence of overlap effect in combination with data segmentation, nor the behavior of k parameter for different theoretical FD values. This work has two aims, first to analyze the influence of the overlap, segmentation, different values of theoretical FD over noise data. The goal is to obtain a k value for each case and analyze its behavior. Second, to calculate the FD over EEG signals when the subject is development cognitive task and to analyze the statistical differences between the different cognitive tasks. This work is divided as follows: Section II presents the materials and methodology. Section III presents the experimental results and discussion. Finally, we will present the conclusions of this work in Section IV.

II. Materials and methods

A. Subjects and EEG recording

The EEG data used here is available to researchers and has been provided by the Colorado State University [10] and were collected by Keirn and Aunon [11].

An elastic electrode cap was used, where its electrodes were placed in the central part of the left hemisphere ($C3$), central part of the right hemisphere ($C4$), parietal lobe of the left hemisphere ($P3$), parietal lobe of the right hemisphere ($P4$), occipital lobe of the left hemisphere ($O1$), and occipital lobe of the right hemisphere ($O2$) in order to record the six-channel EEG signals. One-channel EOG (Electrooculography) was defined by the $10-20$ system of electrode placement. The EEG signals were sampled at $250Hz$ and filtered from 0.1 to 100 Hz. The EEG database contains records of 7 subjects for 10 seconds of 5 mental tasks: Baseline (B), relaxed state; Multiplication (M), the subjects are making a multiplication; Letter-composing (LC), imagining writing a letter; Rotation (R), imagining rotation of object; Visual-Counting (C), erasing and redrawing figures. Data were recorded for 10 seconds and each task was repeated five times per session. In the original EEG dataset, there were seven subjects in the study, but only four subjects were chosen (subject 1, subject 3, subject 4, and subject 5) because the other three had fewer than ten sessions or some errors in the recording.

B. Surrogate data

The method of surrogate data is a technique that allows to test the nonlinearity over data. Scientific literature reported that surrogate data is generated by means of a linear Gaussian stochastic process, these data series are considered like the null hypothesis [15]. Thereby, the surrogate data keeps some statistical characteristics of an original time series such as power spectrum and auto-correlation function. The surrogate data are compared with the original data by means of a discriminating nonlinear measure. After that, if the comparative differences are reported, the null hypothesis is rejected and the original series is considered nonlinear.

Also in this paper, the time reversibility was used to nonlinearity detection [15]. This method was applied over brain signals during epilepsy [18], childrens sleep [2], uterine electromyogram signals during pregnancy and labor signals [7] to analyzed the nonlinear characteristics. Furthermore, the iterative amplitude adjusted Fourier transform (IAAFT) method was selected to generated surrogate data and the rank test was used to reject or accept the null hypothesis [15].

C. Higuchi's fractal dimension

Fractal dimension is a statistical complex element that consists in comparing pattern changes; it refers to a non-integer or Fractional Dimension of a geometric object. Various algorithms have been proposed (Katzs algorithm [9], Higuchi's method [8], Petrosians method [13]). However in this work, the Higuchi's algorithm was used because comparing results make it an easier tool [5], [22], [3], [1], [23], [17], [16]. Also, it is a very efficient method for measuring the FD of discrete time sequences and calculates the FD directly from the time series.The algorithm approximates the mean length of the curve using segments of k samples and estimates the dimension of a time-varying signal directly in the time domain, that help us saving time in running the data. Higuchi's fractal dimension estimation technique is resumed as follows [8]

1) Define the values of a finite set of time series that are taken within a regular interval as $y_1, y_2, ..., y_i, ..y_N = y(1), y(2), ..., y(i), .., y(N)$ where$i = 1, 2, .., N$ (N: number of points of the time series). Here, y are the successive EEG values. For a range of k values ranging from 1 to $kmax$, construct k new times series y_k^m defined as follows:
$$y_k^m : y(m), y(m+k), y(m+2k), .., y\left(m + \left(\frac{N-m}{k}\right)k\right) \quad (1)$$
where $m = 1, 2, ..., k$
2) Calculate the length $L_m(k)$ of each curve y_k^m :
$$L_m(k) = \frac{\sum_{i=1}^{N-m/k} |y(m+ik) - y(m+(i-1)k)| (n-1)}{\left(\frac{N-m}{k}\right)k} \quad (2)$$
where N is the total length of the data sequence $(N-1)/[(N-m)/k]k$ is the normalization factor for the curve length of y_k^m
3) Calculate the mean length of the curve for each k, $L(k)$, as the average value over k sets of $L_m(k)$, for $m = 1, 2, .., k$, as $L(k) = \frac{1}{k}\sum_{m-1}^{k}(L_m(k))$ Repeat the calculation for k ranging from 1 tok_max.
4) If $L(k) \propto k^{-FD}$, then the curve is fractal with dimension FD. In that case, the plot of $ln(L(k))$ against $ln(k)$ should fall on a straight line with slope equal to FD. Therefore, FD can be calculated by means of a least-squares linear best-fitting procedure.

D. Calculation of optimal k

To analyze and calculate the optimal value k, we studied the impact of segmentation and overlap on the data with white Gaussian noise. The deterministic Weierstrauss cosine function was implemented in MATLAB, for the synthetic signal generation, [3] given as follows:
$$W_H(t) = \sum_{i=1}^{M}(\gamma^{-iH} cos(2\pi\gamma^i t)), 0 < H < 1 \quad (3)$$
where $\gamma > 1$, and we fixed $\gamma = 5$ and $M = 26$. The fractal dimension of signal is given by $D = H - 2$. A set of several sequences was created with a Weierstrauss function with a length of 2000 points and each synthetic data was added with white Gaussian, yielding a signal to noise ratio (SNR) of $10db$. The data segmentation was 10% until 50% of total length data and the overlap was since 0% until 90% of each segmentation. In this paper, we analyzed for three values of theoretical FD such as 1.3, 1.5 and 1.7.

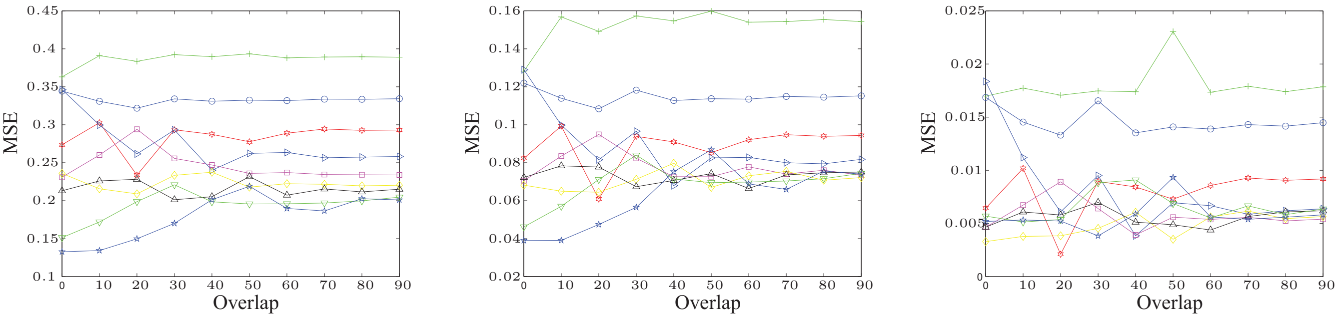
E. EEG data processing

Fractal Dimension was calculated using the Higuchi's algorithm for each mental task and was analyzed over the EEG signal. The k parameter was selected according II.D and the surrogate data method was applied to test the nonlinearity of EEG data. The data were filtered with a notch filter to reject the

TABLE I: Optimal k values

seg	Overlap 0%			10%			20%			30%			40%		
FD	1.3	1.5	1.7	1.3	1.5	1.7	1.3	1.5	1.7	1.3	1.5	1.7	1.3	1.5	1.7
10%	100	100	100	100	100	44	100	100	44	100	100	44	100	100	44
15%	150	150	140	150	150	150	150	150	150	150	150	150	150	150	150
20%	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
25%	185	186	204	250	219	219	250	250	222	250	210	216	250	250	250
30%	300	284	235	300	221	219	219	214	216	300	233	221	300	220	217
35%	350	268	235	350	283	235	350	301	284	350	237	222	350	236	221
40%	400	234	220	400	296	220	400	301	235	400	400	296	400	400	232
45%	450	450	286	450	450	223	450	450	222	450	450	220	450	450	221
50%	500	500	500	500	500	500	500	500	222	500	500	223	500	500	219

seg	Overlap 50%			60%			70%			80%			90%		
FD	1.3	1.5	1.7	1.3	1.5	1.7	1.3	1.5	1.7	1.3	1.5	1.7	1.3	1.5	1.7
10%	100	100	44	100	100	44	100	100	44	100	100	44	100	100	44
15%	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
20%	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
25%	250	250	232	250	250	222	250	250	221	250	250	223	250	250	221
30%	300	298	232	300	300	222	300	300	223	300	300	222	300	298	223
35%	350	268	235	350	299	232	350	350	223	350	299	232	350	300	232
40%	400	228	219	400	290	235	400	296	232	400	400	223	400	299	223
45%	450	450	222	450	450	222	450	450	223	450	450	232	450	450	223
50%	500	500	219	500	500	222	500	500	222	500	500	222	500	500	223



(a) Theoretical $FD = 1.3$ (b) Theoretical $FD = 1.5$ (c) Theoretical $FD = 1.7$

Fig. 1: MSE by each length segmentation versus length overlap. (a) Theoretical $FD = 1.3$. (b)Theoretical $FD = 1.5$. (c) Theoretical $FD = 1.7$. Segmentation, −+− 10%, −ø− 20%,−✡− 30%,− ▷ − 40%,− □ − 50%,− ⋄ − 60%,−Δ− 70%,−∇− 80%,−★− 90%

power line interference $(60Hz)$ and the continuum component (DC) was removed. EEG signals were filtered with a fifth-order Elliptic filter twice (filtering forward and backward) in order to remove the effects of phase distortion and to eliminate the artifacts of the signals to all electrodes. Furthermore, the filters were designed to obtain a minimum attenuation of $-30dB$ for frequencies of 0.5 Hz. The hypothesis test for non-parametric statistical data related, Wilcoxon signed-rank test, was applied to analyze the difference between each of the tasks because the same subjects were part of the whole study. The data of the two approaches have shown a non-normal distribution that has been confirmed with the Kolmogorov-Smirnov test. When the parameters of the statistical significance (p) are smaller than 0.05 $(p < 0.05)$, this means that the statistical difference between the two tasks are within a 5% significance.

III. Results and Discussion

A. Influence of segmentation and overlap over k parameter

As already discussed, based on scientific literature the selection of the k parameter plays a important role to calculate FD by means of the Higuchi's algorithm. In section II.D, we explained how to obtain the better k parameters to different length segmentation (segm) and overlap (over) on data that it was contaminated with normal Gaussian noise. After several simulations, the better k parameters are summarized in the Table I. The k parameters obtained with theoretical $FD = 1.3$ reported that better parameter k was half of length segment in each case, except for two case: (segm:25% and over:0%) and (segm:30% and over:20%) for k value of 185 and 219 respectively. The results with theoretical $FD = 1.5$ reports that better parameter k was half of segment in each case, except for the length segmentation of 25% until 40%. The theoretical $FD = 1.7$ shows different k parameter except for the length segmentation of 15% and 20% where the k value was half of the total segment. The analysis of k parameters in different ranges showed the optimal k parameter for different conditions (segmentation and overlap). These k parameters reported in this paper were in disagreement with [1], [3], [14], the reason can be explain due to they did not consider different cases like it has been demonstrated. Theses results can be used as a reference to calculate the k parameters for different length segmentation and overlap.

Also, we analyzed the mean squared error (MSE) values for each case to discovered the better combination between minimum MSE, theoretical FD, length segmentation and overlap. Theses results were presented in the Figure 1a, Figure 1b and Figure 1c to theoretical FD value of 1.3, 1.5 and 1.7 respectively. Figure 1a shows that overlap length does not have a positive effect over the calculate FD when increased the overlap length, the MSE increase as well. In the case of the 80% and 90% segmentation, the MSE get worse when we increased the overlap. In another hand, the minimum MSE value was reported with the 90% segmentation and without overlap. Figure 1b shows that overlap length not have a effect over the calculate of FD, because the MSE hold constant. In the case of the 80% and 90% segmentation, the MSE get worse when we increased the overlap but both reported the minimum MSE without overlap. Figure 1c shows that overlap length have a positive effect only over the 40% of segmentation, in another segmentation length, the overlap effect hold the same

MSE.

B. EEG analysis with better k parameter

In this section, the results of FD over brain signals are presented. In order to obtain the betters FD by means of the Higuchi's algorithm, we selected different parameters (k, segmentation, overlap) according of the results of section III.A. After several simulations, we selected a segmentation (10%), overlap (30%) and parameter $k = 44$.

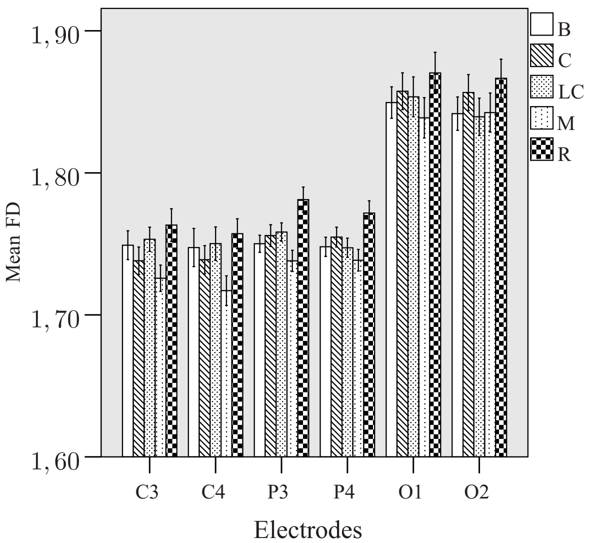


Fig. 2: The mean Fractal Dimension with $k = 44$ to the EEG signal versus the electrodes to the analysis of cognitive tasks. The bar chart show error bar that represent confidence interval (CI) of 95%.

According to the experimental instruction, the EEG database was analyzed and the Fractal dimension was computed and recorded with five different mental tasks. The results of the EEG are illustrated in a bar chart figure 2 that represents the average Fractal Dimension for each of the electrode. The x-axis represents each electrode: $C3$, $C4$, $P3$, $P4$, $O1$, $O2$, with bar to different texture related to a mental task. Also the bar chart show error bar that represent confidence interval (CI) of 95%.

The bar chart showed a FD into the range of 1.7 to 1.9 for all cognitive tasks and electrodes. The electrodes in the central part ($C3$, $C4$) and the parietal lobe ($P3$, $P4$) have the similar mean FD values. On the other hand, the electrodes in occipital lobe ($O1$, $O2$) have different mean FD values compared with the other electrodes. These results report that for the five mental cognitive tasks that the occipital lobe has brain oscillations more complex than the other brain areas. The highest dimensions observed in occipital channels ($O1$, $O2$) can also be a consequence of a lower SNR. Occipital electrodes are typically dominated by alpha rhythms; however, are strongly suppressed during cognitive attention tasks. The suppression of alpha rhythms is the principal contribution in the occipital channels, thus achieving lower SNR and providing an alternative explanation for high FD.

The statistical difference of different pair tasks calculated by analyzing the EEG signals are summarized in figure 3. In order to pursue the goal, we designed bar charts for each mental task pair. We added the symbol $*$ on the top of each pair bar when a statistical significance was smaller than $0.05 (p < 0.05)$, this means that it represent a statistical difference between two mental tasks.

The comparative analysis of the pair tasks (B, M), (B, C) and (LC, C) are shown in figure 3a, figure 3d and figure 3i respectively and only reported one differential statistic in electrodes $C3$, $O2$ and $O2$, meanwhile the pair task (B, LC) shown in figure 3b did not report any statistical difference. Three statistical differences were reported in the pair tasks (M, LC), (M, C) and (LC, R), shown in figure 3e, figure 3g and figure 3h. It can be seen in figure 3c that four statistical differences are reported in the pair task (B, R) in electrodes $C3$, $P3$, $P4$ and $O2$. Finally, the pair task (M, R) shown in figure 3f reported the best statistical difference to all six electrodes. Moreover, the task B showed a signicant difference from tasks M, R and C compared to other mental tasks.

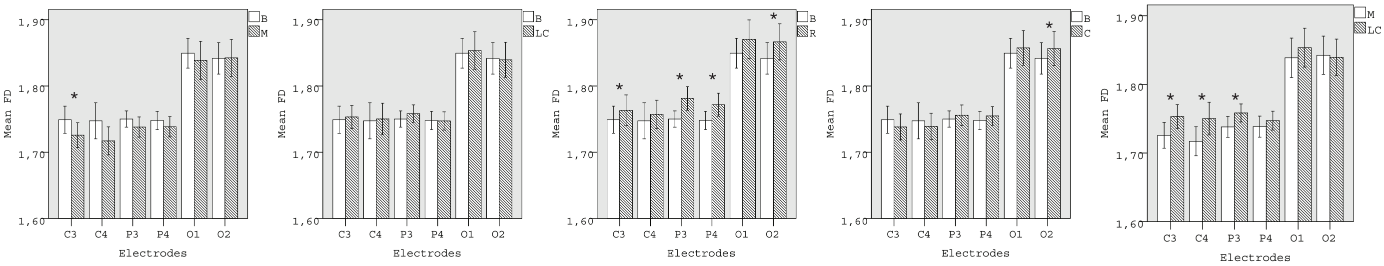
In summary, the EEG signals presented signicance difference among 9 pairs of mental tasks, getting more information from $P3$ to discriminate among 6 tasks pairs. The pairs of tasks MxR were more discriminated with 6 electrodes. Our results highlighted a signicance difference between 9 of 10 pairs of mental tasks, consequently the correct selection of k parameter allow us to obtain an increased signicant difference between the cognitive tasks.

IV. Conclusions

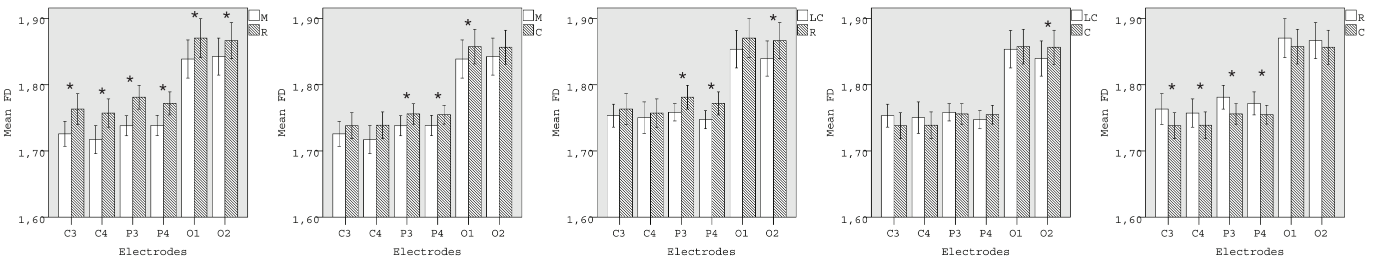
In summary, this study provides the better k parameters for different effects such as segmentation, overlap, add noise. These results can be used how guide for different studies to calculate the FD by means of the Higuchi's algorithm. To prove these results, we analyzed the EEG brain oscillations during the development of cognitive tasks, with the better k parameters. The EEG signals reported the presence of FD between 1 and 2, this result agrees with [1] is due to the dimension of a plane is equal to 2 and the dimension of a line is equal to 1. The EEG signal reported significance difference among 9 out of 10 pairs of mental tasks. In conclusion, the Fractal Dimension of the EEG signal is a useful tool for discriminating mental tasks. This approach can be applied to study different diseases effect above the neural brain like Alzheimer, Parkinson, and ObsessiveCompulsive disorder. Also, we could research the effect such as drugs, meditation, hypnosis and dream to build a brain-computer interface for subjects with motor disabilities.

Acknowledgment

The authors would like to thank PhD. R. Canahuire, PhD. H. Montes and MSc. D. Akamine for their help.



(a) EEG signal BxM (b) EEG signal BxLC (c) EEG signal BxR (d) EEG signal BxC (e) EEG signal MxLC



(f) EEG signal MxR (g) EEG signal MxC (h) EEG signal LCxR (i) EEG signal LCxC (j) EEG signal RxC

Fig. 3: The statistical analysis of the wilcoxon signed rank hypotesis to different pair tasks. The symbol $*$ on the top of each pair bar, represent the statistical significance when it is smaller than 0.05 ($p < 0.05$). The bar chart show error bar that represent confidence interval (CI) of 95%.

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