

Read: - Sec 1.1 - Sec 1.2 - Sec 1.4 - Sec 8.1

1. Let V be the

$$\mathbb{R}$$

- vector space of real valued continuous functions on the interval [

$$-1, 1$$

]. Which functionals are linear? Which are not and why? (Functionals just send a vector from the vector space to a value in the scalar field)

• a.

$$\int_{-1}^1 f(x) dx$$

• b.

$$\int_{-1}^1 f(x)^2 dx$$

• c.

$$f \mapsto f(0)$$

• d.

$$\int_{-1}^1 f(x) dx$$

• solutions:

• **linear transformation def:**

$$T(ax + by) = aT(x) + bT(y)$$

for all

$$x, y \in V$$

(vector space) and

$$\forall a, b \in$$

scalar field

• **affine def** $f(x) = ax + b$ is *affine* - not linear, it's a translation of something linear. **1a**

• S1: notice that this would be a linear functional because it sends a vector from the vector space (

$$f \in$$

) set of all continuous functions in $[-1, 1]$ to a *definite integral*, which is a scalar value in

$$\mathbb{R}$$

- if it were not a definite integral we would run into problems.

- QUESTION: is an indef integral a linear functional?

• S2: check additivity: f, g , in Vector space. what is $f+g$

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$$f + g = \int_{-1}^1 (f + g)(x) dx = \int_{-1}^1 f(x) + g(x) dx$$

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$$\int_{-1}^1 f(x) dx + \int_{-1}^1 f(x) dx$$