Read: - Sec 1.1 - Sec 1.2 - Sec 1.4 - Sec 8.1

1. Let V be the

$$\mathbb{R}$$

- vector space of real valued continuous functions on the interval [

$$-1, 1$$

]. Which functionals are linear? Which are not and why? (Functionals just send a vector from the vector space to a value in the scalar field)

- a.

$$\int_{-1}^{1} f(x)dx$$

- b.

$$\int_{-1}^{1} f(x)^2 dx$$

- c.

$$f \mapsto f(0)dx$$

- d.

$$\int_{-1}^{1} f(x)dx$$

- solutions:
- **linear transformation def:**

$$T(ax + by) = aT(x) + bT(y)$$

for all

$$x, y \in V$$

(vector space) and

$$\forall a, b \in$$

scalar field
- **affine def** f(x) = ax + b is *affine* - not linear, it's a translation of something linear. **1a**
- S1: notice that this would be a linear functional because it sends a vector from the vector space (

$$f \in$$

) set of all continuous functions in [-1,1] to a *definite integral* , which is a scalar value in

$$\mathbb{R}$$

  - if it were not a definite integral we would run into problems.
  - QUESTION: is an indef integral a linear functional?
- S2: check additivity: f,g, in Vector space. what is f+g

- $$f + g = \int_{-1}^{1} (f+g)(x)dx = \int_{-1}^{1} f(x) + g(x)dx$$
- $$\int_{-1}^{1} f(x)dx + \int_{-1}^{1} g(x)dx$$

* S3: check homogeneity: aT(v) = T(av)
  - $$a$$
    is a scalar.
    $$f$$
    is a vector in vector space.
  - $$a * T(f) = a \int_{-1}^{1} f(x)dx = \int_{-1}^{1} af(x)dx$$