Time series notes

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1 Chapter 1: Characteristics of Time Series

General Notes:

- Obvious correlation is introduced by the sampling of adjacent points in time can restrict the applicability of many conventional statistical methods
- Primary objective of time series analysis is to develop mathematical models that provide plausible descriptions for sample data
- If stochastic behavior of a time series can be explained by the white noise model $\rightarrow$ Classical statistical methods will suffice
- Moving average models 'smooths' out the time series
- Autoregression stems from a regression of x_t on its past or future values; hence, 'auto' regression
- Complete description of time series:
  - n random variables at arbitrary integer time points $t_1, t_2, \ldots t_n$, for any positive integer
  - Provided by the joint distribution: $F(c_1, c_2, \ldots, c_n) = P(x_{t_1} \leq c_1, x_{t_2} \leq c_2, \ldots, x_{t_n} \leq c_n)$
  - Evaluated as the probability that the values of the series are jointly less than the n constants, $c_1, c_2, \ldots, c_n$

Time Series Statistical Models

White Noise:

- Uncorrelated random variable, w_t, with mean 0 and finite variance σ_w^2
- $w_t \sim wn(0, \sigma_w^2)$

Moving Averages:

- Average of current value and its immediate neighbors in the past and future
- $v_t = \frac{1}{3}(w_{t-1} + w_t + w_{t+1})$

Autoregressions

- w_t as an input, x_t as an output of second-order equation
- $x_t = x_{t-1} - 0.9x_{t-2} + w_t$

Random walk w/ drift

- $x_t = \delta + x_{t-1} + w_t$; where δ is drift
- Cumulative sum: $x_t = \delta t + \sum\limits_{j=1}^{t} w_j$

Measures of Dependence: Autocorrelation and Cross-Correlation

Definition 1.1 The **mean function** is defined as:

$$\mu_{xt} = E(x_t) = \int_{-\infty}^{\infty} x f_t(x) dx$$

provided it exists, where E denotes the usual expected value operator.

- **Example 1.13** Mean function of a white noise series is 0
- **Example 1.14** Mean function of a random walk with drift is δt as r.w. w/ drift is $x_t = \delta t + \sum_{j=1}^{t} w_j$. The mean function is a straight line with a slope of δ

Definition 1.2 The **autocovariance function** is defined as the second moment product

$$\gamma_x(s,t) = cov(x_s, x_t) = E[(x_s - \mu_s)(x_t - \mu_t)]$$

for all s and t.

- Note that $\gamma_x(s,t) = \gamma_x(t,s)$ for all time points s and t
- Autocovariance measures the linear dependence between two points on the same series observed at different times
- Smooth time series = large autocov even when t and s are far apart; choppy series tend to have autocov functions that are nearly zero for large separations
- $\gamma_x(s,t) = 0$, x_s and x_t are not linearly related but still may be some dependence structure between them
- If they are bivariate normal then $\gamma_x(s,t) = 0$ ensures their independence TEST