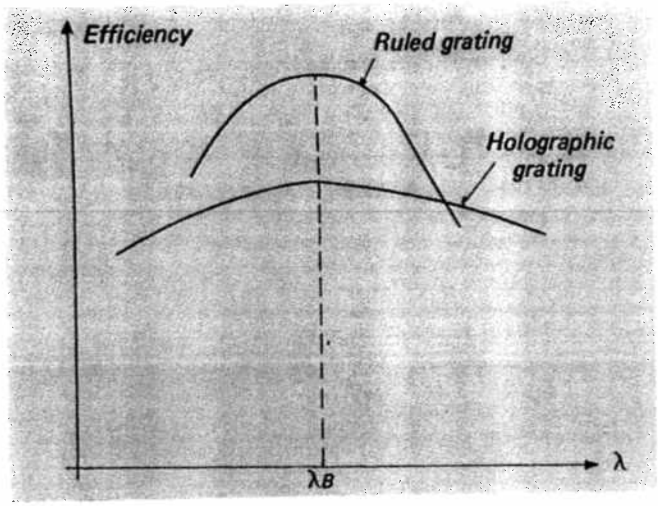

Fig. 3-5 The efficiency curve of a holographic grating is generally lower but flatter than that of a ruled grating if $0.2 \leqslant \lambda/a \leqslant 0.8$.

considered as the most important factor for an instrument, (see chapter IV paragraph I).

The presence of ghosts in ruled gratings is due to an error, periodical or random, in the groove spacing, (see chapter IV for discussion). However, **since holographic gratings are obtained by recording a perfect optical phenomenon with groove spacing absolutely constant, they cannot have ghosts.**

For the same reason, that portion of stray light due to a random error in groove position is non existant. The photograph 4-18 shows a comparison of stray light levels and ghosts levels for a ruled grating and a holographic grating. As a consequence of the absence of ghosts and the extremely lower levels of stray light, holographic gratings have a higher signal to noise ratio than ruled gratings (see chapter IV, K for discussion).

IV DIFFRACTION GRATING PHYSICS

A Diffraction grating formula

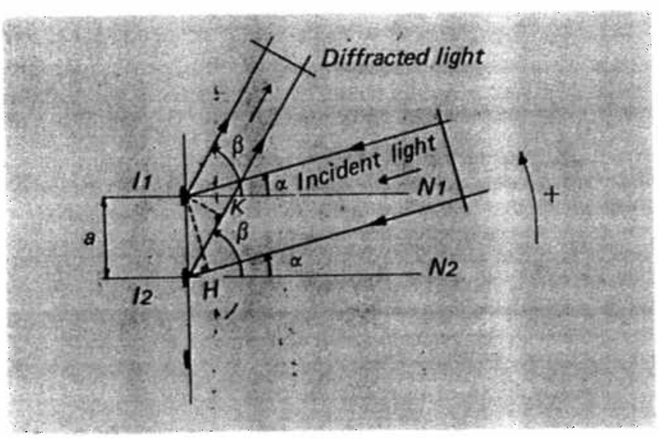

Fig. 4-1 Diffraction grating formula.

Consider the case of reflection gratings, (the angles are positive in the counterclockwise direction).

Shine a parallel beam of monochromatic light of wavelength λ.

Let α be the angle of this beam with respect to the normal.

Let β be the direction at which we observe the diffraction phenomenon.

Let a be the groove spacing.

Compute the path difference along two rays of light diffracted by successive reflective surfaces.

$$\Delta = \overline{I_2H} + \overline{I_2K} = a\,(\sin\alpha + \sin\beta) \quad (1)$$

Constructive interference phenomenon will be observed in the directions where $\Delta = k\lambda$, (k being a positive or negative integer).

Then the formula $\Delta = a\,(\sin\alpha + \sin\beta) = k\lambda$ gives, for each α, all the possible values of β where a maximum of intensity for wavelength λ is found (one maximum for each value of k).

$\sin\alpha + \sin\beta = \frac{k\lambda}{a}$ (2) is the diffraction grating equation.

(2) can be written $\sin\alpha + \sin\beta = k\,n\,\lambda$ (3) where $n = \frac{1}{a}$ is the number of grooves per unit length.

This formula shows that β depends on λ and, as a consequence, the light is dispersed by the grating. For each possible value of k we obtain a spectrum.

For $k = 0$ we obtain all the wavelengths in the same direction; there is no dispersion in order 0.

A grating is said to be used in the Littrow configuration when $\beta = \alpha$ (i.e. if we observe in the direction of illumination).

In this case the grating formula becomes $2\sin\beta = \frac{k\lambda}{a}$ (4)

In the case of transmission gratings the grating formula (with the same sign convention), is $\sin\alpha - \sin\beta = \frac{k\lambda}{a}$

B Free spectral range

For the same angle α and at the same angle of observation β several wavelengths are superimposed. These are all wavelengths for which $k\,n\,\lambda$ = constant or $k\lambda$ = constant.

Relation (3) could be written $\sigma = \frac{1}{\lambda} = \frac{k\,n}{\sin\alpha + \sin\beta}$

σ is called wave number $\sigma = |k|\,\sigma_1$ with $\sigma_1 = \frac{n}{\sin\alpha + \sin\beta}$

The superimposed radiations at angle β have wave numbers in an arithmetic progression: σ_1, $2\sigma_1$, $3\sigma_1$.

If we want to prevent the superimposition of the spectrums, it is necessary that the incident radiation be composed of vibrations of wave numbers contained in a maximum interval $\Delta\sigma_o$. $\Delta\sigma_o$ is called the free spectral range.

We see that the free spectral range is not exactly a characteristic of a grating because it depends on the working angular conditions, but the finer the grating spacing, the larger is the free spectral range.

C Resolution

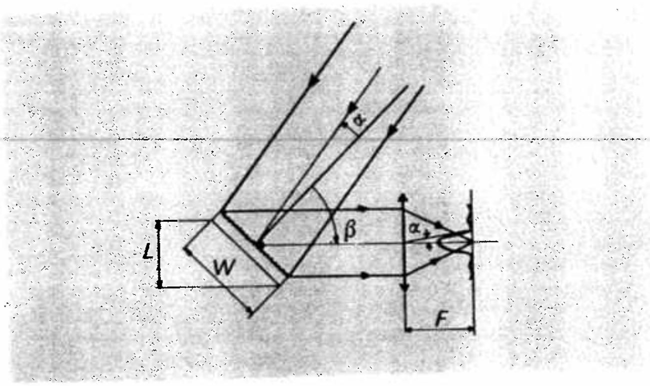

Fig. 4-2 Resolution.

The resolution of an optical instrument measures its ability to separate adjacent spectrum lines. It is generally defined by

$$R = \frac{\lambda}{\Delta\lambda}$$

d λ being the difference in wavelength between two equal intensity spectrum lines that are just separated.

Two peaks are considered resolved if the distance between them is at least such that the maximum of one falls at the first minimum of the other (Fig. 4-3). This is called Rayleigh criterion. The angular width of the diffraction image is d θ

$$d\theta = \frac{\lambda}{L} = \frac{\lambda}{W \cos\beta}$$

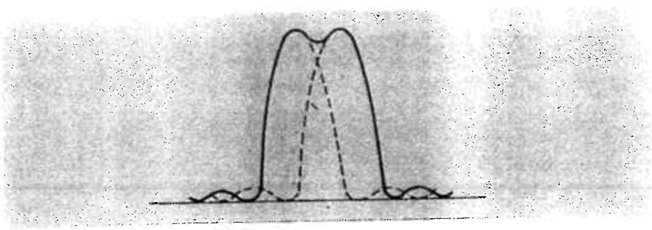

Fig. 4-3 Two peaks which are just resolved.

From (2) by differentiation one obtains

$$\cos\beta \, d\beta = \frac{k}{a} d\lambda$$

two wavelengths are separated if $d\beta = d\theta$

$$\frac{\lambda}{W \cos\beta} = \frac{k \, d\lambda}{a \cos\beta}$$

This gives $\frac{\lambda}{d\lambda} = \frac{k W}{a} = = k N$

N being the total number of grooves on the grating.

$$R = kN \quad (5)$$

This relation should be used very carefully.

Let us point out one of the dangers of this formula. Consider that, (because of instrument requirement), we have the width of the grating fixed in advance.

In order to increase the resolution one could think of increasing N, (a finer ruled grating), but one has to be aware that by doing this, certain orders disappear and the maximum possible value for k diminishes.

Resolution could also be expressed as follows:

$$R = \frac{a}{\lambda}(\sin\alpha + \sin\beta) N$$

$$R = \frac{W}{\lambda}(\sin\alpha + \sin\beta) \quad (6)$$

Formulas (5) and (6) are indeed identical. But (6) is by far safer if not easier to use.

We see that the resolution depends upon the width of the grating, upon the working angle conditions, and upon the wavelength.

Another possible expression of resolution is obtained by expressing kN with the aid of (1).

$$R = \frac{a(\sin\alpha + \sin\beta)}{\lambda} N = \frac{N\Delta}{\lambda} = \frac{\Delta \text{ total}}{\lambda}$$

$$R = \frac{\Delta \text{ total}}{\lambda}$$

Δ total being the optical path difference between the extreme rays on the grating.

The preceeding results assumed that the wave diffracted by the grating was a perfect plane wave. If the wave departs from a plane wave by more than $\lambda/4$ the diffraction image angular width will increase and the resolution will drop sharply. This is the reason why Jobin-Yvon gratings are all controlled on a Michelson interferometer. All gratings departing by more than $\lambda/4$ from the ideal wavefront are rejected. The photo (Fig. 4-4), shows an interferogram.

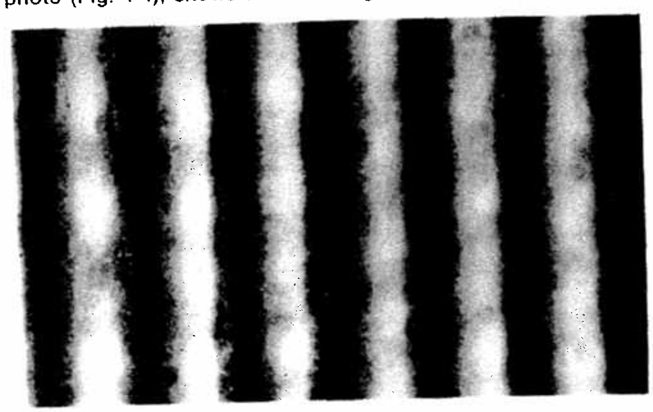

Fig. 4-4 Interferogram obtained on the Michelson interferometer.

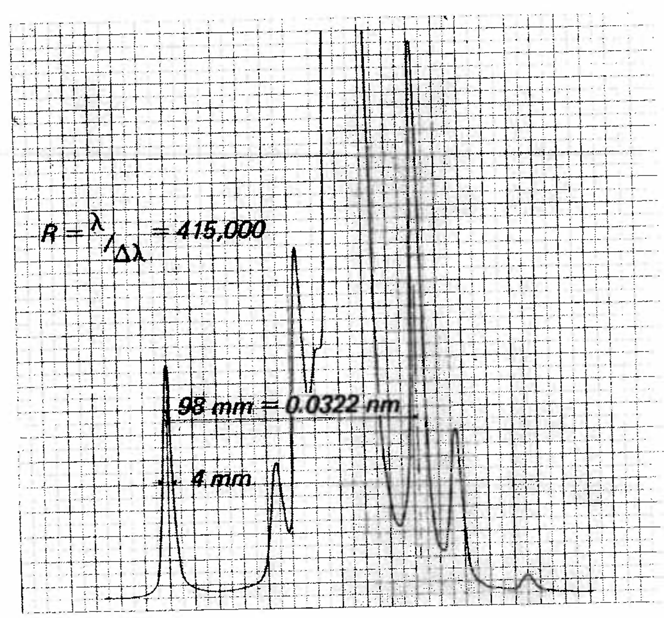

Fig. 4-5 A THR (1.5 meter J-Y monochromator) scan of the 546 nm, mercury line, with a resolution of 415,000 (Non cooled, low pressure, Phillips lamp).

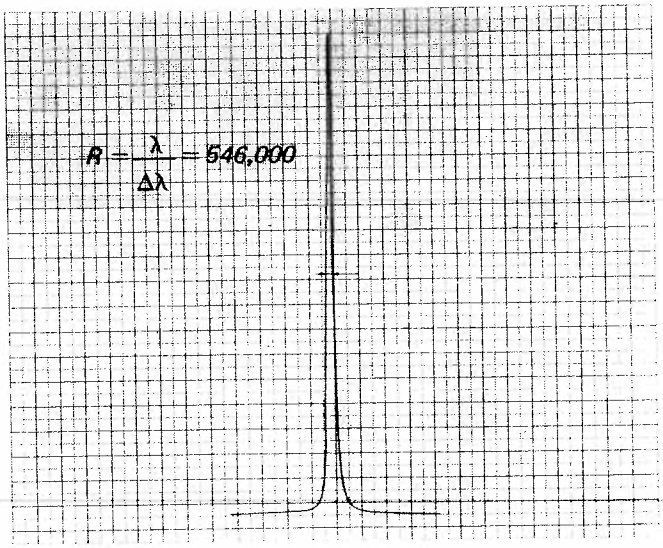

Fig. 4-5 B THR monochromator scan of the 546 nm mercury line with a resolution of 546.000 (isotopic source).

Resolution can also be actually measured by obtaining a direct recording of a well known complex spectral line (for example the 546 nm mercury line) and literally computing

$R = \lambda / \Delta\lambda$ as in Fig. 4-5.

D Angular Dispersion

Angular Dispersion is the angular separation $d\beta$ obtained for two different radiations separated by $d\lambda$.

From differentiation of equation (2), α being a constant, one obtains

$$\cos\beta \, d\beta = \frac{k}{a} d\lambda$$

and

$$\frac{d\beta}{d\lambda} = \frac{k}{a\cos\beta} = \frac{kn}{\cos\beta} \quad (7)$$

Angular dispersion could also be written

$$\frac{d\beta}{d\lambda} = \frac{1}{\lambda} \frac{(\sin\alpha + \sin\beta)}{\cos\beta} \quad (8)$$

As in paragraph C it should be pointed out that (7) should be used carefully because k and n are not independant variables. Formula (8) shows clearly that for a given wavelength dispersion depends only on the working angular conditions.

High dispersions are obtained for large diffraction angles and these are associated with large blaze angles. For a given order and given wavelength this leads to finely ruled gratings. One has to be careful because if the spacing is sufficiently fine the wavelength to be studied may vanish in the order considered.

E Linear Dispersion

The linear dispersion of a grating system is the reciprocal of the product of the angular dispersion by the effective focal length. It measures the number of nanometers that will be found in the unit length of the spectrum.

$$\frac{d\lambda}{dx} = \frac{a\cos\beta}{kF}$$

$$\frac{d\lambda}{dx} \text{ nm/mm} = \frac{10^6 \cos\beta}{k\,n\,F}$$

It is to be noted that an instrument with a linear dispersion of 0.4 nm/mm has a better linear dispersion than an instrument with linear dispersion of 0.6 nm/mm: if the number (in nm/mm) measuring the linear dispersion decreases, the linear dispersion improves.

F Energy distribution

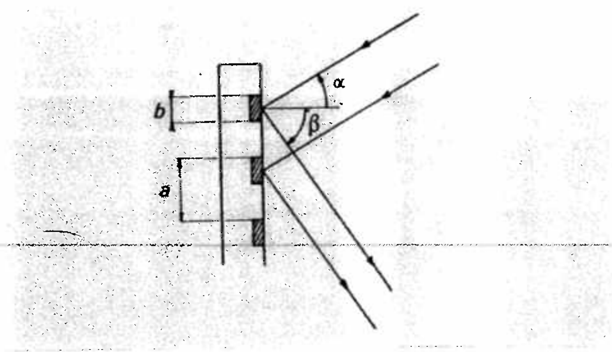

Fig. 4-6 Parameters for energy distribution computation.

We want to compute the intensity of the diffracted wave in all possible directions. In order to obtain the amplitude of the diffracted wave we have to add all the elementary waves diffracted by each reflector, (groove).

The phase difference between two consecutive diffracted waves is, (see chapter IV A)

$$\varphi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi a}{\lambda}(\sin\alpha + \sin\beta)$$

If we assume all the elementary waves to have the same amplitude, then total amplitude becomes:

$$A = \sum_{p=1}^{p=N} A_0 \, e^{-ip\varphi} = A_0' \frac{\sin\frac{N\varphi}{2}}{\sin\frac{\varphi}{2}}$$

The intensity of the diffracted wave is

$$I(\alpha, \beta) = I_0 \frac{\sin^2\left[\frac{W\pi}{\lambda}(\sin\alpha + \sin\beta)\right]}{\sin^2\left[\frac{\pi a}{\lambda}(\sin\alpha + \sin\beta)\right]}$$

with W = Na, the width of the grating.

We see that we have for a fixed α a certain number of directions β for which the intensity is maximum. These directions are obtained for $\varphi = 2k\pi$ or for $\sin\alpha + \sin\beta = \frac{k\lambda}{a}$ and lead us to the grating formula.

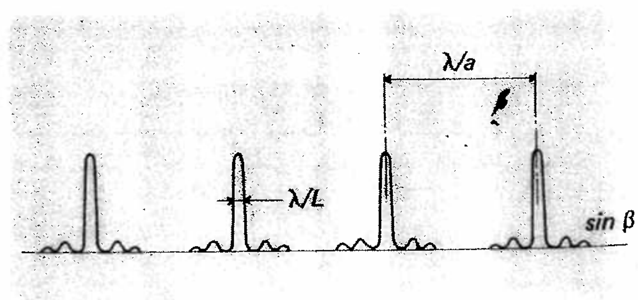

Fig. 4-7 Energy distribution for monochromatic incident radiation.

Fig. 4-7 shows for a fixed α the appearance of the energy distribution if the incident radiation is monochromatic. Each peak corresponds to an order.

If we now introduce the fact that each reflector, (groove), has a width 'b' we find

$$I(\alpha,\beta) = I_0 \frac{\sin^2\left[\frac{\pi b}{\lambda}(\sin\alpha + \sin\beta)\right]}{\left[\frac{\pi b}{\lambda}(\sin\alpha + \sin\beta)\right]^2} \cdot \frac{\sin^2\left[\frac{W\pi}{\lambda}(\sin\alpha + \sin\beta)\right]}{\sin^2\left[\frac{\pi a}{\lambda}(\sin\alpha + \sin\beta)\right]}$$

These calculations are valid only for scalar approximation, that is for λ/a and $\lambda/b < 0.2$.

We see in this way that the diffraction peaks corresponding to each order are going to be within one envelope whose width is $\frac{\lambda}{b}$ (Fig. 4-8) $\left(\frac{\lambda}{b}\text{ is much larger than }\frac{\lambda}{W}\right)$

All orders haves a part of the energy. The order 0 (the one that does not disperse light) has most of the energy.

G Blaze

In order to obtain a better use of the grating one can think of concentrating spectral energy into any one of the orders (excepted the order 0). It has been known for a long time that the distribution of energy among orders depended on groove shape. The principle is to rule the grating so that the reflecting elements, (grooves), are tilted with respect to the grating surface (Fig. 4-9). In this way the envelope curve of Fig. 4-8 shifts.

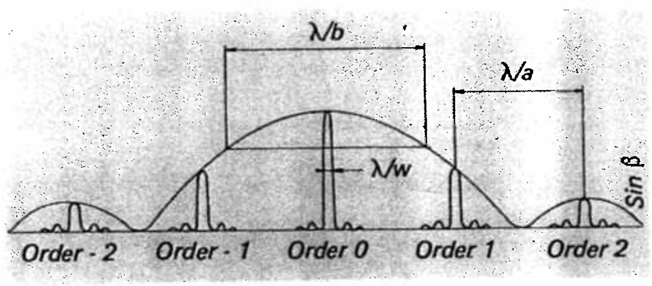

Fig. 4-8 Blazing of a reflection grating.

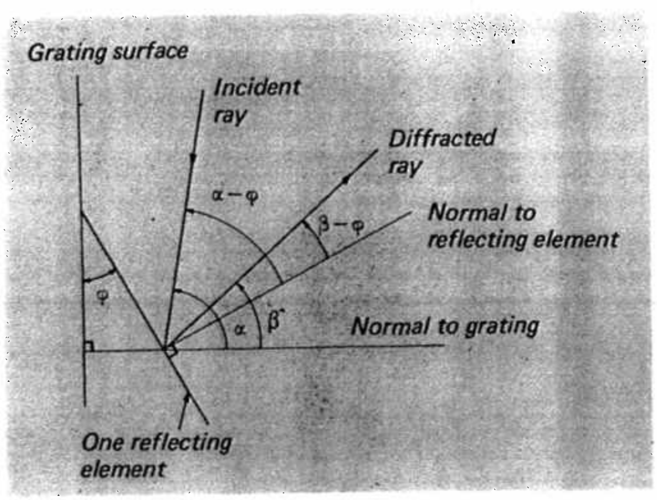

Fig. 4-9 Angular parameters for computation of blaze efficiency.

Let φ be the angle of the reflecting elements with respect to the grating surface. Under these conditions

$$I(\alpha,\beta) \quad I_0 \frac{\sin^2\left[\frac{\pi b}{\lambda}(\sin(\alpha-\varphi) + \sin(\beta-\varphi))\right]}{\left(\frac{\pi b}{\lambda}\right)^2 [(\sin(\alpha-\varphi) + \sin(\beta-\varphi)]^2} \cdot \frac{\sin^2\left[\frac{W\pi}{\lambda}(\sin\alpha + \sin\beta)\right]}{\sin^2\left[\frac{\pi a}{\lambda}(\sin\alpha + \sin\beta)\right]}$$

In the case where $b = a\cos\varphi$, (i.e. the groove profile is rectangular), and in the case of the Littrow configuration, if φ is selected in the proper manner, the energy distribution is indicated on Fig. 4-10. This assumes that $\beta - \varphi$ and $\alpha - \varphi$ are small angles. All orders extinguish and the total available energy is concentrated in one of the orders. Fig. 4-11 shows the corresponding working conditions and groove shape ($b = a\cos\varphi$, $\alpha = \beta = \varphi$). All previous results are true for only one wavelength. This wavelength is called the blaze wavelength. The angle φ depends on the blaze wavelength.

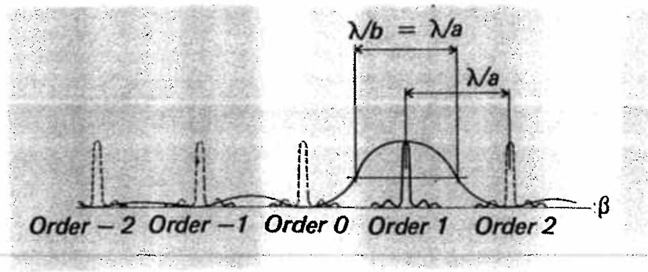

Fig. 4-10 Blaze in first order.

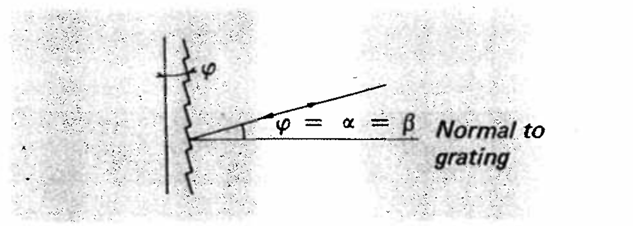

Fig. 4-11 Working angle conditions for Littrow use of a grating at blaze wavelength

φ is called the blaze angle. This angle is the angle of the reflecting elements with respect to the grating surface. It is also the angle of incidence at which one has to work in order to get the blaze effect.

The relation between φ and λ is simply obtained by applying the grating formula

$$\sin\varphi + \sin\varphi = \frac{k\lambda_B}{a}; \qquad 2a\sin\varphi = k\lambda_B \qquad (8)$$

λ_B being the blaze wavelength in order k. This relation gives the angle φ corresponding to λ_B. It is obvious from relation (8) that a grating blazed for λ_B in first order is also blazed for $\frac{\lambda_B}{2}, \frac{\lambda_B}{3}, \ldots$ in the 2nd, 3rd ... orders.

General convention: when speaking of a grating blazed at λ_B it is implied that the grating is blazed in first order for this wavelength and for Littrow use.

H Efficiency

$$\text{Absolute efficiency} = \frac{\text{energy diffracted by the grating at wavelength } \lambda \text{ in the order of interest}}{\text{energy into the grating at wavelength } \lambda}$$

$$\text{efficiency} = \frac{\text{energy diffracted by the grating at wavelength } \lambda \text{ in order of interest}}{\text{energy reflected by a mirror under the same working conditions}}$$

The number measuring absolute efficiency is always smaller than the number measuring efficiency. In fact as a rough estimate we can write:

Absolute efficiency = efficiency × reflecting power of mirror (with the same coating as the grating and working in the "same" angular conditions).

When speaking of the efficiency of a grating at blaze wavelength one implies working under Littrow conditions. The certificate issued with each J.Y. grating gives efficiency of the