# Tunneling decay of false kinks

Éric Dupuis[a],[*] Yan Gobeil[a],[†] Richard MacKenzie[a],[‡]

Luc Marleau[b],[§] M. B. Paranjape[a],[¶] and Yvan Ung[a**]

[a] *Groupe de physique des particules, Université de Montréal,*

*C. P. 6128, Succursale Centre-ville,*

*Montreal, QC, Canada, H3C 3J7 and*

[b] *Département de physique, de génie physique et d'optique,*

*Université Laval, Québec, QC, Canada G1K 7P4*

## Abstract

We consider the decay of "false kinks," that is, kinks formed in a scalar field theory with a pair of degenerate symmetry-breaking false vacua in 1+1 dimensions. The true vacuum is symmetric. A second scalar field and a peculiar potential are added in order for the kink to be classically stable. We find an expression for the decay rate of a false kink. As with any tunneling event, the rate is proportional to $\exp(-S_E)$ where $S_E$ is the Euclidean action of the bounce describing the tunneling event. This factor varies wildly depending on the parameters of the model. Of interest is the fact that for certain parameters $S_E$ can get arbitrarily small, meaning the kink is only barely stable. Thus, while the translation-invariant false vacuum may be very long-lived, the presence of kinks can give rise to rapid vacuum decay.

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[*] eric.dupuis.1@umontreal.ca

[†] yan.gobeil@umontreal.ca

[‡] richard.mackenzie@umontreal.ca

[§] luc.marleau@phy.ulaval.ca

[¶] paranj@lps.umontreal.ca

[**] yvan.ung@umontreal.ca

## I. INTRODUCTION

Kinks are topological solitons in 1+1-dimensional field theories with a real scalar field $\phi$ and spontaneously-broken discrete symmetry $\phi \to -\phi$. The potential has degenerate vacua at field values $\phi = \pm v$; the kink interpolates between the two. The same model in higher dimensions gives rise to extended objects: linelike defects in 2+1 dimensions, domain walls in 3+1 dimensions, and so on.

We are interested in models which "demote" the vacua at $\pm v$ to *false vacuum* status, there being a lower-energy *true vacuum* at $\phi = 0$. A specific example is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V_1(\phi)$$

where (see Fig. 1)

$$V_1(\phi) = \lambda(\phi^2 - \delta v^2)(\phi^2 - v^2)^2 \tag{1}$$

with $0 < \delta < 1$. In the true vacuum the symmetry is of course restored. The motivation is that the presence of topological defects can have a dramatic effect on the quantum mechanical stability of the false vacuum. Using cosmological language for convenience, if the universe is in a false vacuum throughout space, it will decay through quantum tunneling [1], yet the decay rate per unit volume can be exceedingly small, to the point where the observable universe could be trapped in a false vacuum for times exceeding the age of the universe. Such a scenario is invoked in certain models of fundamental physics; see for instance [2]. If so, the presence of topological defects can have a dramatic effect on the decay. This possibility has been examined previously for magnetic monopoles [3], vortices in 2+1 dimensions [4], and cosmic strings [5].

Since there is a unique true vacuum in this model, there are no classically stable nontrivial static solutions. For instance, a kinklike configuration interpolating between the two false vacua would bifurcate into two halves, one interpolating between $\phi = -v$ and $\phi = 0$ and the other between $\phi = 0$ and $\phi = v$. The potential energy density being lower between the two halves of the kink than in the exterior region, the kinks would essentially repel each other; they would fly off to spatial infinity at speeds approaching that of light, leaving a gas of particles in their wake.

Thus we must consider a more complicated model if we wish to study the decay of classically stable kinks with symmetry-breaking false vacua. The paper is outlined as follows.

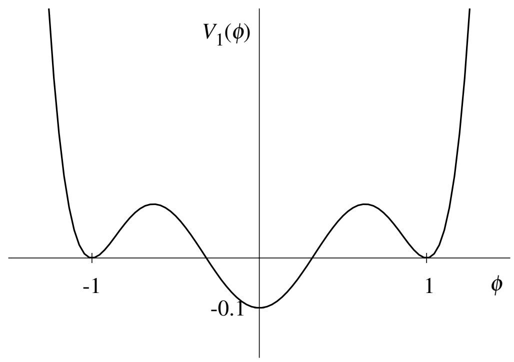


FIG. 1. Example potential (1) with symmetric true vacuum and symmetry-breaking false vacua. Here $\lambda = v = 1,\ \delta = 0.1$.

In Section 2, we introduce a generalized model and argue that it does have classically stable kinks. In Section 3, we find numerical solutions for a variety of parameters. These solutions, although classically stable, will tunnel to the true vacuum, a process analyzed in Section 4. The comparison between kink-mediated vacuum decay and ordinary vacuum decay is analyzed in Section 5. We then present conclusions and suggestions for further work.

## II. A MODEL WITH CLASSICALLY STABLE KINKS

As argued above, classically stable kinks do not exist in the simplest model in which the vacuum structure is as outlined above (two symmetry-breaking false vacua, symmetric true vacuum). One way to obtain stable kinks is to add a second scalar field to the model with an unusual potential. Consider the following model, which is to be viewed more as an example having the desired vacuum structure and stable kinks rather than as a realistic model for a particular physical system.

$$\mathcal{L} = \frac{1}{2}\left((\partial_\mu\phi)^2 + (\partial_\mu\chi)^2\right) - V(\phi,\chi)$$

where

$$V(\phi,\chi) = \lambda_1(\phi^2 - \delta_1 {v_1}^2)(\phi^2 - {v_1}^2)^2 + \frac{\lambda_2}{\phi^2 + \gamma v_1^2}\left[(\chi^2 - v_2)^2 - (\delta_2/4)(\chi - 2v_2)(\chi + v_2)^2\right]$$

The potential has seven parameters, two of which can be eliminated by rescaling the fields and $x$ to dimensionless variables. Doing so, we can rewrite the (dimensionless) Lagrangian as follows, having chosen the constants in such a way as to simplify the equations of motion:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\alpha}{2\beta}(\partial_\mu \chi)^2 - V(\phi, \chi) \tag{2}$$

where

$$V(\phi,\chi) = (\phi^2 - \delta_1)(\phi^2 - 1)^2 + \frac{\alpha}{\phi^2 + \gamma}\left[(\chi^2 - 1)^2 - \frac{\delta_2}{4}(\chi - 2)(\chi + 1)^2\right]. \tag{3}$$

There are now five parameters, $\alpha, \beta, \gamma, \delta_1, \delta_2$, which we take to be positive; furthermore, we suppose $0 < \delta_1 < 1$ and $0 < \delta_2 < 16/3$ in order for the potential to have the properties we desire.

The potential is the sum of two terms. The first term depends on $\phi$ only and is, apart from rescaling, the potential (1) of the original model (see Fig. 1). The second term is a product of two factors. The second of these (in square parentheses, written $V_2(\chi)$ below; see Fig. 2) depends on $\chi$ only and has two minima: a global minimum, of zero energy density, at $\chi = -1$ and a local minimum, of energy density $\delta_2$, at $\chi = +1$. The first factor can be viewed as a modulating function which varies the "strength" of the second factor depending on the value of $\phi$. In particular, if $\phi = 0$ (its true vacuum), the modulating factor is maximal, so that if $\chi$ passes from one minimum to the other, the cost in potential energy where $\chi \simeq 0$ will be large.

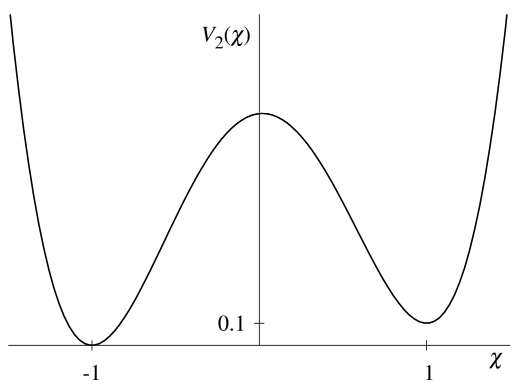


FIG. 2. Last factor of the potential (3). Here $\delta_2 = 0.1$.

The relevant features of the potential are as follows. The true vacuum of the model is $(\phi, \chi) = (0, -1)$; its energy density is $V(0, -1) = -\delta_1$. There are false vacua at $(\phi, \chi) = (\pm 1, -1)$, of energy density $V(\pm 1, -1) = 0$, as well as a number of other false vacua which do not concern us. There is also a maximum in the vicinity of $(\phi, \chi) = (0, 0)$ which can be quite pronounced. These features are displayed in Fig. 3.

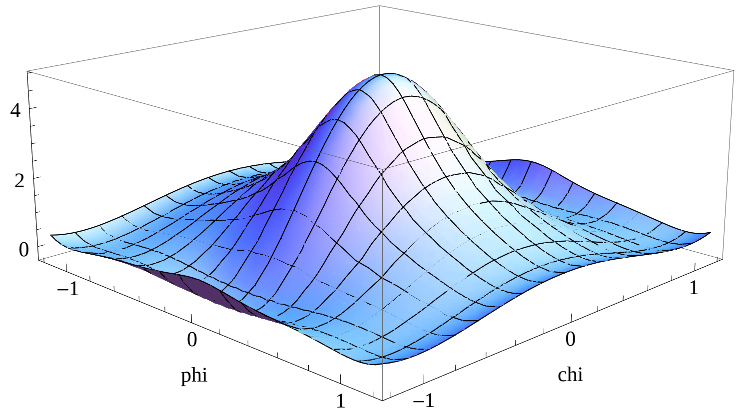


FIG. 3. (color online) 3-dimensional plot of $V(\phi, \chi)$ for $\alpha = 1, \delta_1 = 0.3, \delta_2 = \gamma = 0.1$. The true vacuum is at $(\phi, \chi) = (0, -1)$; false vacua are at $(\phi, \chi) = (\pm 1, -1)$. The energy is maximal, and can be very large, near $(\phi, \chi) = (0, 0)$.

We can understand qualitatively why there could be stable solitons interpolating between the two false vacua $(\phi, \chi) = (\pm 1, -1)$. Consider a configuration of the form illustrated in Fig. 4, where $\phi$ has half-kinks at $\pm l_\phi$, one interpolating between $\phi = -v$ and $\phi = 0$ and the other between $\phi = 0$ and $\phi = v$, these being "enveloped" by a kink-antikink of $\chi$ at $\pm l_\chi$. (To simplify the discussion we will call all of these objects kinks.) There are five regions where the fields are approximately constant, two pairs of which are related by symmetry; these regions are denoted (a) (between the two $\phi$ kinks), (b) (the regions between the $\phi$ and $\chi$ kinks), and (c) (exterior to the $\chi$ kinks). In these regions the energy density comes entirely from the potential energy and is easily evaluated:

$$V_{(a)} = -\delta_1 + \frac{\alpha}{\gamma}\delta_2, \qquad V_{(b)} = \frac{\alpha}{1+\gamma}\delta_2, \qquad V_{(c)} = 0.$$

It is easy to see that the configuration depicted in Fig. 4 cannot be a solution. Consider a family of such configurations parameterized by $l_\phi,\ l_\chi$, where the positions but not the shapes of the transitions vary. For the configuration to be a solution, its energy must be stationary as a function of $l_\phi,\ l_\chi$. For small displacements $\Delta l_\phi,\ \Delta l_\chi$, the variation in energy is

$$\Delta E = 2(V_{(a)} - V_{(b)})\Delta l_\phi + 2(V_{(b)} - V_{(c)})\Delta l_\chi.$$

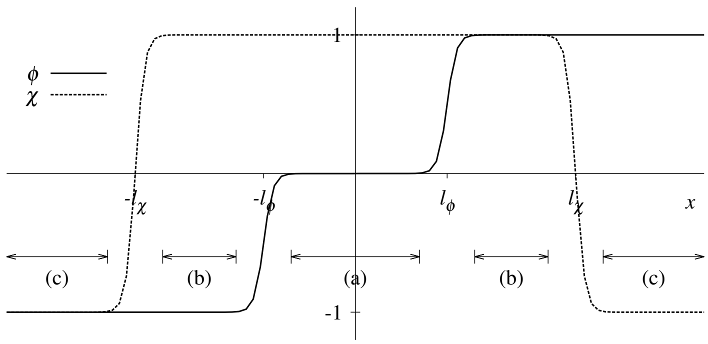


FIG. 4. Configuration to illustrate intuitively the existence of stable solitons.

Since $V_{(c)} < V_{(b)}$, the energy is *not* stationary in $l_\chi$; indeed, if left to evolve dynamically, the two $\chi$ kinks would move towards the origin to reduce the static energy. The situation with the $\phi$ kinks is less clear because, depending on the parameters of the model, $V_{(b)}$ could be larger or smaller than $V_{(a)}$.

This variational argument is not powerful enough in itself to determine if a configuration such as that depicted in Fig. 4 would evolve into a stable one because as soon as the kinks overlap they interact and the energy is no longer a straightforward function of $l_\phi,\ l_\chi$. If the configuration can be deformed to an unstable one without encountering an energy barrier along the way, it will not evolve into a stable one. One obvious way to go from Fig. 4 to an unstable configuration would be to deform $\chi$ to a constant, $\chi(x) = -1$ (after which the $\phi$ kinks will fly apart, as described above). This could be done by moving the $\phi$ kinks towards one another so that they annihilate, or by deforming the value of $\chi$ between the kinks from $+1$ to $-1$. In either case, the fields pass through the large potential energy barrier at $(\phi, \chi) \simeq (0, 0)$.

Of course, this argument is merely suggestive of the existence of stable solitons. We have solved the equations of motion numerically and find that stable solutions do indeed exist. This is described in the following section.

## III. NUMERICAL SOLUTIONS

The static equations of motion that follow from (2) are:

$$\phi'' - 2\phi(\phi^2-1)(3\phi^2-1-2\delta_1) + \frac{2\alpha\phi}{(\phi^2+\gamma)^2}\left[(\chi^2-1)^2 - \frac{\delta_2}{4}(\chi-2)(\chi+1)^2\right] = 0 \quad (4)$$

$$\chi'' - \frac{\beta}{(\phi^2+\gamma)}(\chi^2-1)(4\chi - 3\delta_2/4) = 0 \quad (5)$$

We look for solutions interpolating between the false vacua $(\phi,\chi) = (\pm 1, -1)$. We expect $\phi$ to be odd and $\chi$ even under space reflection, so we can solve the equations on the half-line using the following boundary conditions:

$$\phi(x) \to 0, \quad \chi'(x) \to 0 \quad \text{as} \quad x \to 0, \quad (6)$$

$$\phi(x) \to 1, \quad \chi(x) \to -1 \quad \text{as} \quad x \to \infty. \quad (7)$$

## IV. TUNNELING

## V. KINK-MEDIATED VACUUM DECAY?

## VI. CONCLUSIONS

### ACKNOWLEDGEMENTS

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