Summary: Types of Error

- Unitary errors (including leakage and cross-talk) due to gates, interactions.

How does this scale up (meet resonance conditions for misc. higher-order photon exchange processes of multiple qubits $\rightarrow$ higher-weight errors).

- Stochastic errors: amplitude-damping T_1 and dephasing T_2. Dimensionless figure of

merit: $\frac{T_{gate}}{T_1}$ and $\frac{T_{gate}}{T_2}$ with $T_{gate} = 100\ nanosec, T_i = 10\mu sec$. gives 1% error rate or less due to stochastic noise during gates.

- Measurement error (e.g. 92.5% fidelity) and measurement duration ($O(100) nanosec$) and resonator reset.

One read-out resonator per qubit.

- Leakage not immediately dealt with by QEC.

- Unitary errors, merely systematic...? What is their figure of merit 'error rate'?

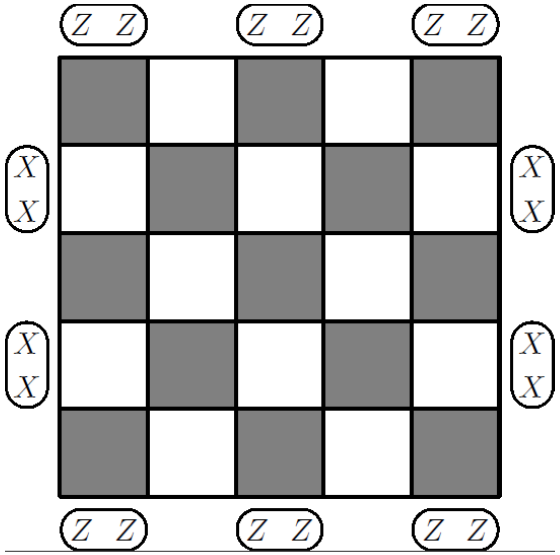
Overrotation such that $F = |\langle\varphi_{error}|\varphi\rangle| = \cos(\theta) \approx 1 - \frac{\theta^2}{2}$ and θ is error amplitude. Noise threshold value in principle pertains to error amplitudes (not amplitude^2 = probability), e.g. Terhal & Burkard, PRA 2005.

- Non-Pauli stochastic or non-stochastic errors approximated by Pauli errors (easy in simulation).

See e.g. Terhal, Quantum Error Correction for Quantum Memories, Rev. Mod. Phys. (2015)

Quantum Error Correction

Surface code for storing a logical qubit

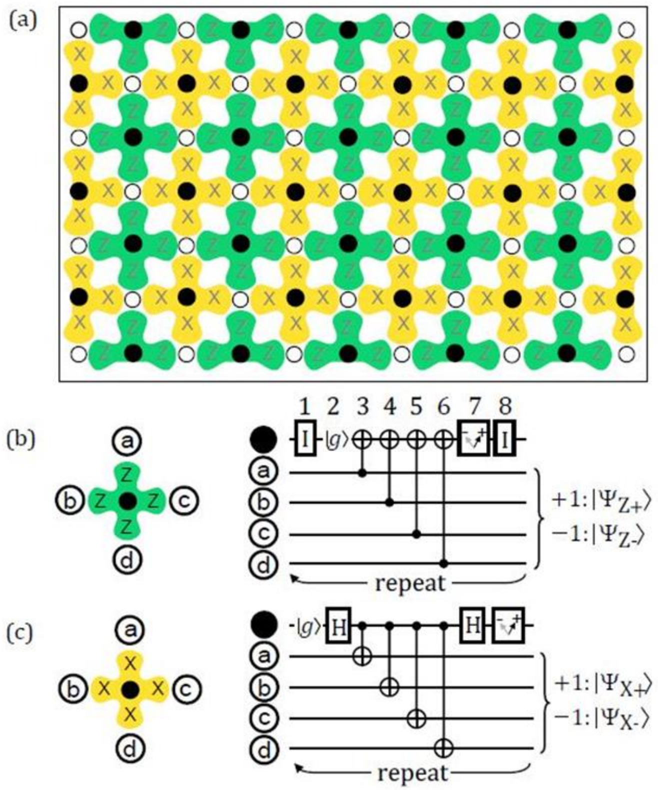


$$[[n = d^2, k = 1, d]]$$

code

d=3 Surface-17

Here d=6



Measure of success?

- Get a encoded qubit with a longer lifetime τ ($F(t) \approx e^{-t/\tau}$). But how fast are the encoded gates, t_{gate} (QEC slows things down)? Improve $\frac{t_{gate}}{\tau}$!
- Show (for surface code) that encoded qubit has $P_{error}\big(t = O(d)\big) = c_1 e^{-c_2 d}$

Noise Threshold for Toric Code

Assume we measure syndrome

by noisefree circuits.

Assume independent X

and Z errors (probability

p of Z error on each qubit and

prob. p of X errors in one time-step)

Decode after every step (instantly).

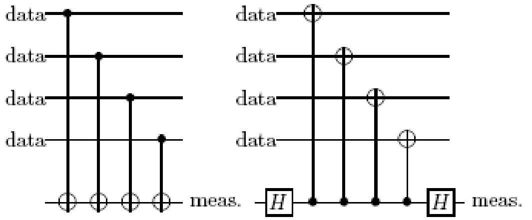


FIG. 14. Circuits for measurement of the plaquette ($Z^{\otimes 4}$) and site ($X^{\otimes 4}$) stabilizer operators.

Critical or threshold noise rate p_c is 11% using minimum weight decoding. Very high!

Noisy Error Correction

But errors can occur in CNOT gates, H gate, measurement and preparation.

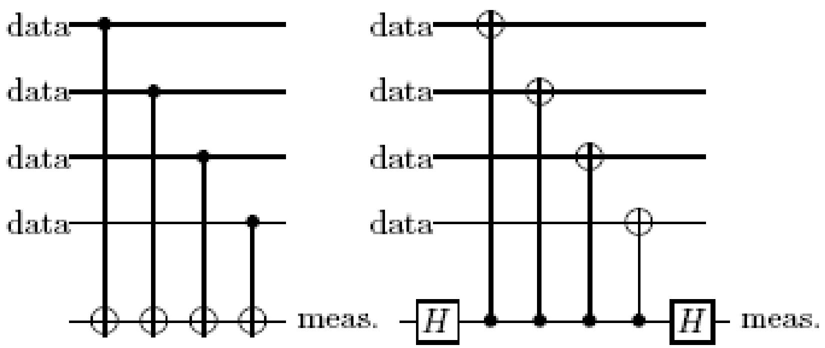


FIG. 14. Circuits for measurement of the plaquette ($Z^{\otimes 4}$) and site ($X^{\otimes 4}$) stabilizer operators.

Assume error probability p per step

Two effects:

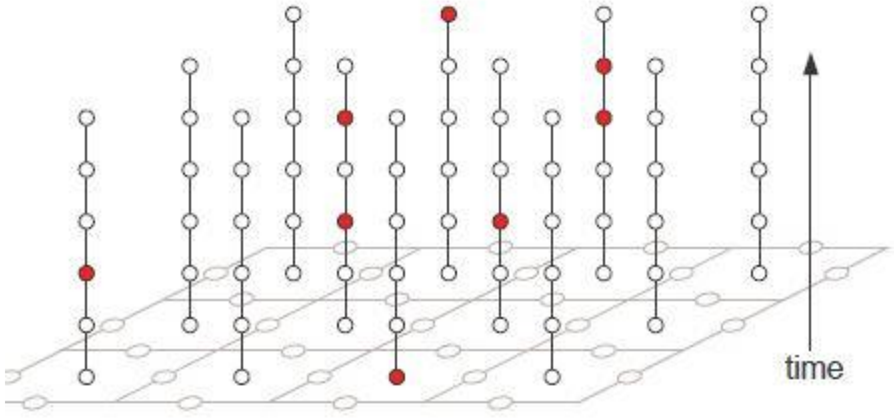
- Error on ancilla can propagate to 4 data qubits.

(Leaked ancilla qubit can spread damage to data qubits).

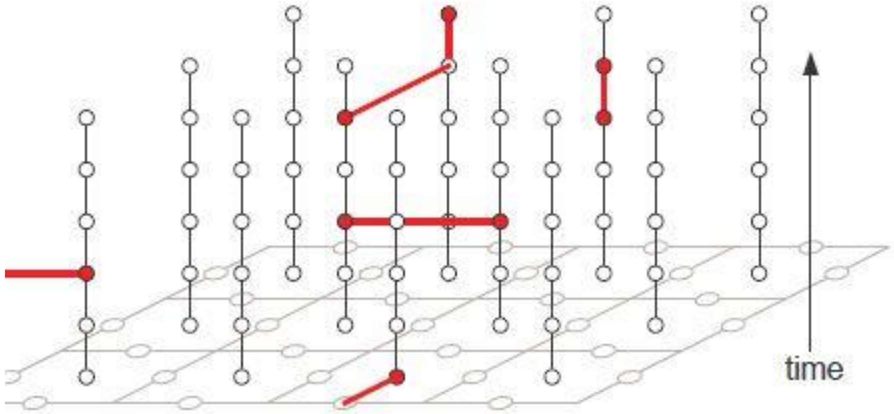
- Erroneous CNOTs, measurement etc. can lead to faulty error syndrome. (Leaked data qubit can mess up syndrome for error correction cycles until data qubit is put back in code space).

Solution: repeat measurement L times and decode using L measurements….How does this work? (Leakage requires different treatment).

Decoding in 3D



Measurement of plaquette operators.
At red dots the syndrome value changes



Errors that could have caused such syndrome.
Vertical errors are syndrome measurement errors. So again we need to find a minimum-weighted (error) chain which connects the red defects.

Surface Code with Noisy EC

Easy scaling behavior of logical error probability p_L (probability of logical error per QEC cycle)

- Logical error $p_L \sim N\, p^{t+1}$ when code can correct t errors (distance d=2t+1), *N* is some (combinatorial) factor

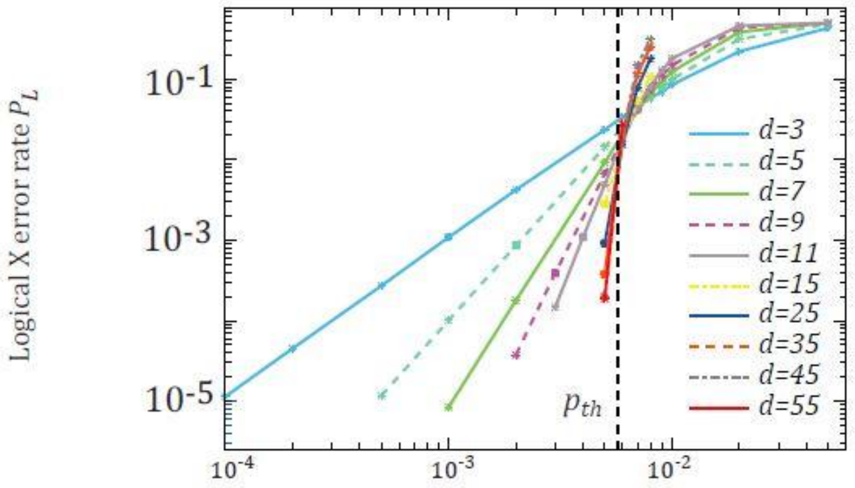
- Threshold p_c set by $p_c = N\, {p_c}^{t+1}$.

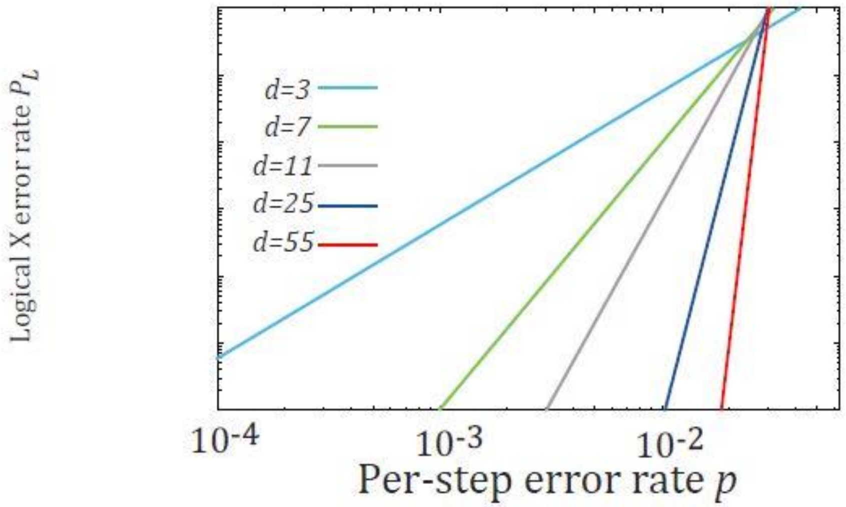
Thus $p_L \sim p_c \left(\frac{p}{p_c}\right)^{(d+1)/2}$ for odd distance d

Numerical fit for surface code (Fowler, Mariantoni *et al.*, PRA 2012*):*

$p_L \cong 0.03\, (\frac{p}{p_c})^{(d+1)/2}$ for odd distance *d=L*

with $p_c \approx 0.57\%$





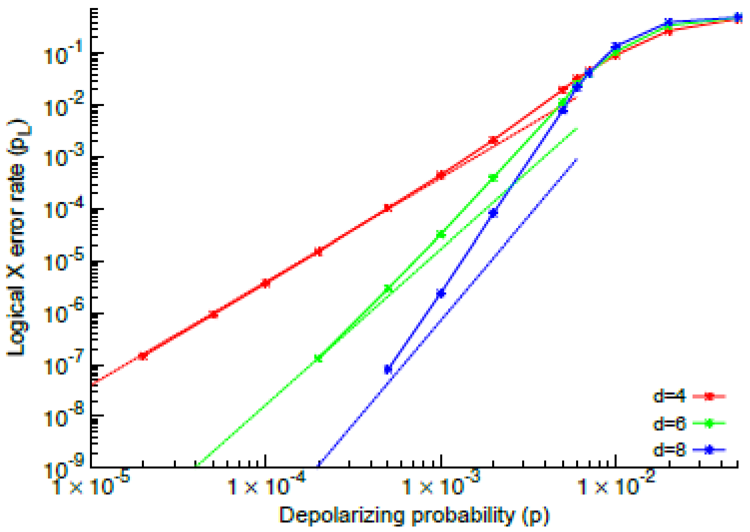
Recent experiments on bit-flip code in circuit-QED (Kelly *et al.)*

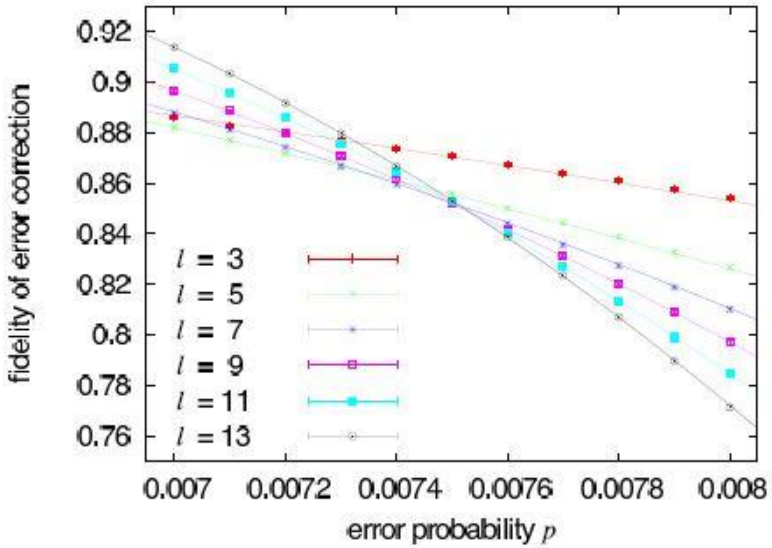
Logical error rate $\varepsilon \sim 1/\Lambda^{n+1}$ for n-th order fault-tolerance (can correct n errors), compare with above: $\Lambda = \frac{p_c}{p}$.

Logical error rate does not refer to encoded qubit life-time:

A. Life-time break-even point or B. QEC unit break-even point (harder!). B. is correct

Surface Code with Noisy EC





The memory noise threshold (no gates) has also been estimated as being between 7.5×10^{-3} and 1.1×10^{-2}. (depending on decoding cleverness).

Logical error rate $P_L \backsim \exp(-\kappa(p)L)\,, \kappa(p) = 0.8\ at\ p = p_c/3$.

Surface Code: At L=6, for a depolarizing probability p=2 x 10^{-4}, one can have a logical $\bar{X}$ error rate of 10^{-7}.

There is no known code with a better threshold (except concatenated scheme by Knill): find one!?

Surface Code Memory

Different break-even points

ABLE III. Qubit relaxation, dephasing, and gate times assumed for different architectures. DiVincenzo and Helmer parameters e taken from [21]. SC denotes superconductor; IT denotes ion trap architecture.

Parameter	Description/Location	SC_S (Slow)	SC_F (Fast)	SC_D (DiVincenzo)	SC_H (Helmer)	IT_S (Slow)	IT_F (Fast)
T_1	qubit relaxation time	T_1	T_1	T_1	T_1	T_1	T_1
T_2	qubit dephasing time	T_1	T_1	2 T_1	T_1	0.1 T_1	0.1 T_1
$t$prep	state preparation	5 μs	1 μs	40 ns	40 ns	100 μs	30 μs
t_1	single-qubit rotation	100 ns	10 ns	5 ns	5 ns	1 μs	1 μs
$t$meas	measurement	5 μs	1 μs	35 ns	35 ns	100 μs	30 μs
t_{CNOT}	CNOT	1 μs	100 ns	80 ns	20 ns	100 μs	10 μs
$t_{r,13}$	one round (S-13)	28.2 μs	4.82 μs	800 ns	320 ns	1202 μs	202 μs
$t_{r,17\&25}$	one round (S-17, S-25)	14.2 μs	2.42 μs	405 ns	165 ns	602 μs	102 μs

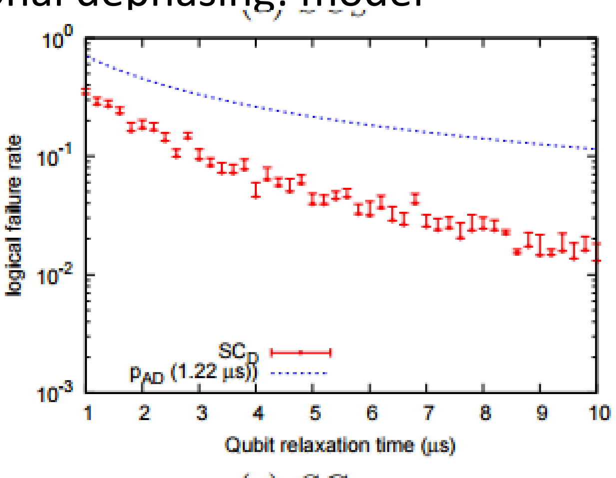
TABLE IV. Comparison of thresholds and pseudothresholds for the surface code under symmetric depolarizing noise.

Code	Threshold	$P^{th}_{1,X}$	$P^{th}_{3,X}$
Surface-13	-	3.0×10^{-4}	1.2×10^{-4}
Surface-17	-	8.0×10^{-4}	2.0×10^{-4}
Surface-25	-	5.0×10^{-4}	1.4×10^{-4}
Wang (2011) [8]	1×10^{-2}	-	-
Fowler (2012) [7, 45]	9×10^{-3}	$\sim 2 \times 10^{-3}$	-

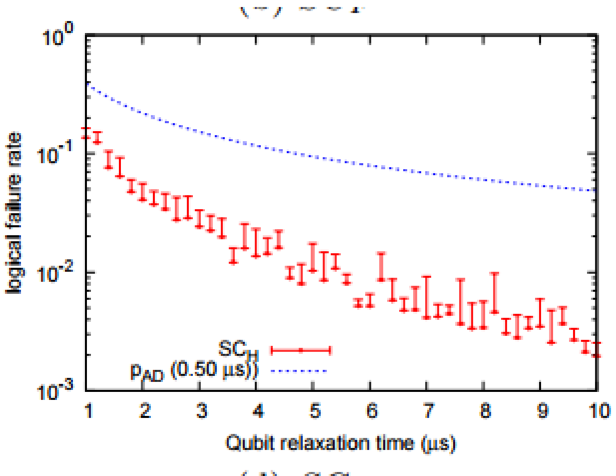
Tomita, Svore, PRA 2014

- Use amplitude damping model to model T_1 relaxation on qubit (transmon qubit coherence time is $O(10)\mu sec$)
- Additional dephasing: model T_1 and T_2

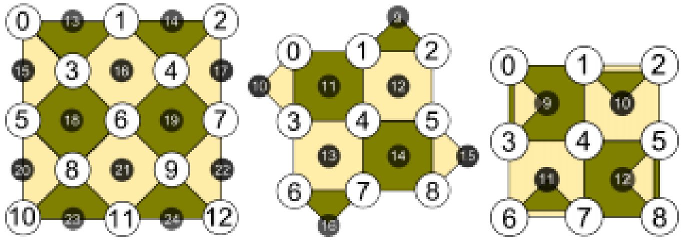
Using LIQ*Ui* for simulation
Data for Surface-17 below



(c) SC_D



(d) SC_H

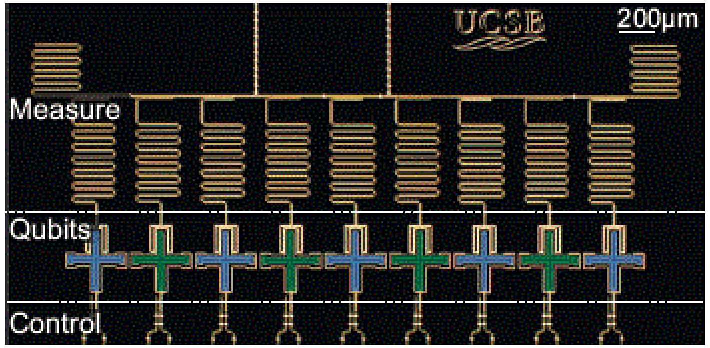


(a) Surface-25 (b) Surface-17 (c) Surface-13

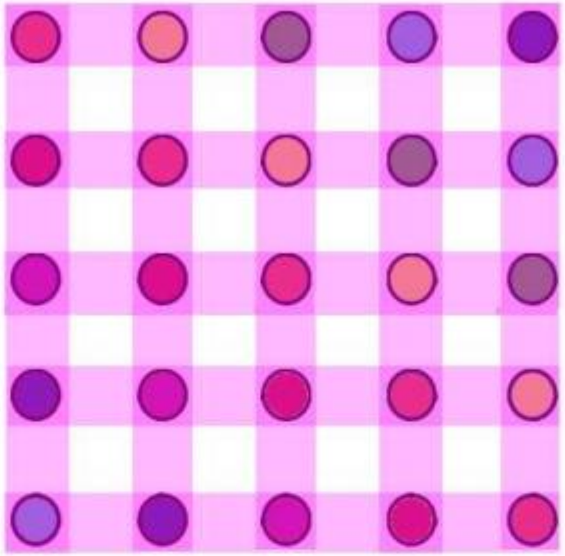
Each qubit is involved in 4 parity check measurements: each qubit needs to interact with 4 ancilla qubits (via 4 resonators e.g.)

Fixed-Coupling Architectures

- Scalable frequency use (multiplexing) and minimal coupling schemes (avoidance of cross-talk). Time-multiplexing of read-out.
- Read-out pulses (and flux-lines?) from the top



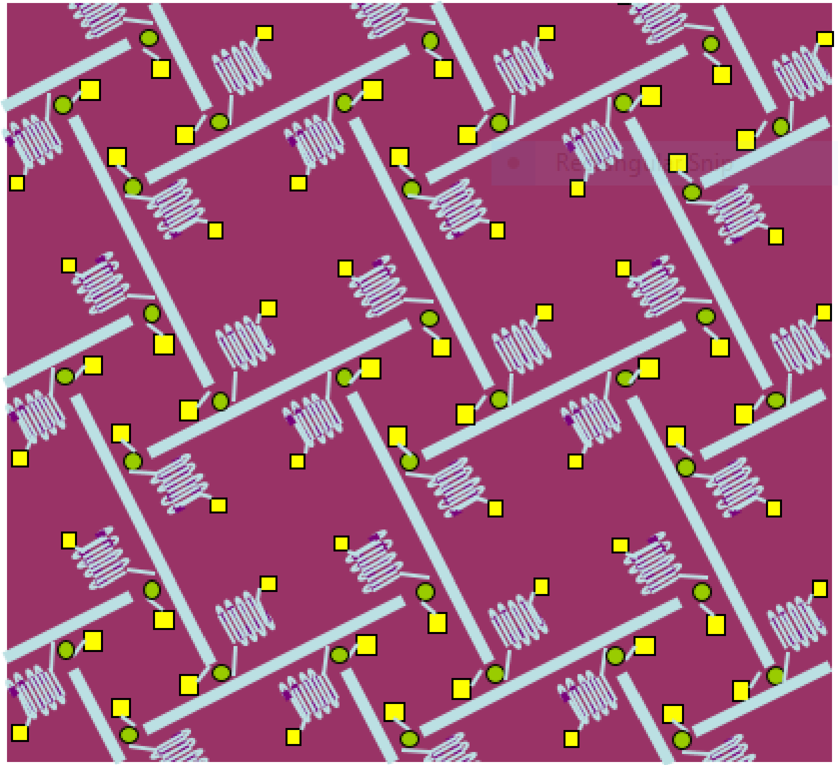
UCSB/Google:
1D repetition code with ZZ checks on a line
Where to put the read-out resonators
when going to a full 2D design?

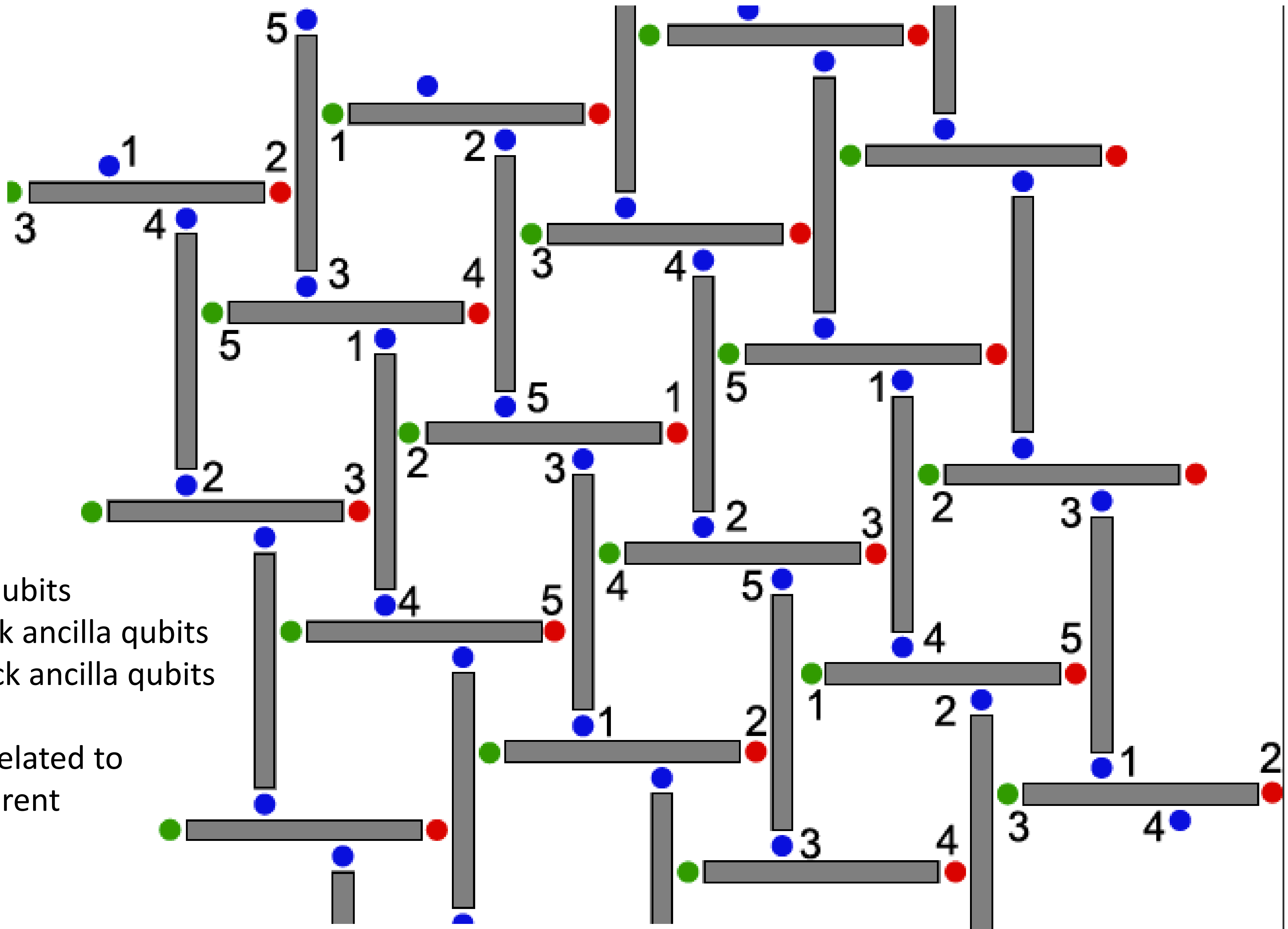


Non-scalable Helmer architecture
(including ancillas, distance-3 code)
Vertical and horizontal lines are
resonators to which several transmon
qubits at different frequencies (color)
are coupled. No. of frequencies grows with lattice size.

DiVincenzo Architecture

- Qubits (green) coupled via high-Q superconducting resonators (gray). Assume CNOT gates done via resonator. Each qubit coupled to 2 resonators.
- Every qubit has a number of controller and sensor lines to be connected to the outside world (gold pads)
- Where is the surface code in this?

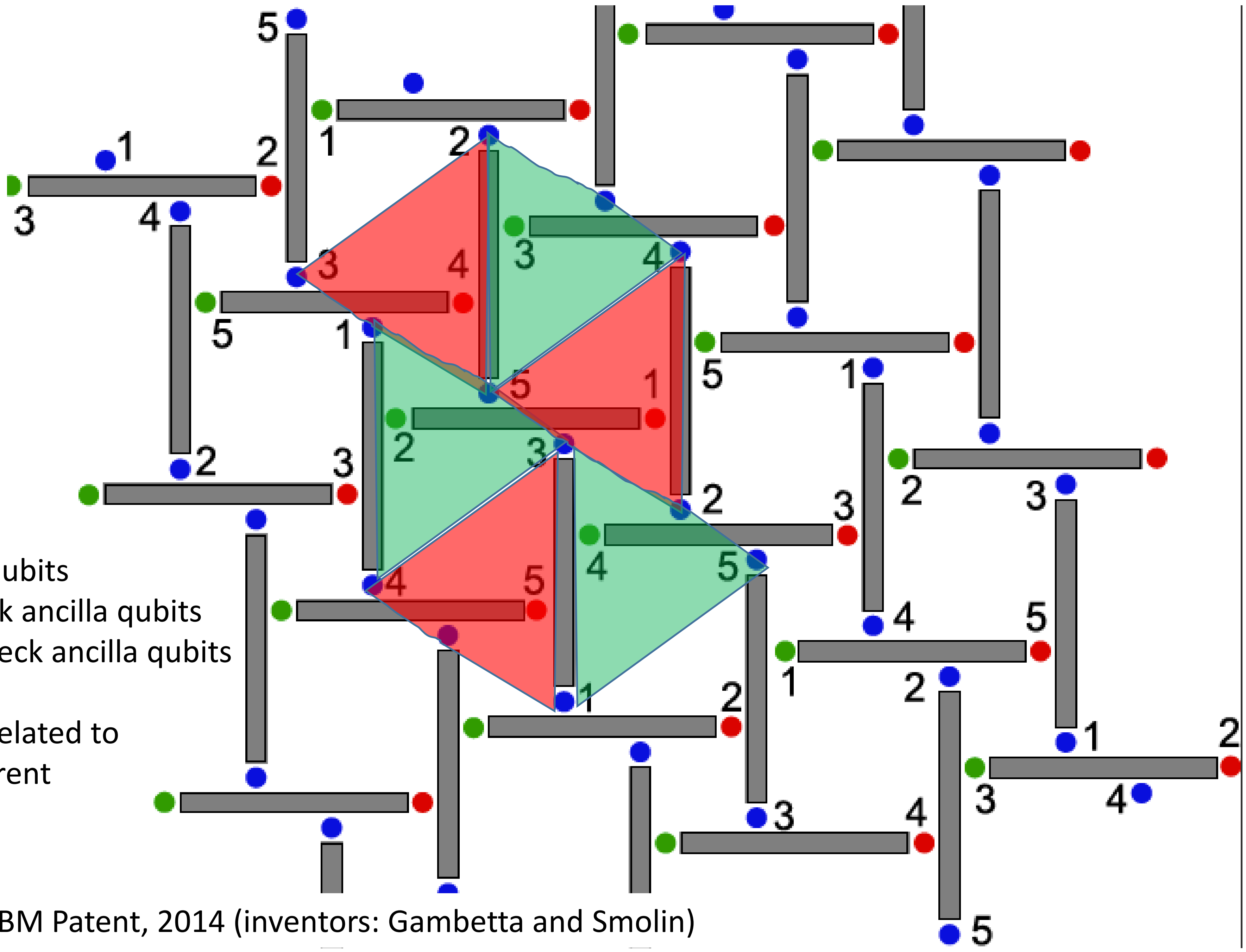




Blue are data qubits
Red are X-check ancilla qubits
Blue are Z-check ancilla qubits

Numbering is related to using four different frequencies

IBM Patent, 2014 (inventors: Gambetta and Smolin)



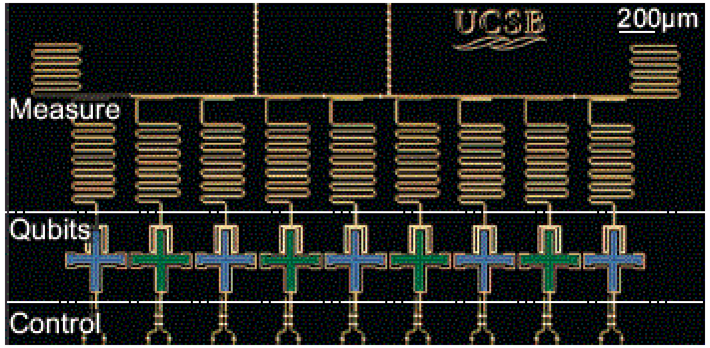
Blue are data qubits
Red are X-check ancilla qubits
Green are Z-check ancilla qubits

Numbering is related to using five different frequencies

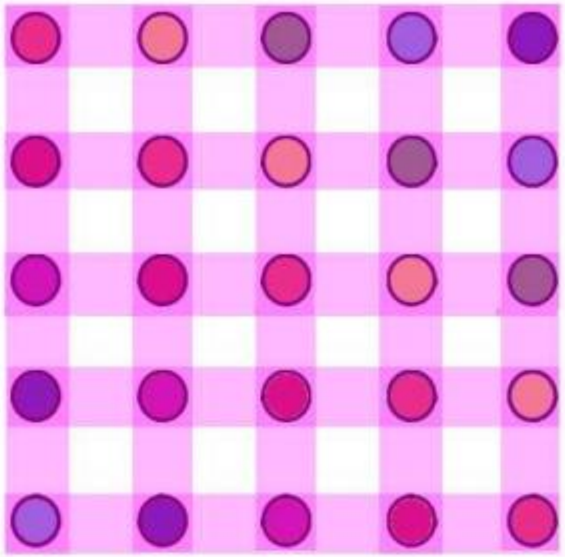
IBM Patent, 2014 (inventors: Gambetta and Smolin)

Fixed-Coupling Architectures

- Scalable frequency use (multiplexing) and minimal coupling schemes (avoidance of cross-talk).
- Read-out pulses (and flux-lines?) from the top



UCSB/Google:
1D repetition code with ZZ checks on a line
Where to put the read-out resonators when going to a full 2D design?



Non-scalable Helmer architecture
(including ancillas, distance 3 code)
Vertical and horizontal lines are resonators to which several transmon qubits at different frequencies (color) are coupled.

Surface Code in Progress

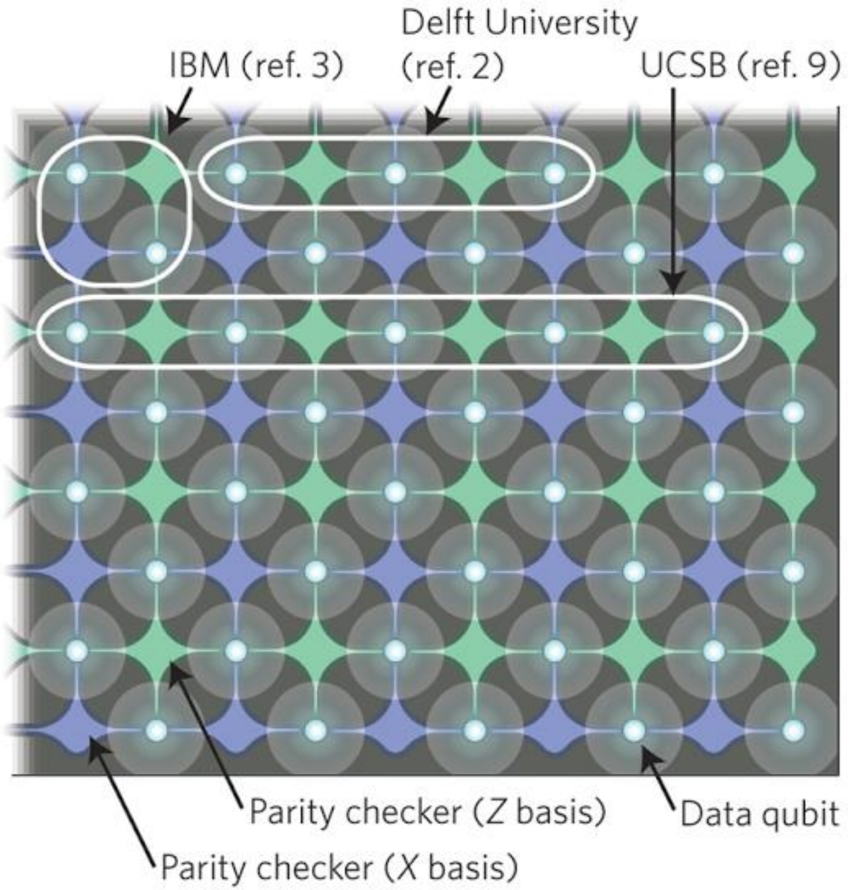
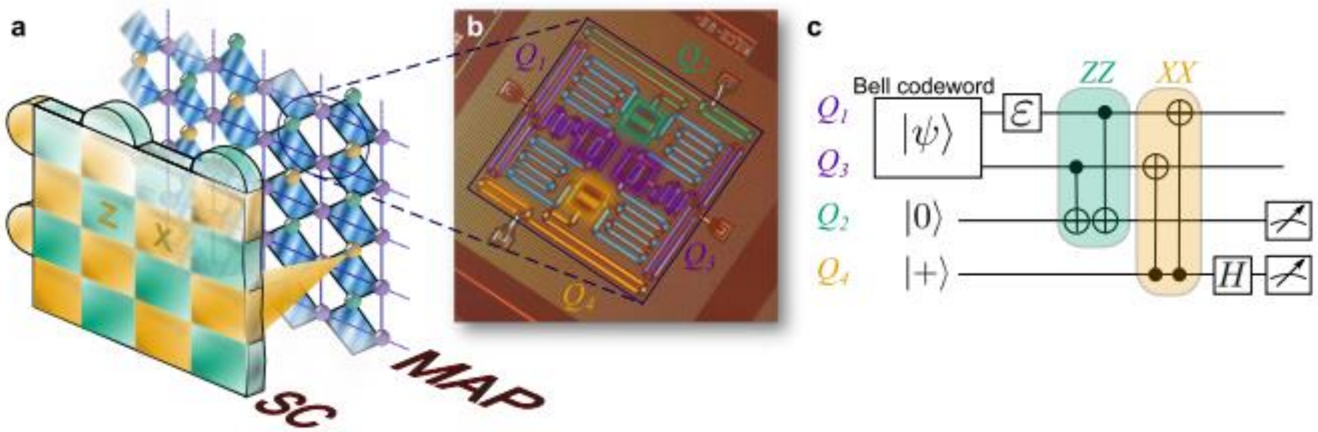
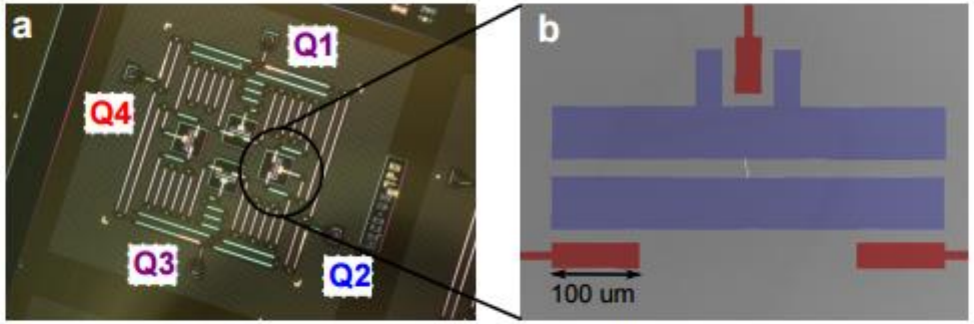


Fig. from S. Benjamin & J. Kelly, *Superconducting Qubits: Solving a wonderful problem*.
News & Views, Nature Materials 14, 561–563 (2015)

Surface Code in Progress





Chow *et al*, Proc. SPIE 9500 (2015)
& Nat. Comm.

Quantity	Targeted	Q1	Q2	Q3	Q4
qubit transition frequency (GHz)	5.3	5.303	5.101	5.291	5.415
anharmonicity (MHz)	-339.9	-340 ± 3	-340 ± 3	-341 ± 3	-340 ± 3
critical current (nA)	27	26.8 (27.2)	25.1 (25.4)	26.7 (27)	27.8 (28.2)
qubit capacitance (fF)	$62 + C_J$	65.5 (66.5)	65.9 (66.9)	65.3 (66.3)	65.3 (66.3)
E_J/E_C	45.7	45.0	42.4	44.7	46.5
charge dispersion (kHz)	24.9	28.3	45.3	30.0	21.6
T_ϕ from charge (ms)	41	35.8	22.4	33.8	47.0
readout resonator (GHz)	6.5/6.7	6.494	6.695	6.491	6.693
readout Q factor	15000	10560	15200	22600	5530
dispersive shift χ (MHz)	-1.6	-1.5	-1.0	-1.25	-1.4
g_R coupling to readout (MHz)	94	89	94	82	92
coupling capacitance C_R (fF)	5.5	5.4	5.7	5.0	5.3
Purcell limited T_1 (μs)	69	70	182	182	40

Surface Code in Progress

	CR_1 $D_1D_2D_3D_4$ (kHz)	CR_2 $D_2D_1D_3D_4$ (kHz)	CR_3 $S_1D_1D_2D_4$ (kHz)	CR_4 $S_1D_1D_2D_3$ (kHz)
$IIIZ$	-298	-688	-178	640
$IIZI$	-348	ϵ	130	ϵ
$IZII$	-129	-140	ϵ	ϵ
$ZIIZ$	ϵ	ϵ	113	105
$ZIZI$	ϵ	-129	ϵ	ϵ

TABLE III. **CR-activated Z interactions** Approximate Z-strengths on spectator qubits brought on by CR tones, in kHz. ϵ indicates a $Z-$strength below our sensitivity limit of ~ 100 kHz and only terms with values above this limit from any of the cross-resonance tones are shown.

	CR_1	CR_2	CR_3	CR_4
$\mathrm{ECR}_{\text{2-pulse}}$ fidelity (gate length, ns)	0.9637 ± 0.0014 (660)	0.9572 ± 0.0014 (340)	0.9523 ± 0.0001 (720)	0.9469 ± 0.0007 (1010)
$\mathrm{ECR}_{\text{4-pulse}}$ fidelity (gate length, ns)	0.9405 ± 0.0011 (740)	0.9458 ± 0.0016 (580)	0.9469 ± 0.0015 (820)	0.9384 ± 0.0013 (940)

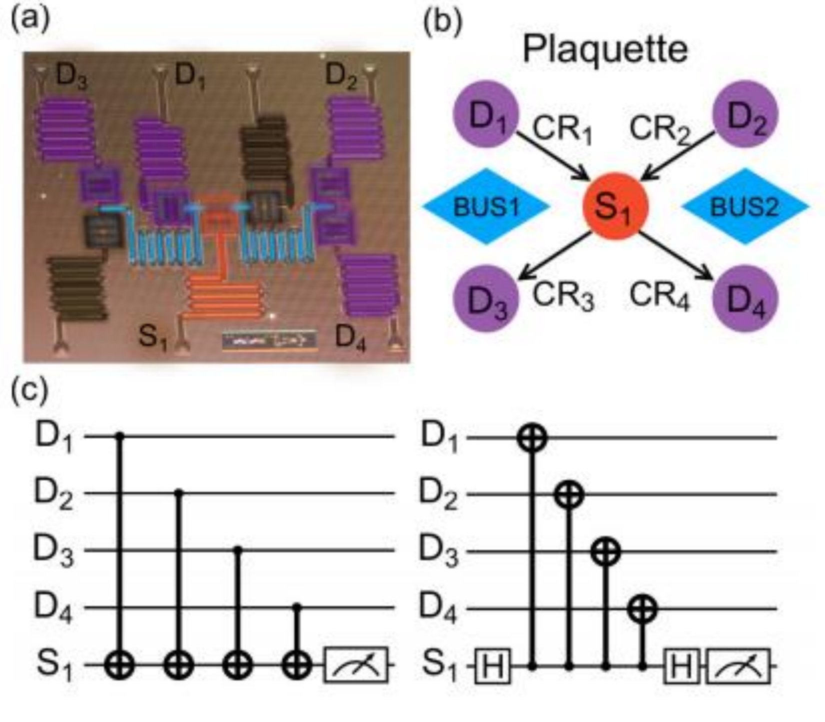
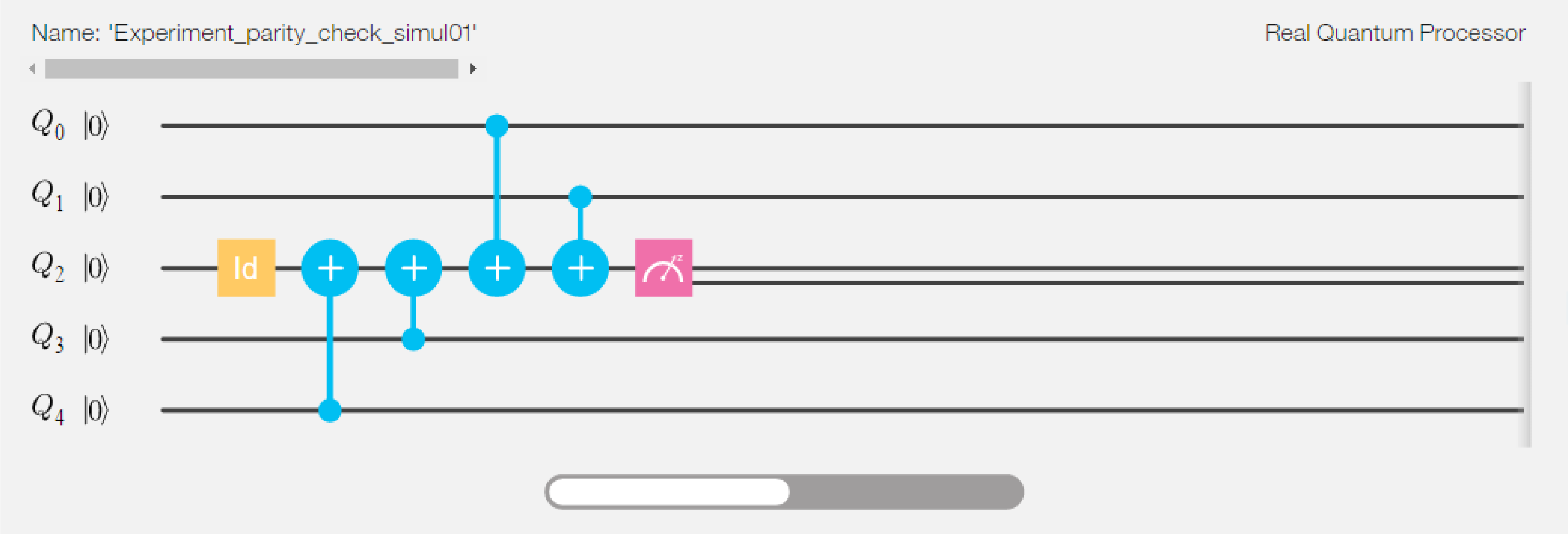


FIG. 1. (color online) (a) False-colored picture of a 7-qubit lattice. The five specific qubits used for the plaquette experiment are highlighted and labeled as data qubits (D_i, $i \in [1,4]$) and syndrome qubit S_1. (b) Cartoon representation of a plaquette. The arrows represent the cross-resonance two-qubit gate directions between the syndrome qubit and the four data qubits, with the convention of pointing from control to target qubit. The two quantum bus resonators, bus 1 and bus 2, are measured to be $\omega_{B1}/2\pi = 6.562$ GHz and $\omega_{B2}/2\pi = 6.810$ GHz respectively. (c) The $ZZZZ$ and $XXXX$-parity measurement quantum circuits.

Parity check measurement fidelity is ∽0.774
Much more than 3% for p=q (qubit error rate is parity check measurement error rate) model in surface code
2.4μsec integration time for measurement.

Takita *et al*, arXiv.org:1605.01351

Standard User, Units: 6/15



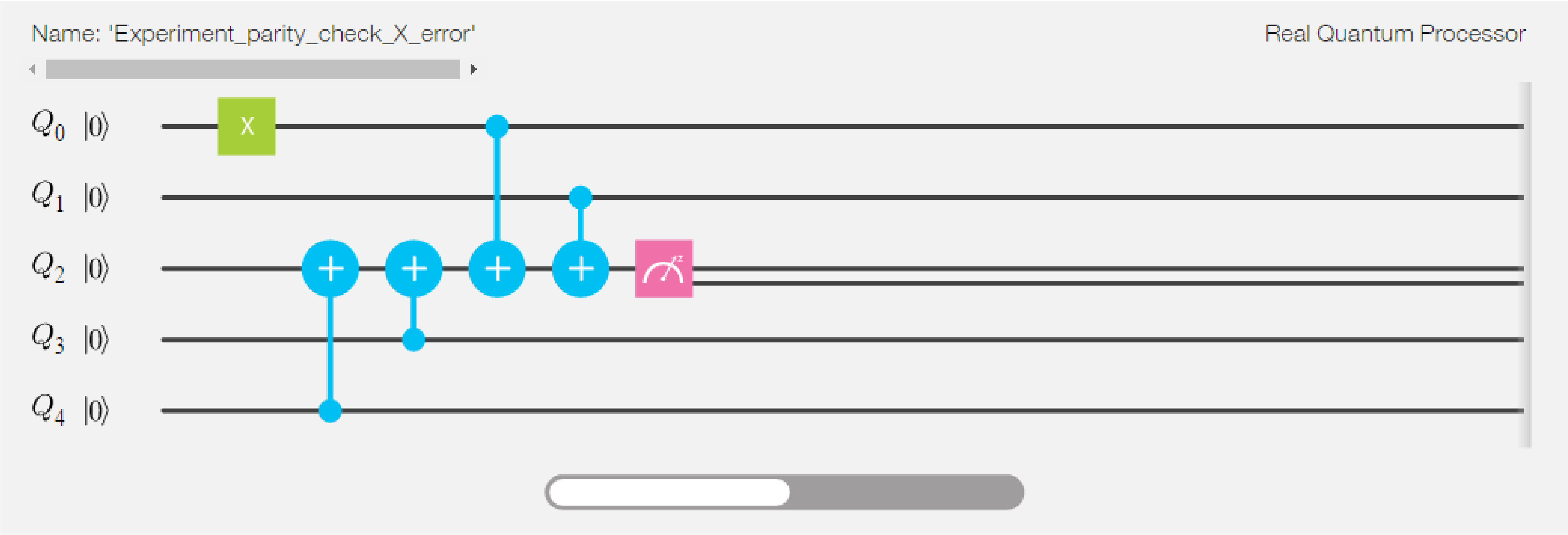
Executed on: May 9, 2016 2:55:14 AM

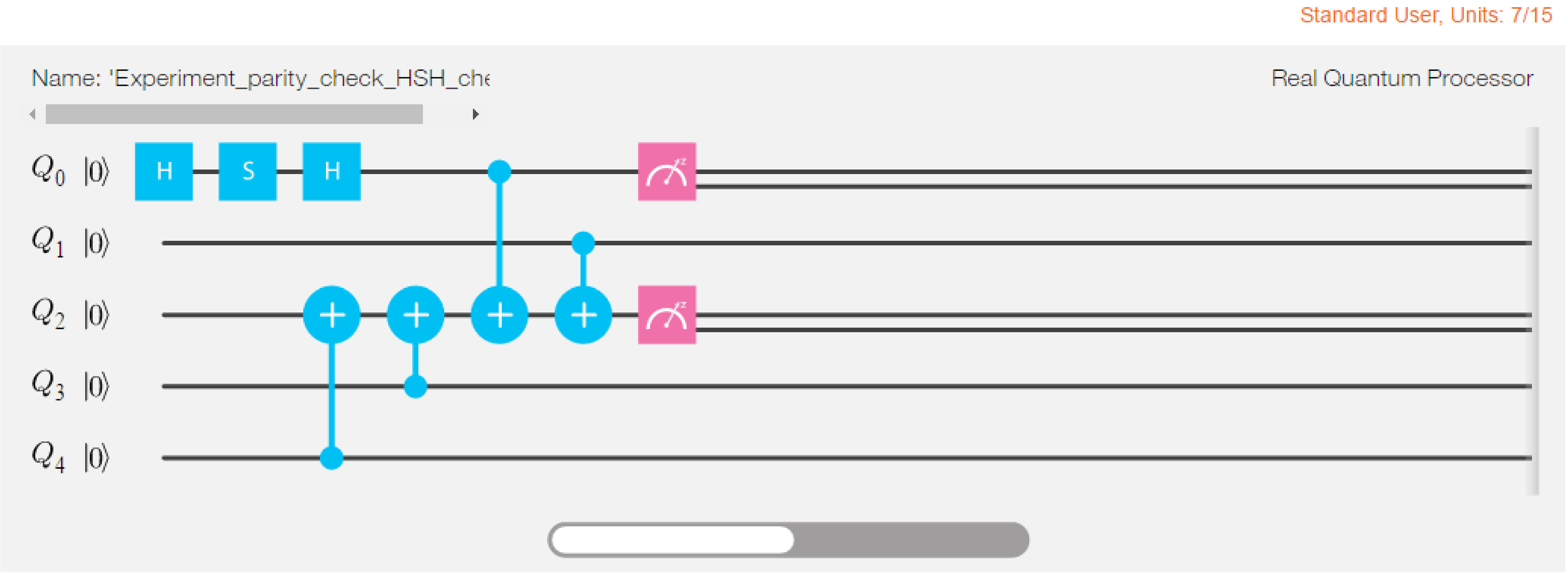
Results date: May 9, 2016 2:55:30 AM

Number of shots: 1024

Distribution

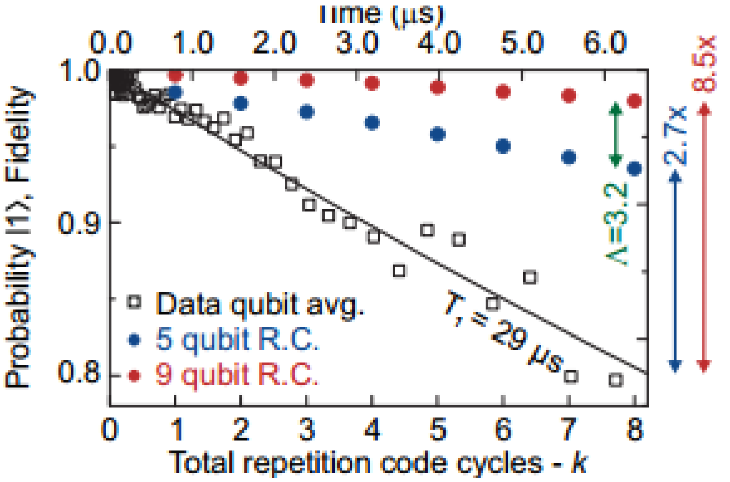
Standard User, Units: 0/15





Statistics of 4000 shots

Surface Code in Progress

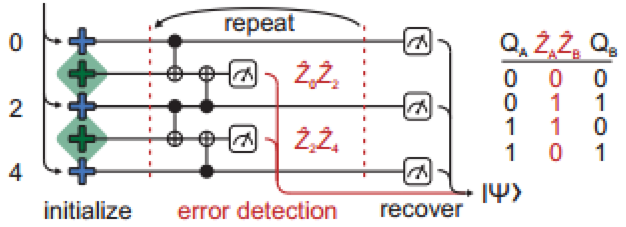


Failure Probability $\sim 1/\Lambda^{n+1}$
Is Λ larger than 1?

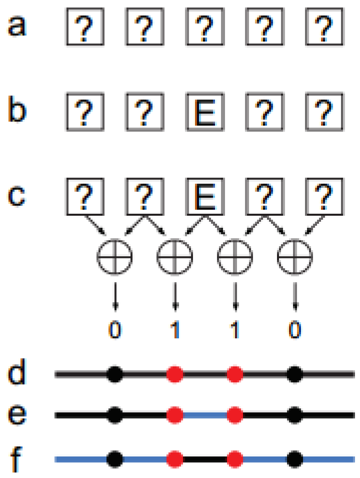
Kelly *et al.* data

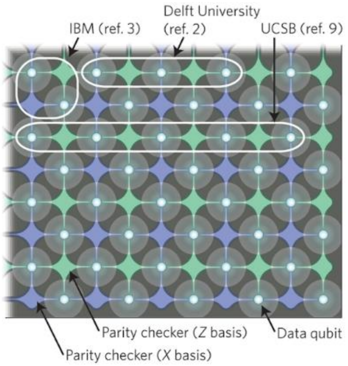
TABLE S1. Input error model.

Gate	Error
CZ	1%
X	0.1%
Idle (20 ns)	0.05%
Initialization	2.5%
Readout (measure qubit)	1.5%
Readout (data qubit)	3%.

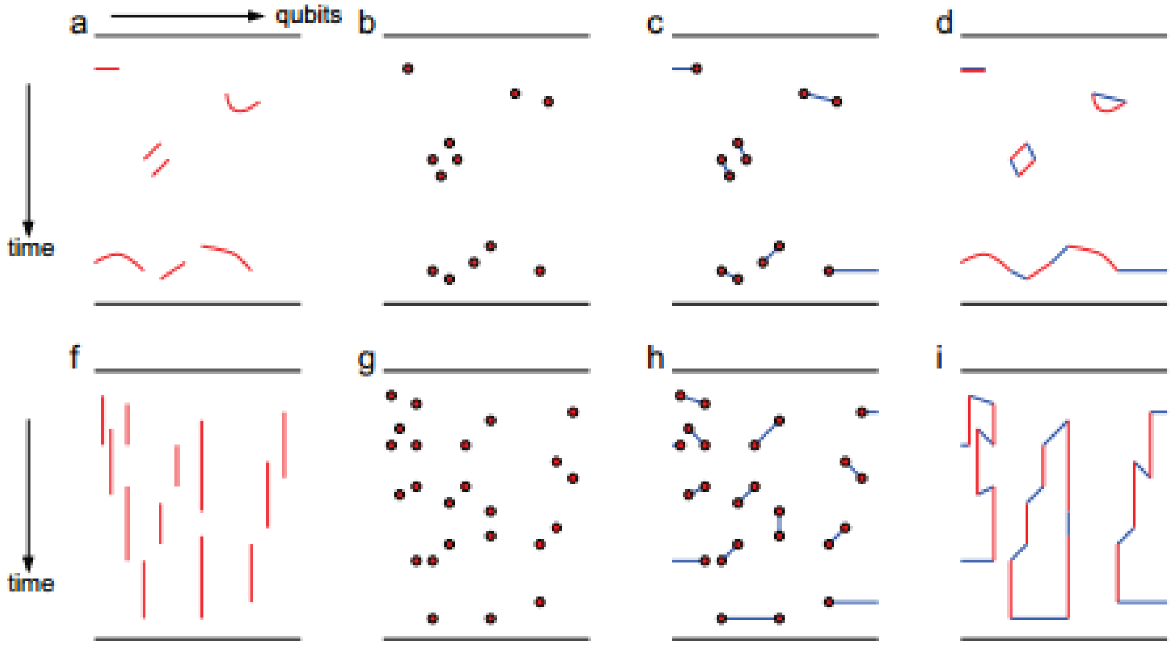


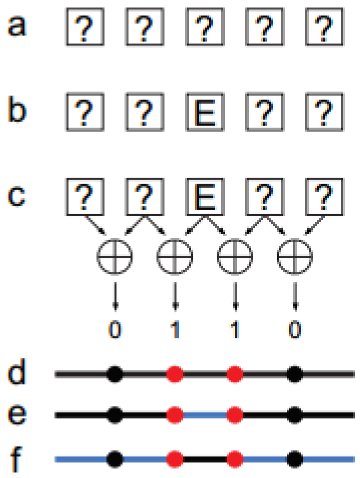
	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Qubit frequencies and coupling strengths									
f_{10}^{max} (GHz)	5.30	5.93	5.39	5.90	5.36	5.94	5.33	5.91	5.39
$\eta/2\pi$ (GHz)	-0.230	-0.216	-0.229	-0.214	-0.227	-0.214	-0.242	-0.212	-0.225
f_{10}^{idle} (GHz)	4.3	5.18	4.43	5.28	4.49	5.40	4.60	5.46	4.7
f_{res} (GHz)	6.748	6.626	6.778	6.658	6.601	6.687	6.540	6.718	6.567
$g_{res}/2\pi$ (GHz)	0.110	0.128	0.111	0.109	0.110	0.110	0.098	0.111	0.104
$g_{qubit}/2\pi$ (MHz)	13.8		14.1		15.4		14.4		
$g_{qubit}/2\pi$ (MHz)		14.5		14.4		14.6		15.6	
$1/\kappa_{res}$ (ns)	675	69	555	30	1144	36	590	28	473
Readout (RO) parameters									
RO error	0.015	0.004	0.067	0.007	0.048	0.013	0.017	0.011	0.018
simult. RO error		0.004		0.012		0.022		0.013	
separation error		$4\cdot10^{-6}$		$2\cdot10^{-5}$		$2\cdot10^{-3}$		$2\cdot10^{-3}$	
Thermal |1⟩ pop.	0.013	0.007	0.028	0.01	0.037	0.018	0.012	0.009	0.012
RO pulse length (ns)	800	160	800	300	800	300	800	300	800
RO demodulation length (ns)	800	560	800	460	800	460	800	460	800
$f_{10,RO}$ (GHz)		5.46		5.31		5.40		5.54	
resonator $n_{photons}$		37		18		10		14	
Gate parameters									
Single qubit gate error		0.0006		0.0009		0.001		0.001	
X_π length (ns)	25	20	25	20	25	20	25	20	25
CZ length (ns)	45		45		45		45		
CZ length (ns)		45		45		45		45	
Qubit lifetime at idling point									
T_1 (μs)	26.3	24.7	39.2	21.3	41.1	19.1	22.0	28.1	18.6

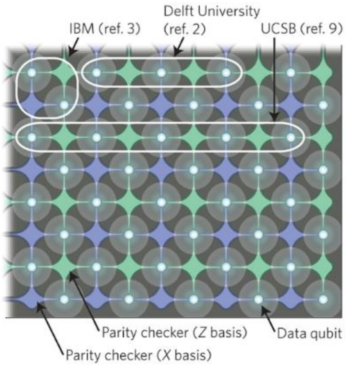




Surface Code in Progress







Surface Code in Progress

Issues:

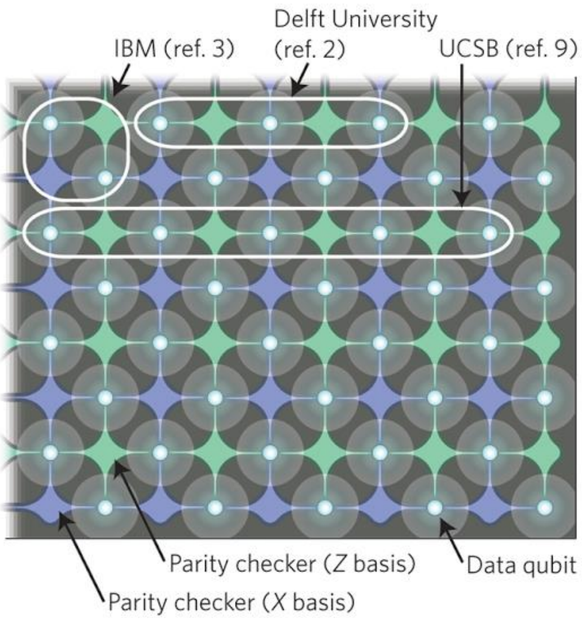
- Qubit leakage
- Are gate fidelities (two-qubit gate) high enough
- Time duration of measurement (reset resonator) while qubits decohere
- Scaling-up: cross-talk, stray EM modes

Theorists work:

- Better models of noise using superoperators & Lindblad equations.

Effect on noise threshold.

- Best fast decoders for surface code
- Different codes (concatenation of [[4,2,2]] with surface code, color code)



Alternative Routes and Means

- **Higher fidelity parity measurement needed!**

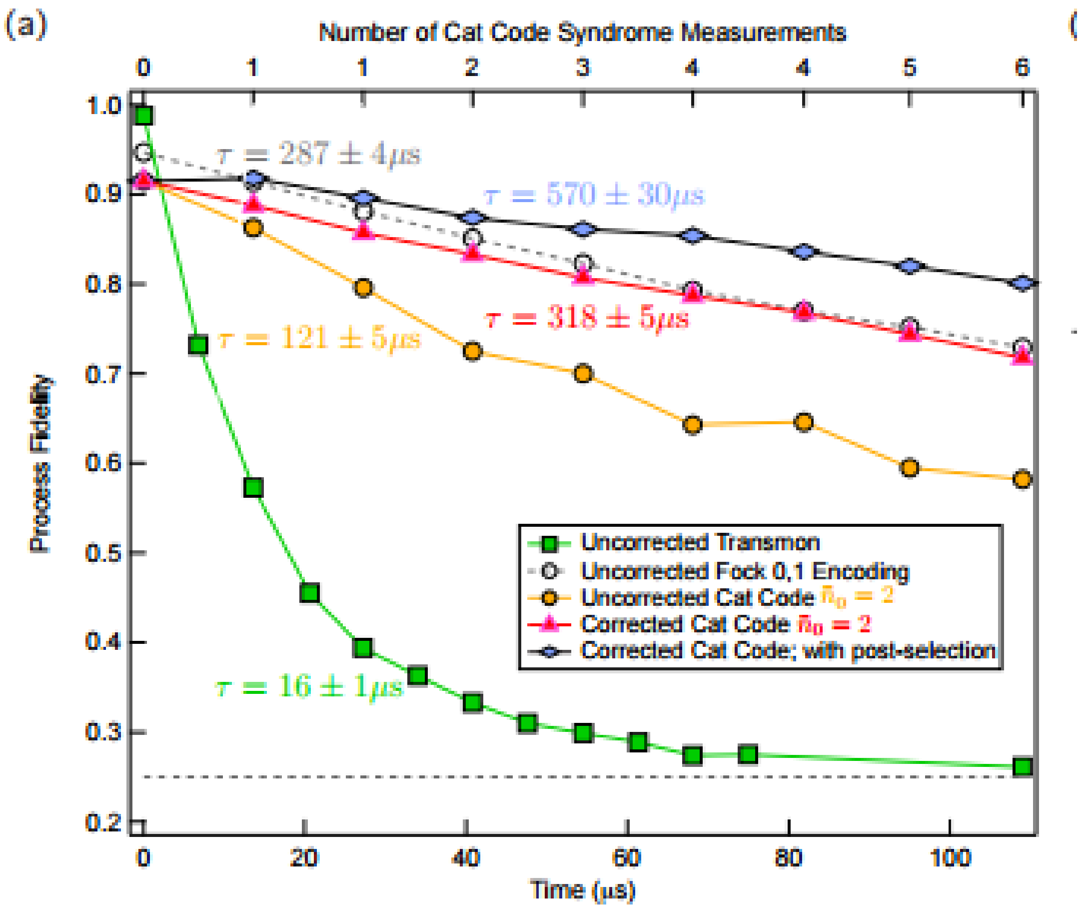
Direct parity measurement: no ancilla transmon used, directly focus on getting the interference right on outgoing microwave field. $\alpha(t) = \alpha(t)e^{i\pi P}$ where P=0,1 is parity of set of qubits (pick up phase shift of $e^{i\pi}$ at each qubit when it is in state $|1\rangle$, and no phase shift when it is in $|0\rangle$.)

- Use of longitudinal coupling $Z(a + a^{\dagger})$ between resonator and qubit.

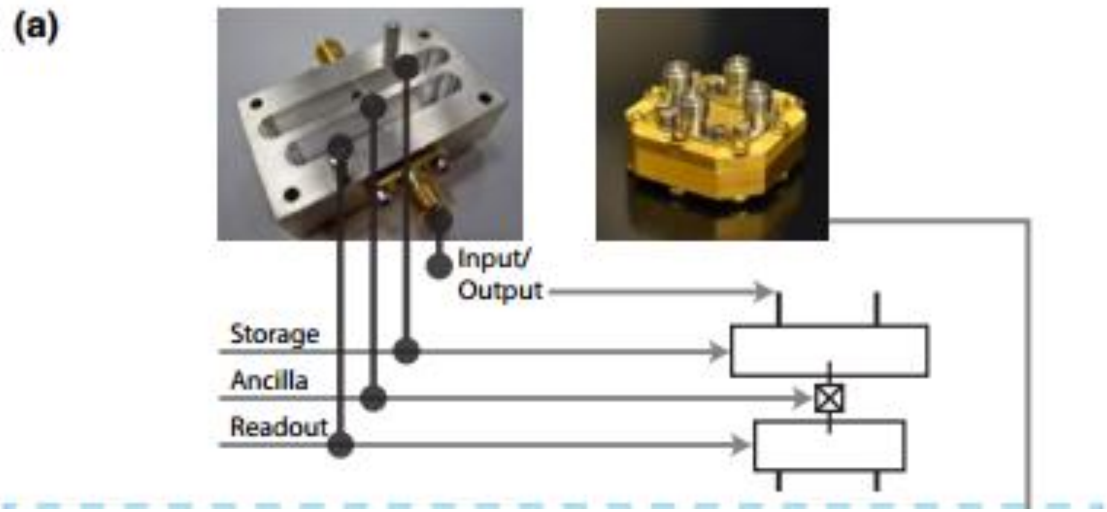
- The use of other qubits e.g. fluxonium, new flux or phase qubits coupled to resonators.

- Encoding qubits into resonators

Encoding a qubit into a resonator

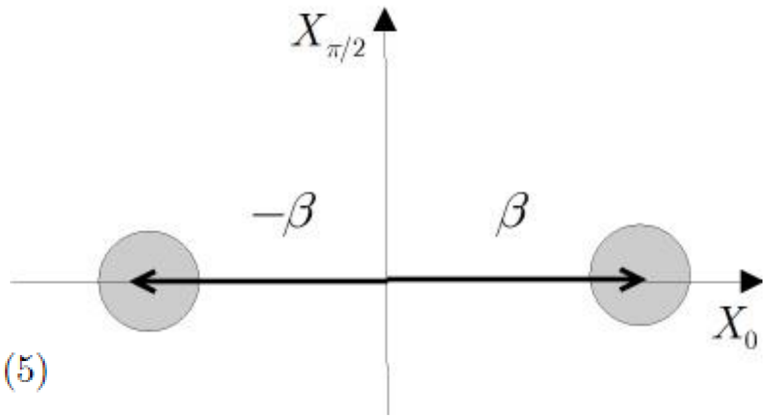


- Resonator: mostly photon loss
- Encode a better qubit in resonator (then concatenate)



Ofek *et al.*, arXiv.org:1602.04768

Cat-State Code



A cat state is an equal superposition of 2 quasi-orthogonal coherent states.

$$|\overline{0}_+\rangle = \frac{1}{\sqrt{N_+}}(|\alpha\rangle + |\text{-}\alpha\rangle), \quad |\overline{1}_+\rangle = \frac{1}{\sqrt{N_+}}(|i\alpha\rangle + |\text{-}i\alpha\rangle). \tag{5}$$

Here $|\alpha\rangle$ is a coherent state $|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$ and $N_\pm = 2(1 \pm \exp(-2|\alpha|^2)) \approx 2$. For sufficiently large photon number $\langle n\rangle = |\alpha|^2$, the states $|\pm\alpha\rangle, |\pm i\alpha\rangle$ (and thus $|\overline{0}_+\rangle$ and $|\overline{1}_+\rangle$) are approximately orthogonal. The creation and manipulation of such cat states has been actively explored for cavity modes in micro-wave cavities, see e.g. [18]. The code states are chosen such that loss of a photon from the cavity maps the states onto (approximately) orthogonal states. As $a|\alpha\rangle = \alpha|\alpha\rangle$, we have

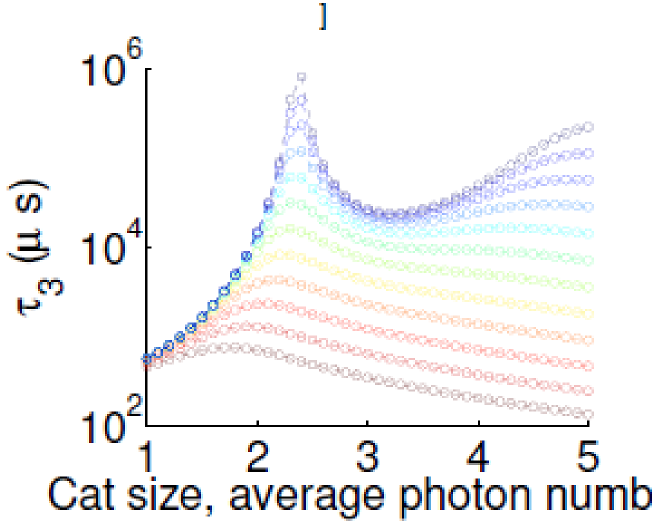
$$a|\overline{0}_+\rangle = \alpha\sqrt{N_+/N_-}\,|\overline{0}_-\rangle, \; a|\overline{1}_+\rangle = i\alpha\sqrt{N_+/N_-}\,|\overline{1}_-\rangle, \tag{6}$$

with $|\overline{0}_-\rangle = \frac{1}{\sqrt{N_-}}(|\alpha\rangle - |\text{-}\alpha\rangle)$ and $|\overline{1}_-\rangle = \frac{1}{\sqrt{N_-}}(|i\alpha\rangle - |\text{-}i\alpha\rangle)$. As we know the preservation of orthog-

The states $|\overline{0}/\overline{1}_+\rangle$ are even photon number states while $|\overline{0}/\overline{1}_-\rangle$ have an odd # photons. QEC conditions are met for $E_0 = \sqrt{\tau\kappa_-}\,a$ (even $\leftrightarrow$ odd, detect by measuring $P = e^{i\pi a^\dagger a}$) and $E_1 = I - \frac{\tau\kappa_-}{2}\,a^\dagger a$ ($\alpha \to \alpha e^{-\tau\kappa_-/2}$) when $|\alpha| \to \infty$. For small $\bar{n} = |\alpha|^2$ there is a sweet spot $\bar{n} \approx 2.4$ where E_0 and $E_1 = I$ meet the 'QEC conditions'

Duivenvoorden, Terhal (unpublished)

Best possible performance of cat state code



$$\eta_{\mathcal{R}} = 1 - \min_{|\psi\rangle \in C} F^2\left[|\psi\rangle, \mathcal{R}\cdot\mathcal{E}(|\psi\rangle\langle\psi|)\right]$$

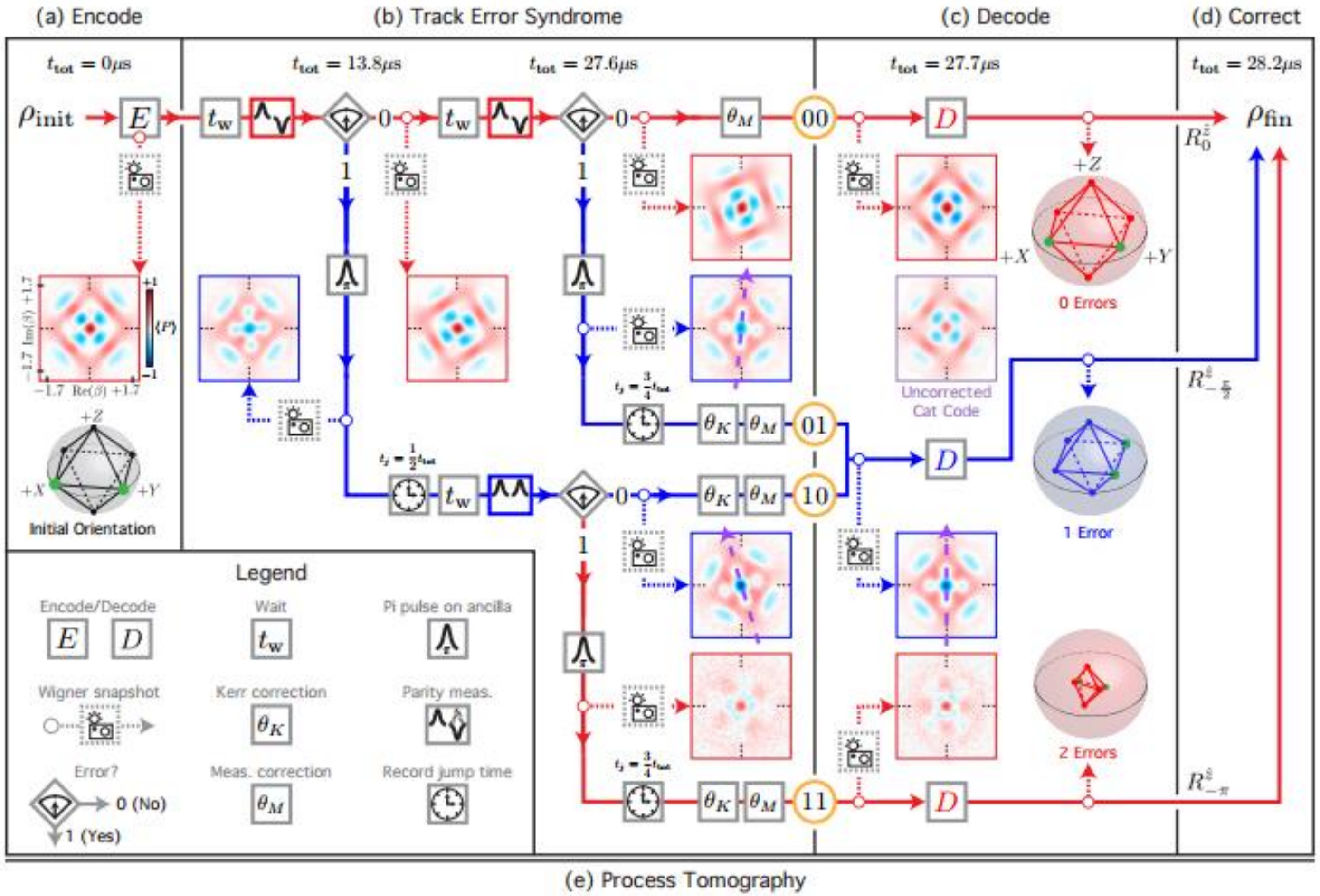
Figure 1: Behavior of the decay time τ_3 of the qubit during the error-correction procedure depending on CAT size. The different lines refer to different time-to-correction $T \in [2^{-7}, 2^{-6}, \dots 2^{4}]\mu s$. Red is large T, blue is small T.

- Only photon loss (Lindblad equations), $\kappa^{-1} = 300\mu$sec interspersed with the best possible quantum error correction at intervals T (time-to-correction).
- Best possible QEC minimizes has fidelity loss η close to $\eta_{op} = \min_R \eta_R$, i.e. $\eta \leq 3\eta_{op}$
- Sweet spot at $\bar{n} \approx 2.4$ (right where experiment is done)

Using ‘transpose channel QEC’ see Ng, Mandayam, PRA (2010)

Ofek *et al.*, arXiv.org:1602.04768

Experimental cat code



Term	Measured (Prediction)
$\omega_a/2\pi$	6.2815 GHz
$\omega_s/2\pi$	8.3056 GHz
$\omega_r/2\pi$	9.3149 GHz
$K_a/2\pi$	297 MHz
$K_s/2\pi$	4.5 kHz
$K_r/2\pi$	(0.5 kHz)
$\chi_{sa}/2\pi$	1.97 MHz
$\chi_{ra}/2\pi$	1 MHz
$\chi_{sr}/2\pi$	(2 kHz)

Table 1: **Hamiltonian parameters**

	Ancilla	Storage	Readout
T_1	$35\mu s$	-	-
T_2	$12\mu s$	-	-
τ_s	-	$250\mu s$	100ns
T_2^s	-	$330\mu s$	—
ground state (%)	96%	> 98%	> 99.3%

Weakest link: qubit T_1 during photon parity measurement. Non-fault tolerant. Why?

Taking $\bar{n}$=3. Two rounds of photon parity measurements (via a qubit-ancilla dispersively coupled: $Za^\dagger a$ coupling)

QEC Conditions

Useful when we consider more general codes (e.g. bosonic codes), specific, non-Pauli errors and approximate error correction.

Assume super operator description of noise:

$$S(\rho) = \sum_k E_k \rho \, {E_k}^\dagger$$

i.e. depolarizing channel, independent X and Z errors.

QEC Conditions. We encode bit strings $i \rightarrow |\bar{\imath}\rangle$.
One can correct the set of errors $\{E_k\}$ (or any linear combination of these errors) if and only if, for all k, l, we have

$$\forall i, j, \langle \bar{\imath} | {E_k}^\dagger E_l | \bar{\jmath} \rangle = c_{kl} \delta_{ij}$$

QEC Conditions

A. $$\forall i, \langle \bar{\imath} | E_k{}^{\dagger} E_l | \bar{\imath} \rangle = c_{kl}$$

where c_{kl} does not depend on i. If $c_{kl} = \delta_{kl}$, errors send a codeword to orthogonal error-spaces (labeled by k), but errors may have identical effect on codewords.

B. $$\langle \bar{\imath} | E_k{}^{\dagger} E_l | \bar{\jmath} \rangle = 0,$$

orthogonal codewords are mapped by different (or the same) errors onto orthogonal states so that we can reverse the error
(by a unitary transformation on codespace + ancilla).

One could obey these conditions approximately, so that only approximate reversal is possible, leading to a reduction in error rate.

$e^A e^B = e^B e^A e^{[A,B]}, for\ A, B\ lin. comb\ of\ p\ and\ q$

Qubit into an oscillator

Gottesman, Kitaev, Preskill 2001:

Common +1 eigenstates of $S_p = \exp(-i\,\hat{p}\,2\sqrt{\pi})$ and $S_q = \exp(i\,\hat{q}\,2\sqrt{\pi})$ with $[\hat{q}, \hat{p}] = i$.

Thus

$$p = 0\ mod\ \sqrt{\pi}, q = 0\ mod\ \sqrt{\pi}.$$

We have $Z = \exp(i\,\hat{q}\,\sqrt{\pi})\,, X = \exp(-i\,\hat{p}\,\sqrt{\pi}),$

$$Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle$$

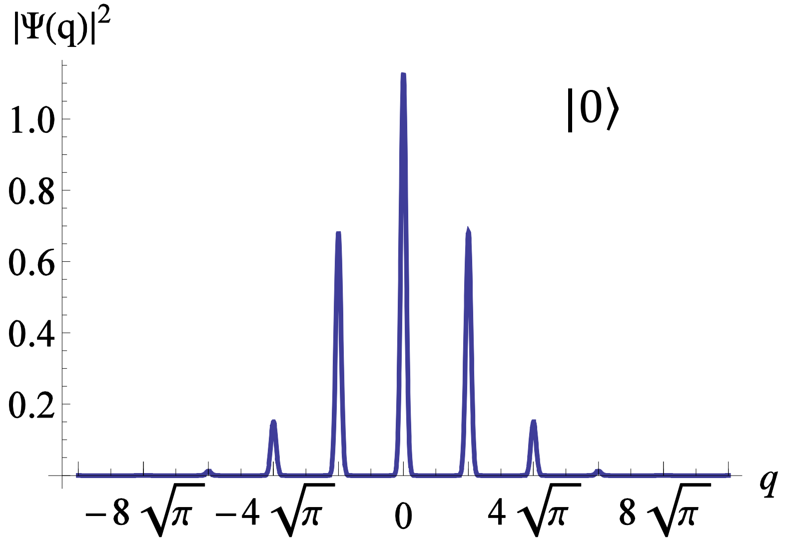
How to prepare a finite-photon number version of these states (using coupling of bosonic mode to a qubit)?

Terhal, Weigand, PRA (2015)

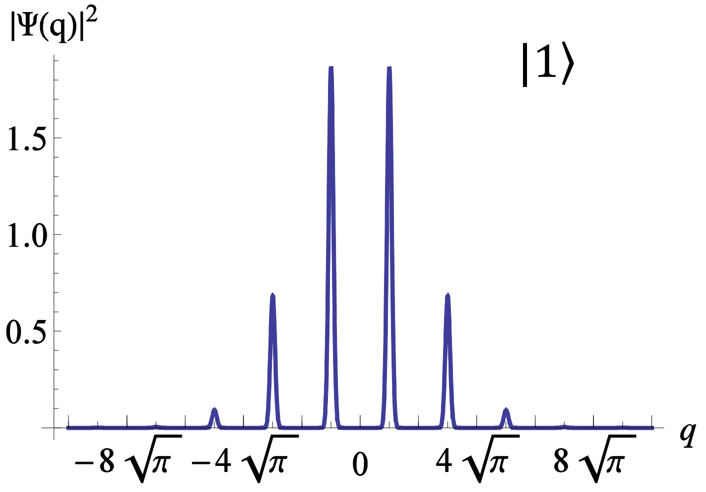
Approximate States

Common +1 eigenstates of $S_p = \exp(-i\,\hat{p}\,2\sqrt{\pi})$ and $S_q = \exp(i\,\hat{q}\,2\sqrt{\pi})$ are $|0\rangle$ and $|1\rangle$ so that

$\mathrm{Z} = \exp(i\,\hat{q}\,\sqrt{\pi})$ with $Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle$.



Squeezed peaks at even multiples of $\sqrt{\pi}$

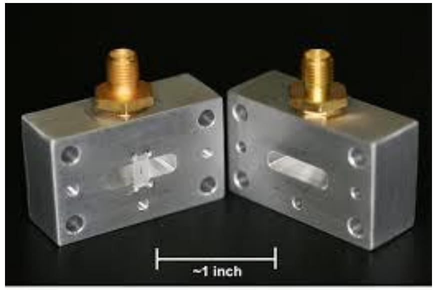


Squeezed peaks at odd multiples of $\sqrt{\pi}$

Error correction means detecting small displacements and reversing these: this works for with $|u|, |v| \leq \sqrt{\pi}/2$.

Cats in cavities, e.g. Vlastakis *et al.*, Science 2013, Ofek *et al.:* arXiv.org:1602.04768

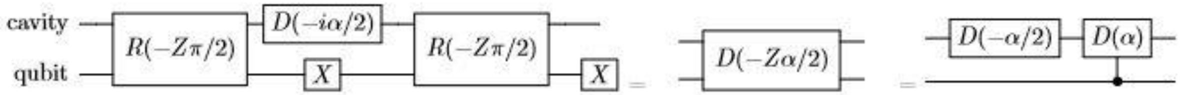
GKP Code Implemention

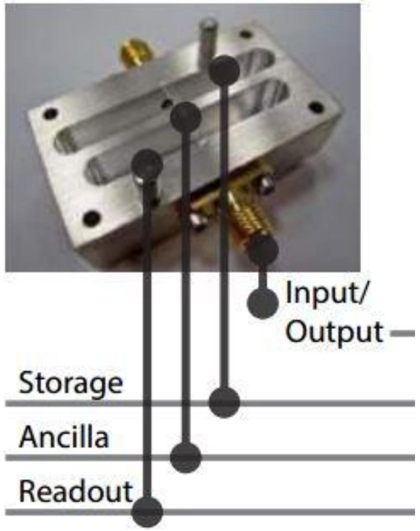


- High-Q micro-cavity, say, $1\ msec$ or more.
- High quality qubit, say, $T_1, T_2 \approx O(10)\ \mu sec$
- Strong dispersive qubit-cavity coupling $\chi Z a^\dagger a$

(e.g. $\frac{\chi}{2\pi} = 2.5 MHz$, cavity/qubit detuning *1 GHz*, nonlinearities *O(1) kHz*)

- Dispersive coupling allows for qubit-controlled cavity rotation $(R(\theta Z) = \exp\left(-i\theta\ a^\dagger a\ Z\right))$ which can be directly used for qubit-controlled displacement.





- Controlled-rotations take $T = {}^{\pi}/_{\chi} = 200\ nanosec$.
- Use no more than 50 photons

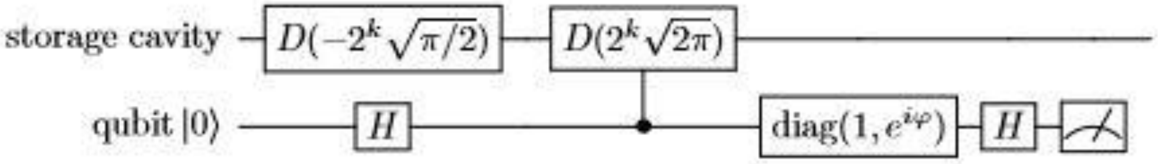
Displacement notation $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$

Preparation of |0>

Approximate +1 eigenstate of $S_q = \exp(i\,\hat{q}\,2\sqrt{\pi})$
(and $\mathrm{Z} = \exp(i\,\hat{q}\,\sqrt{\pi})$) is squeezed vacuum $q \approx 0$.
How to make this into an +1 eigenstate of $S_p = \exp(-i\,\hat{p}\,2\sqrt{\pi})$?

Measure the eigenvalue $e^{i\theta}$ of $S_p = D(\sqrt{2\pi})$!

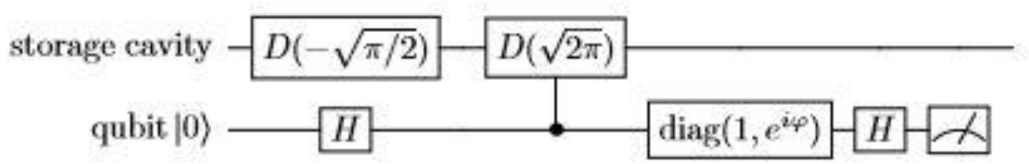
Phase Estimation



Repeat such circuit M times, with k=0 or higher k, with or without varying ‘feedback phase’ φ, to estimate θ (or better said: to project onto state with a certain value for θ).

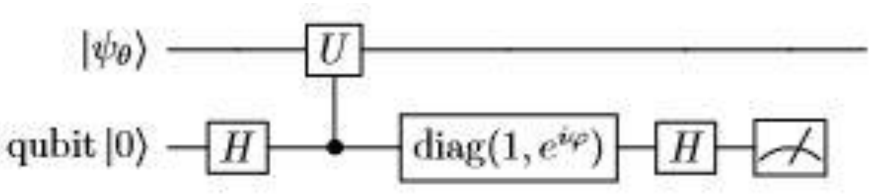
Phase Estimation Protocols

- Start with squeezed vacuum state
- Apply M rounds of this circuit:



1. Non-adaptive protocol: M/2 times with phase $\varphi = 0$, M/2 times with phase $\varphi = \pi/2$.

2. Adaptive protocol with feedback (Berry, Wiseman, Breslin, PRA 2001): phase φ changed/adapted in each round, depending on qubit measurement outcomes.



$$U|\psi_\theta\rangle = e^{i\theta}|\psi_\theta\rangle$$
$$P(0) = \frac{1}{2}(1 + \cos(\theta + \varphi))$$

Numerical Simulation Results

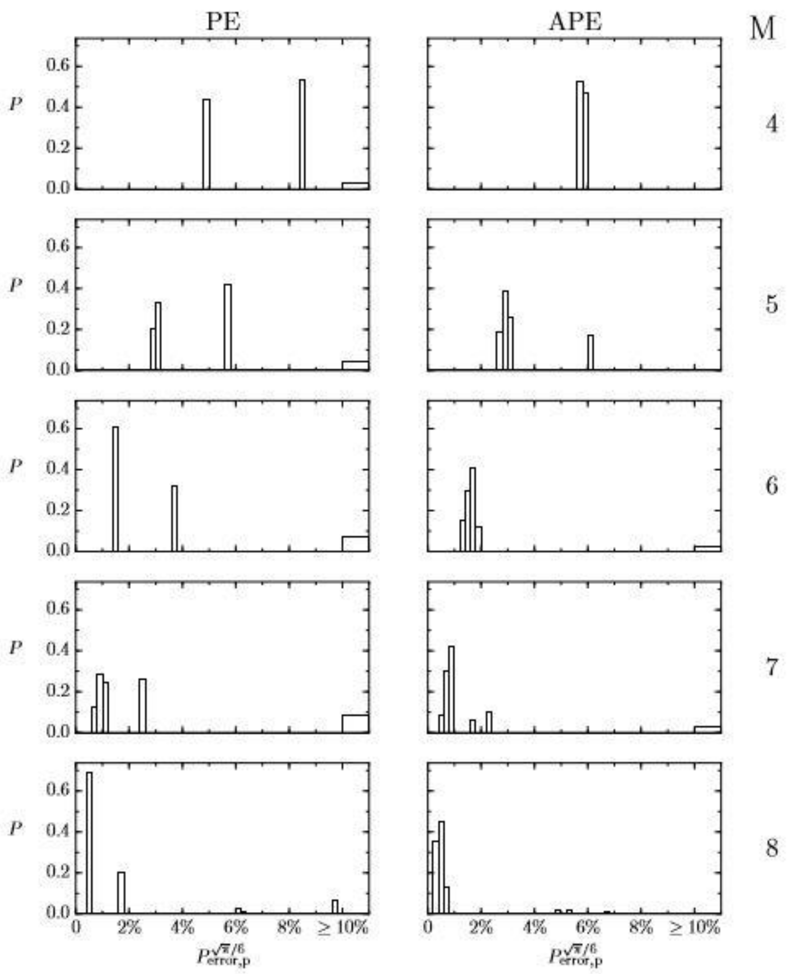
- Start with squeezed vacuum with 8.3 dB of squeezing.

- M=8 protocol is executed in $4\ \mu sec$.
(number of photons in state $\bar{n} \approx 25 \pm 25$)

Adaptive protocol with M=8 is best

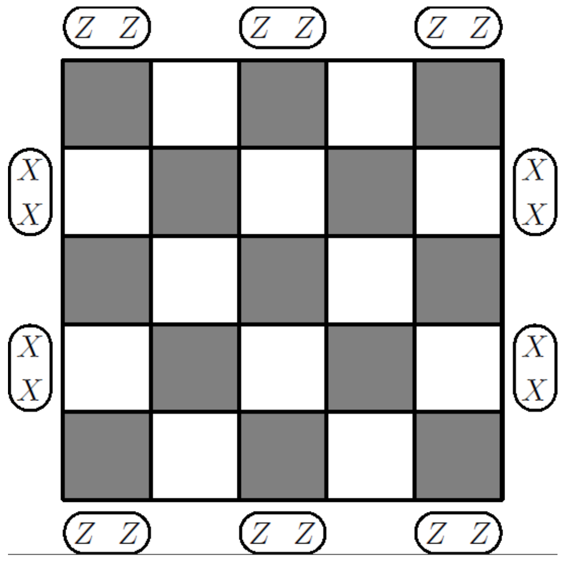
Gives a 94% (heralded) chance of preparing a state for which ‘probability for p-shift errors beyond $\sqrt{\pi}/6$’ is less than 1%.

Biggest source of concern are nonlinearities
$$K\ (a^{\dagger}a)^2, \chi' Z (a^{\dagger}a)^2.$$



Surface Code of Oscillators

Concatenate surface code with bosonic GKP code: surface code of oscillators (cavities coupled with transmon qubits)



- Plaquette ZZZZ $\Rightarrow$ measurement

around plaquette u of the sum

$q_{u-\hat{x}} - q_{u+\hat{x}} + q_{u-\hat{y}} - q_{u+\hat{y}}$

- Star XXXX $\Rightarrow$

measurement around star v

of the sum $-p_{v-\hat{x}} + p_{v+\hat{x}} + p_{v-\hat{y}} - p_{v+\hat{y}}$

(Using $[p_1 + p_2, q_1 - q_2] = 0$)

- Cliffford (CNOT, Hadamard etc.) gates on encoded qubits are done by linear optics (squeezing needed)

Discussion