PHYS 251 - Homework 8

8 Homework 8: Ordinary Differential Equations (ODE)

Please upload your answers to Bitbucket.

The documentation provided in the Python web page should be the first stop to get help. **Python documentation**.

8.1 Problem 1: Forward Euler

Find the numerical solution for each of the following ODE's using the Forward Euler method.

a)

$$\begin{aligned} \text{ODE:} \qquad & y^{'} = t\, e^{3t} - 2y \\ & 0 \leq t \leq 1 \\ \text{inital condition} \qquad & y(t=0) = 0 \end{aligned} \tag{1}$$

Find the numerical solution of the ODE using the following Δt's: 0.5, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's in the same plot and add labels, a grid and a legend to the plot.

b)

$$\begin{aligned} \text{ODE:} \qquad & y^{'} = 1 + (t-y)^2 \\ & 2 \leq t \leq 3 \\ \text{inital condition} \qquad & y(t=2) = 1 \end{aligned} \tag{2}$$

Find the numerical solution of the ODE using the following Δt's: 0.5, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's in the same plot and add labels, a grid and a legend to the plot.

c)

$$\begin{aligned} \text{ODE:} \qquad & y^{'} = 1 + \frac{y}{t} \\ & 1 \leq t \leq 2 \\ \text{inital condition} \qquad & y(t=1) = 1 \end{aligned} \tag{3}$$

Find the numerical solution of the ODE using the following Δt's: 0.25, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's in the same plot and add labels, a grid and a legend to the plot.

d)

$$\begin{aligned} \text{ODE:} \qquad & y' = \cos(2t) + \sin(3t) \\ & 0 \leq t \leq 2\pi \\ \text{inital condition} \qquad & y(t = 0) = 0 \end{aligned} \tag{4}$$

Find the numerical solution of the ODE using the following Δt's: 0.25, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's in the same plot and add labels, a grid and a legend to the plot.

You must write your script using **for** or **while** loops.

Please place comments in your code.

Save your script as **hw8_1.py**. Place your figures in a document using Word or PDF, add a captions to your figures. Save the document with the figures as **hw8_1_figs**. Upload the files to your Bitbucket account.

8.2 Problem 2: Forward Euler and scipy.integrate.odeint

Find the numerical solution for each of the following ODE's using the Forward Euler method and the **scipy.integrate.odeint() function**.

a)

$$\begin{aligned} \text{ODE:} \qquad & y' = e^{t-y} \\ & 0 \leq t \leq 1 \\ \text{inital condition} \qquad & y(t = 0) = 1 \end{aligned} \tag{5}$$

For the Forward Euler method, find the solution using the following Δt's: 0.5, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's and the **odeint()** function in the same plot and add labels, a grid and a legend to the plot.

b)

$$\begin{aligned} \text{ODE:} \qquad & y' = t^2 \left(\sin(2t) - 2ty\right) \\ & 1 \leq t \leq 2 \\ \text{inital condition} \qquad & y(t = 1) = 2 \end{aligned} \tag{6}$$

For the Forward Euler method, find the solution using the following Δt's: 0.5, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's and the **odeint()** function in the same plot and add labels, a grid and a legend to the plot.

c)

$$\begin{aligned} \text{ODE:} \qquad & y' = -y + t\, y^{1/2} \\ & 2 \leq t \leq 3 \\ \text{inital condition} \qquad & y(t = 2) = 2 \end{aligned} \tag{7}$$

For the Forward Euler method, find the solution using the following Δt's: 0.25, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's and the **odeint()** function in the same plot and add labels, a grid and a legend to the plot.

d)

$$\begin{aligned} \text{ODE:} \qquad & y' = \frac{t\,y + y}{t\,y + t} \\ & 2 \leq t \leq 4 \\ \text{inital condition} \qquad & y(t = 2) = 4 \end{aligned} \tag{8}$$

For the Forward Euler method, find the solution using the following Δt's: 0.25, 0.1, 0.05, 0.01. Please plot the solutions for the different Δt's and the **odeint()** function in the same plot and add labels, a grid and a legend to the plot.

You must write your script using **for** or **while** loops.

Please place comments in your code.

Save your script as **hw8_2.py**. Place your figures in a document using Word or PDF, add a captions to your figures. Save the document with the figures as **hw8_2_figs**. Upload the files to your Bitbucket account.