PHYS 251 - Homework 6

6 Homework 6

Please upload your answers to Bitbucket.

The documentation provided in the Python web page should be the first stop to get help. **Python documentation**.

6.1 Problem 1: Numerical Integration using the Rectangle method

Please write a script that will estimate the area under the curve $f(x) = x^2$ in the range $x = [0, 1]$ using the middle rectangle method and n rectangles. Your program should work for any integer value of n greater than zero. The approximated area calculated for n rectangles is called A_n.

Compute the exact area under the curve and save the value in a variable named A_{exact}.

Now, calculate the approximated area of $n = [1\,,\,2\,,\,3\,,\,\cdots\,,\,99\,,\,100]$. Store these areas in an array called A.

We define the difference between the approximation, A_n and the exact area, A_{exact}, as:

$$\delta_n = |(A_n - A_{exact})/A_{exact}| \tag{1}$$

for $n = [1\,,\,2\,,\,3\,,\,\cdots\,,\,99\,,\,100]$.

Plot δ versus n with appropriate legend and axis labels.

You must write your script using **for** or **while** loops.

Please place comments in your code.

Save your script as **hw6_1.py**. Place your figure in a document using Word or PDF, add a caption to your figure. Save the document with the figure as **hw6_1_figs**. Upload the files to your Bitbucket account.

6.2 Problem 2: Numerical Integration using the Composite Trapezoidal method

Write a script to approximate the following integrals using the Composite Trapezoidal method:

1. $$\int_0^4 x^4\,dx \tag{2}$$

2. $$\int_0^3 \frac{2}{x-4}\,dx \tag{3}$$

3. $$\int_1^4 x^2 \ln x\,dx \tag{4}$$

4.

$$\int_0^{2\pi} e^{2x} \sin(2x)\, dx \tag{5}$$

Your script should calculate the approximated area using $n = 1,\ 10,\ 100\ ,1000$.

In addition, calculate the same integrals using the function **quad()** from **scipy.integrate**.

Please print out all the solutions, your Trapezoidal approximations and the **quad()** approximation, in the Python console.

You must write your script using **for** or **while** loops.

Please place comments in your code.

Save your script as **hw6_2.py**. Upload the file to your Bitbucket account.

Hint: you may want to write a function for the Composite Trapezoidal rule and call it **my_trapezoidal()**. *The function* ***my_trapezoidal()*** *should takes two arguments, the function to integrate, f, and the number of trapezoids, n. The function* **my_trapezoidal()** *will return the approximation of the area under the function f.*

6.3 Problem 3: Numerical integration using the Composite Simpson's method

Please repeat the *Problem 2* using the Composite Simpson's rule.

Save your script as **hw6_3.py**. Upload the file to your Bitbucket account.

6.4 Problem 4: Numerical Integration using the Composite Trapezoidal and Composite Simpson's methods

For the curve $y = \frac{x^3}{81} + 1$ in the range of $x = [0, 4]$,

- compute the exact value of the total area,
- use the Composite Simpson's method with two areas to estimate the integral,
- use the Composite Trapezoidal method with two areas to estimate the integral.

We define the relative difference between the approximation of the Simpson's method and the Trapezoidal method as

$$\delta A = \left| \frac{A_{trapezoid} - A_{Simpson}}{A_{Simpson}} \right| \tag{6}$$

If we use the $A_{Simpson}$ of two areas under the curve to calculate δA, now compute the number of trapezoids that are needed to get $\delta A < 10^{-2}$.

You must write your script using **for** or **while** loops.

Please place comments in your code.

Save your script as **hw6_4.py**. Upload the file to your Bitbucket account.