使用蓋亞資料庫探討球狀星團中天琴座RR型變星的組成

Analyzing the Membership Percentage of RR Lyrae Variables in Globular Clusters Using GAIA Data

Background

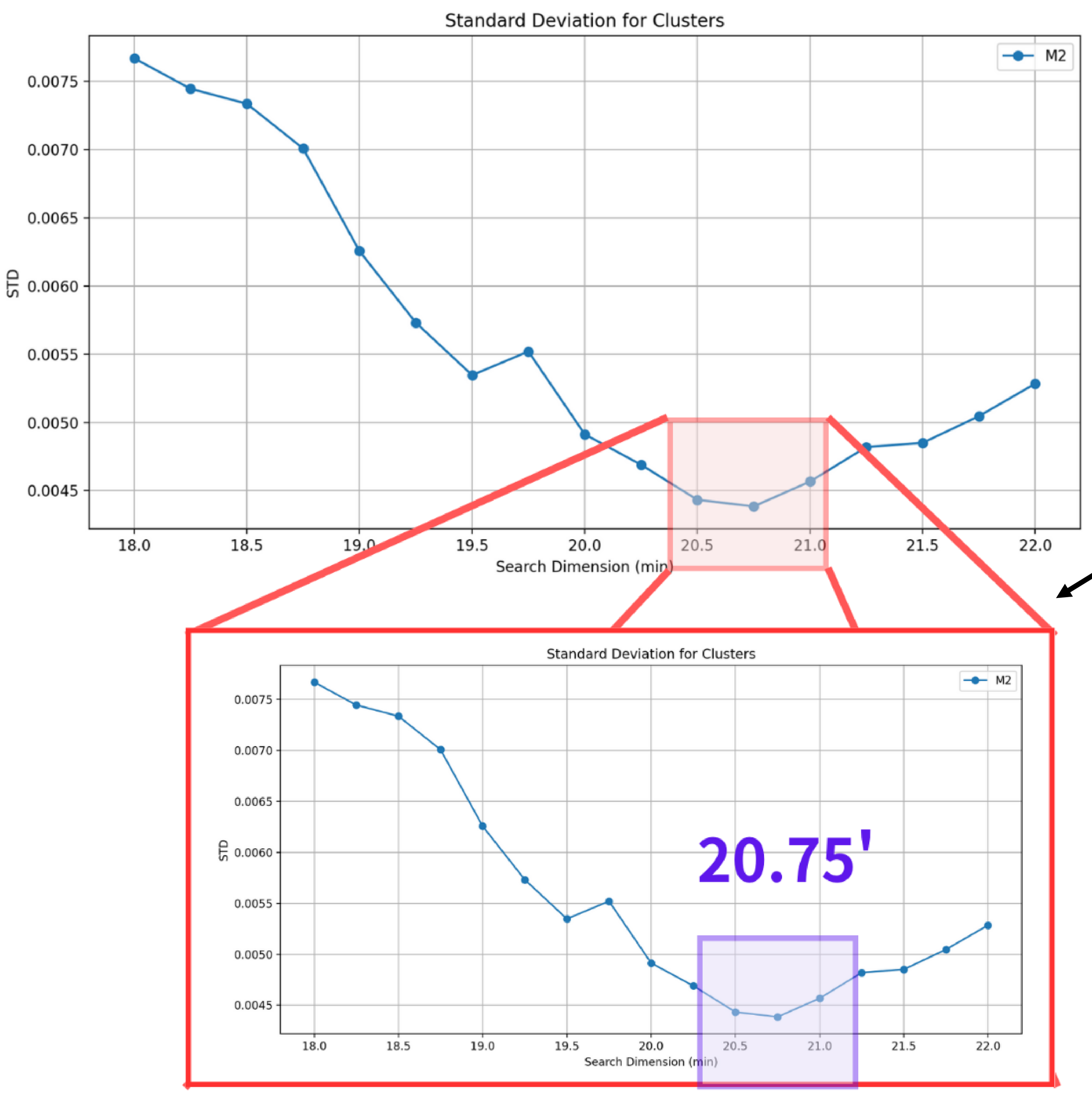
Globular Clusters (GCs) are home to RR Lyrae Variable stars (RRLs), which plays a significant role in determining the evolution of star clusters. Research has been devoted to the classification of RRLs based on brightness curves (S.I. Bailey), pulsation periods (Osterhoff Dichotomy), etc. Due to the relative lack of astronomical data available in the past, membership ratios of variable stars are not yet fully investigated, which is a motivation to find the ratio of a special kind of variable star called RR Lyrae stars in globular clusters with the newly released GAIA database.

Research Goals

- Using GAIA DR3, analyze the **part per million ratio** of RRLs to member stars in GCs (**RRppm**).
- Determine which cluster parameters influences **RRppm** most.
- Propose **hypotheses** regarding the cause of such result.

Methodology

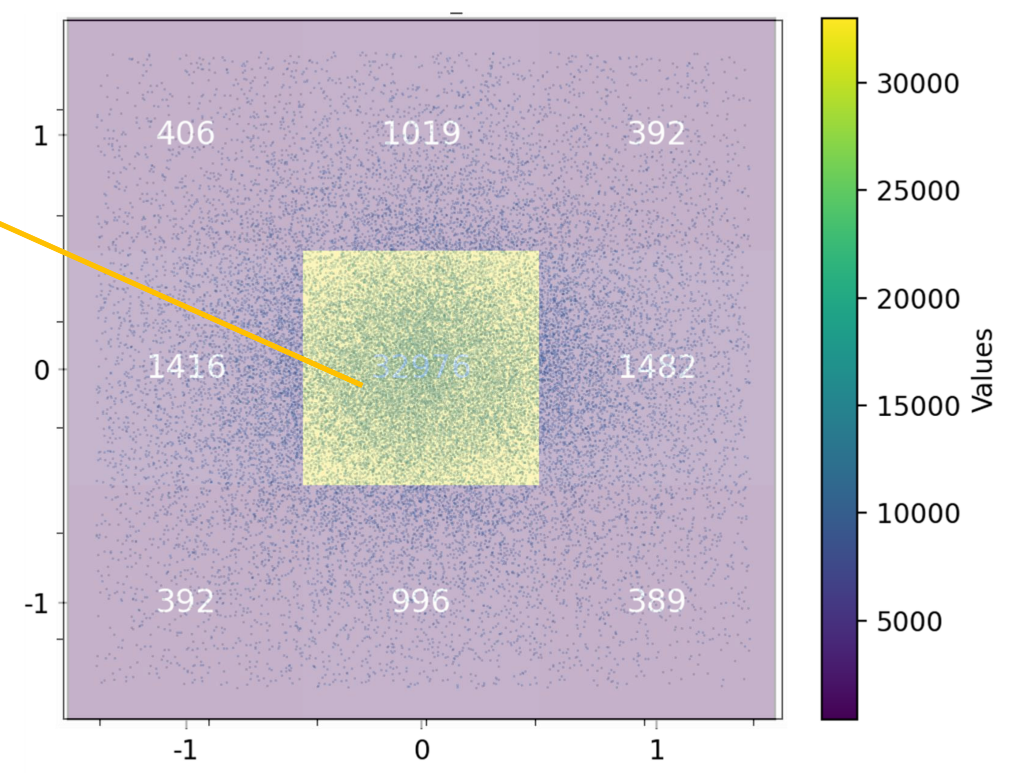
Estimating Number of Stars in Globular Clusters



1. Search the stars in a **square region** of given size covering the target GC.
2. Search 8 equally-sized squares around the GC, forming a 3x3 grid.
3. Count the number of stars in each cell, producing a **heatmap**.
4. Estimated number of stars = Stars number in center cell – average number of stars in the 8 surrounding cells.
5. Calculate STD of 8 surrounding cell. Increase search dimension until STD reaches minimum, indicating the center cell has covered the entire GC.

The center cell has 32976 stars

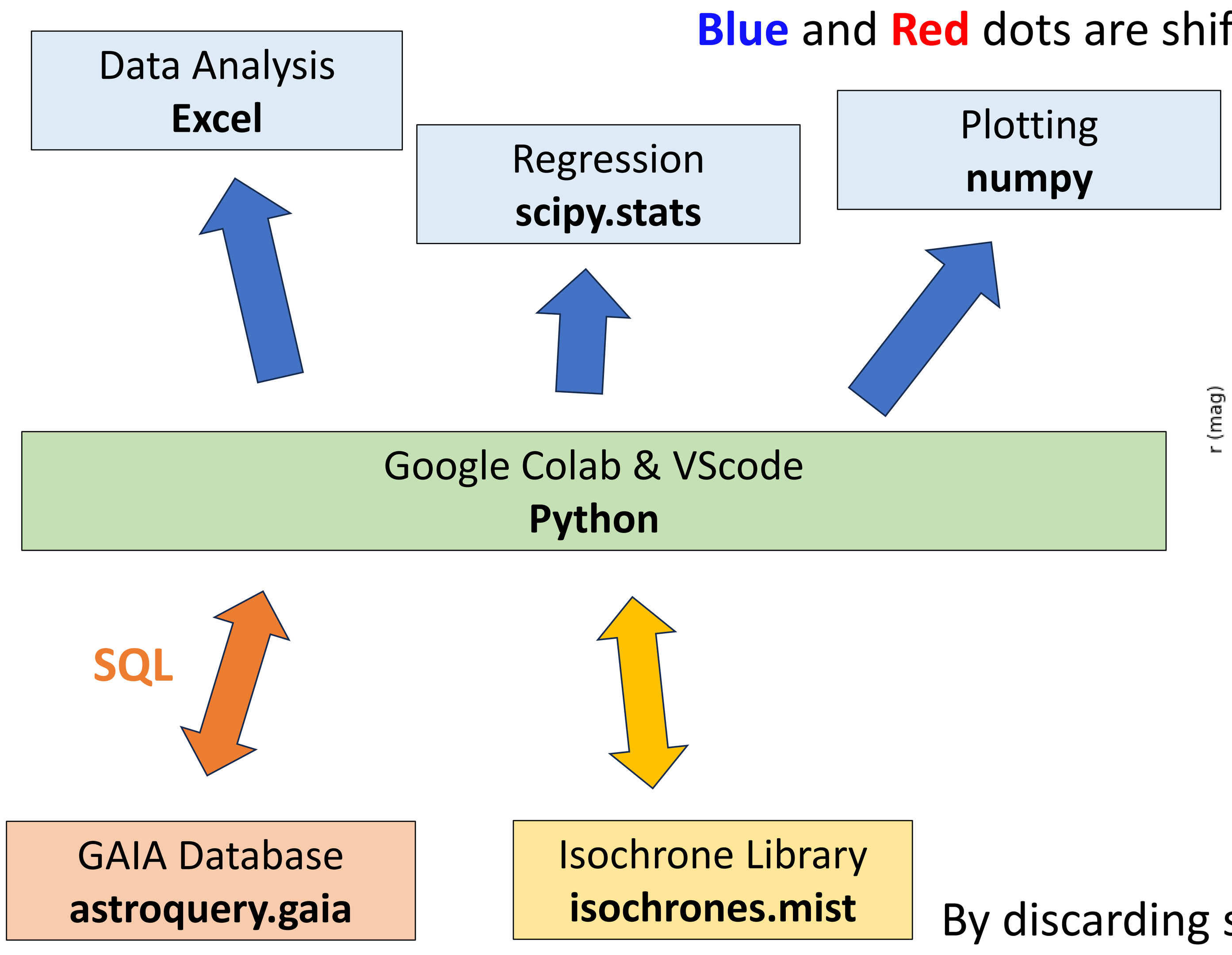
Number in each grid represents **number of stars** in each square region. The center square has significantly more stars as it covers the GC.



Heatmap of Surrounding Number of Stars of M3

Assuming that **foreground** and **background** stars are homogeneous, this method effectively **filters out** stars that are not a member of the target GC.

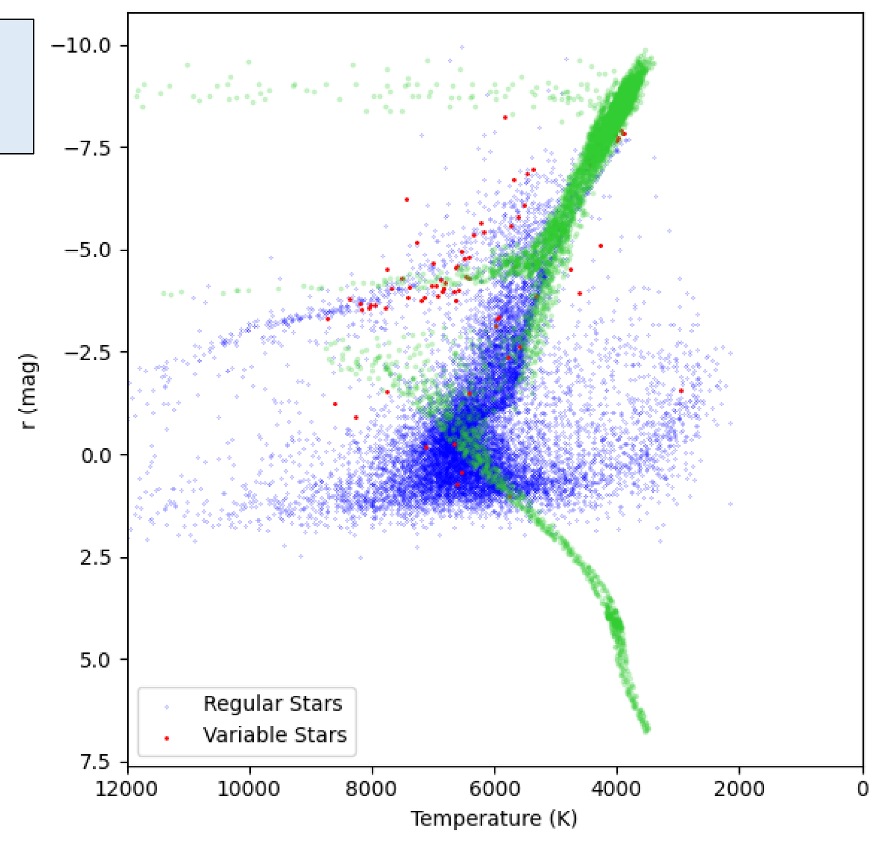
Isochrone Fitting



Isochrone Data Points Obtained with the `isochrone` library

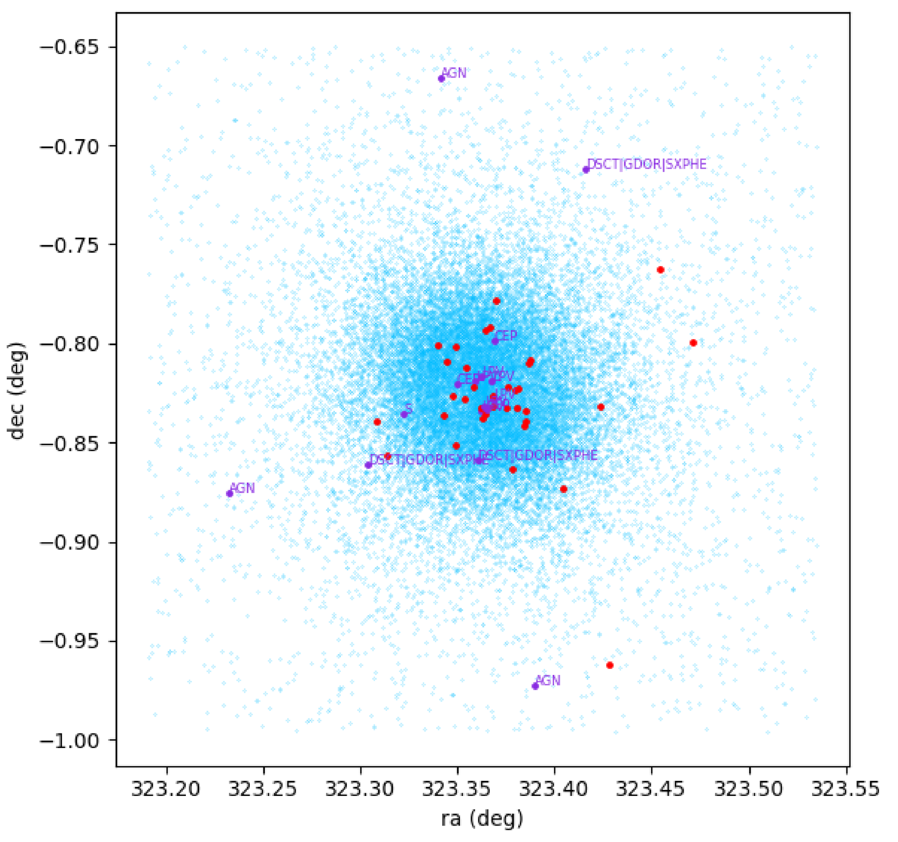
Blue Dots: Regular Stars **Red Dots:** Variable Stars

Blue and **Red** dots are shifted to match the **isochrone Curve.**



(Shifted) Color-Magnitude Diagram of M2

Cyan Dots: Regular Stars
Red Dots: RR Lyrae Variables
Purple Dots: Other Variable Stars



DEC-RA Scatter Plot of M2

By discarding stars that do not have **isochrone points** to the bottom or right, RRLs that do not belong to the GC can be filtered.

Membership Ratio Analysis

$$\text{RRppm} = \frac{\text{Number of RRLs in GC}}{\text{Total Number of Stars in GC}} \times 10^6 \text{ ppm}$$

Finding Correlation Between *RRppm* and Age, [Fe/H]

$$X_{RR} = \log(\text{Age}) + \lambda_1 \cdot [\text{Fe/HJ}$$

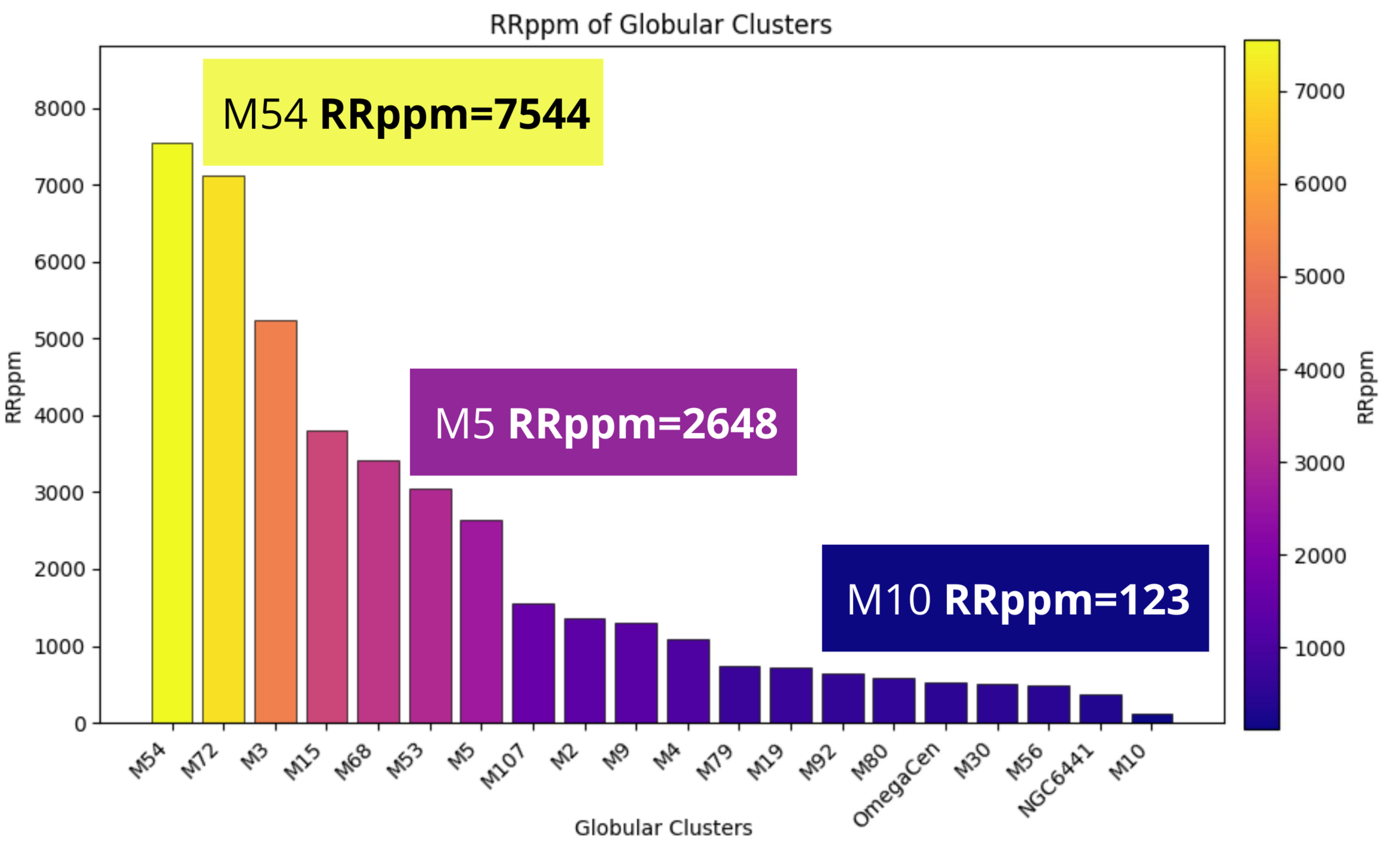
Finding Correlation Between *RRppm* and Age, [Fe/H], Radii

$$Y_{RR,1} = \log(\text{Age}) + \lambda_1 \cdot [\text{Fe/H}] + \mu_1 \cdot \log(\text{Radii})$$

$$Y_{RR,2} = \log(\text{Age}) + \lambda_2 \cdot [\text{Fe/H}] + \mu_2 \cdot \log(\text{Radii})$$

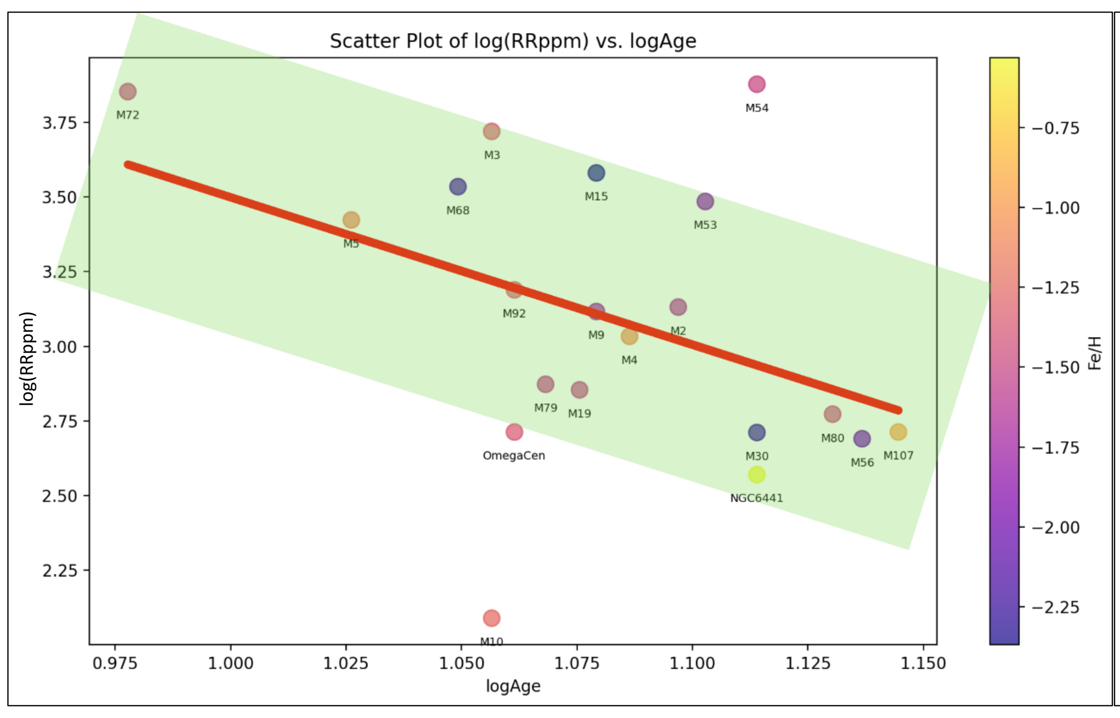
By optimizing $\lambda_{1,2}$ and $\mu_{1,2}$ to find the best **correlation coefficient** between X_{RR}, Y_{RR} and $RRppm$, the influence and relative significance of different parameters can be inferred.

(Half-mass) Radius: Light Years (ly)
Age: Gyr
Metallicity: [Fe/H]

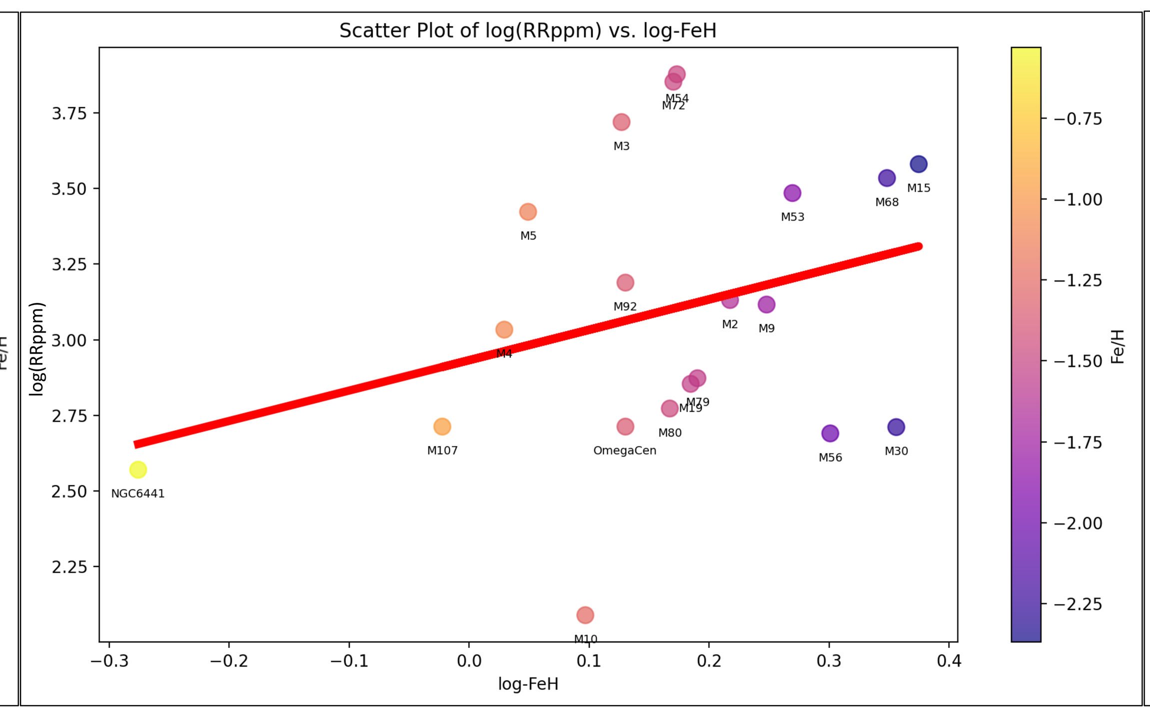


Results

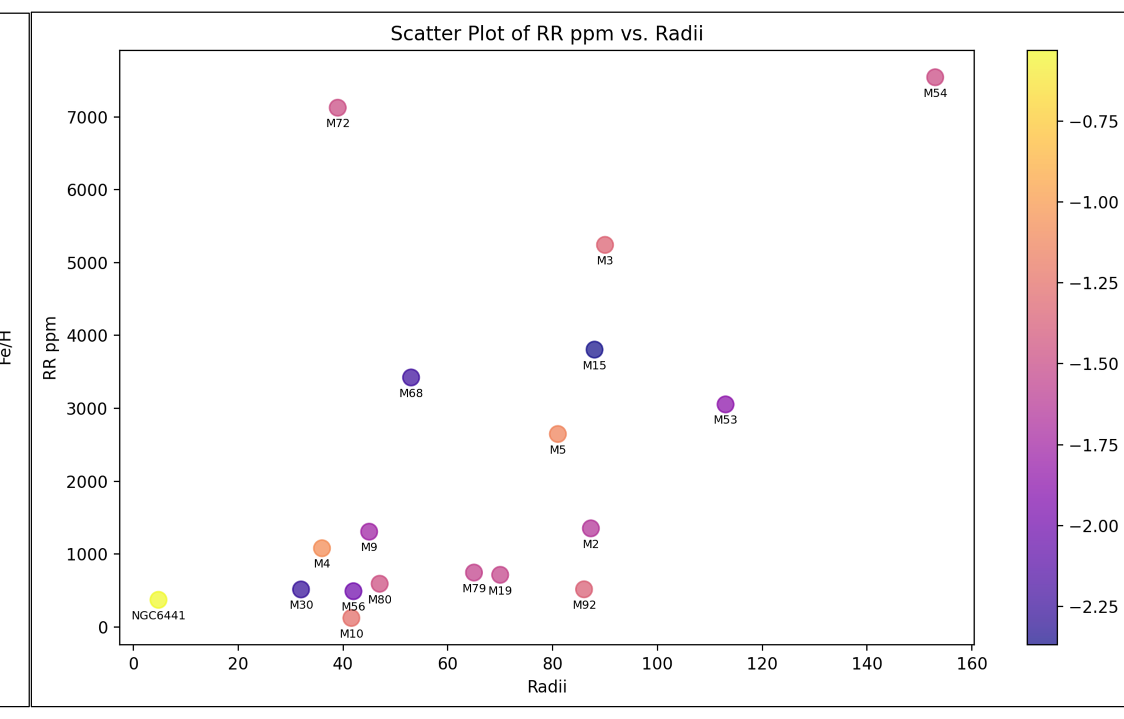
Single-Parameter Trend



Age (Gyr) Negative Correlation
Plot of log(RRppm) vs. log(Age)



Metallicity [Fe/H] Negative Correlation
Plot of log(RRppm) vs. log(-[Fe/H])



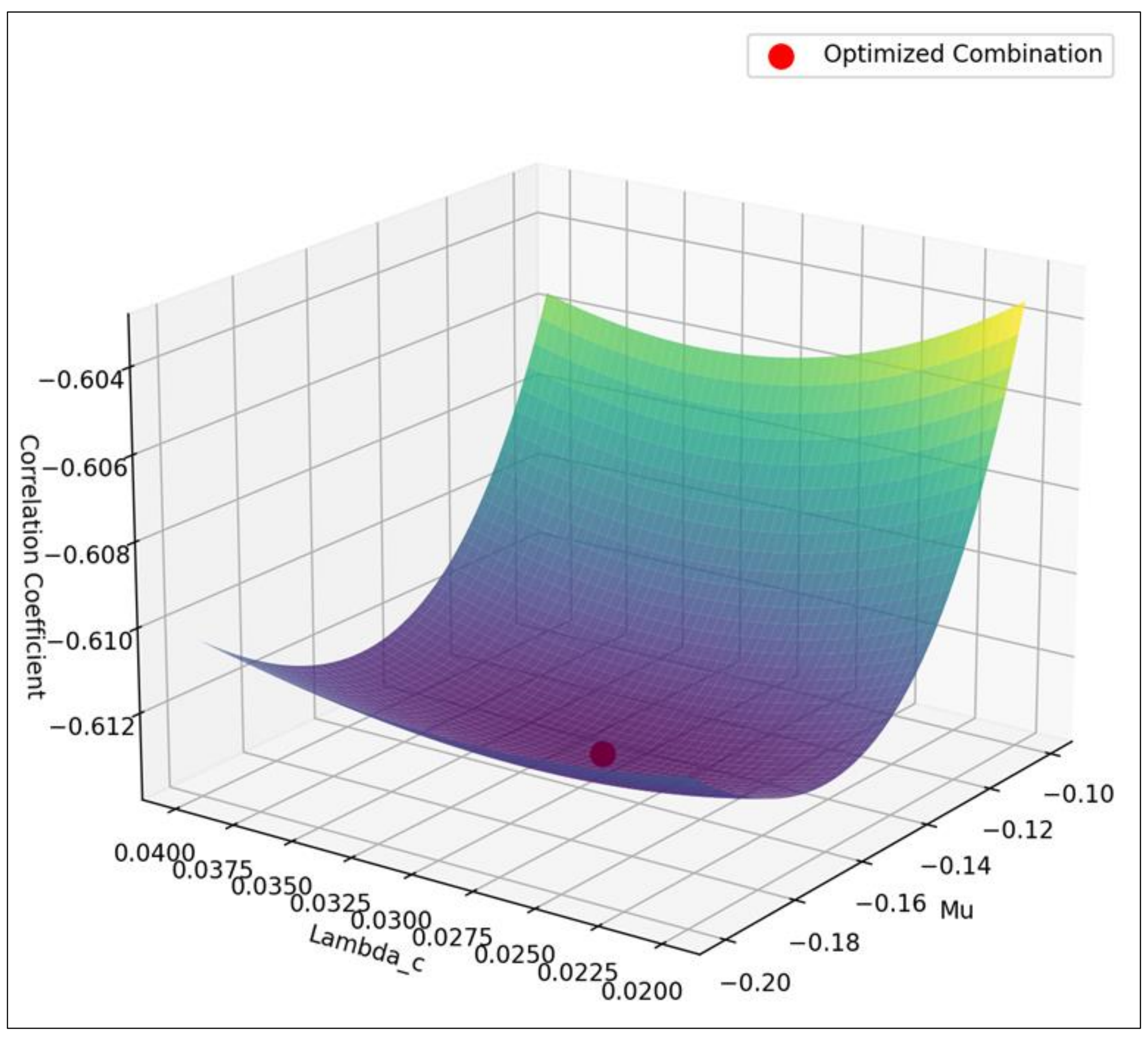
Radius (ly) Positive Correlation
Plot of RRppm vs. Radius

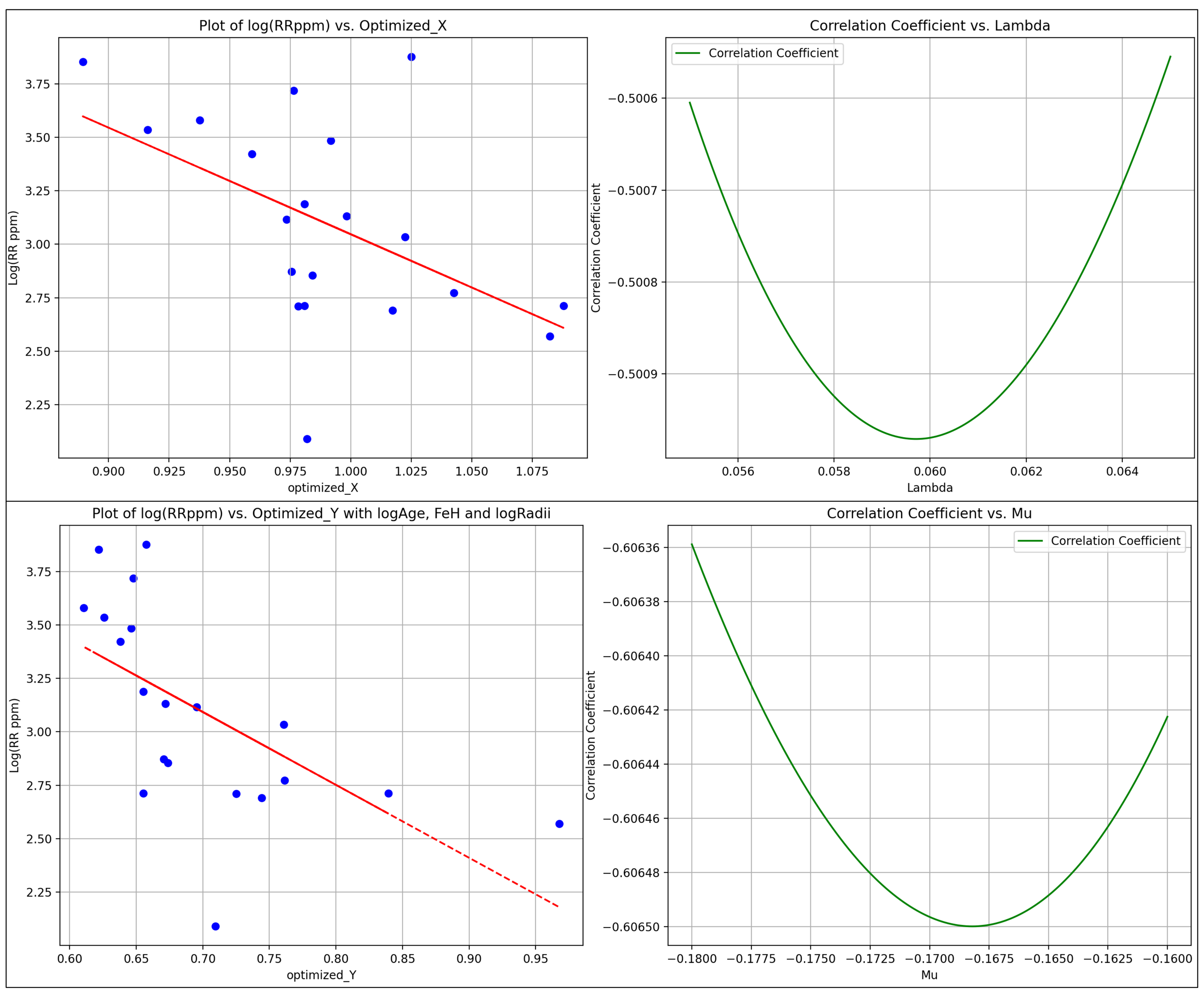
Multi-Parameter Analysis

Quantity	Expected Linear Relationship	Correlation Coefficient
$X_{RR} = \log(\text{Age}) + (\mathbf{0.0597}) \cdot \text{Metallicity}$	$\log(RRppm) = -4.9820 \cdot X_{RR} + 8.0291$	-0.501
$Y_{RR,1} = \log(\text{Age}) + (\mathbf{0.0597}) \cdot \text{Metallicity} + (\mathbf{-0.1682}) \cdot \text{Radius}$	$\log(RRppm) = -3.4129 \cdot Y_{RR,1} + 5.4820$	-0.607
$Y_{RR,2} = \log(\text{Age}) + (\mathbf{0.0296}) \cdot \text{Metallicity} + (\mathbf{-0.1563}) \cdot \text{Radius}$	$\log(RRppm) = -3.9352 \cdot Y_{RR,2} + 6.1098$	-0.613

- Coefficients λ and μ are determined up to 5 significant digits using a Python program.

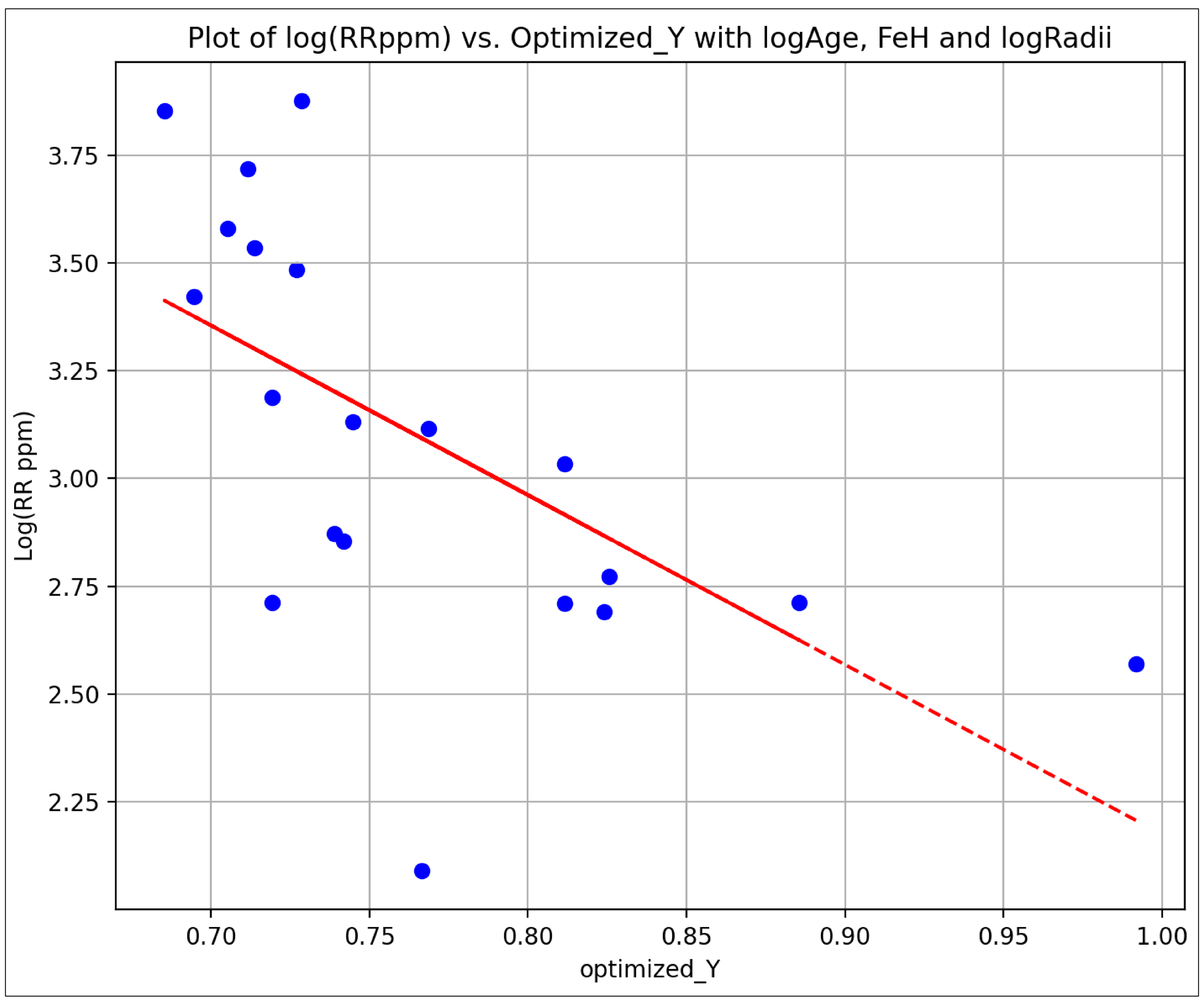
Optimization of λ_2 and μ_2
Optimal coefficients correspond to the lowest point on a 3D diagram.



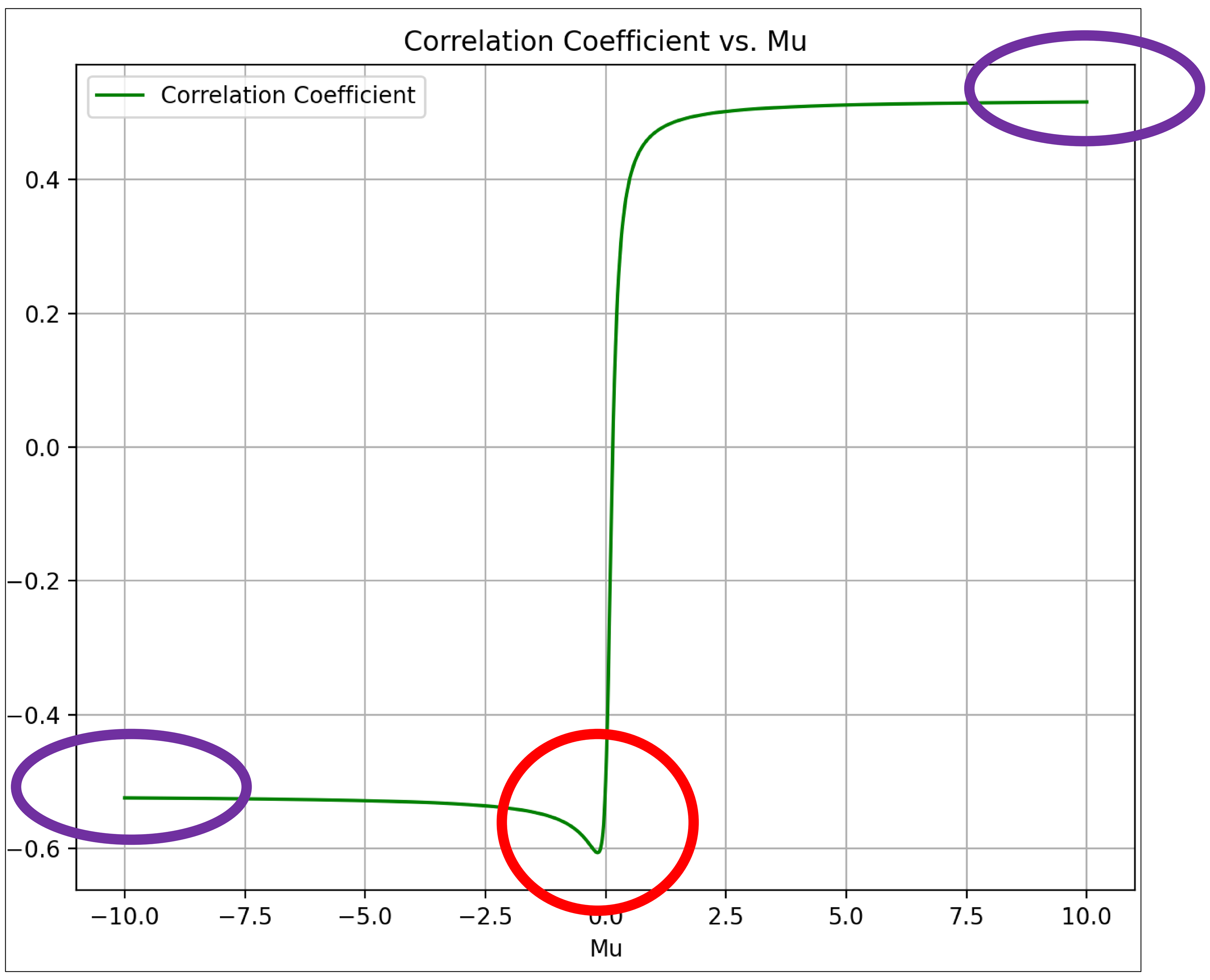


RIGHT
Correlation coefficient as a function of λ and μ. Extremum corresponds to best λ or μ.

LEFT
log(RRppm) versus X_{RR} and Y_{RR} for the λ or μ with highest correlation coefficient.



$\log(RRppm)$ vs. $Y_{RR,2}$ Regression Plot
RRppm is negatively correlated with $Y_{RR,2}$ (and also X_{RR} and $Y_{RR,1}$)



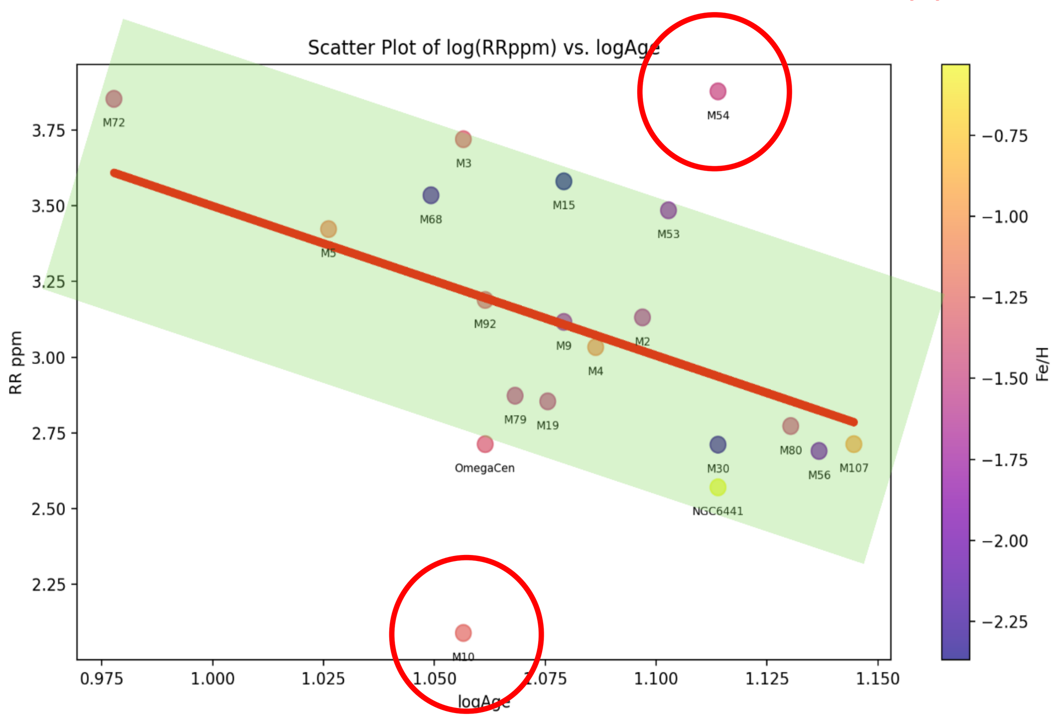
Large-Scale Trend of Correlation Coefficient of $Y_{RR,1} = \log(\text{Age}) + (\mathbf{0.0597}) \cdot \text{Metallicity} + (\mathbf{-0.1682}) \cdot \text{Radius}$
Optimized μ for Lowest Correlation Coefficient $r = -0.607$
Correlation Coefficient with Age & [Fe/H] as Only Variables

Interpretation of Results

Discussion of Special Cases

M54

High *RRppm*



M10

Low *RRppm*

M54: HIGH $RRppm$

M54 deviates from the general trend with no significant age, radius or metallicity

- **M54** belongs to the Sagittarius Dwarf Galaxy while others are in the Milky Way.
- Hence **the formation of RRLs may be influenced by its home galaxy.**

M10: LOW $RRppm$

M10 has a significantly small *RRppm*. In agreement with past research:

- No RRLs in M10 found in 2002 (Braun et al. 2002)
- Only one RRL had been found as of 2020 (Ferro et al. 2020)
- **Hypothesis**:
    1. Ferro et al. claims there may be variations on its Red Giant Branch (RGB) evolution.
    2. M10 may have an unique **mass-loss** due to, for example, collision with another cluster.

[Fe/H] and the Instability Strip

Negative correlation of number of RRLs with [Fe/H] can be explained with the **instability strip:**

- The instability strip is **narrower** when the metallicity is **higher**. (Demarque and Zinn, 1999)
- Therefore **less variable stars** populate the instability strip, causing a smaller *RRppm*.

Correlation with Cluster Radius

Parameter	Value Range	Coefficient in $Y_{RR,2}$	Relative Significance
log (Age (Gyr))	0.978~1.152	1	~1
log (Radius (ly))	0.681~2.185	-0.1563	~0.25
Metallicity ([Fe/H])	-2.37~-0.95	0.0296	~0.05

Factors influencing *RRppm* in decreasing order:

1. **Age** — Primary factor of GC evolution (Gratton et al. 2019)
2. **Radius** — Tidal force & Mass-loss
3. **Metallicity** — Secondary factor of GC evolution (Gratton et al. 2019)

Relative Significance ≈ A Likely Value of the Parameter × Coefficient in $Y_{RR,2}$

Tidal force ∝ **Radius**

Mass-loss plays a role in the formation of RRLs **(Ferro et al. 2020)**

RRppm has **correlation** with Radius

Tidal force influences mass loss of GC

The formation of RRLs may be correlated with **tidal forces.**

Conclusion

- Using **GAIA DR3**, 20 globular clusters are analyzed.
- The number of stars in GCs are found with the method of **heatmaps**. After fitting with **isochrone** curves, *RRppm* can be calculated. By defining $X_{RR}, Y_{RR,1}, Y_{RR,2}$, the influence of cluster parameters are quantitatively analyzed.
- **Age, Radius** and **Metallicity** influences *RRppm* most, listed in decreasing order.
- The role of **Age** and **Metallicity** is in agreement with literature.
- The impact of **Radius** can be explained with **mass-loss** and **tidal forces**.
- From the clusters that deviate from the main trend, it is concluded that **the home galaxy** or **RGB evolution** may influence RRL formation. Moreover, the **instability strip** explains how **Metallicity** affects *RRppm.*

Future Prospects

- Similar analysis could be done on open clusters, given that member stars can be properly distinguished from background and foreground stars.
- Apart from RRL, other types of variable stars could be analyzed.
- Deeper analysis into globular clusters could reveal insights into the early ages of the universe.

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