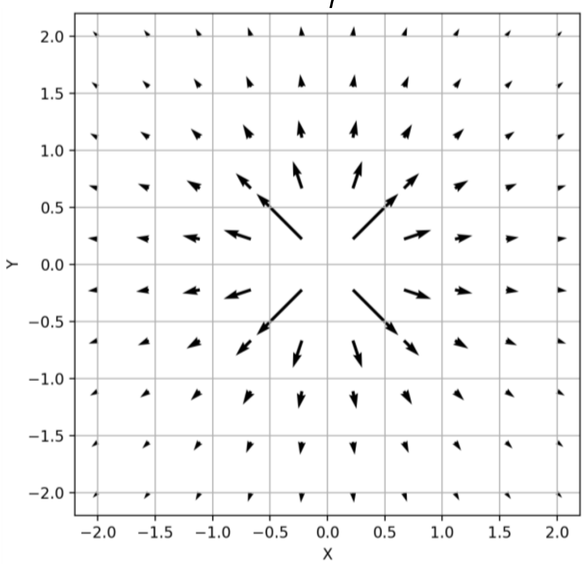
基礎向量分析

向量場 Vector Field

空間中每一點對應到一個**向量**

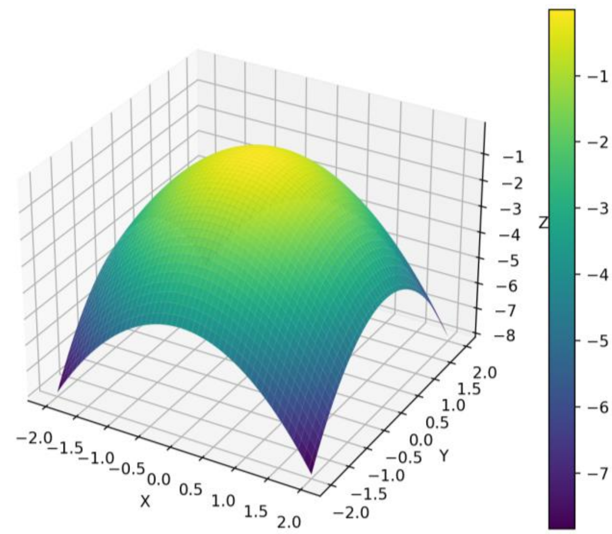
$$\boldsymbol{v}(r) = \frac{0.8}{r}\hat{\boldsymbol{r}}$$



純量場 Scalar Field

空間中每一點對應到一個**純量**

$$V(x,y) = -(x^2+y^2)$$



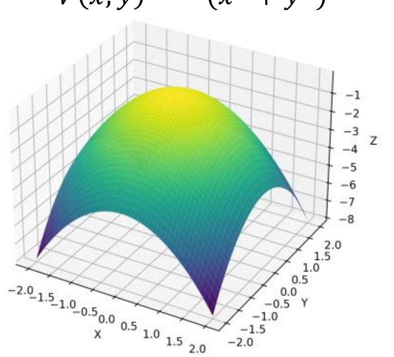
$$f = f_x\hat{\boldsymbol{x}} + f_y\hat{\boldsymbol{y}} + f_z\hat{\boldsymbol{z}} \qquad \nabla \equiv \frac{\partial}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial}{\partial z}\hat{\boldsymbol{z}}$$

梯度 Gradient ∇f

就像一元函數的**導數**表示這個函數圖形的**切線的斜率**，如果多元函數在某點上的梯度不是零向量，則它的方向是這個函數在該點上**最大增長的方向**、而它的量是在這個方向上的**增長率**[1]。

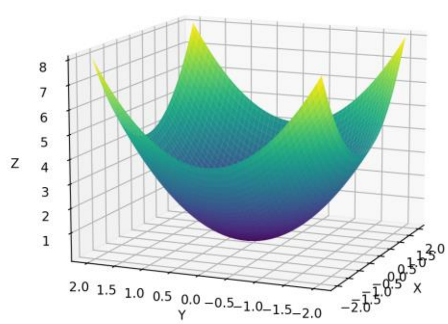
Gradient at (0, 0) >0 =0 <0

$V(x,y) = -(x^2+y^2)$



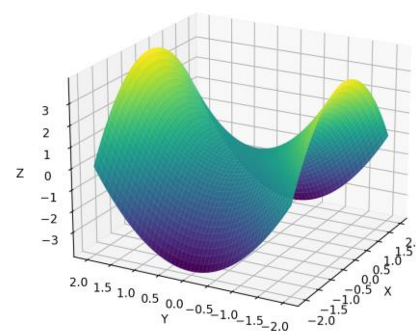
Gradient at (0, 0) >0 =0 <0

$V(x,y) = x^2+y^2$



Gradient at (0, 0) >0 =0 <0

$V(x,y) = -x^2+y^2$



$$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$$

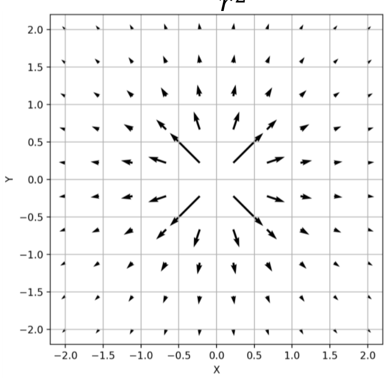
[1] 梯度. (2023, September 17). Retrieved from 维基百科, 自由的百科全書: https://zh.wikipedia.org/w/index.php?title=%E6%A2%AF%E5%BA%A6&oldid=78965959

散度 Divergence $\nabla \cdot \boldsymbol{f}$

散度描述的是向量場裡一個點是**匯聚點**還是**發源點**，就是這包含這一點的一個**微小體元中的向量**是「**向外**」居多還是「**向內**」居多[2]。

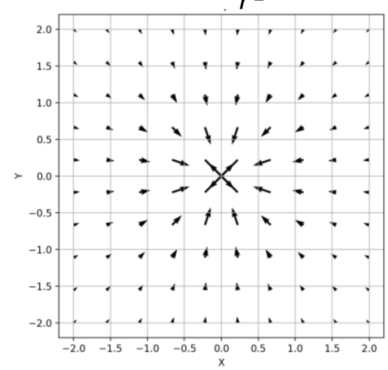
Divergence at (0, 0)　>0　=0　<0

$\boldsymbol{v}(r) = \frac{0.8}{r^2}\hat{\boldsymbol{r}}$



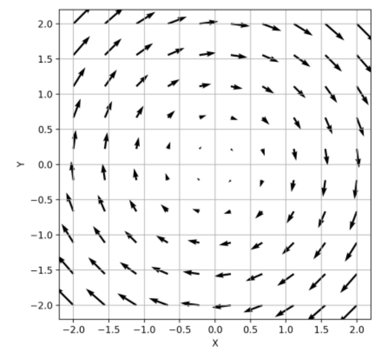
Divergence at (0, 0)　>0　=0　<0

$\boldsymbol{v}(r) = -\frac{0.8}{r^2}\hat{\boldsymbol{r}}$



Divergence at (0, 0)　>0　=0　<0

$\boldsymbol{v}(\boldsymbol{x},\boldsymbol{y}) = -2x\hat{\boldsymbol{x}} + 2y\hat{\boldsymbol{y}}$



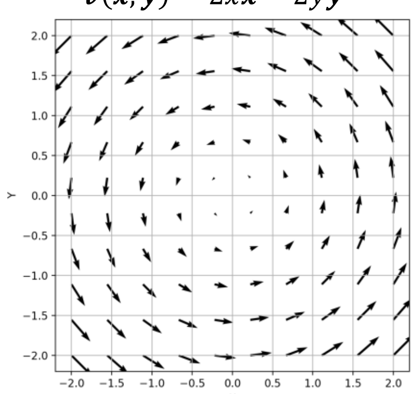
$$\nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial z} + \frac{\partial f_z}{\partial z}$$

旋度 Curl $\nabla \times \boldsymbol{f}$

旋度的方向是旋轉的軸，它由**右手定則**來確定，而旋度的大小是旋轉的量，也就是旋度向量。這個向量的特性（長度和方向）刻畫了在這個點上的**旋轉**[3]。

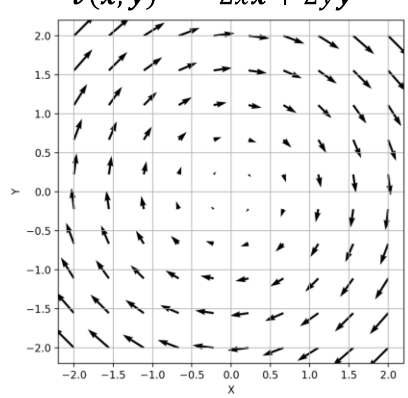
Curl at (0, 0)　>0　=0　<0

$\boldsymbol{v}(\boldsymbol{x},\boldsymbol{y}) = 2x\hat{\boldsymbol{x}} - 2y\hat{\boldsymbol{y}}$



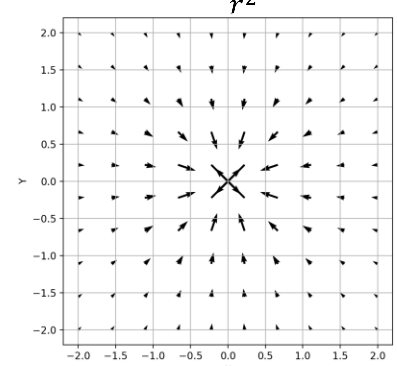
Curl at (0, 0)　>0　=0　<0

$\boldsymbol{v}(\boldsymbol{x},\boldsymbol{y}) = -2x\hat{\boldsymbol{x}} + 2y\hat{\boldsymbol{y}}$



Curl at (0, 0)　>0　=0　<0

$\boldsymbol{v}(r) = \frac{1}{r^2}\hat{\boldsymbol{r}}$



$$\nabla \times f = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)\hat{\boldsymbol{z}} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$$

[2] 散度. (2024, February 8).). Retrieved from 维基百科, 自由的百科全書 https://zh.wikipedia.org/w/index.php?title=%E6%95%A3%E5%BA%A6&oldid=80837177

[3] 旋度. (2024, February 8). Retrieved from 维基百科, 自由的百科全書: https://zh.wikipedia.org/w/index.php?title=%E6%97%8B%E5%BA%A6&oldid=80838012

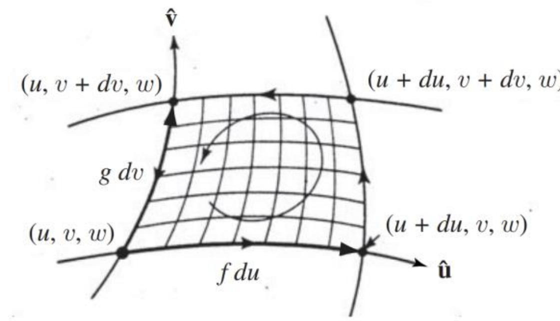
一些定理 Some Theorems

散度定理 Divergence Theorem / 高斯定理 Gauss’s Law

$$\iiint_{\mathrm{V}} \nabla \cdot \boldsymbol{F}\, dV = \oiint_{S} \boldsymbol{F} \cdot d\boldsymbol{A}$$

旋度定理 Curl Theorem / 斯托克斯定理定理 Stokes' theorem

$$\iint_S (\nabla \times \boldsymbol{F}) \cdot d\boldsymbol{A} = \oint_L \boldsymbol{F} \cdot d\boldsymbol{r}$$



[4]

[4] Griffiths, D. J. (2013). Introduction to electrodynamics (4th ed.). Reed College.