Misconception Regarding the Pressure Term in Bernoulli's Equation

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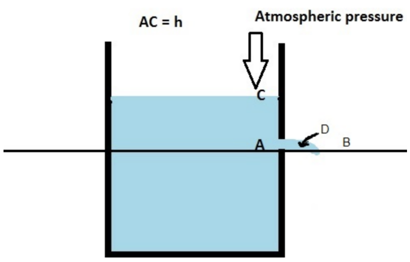
Abstract

This *Misconception* is provided by 吳俊緯, classmate of Chiayi Senior High School's 12th science class.

Argument

Consider a tank filled with water, which has a hole at a distance h below the water surface. The hole size is way smaller than the cross-sectional area of the tank. If we write the Bernoulli equation



$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

for the streamline that passes through a point at the top water surface (point C) and the hole, we obtain

$$P_{atm} + \frac{1}{2}\rho(0)^2 + \rho gh = P' + \frac{1}{2}\rho v^2 + \rho g(0)$$

where P_{atm} indicates the atmospheric pressure and P' is the pressure we write for the point at the hole. The problem is, should it be $P' = P_{atm} + \rho gh$, accounting for the pressure caused by the water at the hole (by $P_{liquid} = \rho g(\Delta h)$), or simply $P' = P_{atm}$? Note that the wrong result will lead to false answers when calculating, for example, the exit velocity of the fluid.

Misconception

The formula for the pressure caused by a height difference in a fluid ($P_{liquid} = \rho g(\Delta h)$) is *only* valid if we're dealing with *fluid mechanics* and not *fluid dynamics*. In other words, the fluid must *not* be flowing for the equation to work. However, fluid is obvious flowing here, so we can't use that equation. However, the pressure at the leftmost end of the tank on the horizontal line depicted in the image is still $P_{atm} + \rho gh$, considering that the tank is large enough and water isn't flowing there.

To determine the pressure, we must analyze the proof of the Bernoulli equation. One is considering the work being done on the fluid by the net force exerted on it, which is caused by the pressure acting on the fluid. Therefore, at the hole you need to be considering the pressure pushing on the fluid at the outside of the hole, which is the atmospheric pressure.[1]

You may also wonder if there's a sudden discontinuity of pressure very near the hole. Fortunately, this is not the case. Consider a packet of fluid from the leftmost edge of the tank, originally on the horizontal line in the picture. As fluid particles accelerate toward the hole, eventually exiting the tank, its pressure decreases by Bernoulli's equation, since ρgh remains constant. Therefore, the pressure will gradually decrease from $P_{atm} + \rho gh$ at the initial position, to only P_{atm} at the hole. [2]

Solution

$P' = P_{atm}$ is correct, since the argument leading to the additional ρgh term is invalid as the fluid is not static.

References

[1] BioPhysicist (username). Bernoulli principle for a leaky water tank. 2020.

[2] Chet Miller (username). If a hole is drilled at the bottom of a vessel, why is the pressure of the liquid leaving the vessel equal to atmospheric pressure? 2016.