# Math Preliminaries for High School Physics Competitions

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$$\sin^2(x) + \cos^2(x) = 1 \tag{1}$$
$$\tan(x) = \frac{\sin(x)}{\cos(x)} \tag{2}$$
$$\sec(x) = \frac{1}{\cos(x)} \tag{3}$$
$$\csc(x) = \frac{1}{\sin(x)} \tag{4}$$
$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)} \tag{5}$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B) \tag{6}$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B) \tag{7}$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B) \tag{8}$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B) \tag{9}$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B) \tag{10}$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B) \tag{11}$$

$$\sin(3x) = 3\sin(x) - 4\sin^3(x) \tag{12}$$
$$\cos(3x) = 4\cos^3(x) - 3\cos(x) \tag{13}$$

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)} \tag{14}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)} \tag{15}$$

Sum to product:

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A) - \sin(B) = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$
$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Product to sum:

$$\sin(A)\cdot\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$
$$\sin(A)\cdot\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$
$$\cos(A)\cdot\sin(B) = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$
$$\cos(A)\cdot\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$A\sin(x) + B\cos(x) = C\sin(x+\phi) \tag{16}$$

$$C = \sqrt{A^2+B^2}$$
$$\tan(\phi) = \frac{B}{A}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}, \tag{17}$$
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{18}$$
$$\operatorname{arcsinh}(x) = \ln(x + \sqrt{x^2+1}), \tag{19}$$
$$\operatorname{arccosh}(x) = \ln(x + \sqrt{x^2-1}), \quad \text{for } x > 1, \tag{20}$$
$$\operatorname{arctanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad \text{for } |x| < 1, \tag{21}$$
$$\operatorname{arccoth}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), \quad \text{for } |x| > 1 \tag{22}$$

$$\frac{d}{dx}[f(x)\cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x) \tag{23}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2} \tag{24}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))\cdot g'(x) \tag{25}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}, \text{ provided } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\infty}{\infty} \tag{26}$$

| | |
|---|---|
| $f(x) = x$ | $f'(x) = 1$ |
| $f(x) = x^2$ | $f'(x) = 2x$ |
| $f(x) = x^n$ | $f'(x) = nx^{n-1}$ |
| $f(x) = \sin(x)$ | $f'(x) = \cos(x)$ |
| $f(x) = \cos(x)$ | $f'(x) = -\sin(x)$ |
| $f(x) = e^x$ | $f'(x) = e^x$ |
| $f(x) = \ln(x)$ | $f'(x) = \frac{1}{x}$ |
| $f(x) = \tan(x)$ | $f'(x) = \sec^2(x)$ |
| $f(x) = \sec(x)$ | $f'(x) = \sec(x)\tan(x)$ |
| $f(x) = \csc(x)$ | $f'(x) = -\csc(x)\cot(x)$ |
| $f(x) = \cot(x)$ | $f'(x) = -\csc^2(x)$ |
| $f(x) = \arcsin(x)$ | $f'(x) = \frac{1}{\sqrt{1-x^2}}$ (for $\lvert x\rvert < 1$) |
| $f(x) = \arccos(x)$ | $f'(x) = -\frac{1}{\sqrt{1-x^2}}$ (for $\lvert x\rvert < 1$) |
| $f(x) = \arctan(x)$ | $f'(x) = \frac{1}{1+x^2}$ |
| $f(x) = \operatorname{arcsec}(x)$ | $f'(x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ for $\lvert x\rvert > 1$ |
| $f(x) = \operatorname{arccsc}(x)$ | $f'(x) = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ for $\lvert x\rvert > 1$ |
| $f(x) = \operatorname{arccot}(x)$ | $f'(x) = -\frac{1}{1+x^2}$ |
| $f(x) = \operatorname{arcsinh}(x)$ | $f'(x) = \frac{1}{\sqrt{x^2+1}}$ |
| $f(x) = \operatorname{arccosh}(x)$ | $f'(x) = \frac{1}{\sqrt{x^2-1}}$ for $x > 1$ |
| $f(x) = \operatorname{arctanh}(x)$ | $f'(x) = \frac{1}{1-x^2}$ for $\lvert x\rvert < 1$ |
| $f(x) = \operatorname{arccoth}(x)$ | $f'(x) = -\frac{1}{x^2-1}$ for $\lvert x\rvert > 1$ |