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Institute of Computer Science
Chair of Communication Networks
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Chapter 3.1

Stochastic Processes

Performance Evaluation of the Internet of Things (IoT)

Module Course: Performance Evaluation of Distributed Systems

Prof. Tobias Hoßfeld, Summer Semester 2022

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The book must be cited and the disclaimer attached when using lectures slides or scripts.

Tran-Gia, P. & Hossfeld, T. (2021).
Performance Modeling and Analysis of Communication Networks - A Lecture Note. Würzburg University Press.
https://doi.org/10.25972/WUP-978-3-95826-153-2

Website to download book, exercises, slides and scripts:
https://modeling.systems/

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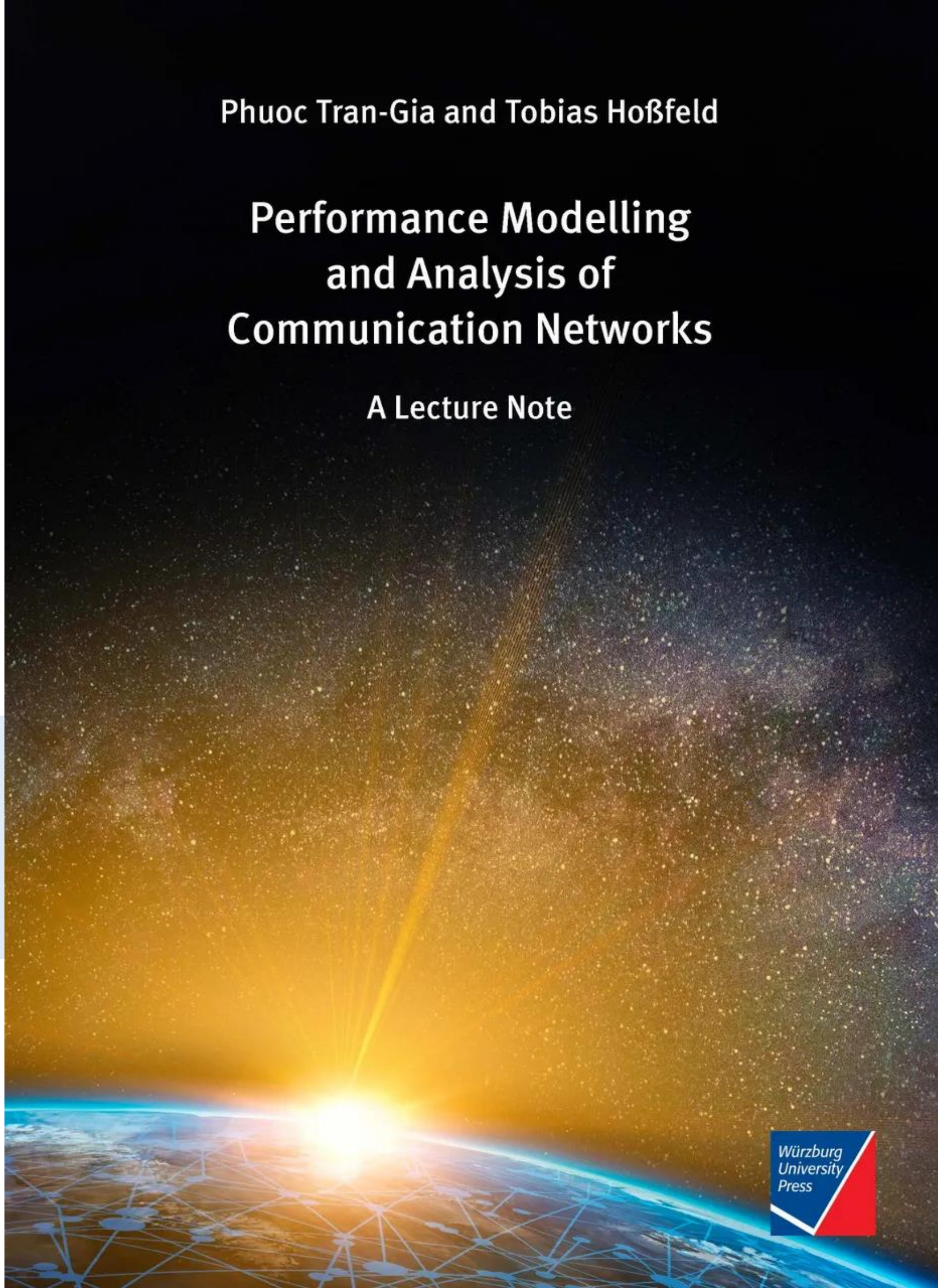


Chapter 3

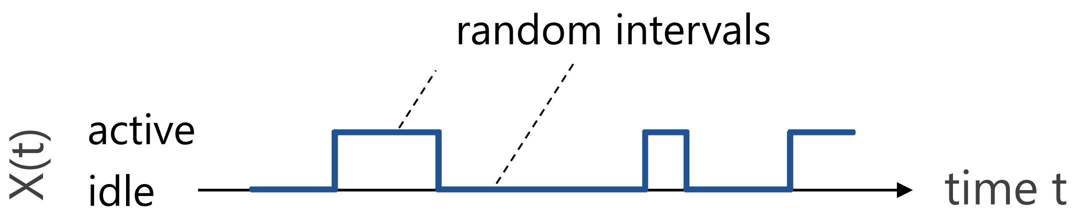
3 Elementary Random Processes

Stochastic Processes

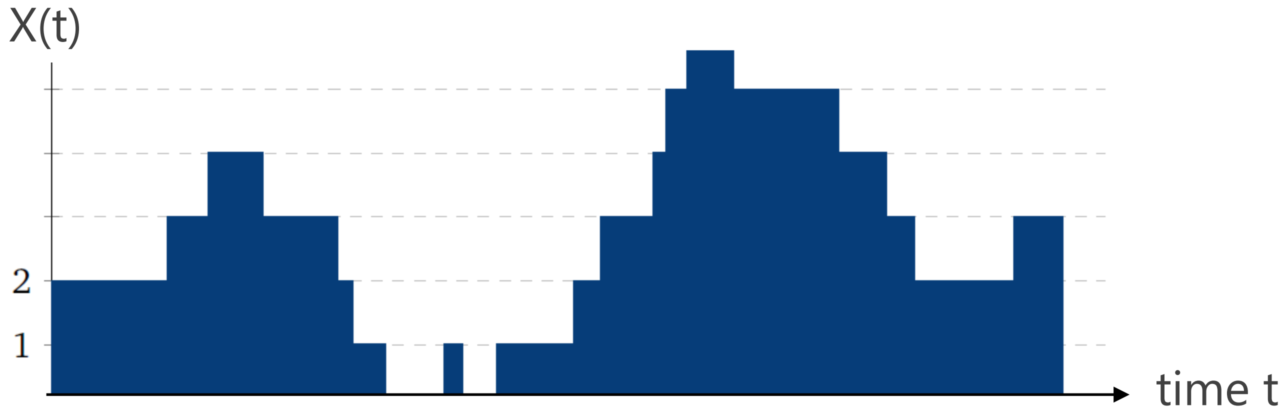
Examples and definition

Examples

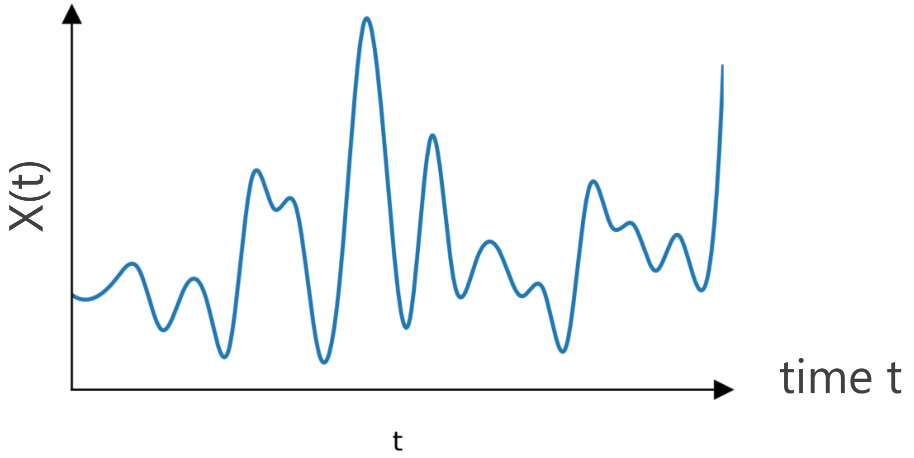
server



buffer state
of a router

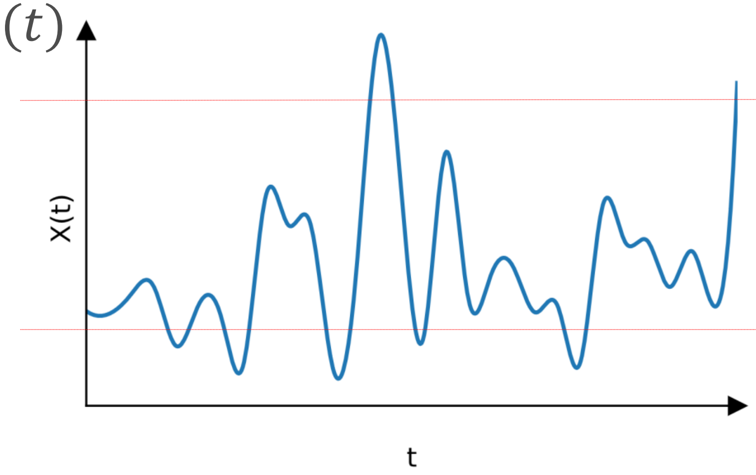


remaining workload
at a server

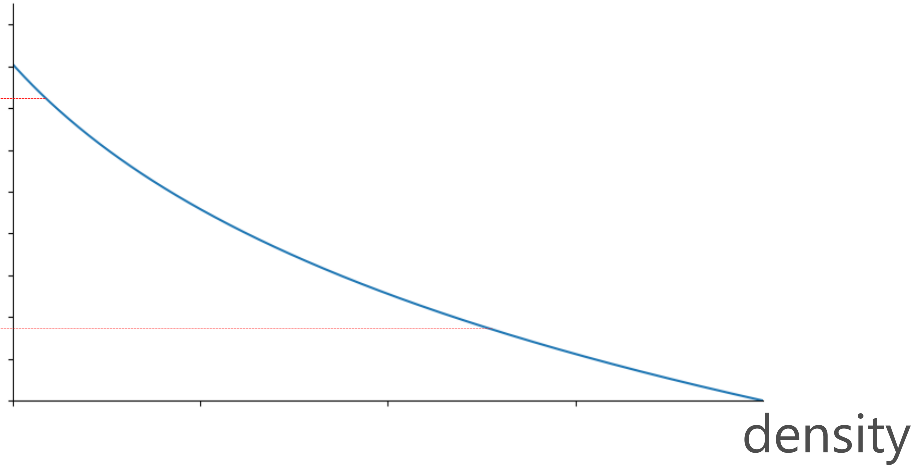


Objective of Stochastic Modeling

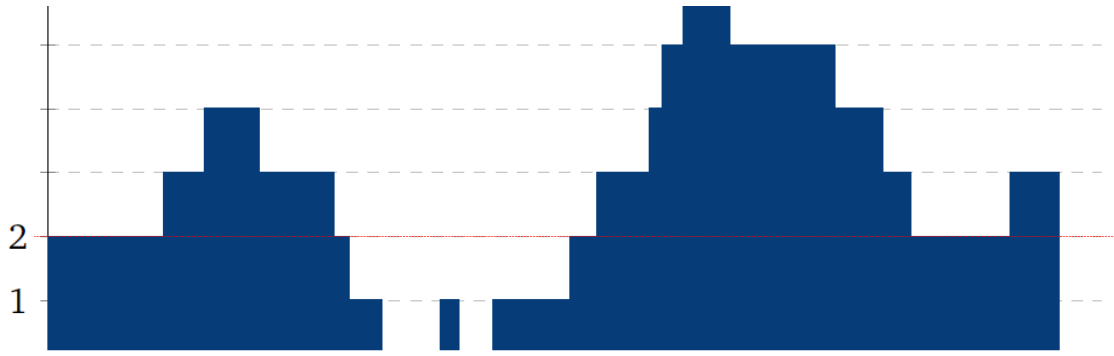
remaining workload
at a server $X(t)$



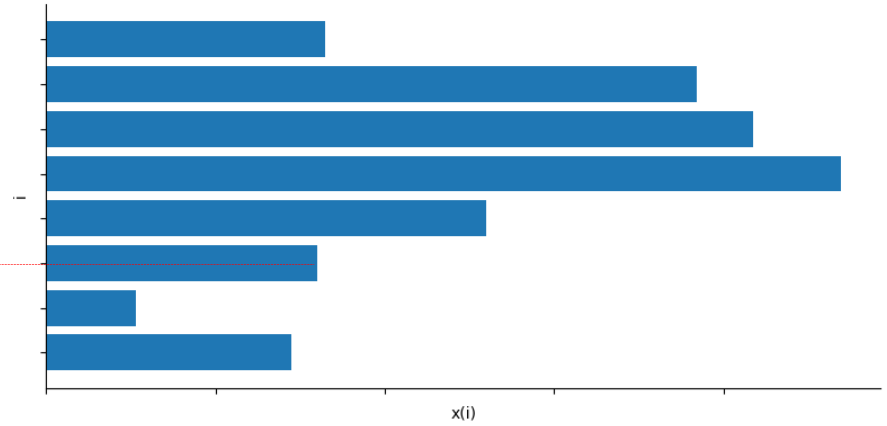
probability density function



system state
$X(t)$

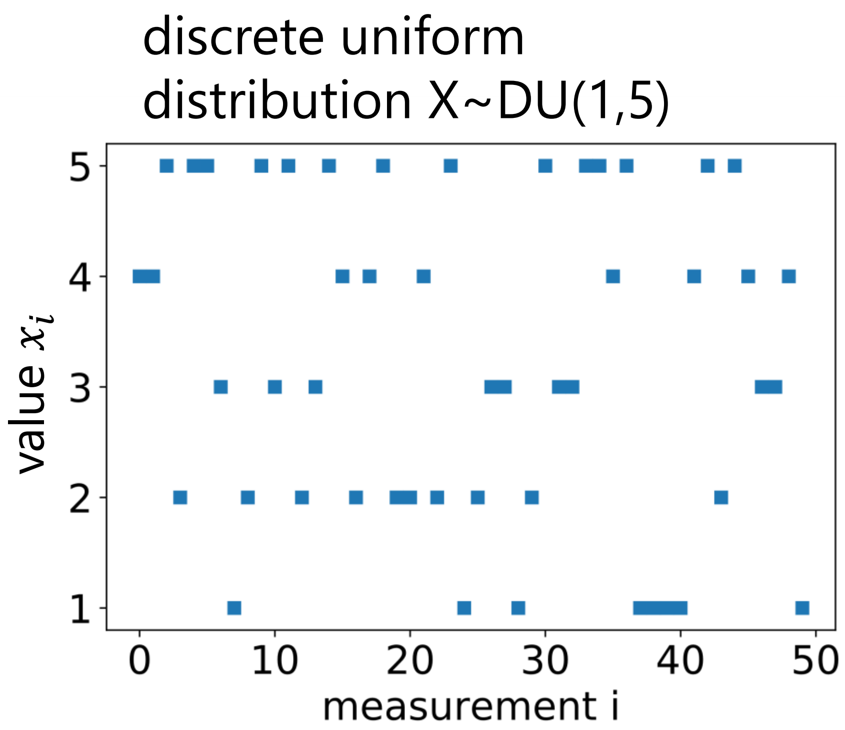


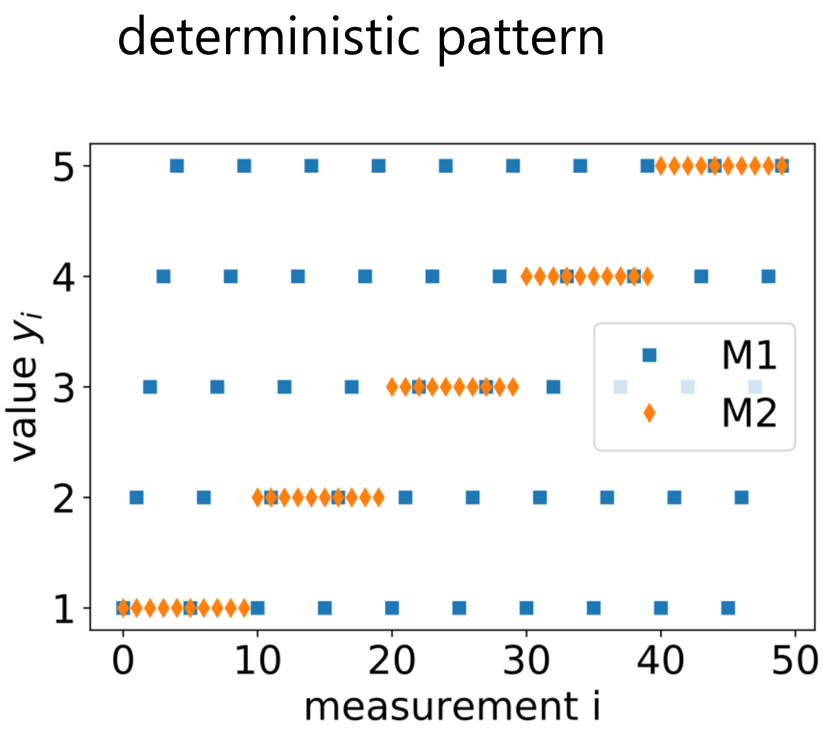
state distribution $x(i) = P(X = i)$



Stochastic Modeling

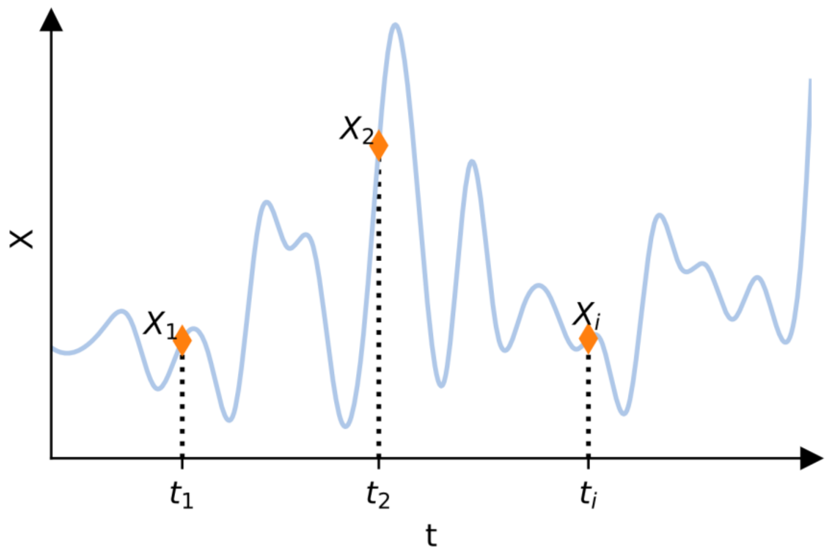
- Same mean values, but different patterns





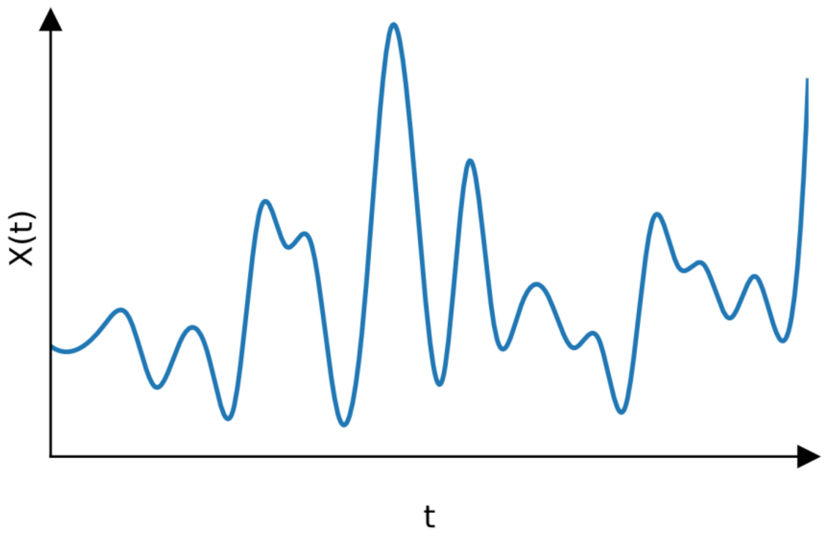
Definition

- Random points in time



- $X_i = X(t_i)$ describes process for $i = 1,2,\dots$
  - $(X_1, t_1), (X_2, t_2), \dots$
  - X_i is r.v. at time t_i

- Family of random variables



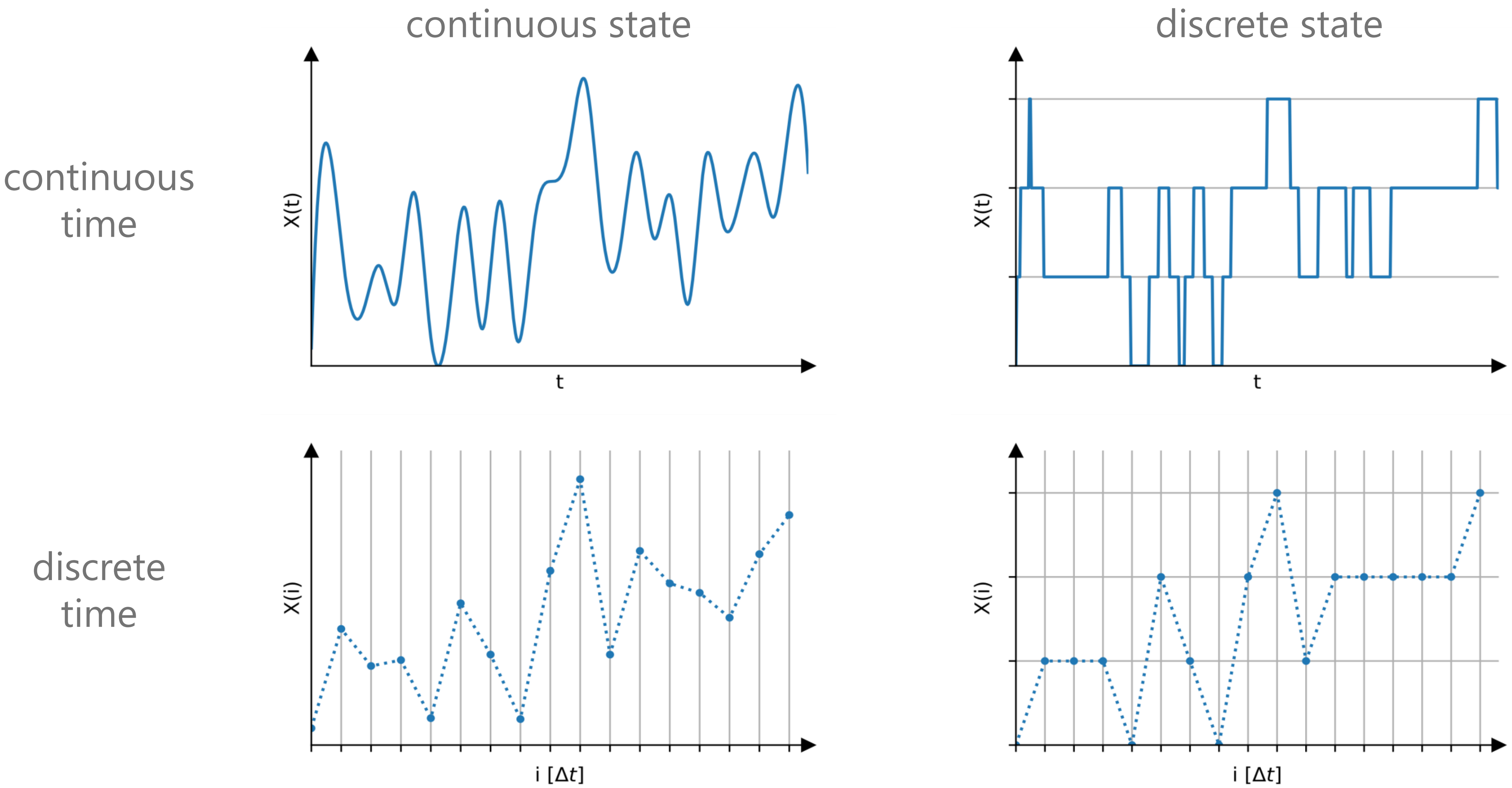
- Generalization with family of r.v.s
  - index set or parameter set Γ, e.g., time
  - state space Ξ: collection of all possible values of the r.v.s
- **Stochastic process:** $\{X(t), t\}$ for $t \in \Gamma$

Example: IoT Gateway

Lecture

- $X(t)$ is the number of data packets waiting in a device of a computer network

Classification of Stochastic State Processes

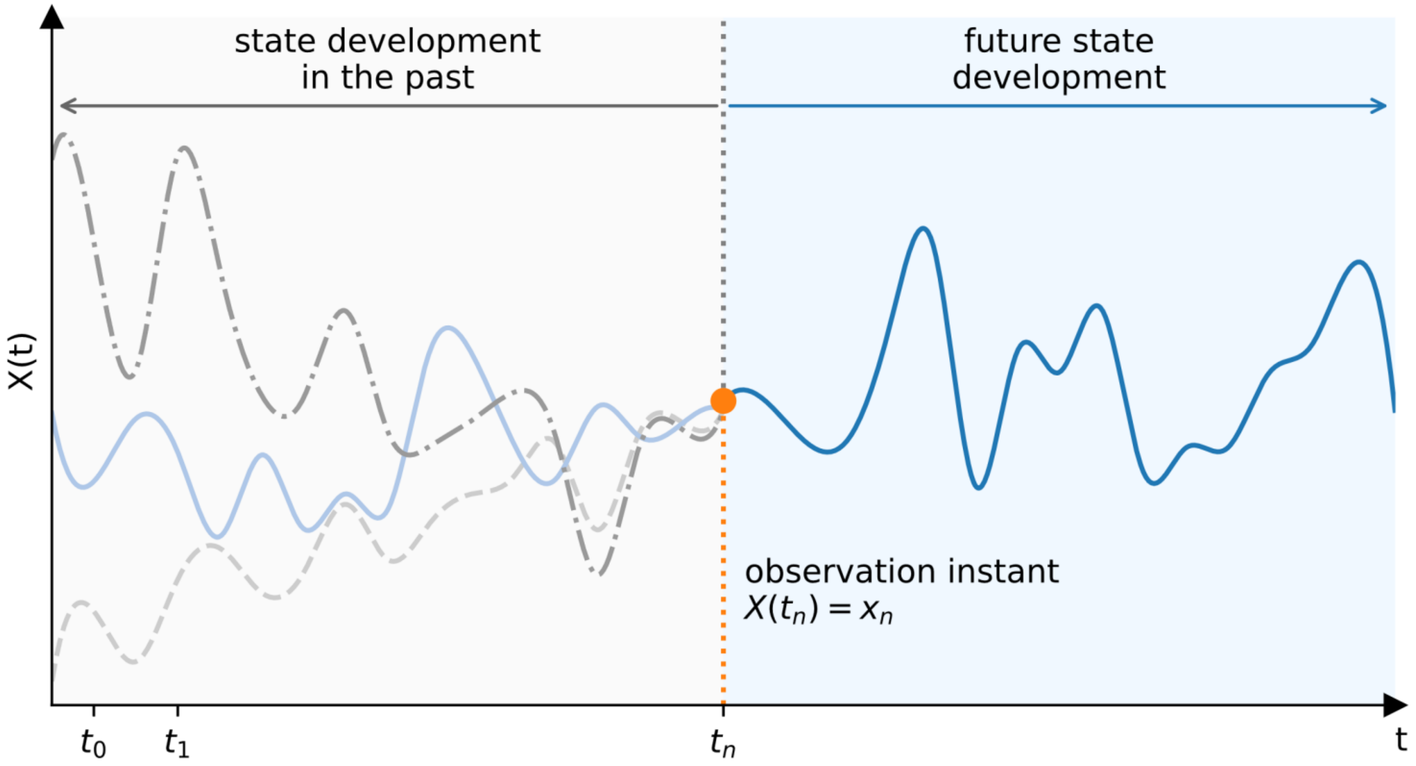


Markov Processes

Markov property

Markov Process

- Future development can be calculated from current situation
- Memoryless process



- Definition of Markov Process

$$
\begin{aligned}
&P(X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n, \dots, X(t_0) = x_0) \\
&\qquad = P(X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n) \;, \quad t_0 < t_1 < \dots < t_n < t_{n+1} \;.
\end{aligned}
$$

Memorylessness of a Random Variable

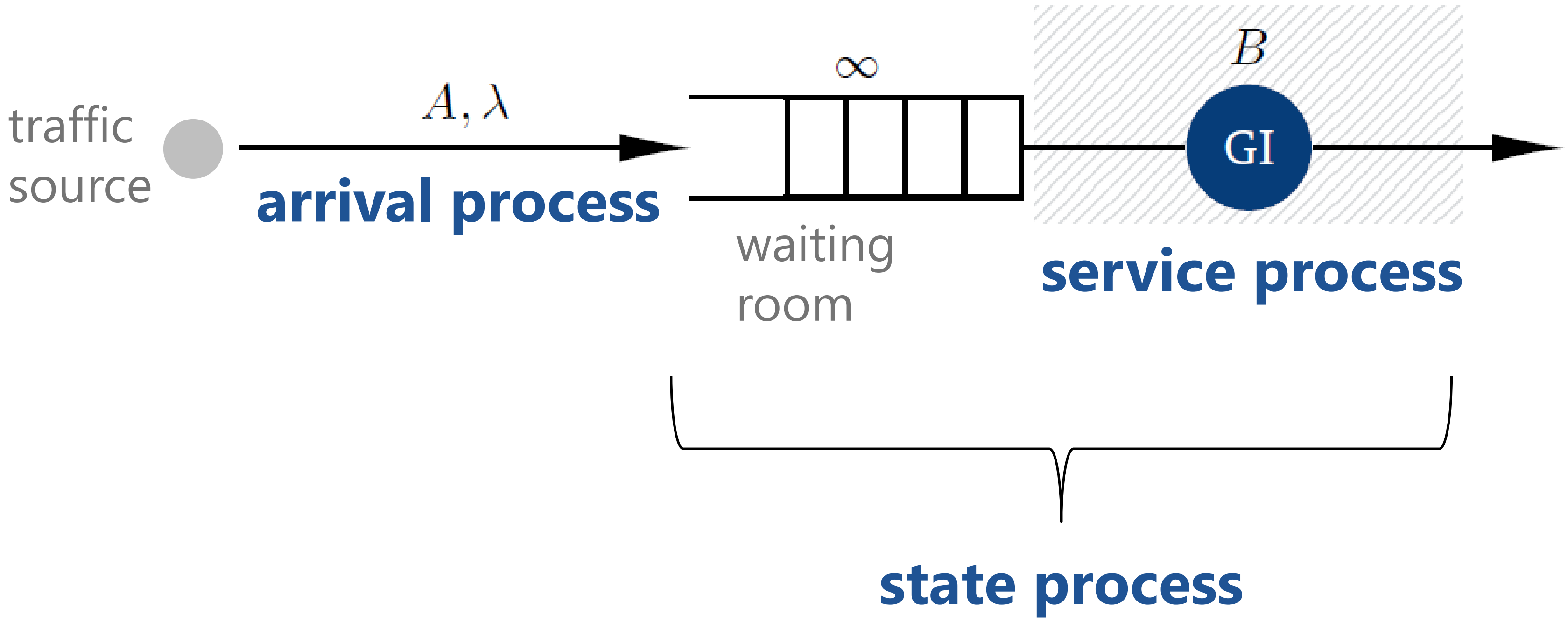
Lecture

Pareto Distribution

Elementary Processes in Performance Models

Arrival process, service process, state process

Random Processes in Traffic Models

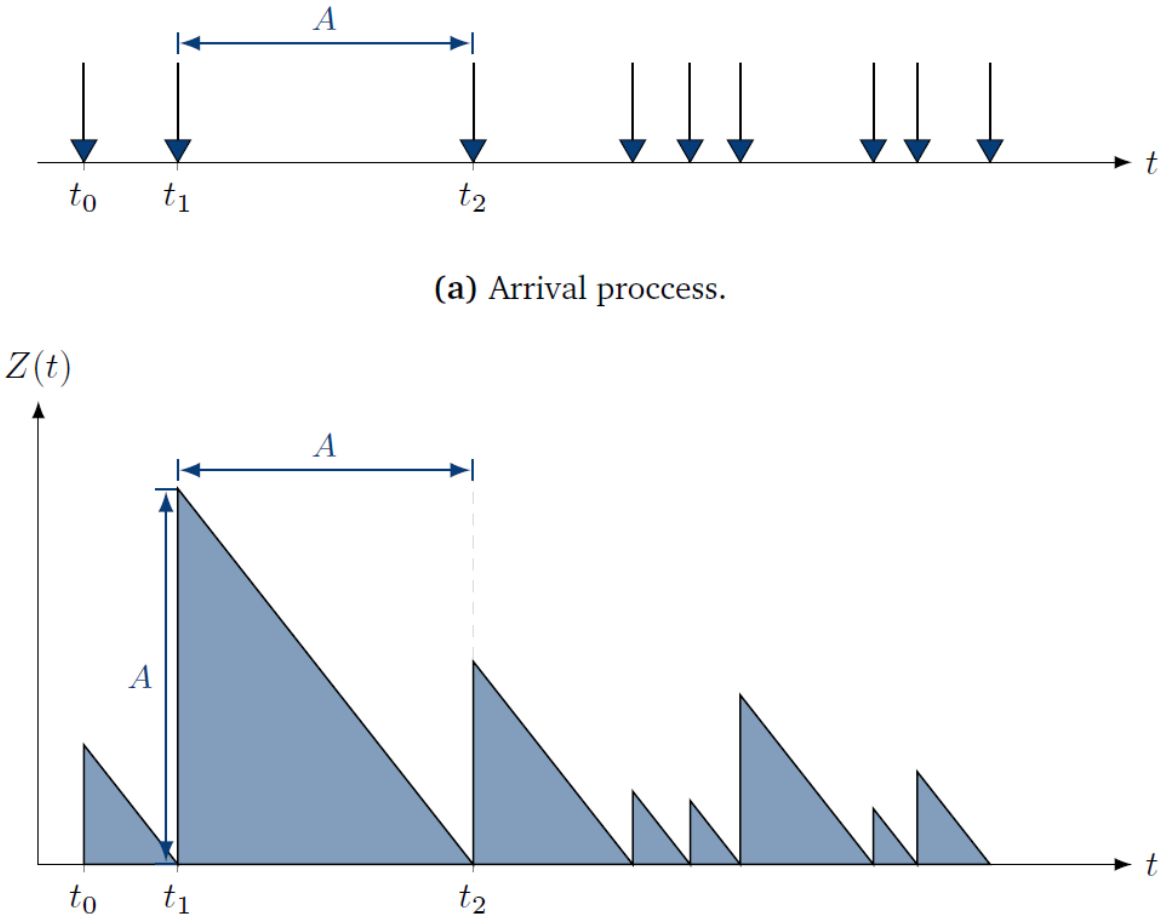


Characterization of Arrival Process

- Interarrival times A_i for i-th arrival
- Here: A_i are iid. and follow same r.v. A (see renewal process)
- Remaining time $Z(t)$ until next arrival
  - $\{Z(t), t\}$ continuous-time and -state

- **Arrival process with Markov property**
  - r.v. $Z(t)$ is independent of the past
  - $Z(t)$ is iid. for any time instant t
  - interarrival time A is memoryless

- **Poisson process** is the only single arrival process with Markov property



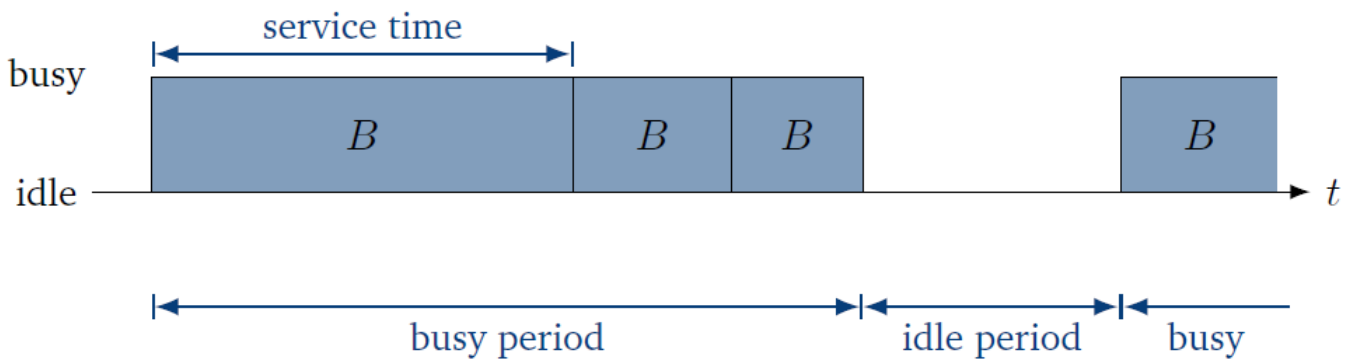
(a) Arrival proccess.

(b) Arrival process as a state process.

Characterization of Service Process

- Service time described with r.v. B_i for i-th job
- Here: service times are iid. for any job and follow r.v. B

- Several consecutive service times constitute a busy period
- Between busy periods there is an idle period



- **Service process with Markov property**
  - memoryless service time
  - duration until end of operation independent of the past
  - exponential distribution is the only continuous distribution with Markov property

State Process

- Depending on traffic model and analysis method, different forms of the state process description are considered
- Number of customers $X(t)$
  - e.g. for M/GI/1 or GI/M/1
- Unfinished work $U(t)$
  - e.g. for analysis of GI/GI/1