Chapter 3.4

Superposition of Independent Renewal Processes

Performance Evaluation of the Internet of Things (IoT)

Module Course: Performance Evaluation of Distributed Systems

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The book must be cited and the disclaimer attached when using lectures slides or scripts.

Tran-Gia, P. & Hossfeld, T. (2021).
Performance Modeling and Analysis of Communication Networks - A Lecture Note. Würzburg University Press.
https://doi.org/10.25972/WUP-978-3-95826-153-2

Website to download book, exercises, slides and scripts:
https://modeling.systems/

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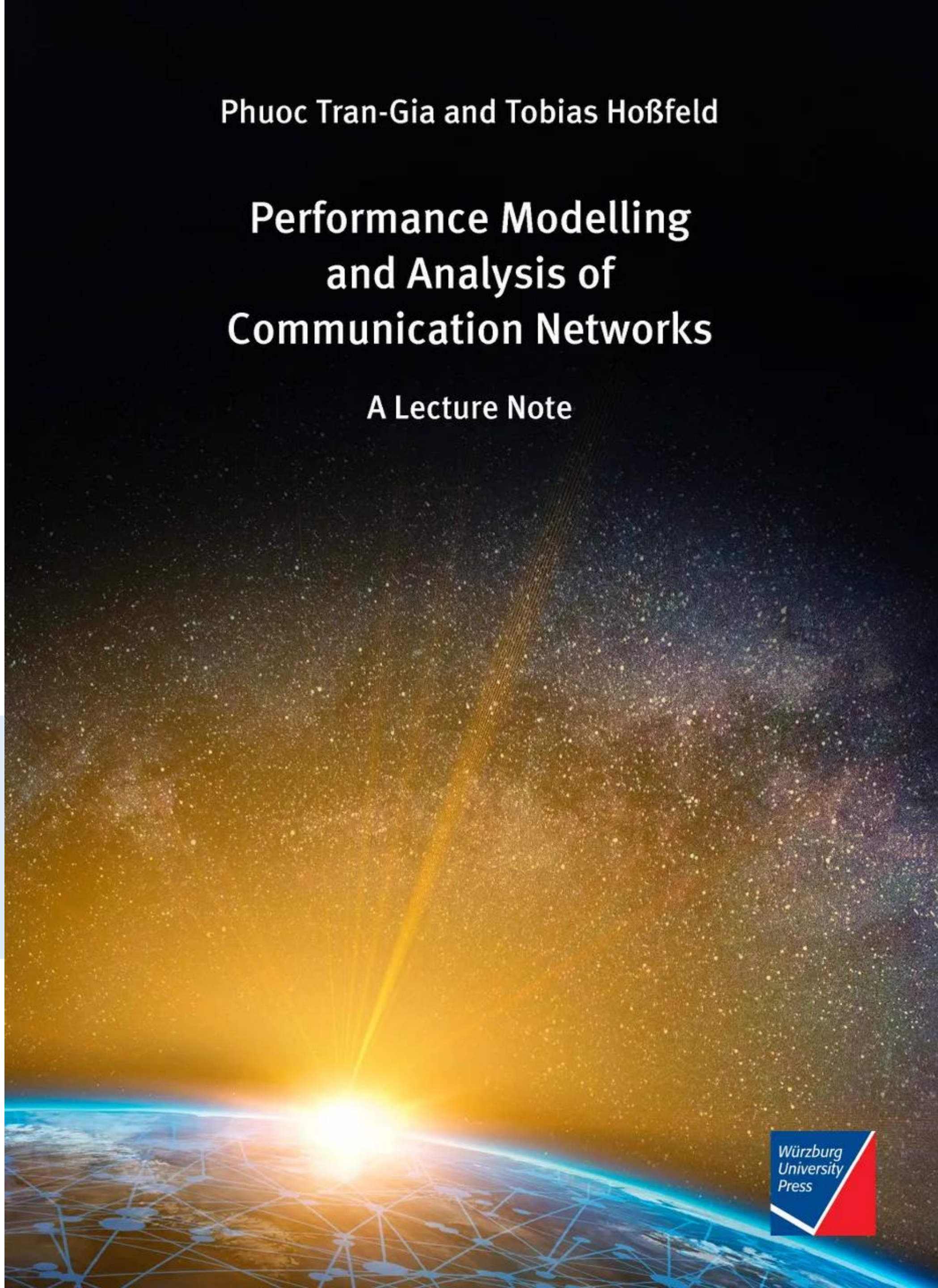
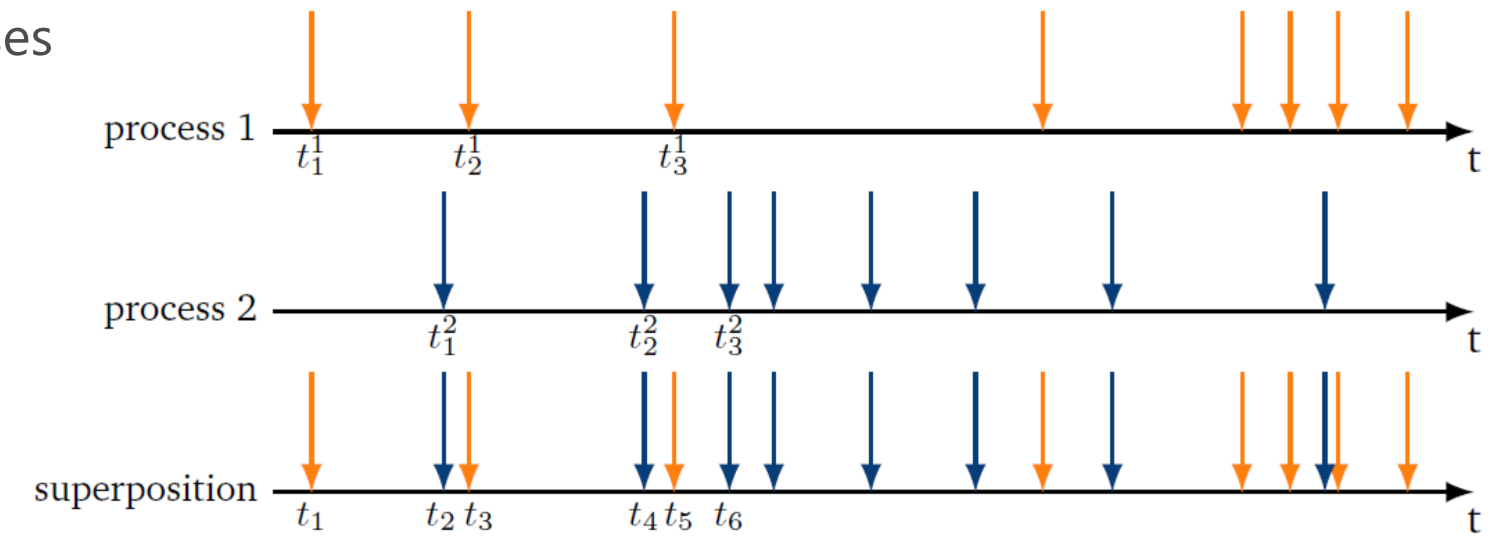


Chapter 3

Merging of Poisson Processes

- Superposition of Poisson processes remains a Poisson process
  - Process 1: $A_1 \sim \text{EXP}(\lambda_1)$
  - Process 2: $A_2 \sim \text{EXP}(\lambda_2)$



- Superposition $A \sim \text{EXP}(\lambda_1 + \lambda_2)$

- Proof: random observer sees minimum of recurrence times of both processes
  - recurrence time is exponentially distributed with same rate
  - $A = \min(A_1, A_2)$ with CDF $A(t) = 1 - \big(1 - A_1(t)\big) \cdot \big(1 - A_2(t)\big) = 1 - e^{-(\lambda_1+\lambda_2)t}$
  - $A = \min(A_1, A_2) \sim \text{EXP}(\lambda_1 + \lambda_2)$

- In general: superposition of n Poisson processes

$$A \sim \text{EXP}\left(\sum_{i=1}^{n} \lambda_i\right)$$

PALM-KHINTCHINE THEOREM

Superposition of renewal processes

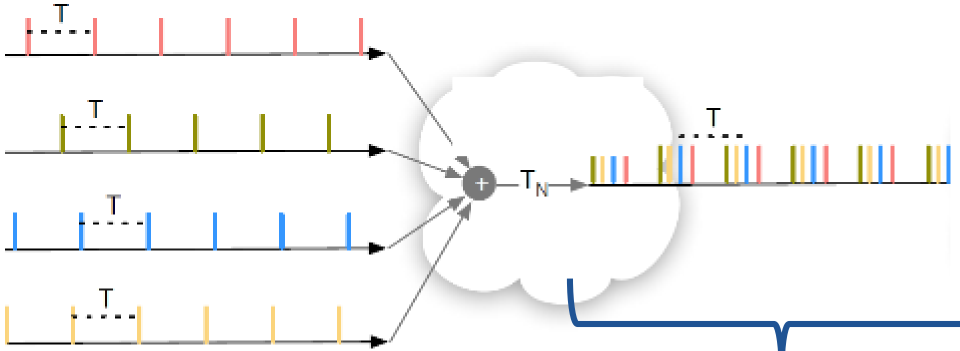
Palm-Khintchine Theorem

- Superposition of a large number n of **independent renewal processes**
  - interarrival times A_i of the renewal processes may follow general distribution
  - each with a finite intensity $\lambda_i = \frac{1}{E[A_i]}$, $i = 1,2,\dots,n$
  - behaves asymptotically like a Poisson process for $n \to \infty$

- Kendall's notation
  - $\sum_i \mathrm{GI}_i/\bullet\ /\bullet$, which must not be confused with the sum of r.v.s
  - $n\ \cdot$GI/● /● if the arrival processes follow the same distribution

- Example: periodic traffic

large number n ...



... Poisson process !

Formally: Palm-Khintchine Theorem

- The superposition of a large number of independent renewal processes, each with a finite intensity, behaves asymptotically like a Poisson process for $n \to \infty$:

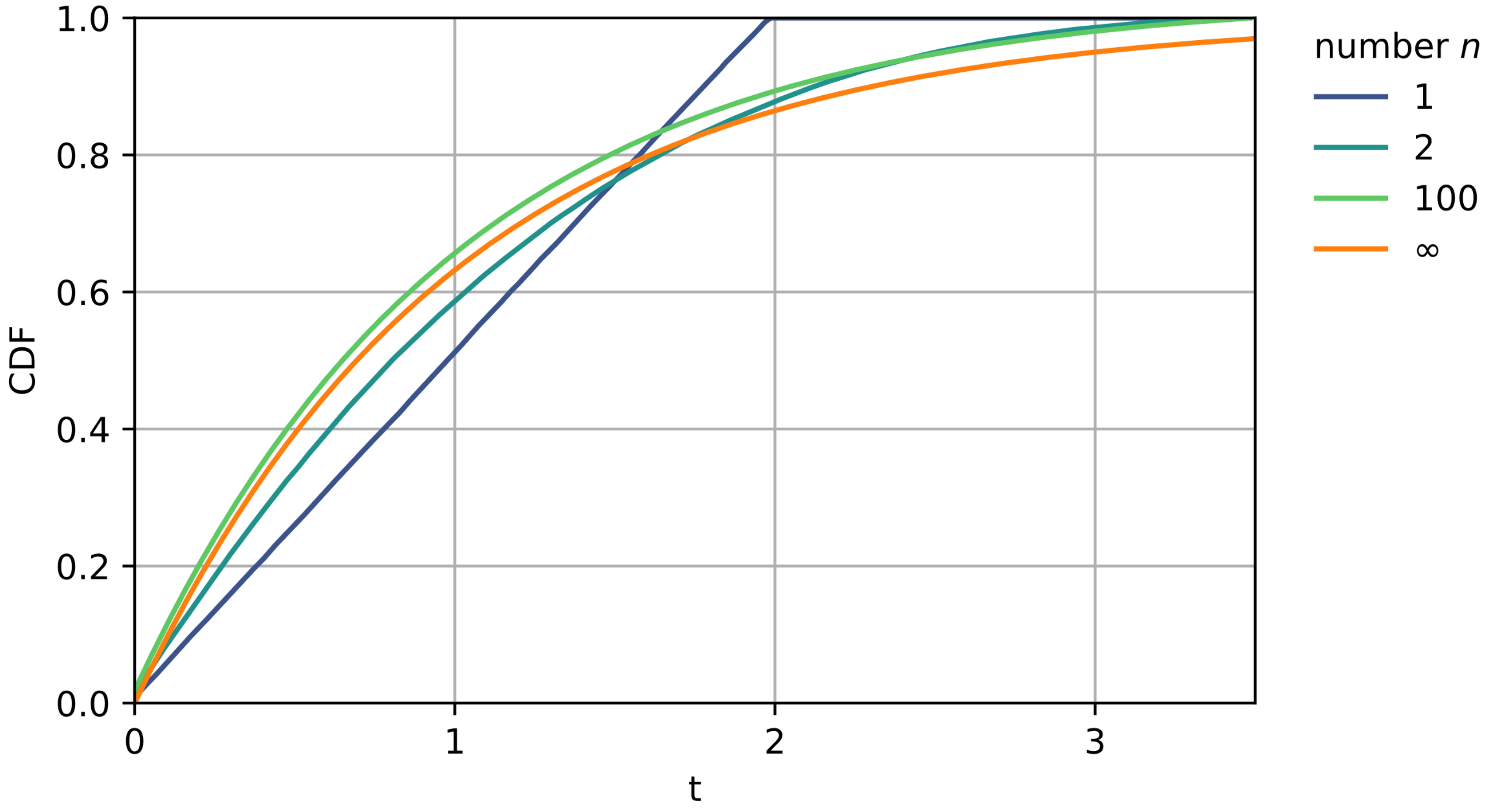
$$A^* \sim \mathrm{EXP}\left(\sum_i \lambda_i\right)$$

- if the following two conditions are fulfilled:
  - 1. the overall load is finite, $\sum_i \lambda_i < \infty$
  - 2. no single process dominates the superposition, $\lambda_k \ll \sum_i \lambda_i$ for any k

- Alternative formulation:
  - We consider renewal process i as a counting process of arrivals $N_i(t)$ in the interval t.
  - Then the superposition of the counting processes $N(t) = N_1(t) + \cdots + N_n(t)$ for $n \to \infty$ tends towards a Poisson process with rate λ.

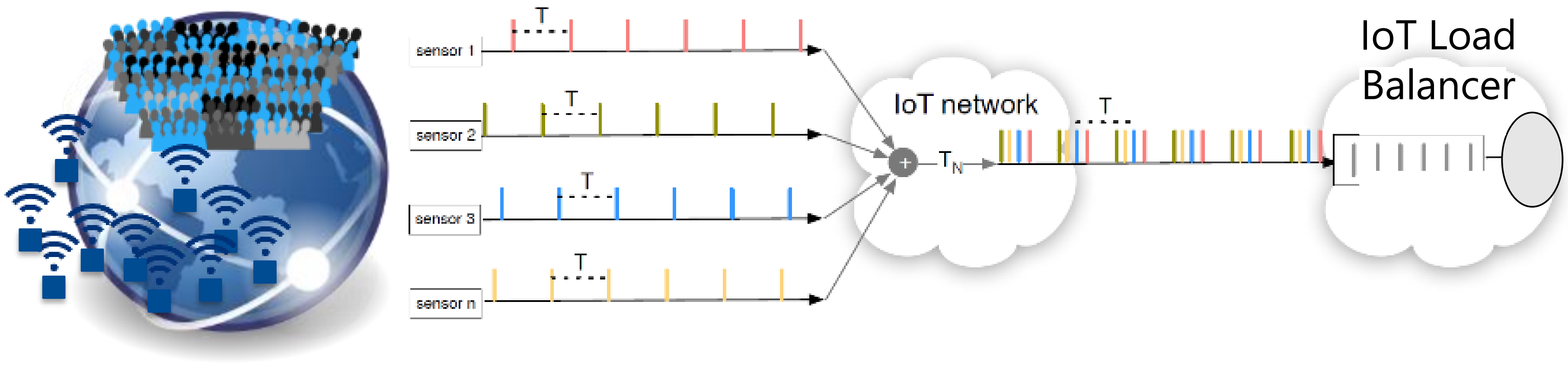
Example: Superposition of Uniformly Distributed IATs

- n independent processes with IAT $A_i \sim U(0{,}2n)$ and $E[A_i] = n$
- Overall load is $\lambda^* = \sum_{i=1}^{n} \frac{1}{n} = 1 < \infty$ for any n



Example: IoT Cloud

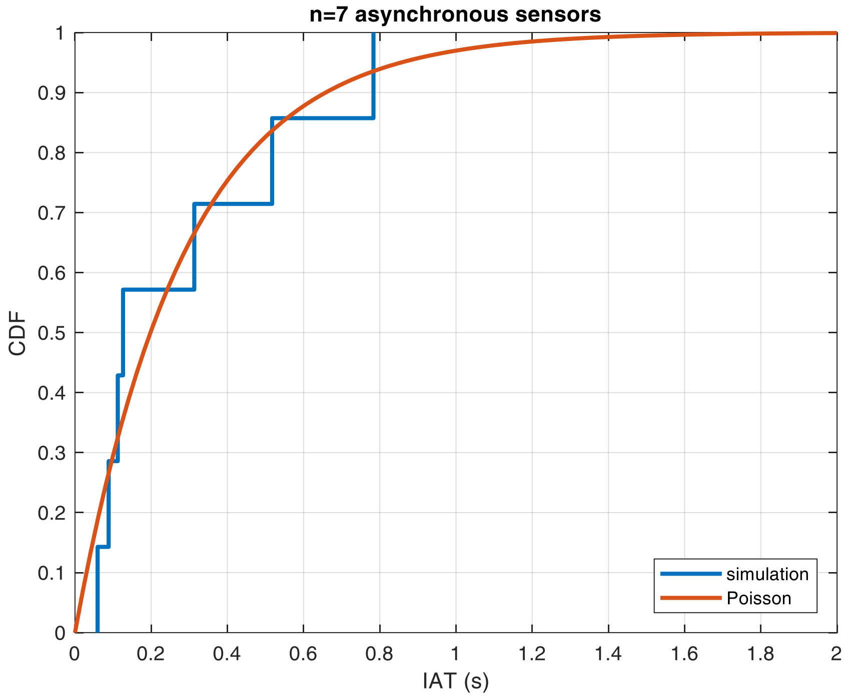
- Many sensors send data to an IoT cloud
- IoT Load Balancer distributes messages to servers in the cloud
- IoT load balancer as a bottleneck
- Deterministic message processing time

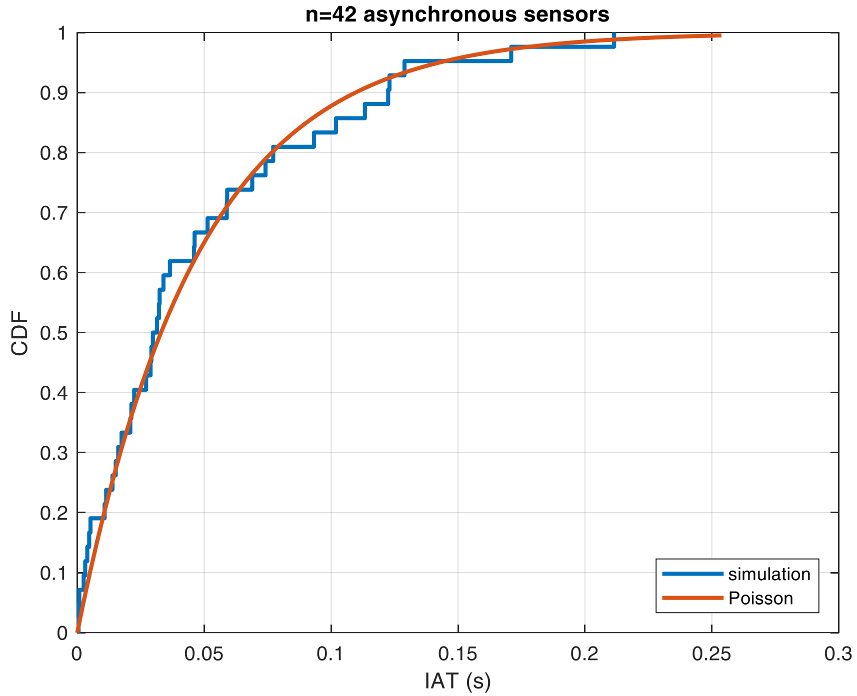


- How to model the load balancer (Kendall notation)?

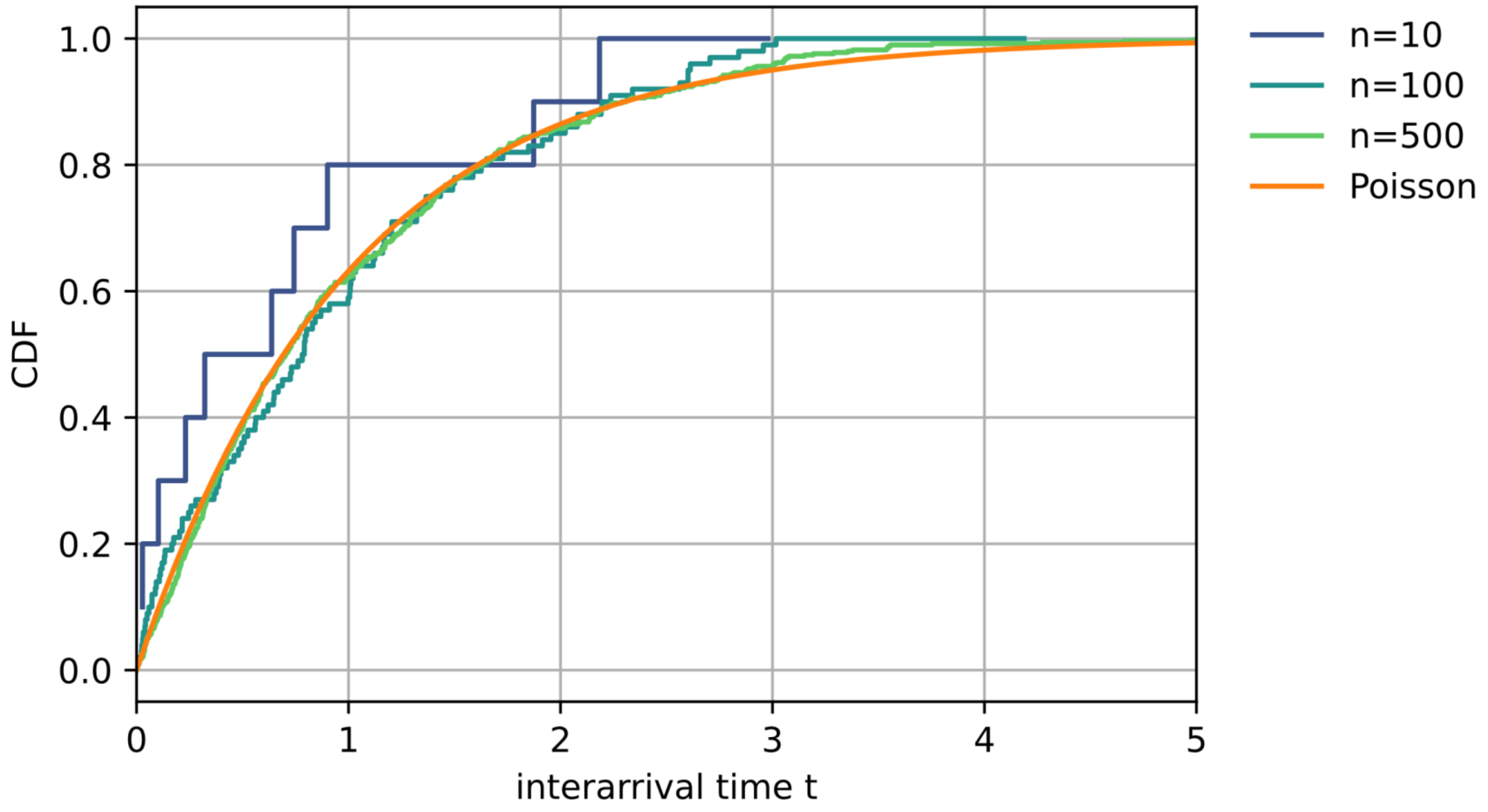
Asynchronous Sensors

- Each node periodically sends every $A = 2\text{s}$
- Single simulation run is considered





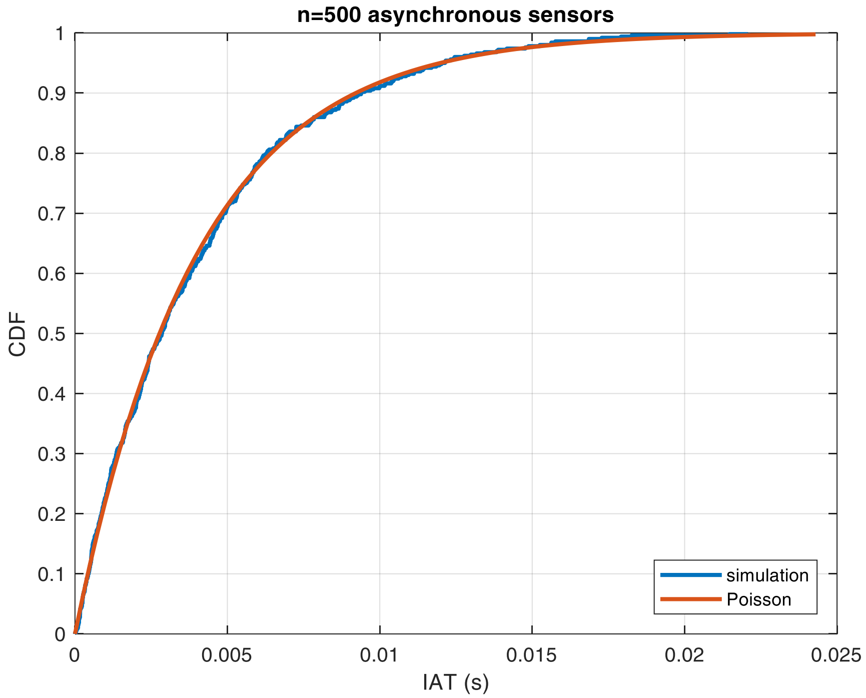
Asynchronous Sensors (f.)



Synchronous Sensors

- How to model the different scenarios?

asynchronous



synchronous