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# Performance Modeling and Analysis of Communication Networks

A Lecture Note

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# 1 Introduction in Performance Modeling

## Question and Answer

**Question 1.1.** A single IoT sensor periodically sends on the hour its current sensor information to an IoT cloud which consists of 100 servers for processing the incoming information. The sensor sends exactly at 1am, 2am, ... a message of fixed size. The cloud provides enough storage information for incoming messages, such that all incoming messages are eventually processed by one of the servers. How can you model the system and describe it in Kendall's notation?

**a.** M/D/1-100

**b.** GI/D/100-0

**c.** D/D/100-$\infty$

**d.** D/D/100-1

**Question 1.2.** The same scenario as in Question 1.1 is considered, but the sensor is now randomly sending once per hour. What is Kendall's notation for the system?

**a.** M/D/1-100

**b.** GI/D/100-$\infty$

**c.** D/D/100-$\infty$

**d.** D/D/100-1

**Question 1.3.** The same scenario as in Question 1.1 is considered. While the sensor is sensing the environment exactly on the hour, the sensor information is sent once per day at noon. Thus, all 24 messages from the last 24 hours are sent at noon. What is Kendall's notation for the system?

**a.** D/D/100-$\infty$

**b.** $D^{[24]}$/D/100-$\infty$

**c.** $D^{[1]}$/D/100-$\infty$

**d.** D/D/24-$\infty$

**Question 1.4.** An IoT load balancer is distributing the incoming messages from sensor nodes to the servers in the cloud. There is a single load balancer which can only process and forward one message, but the IoT load balancer is able to store $1000$ incoming messages while processing one message. We consider a system where millions of sensors are independently and randomly send their sensing information to the cloud. The sensor nodes are not synchronized and each sensor sends once per hour on average. How can the IoT load balancer be modeled?

**a.** D/D/1000-$\infty$

**b.** $M^{[1000]}$/D/1-0

**c.** GI/D/1-1000

**d.** M/D/1-1000

## Exercises and Problems

**Problem 1.1.** Consider the following system in Figure 1.1 which illustrates when the system is busy or idle with processing tasks. The interarrival time of new tasks to be processed is constant, $A = 3\,\mathrm{min}$. The task processing time is constant, $B = 2\,\mathrm{min}$.

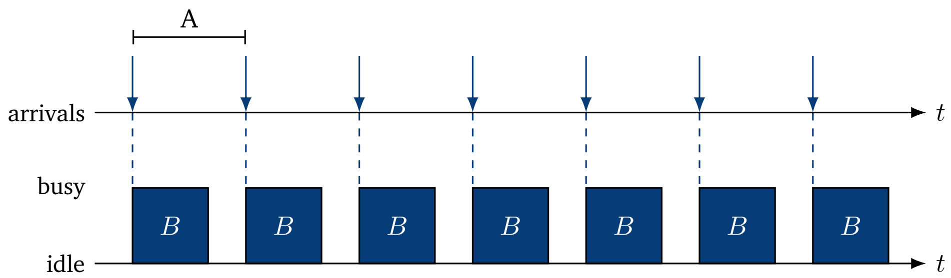


**Figure 1.1:** Illustration of a system with constant interarrival and service times.

**1.1.1.** What is Kendall's notation of the system?
**1.1.2.** What happens if $A = 3\,\mathrm{min}$ and $B = 4\,\mathrm{min}$?

**Problem 1.2.** Consider the system in Problem 1.1, but the task processing time $B$ is now randomly distributed, while the mean task processing time is $\mathrm{E}[\,B\,] = 2\,\mathrm{min}$.

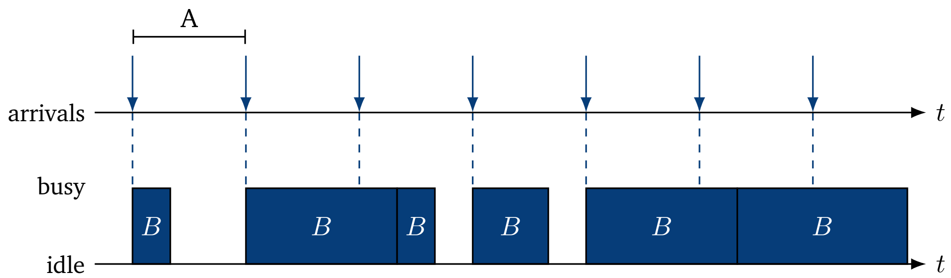


**Figure 1.2:** Illustration of a system with contant interarrival and random service times.

**1.2.1.** What is Kendall's notation of the system?
**1.2.2.** What happens if $A = 3\,\mathrm{min}$ and $\mathrm{E}[\,B\,] = 4\,\mathrm{min}$?
**1.2.3.** Is the utilization of the server in Problem 1.1 (constant processing time) and in this problem (random processing time) identical?

**Problem 1.3.** We model the download times in Wi-Fi 6. Users can access a single Wi-Fi access point at any time and download contents from the web. We assume that any user will download a single file of size $100\,\mathrm{Mbit}$. After the download, the user leaves the Wi-Fi network. The download capacity of the Wi-Fi access point is $700\,\mathrm{Mbps}$ and equally divided among all currently attached users' devices. What is the arrival process? What is the server and the service process? What is Kendall's notation for the system? What is the notation if users are served sequentially? Does it change the performance characteristics?

**Problem 1.4. Queueing Model** What is the arrival process and the service process of the following systems? Describe them in your own words. How can the systems be described as a queueing model in the Kendall's notation? Which are the performance characteristics of interest?

**1.4.1.** A small gas station has two electric charging stations. If both charging stations are occupied, any arriving car will leave the station.

**1.4.2.** At the gas station there are also two gas pumps and three parking spaces for drivers who wait until a gas pump becomes free.

**1.4.3.** An ice cream seller serves the customers which are arriving every $360\,\mathrm{s}$ on average. The interarrival time of customers follows an Erlang-5 distribution. The ice cream seller needs on average $240\,\mathrm{s}$ to serve a single customer. If the seller is already busy with a customer, other customers are waiting.

**1.4.4.** The Dynamic Host Configuration Protocol (DHCP) enables the dynamic assignment of an IP address to computers on a local area network. A computer that requires an IP address asks the associated DHCP server. The DHCP server then automatically assigns the requesting client an IP address for the time the client is connected to the LAN. We consider the following fictitious scenario. The chair of communication networks offers its students Internet access and provides 20 IP addresses via DHCP. On average, every 15 minutes a student arrives at the chair and occupies an IP address for an average of 4 hours. The occupancy time is exponentially distributed and the student arrival stream follows a Poisson process.

**1.4.5.** The radio access of a mobile network is considered which uses time division multiple access (TDMA). The nodes send their data in time slots of fixed length to the base station. Each device is assigned a fixed time slot (dedicated channel). If the traffic load in the mobile cell is too high, several devices may be assigned to the same time slot, but devices are not permanently sending or receiving data. The packets which are transmitted over the dedicated channel have a variable length, but have a maximum size corresponding to the fixed length of the time slot. Packets ready for transmission are temporarily stored and transmitted in the order in which they arrive. The buffer space is limited to 512 packets. Packets which find a full buffer are discarded.

**1.4.6.** A tray conveyor belt in the Hubland cafeteria at the campus of the University of Würzburg.

**1.4.7.** There are only two double-seater bobsleds available on a toboggan run. These are only rented out if two people drive down on them. If a person arrives alone, she has to wait until the next one comes and then they both take a free bobsled. There are two waiting places available.

☞ *Hint:* The extended Kendall notation for a system with batch service and threshold control is GI/GI$^{[\Theta,K]}$/n-S. There are $n$ servers and a single server processes a batch of jobs jointly (batch service). At most $K$ jobs can be processed during batch service. However, the work station is first activated if there are enough jobs available to be processed (threshold control with $\Theta$).

**Problem 1.5. Video Streaming Service** The operator of a video streaming service needs to dimension the system. A complex monitoring tool is used for this and it turns out that the interarrival times of new user requests to watch a video follow an exponential distribution with the rate $\lambda = 110\,\mathrm{h}^{-1}$. The video streaming platform offers videos with an average duration of $7\,\mathrm{min}$. Furthermore, the current system is designed to supply a maximum of 50 users at the same time. Users who attempt to start a stream beyond this limit are rejected by the system. What is the arrival process and the service process? Can you describe the system with Kendall's notation? How can the operator dimension the system?

**Problem 1.6. Call Center** A call center wants to estimate the number of service employees required. For this purpose, the time between two calls and the duration of the service calls are logged. It turns out that the interarrival times of the callers follow an exponential distribution with the rate $56\,\mathrm{h}^{-1}$. The service calls are very different: most inquiries are answered in a short time, but a few issues take a long time to clarify. There are currently 50 employees working in the call center. If all are busy, new callers end up in a queue that can accept any number of callers. Can you describe the system with Kendall's notation? How can the operator dimension the number of required employees?

**Problem 1.7. Weather Map** The sensors of a weather station regularly transmit their weather data to a remote collecting server every $10\,\mathrm{min}$. This data is always processed immediately by the central server and the current weather map is updated. Multiple records can be processed at the same time and no requests will be blocked. The processing is usually very fast, but in individual cases it can take a little longer due to the increased load due to background processes on the server. A total of 17 weather stations send their data to the same server, the sending times are independent of each other. Can you describe the system with Kendall's notation?

**Problem 1.8. Ice Cream Parlor** A local ice cream parlor wants to determine how long the waiting line in front of their sales booth can be. The four salespeople are very experienced, they take about the same amount of time for all orders. The ice cream parlor is very popular in town. However, customers don't like standing in line: when they see that more than customers have recently queued, they stay away for $1\,\mathrm{h}$ and come back. Can you properly describe the system with Kendall's notation?

**Problem 1.9. Canoe Restaurant** A restaurant in the country is right on a river that is very popular with canoeists. The restaurant is only accessible by canoe. Hence, the canoeists are the restaurant's only customers. The canoes pass the restaurant independently of each other and dock there to eat. There are always three people in each canoe. The restaurant only has enough seats for 15 guests, but there is a very (very) large waiting room with a bar, which customers like to use. Can you properly describe the restaurant in the two variants with Kendall's notation?

**1.9.1.** As soon as a customer has finished eating, she immediately leaves the restaurant, goes to the bar and waits for the colleagues to take off together in the canoe.

**1.9.2.** There are five tables with 3 seats each. The three persons in a canoe share food at the table and leave the restaurant together.

**Problem 1.10. Handshaking Protocol** A sender sends messages to a receiver in the form of packets (Figure 1.3). According to a handshaking protocol, packets received without errors are acknowledged with a positive message (ACK: positive acknowledgment). If the transmission is incorrect, the recipient sends a negative acknowledgment (NAK: negative acknowledgment), whereupon the sender repeats the transmission of the corresponding packet. The process is repeated until the packet arrives at the receiver without any errors. Then, the next packet can be transmitted from sender to receiver. This simple handshaking protocol only allows a single packet to currently move over the communication channel. The performance measure of interest is the end-to-end transmission time which is composed of the waiting time of packets and the virtual transmission time. The sender is able to buffer 500 outgoing data packets. What is the Kendall notation of the system?

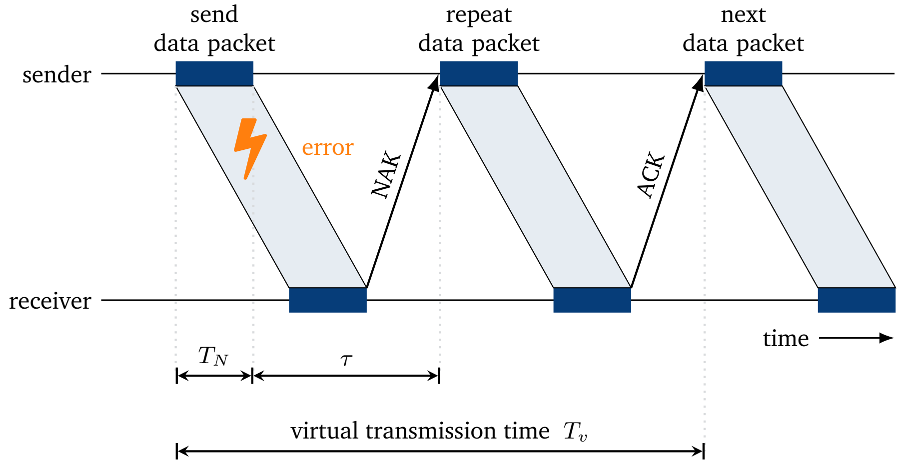


**Figure 1.3:** Handshaking protocol: illustration how it works.