

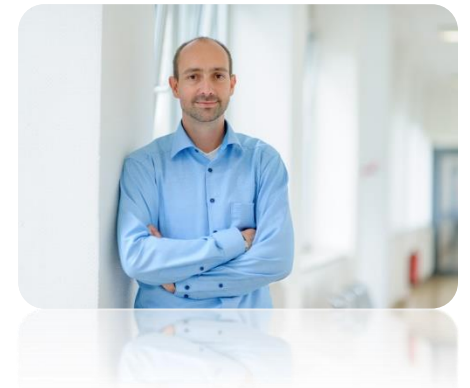
# Chapter 3.1

## Stochastic Processes

### **Performance Evaluation of the Internet of Things (IoT)**

Module Course: Performance Evaluation of Distributed Systems

Prof. Tobias Hoßfeld, Summer Semester 2022



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*Tran-Gia, P. & Hossfeld, T. (2021).  
Performance Modeling and Analysis of Communication  
Networks - A Lecture Note. Würzburg University Press.  
<https://doi.org/10.25972/WUP-978-3-95826-153-2>*

Website to download book, exercises, slides and scripts:  
<https://modeling.systems/>

# Chapter 3

## 3 Elementary Random Processes

### 3.1 Stochastic Processes

#### 3.1.1 Definition

#### 3.1.2 Markov Processes

#### 3.1.3 Elementary Processes in Performance Models

### 3.2 Renewal Processes

#### 3.2.1 Definition

#### 3.2.2 Analysis of Recurrence Time

### 3.3 Poisson Process

#### 3.3.1 Definition of a Poisson Process

#### 3.3.2 Properties of the Poisson Process

#### 3.3.3 Poisson Arrivals during Arbitrarily Distributed Interval

### 3.4 Superposition of Independent Renewal Processes

#### 3.4.1 Superposition of Poisson Processes

#### 3.4.2 Palm-Khintchine Theorem

### 3.5 Markov State Process

#### 3.5.1 Definition of Continuous-Time Markov Chain

#### 3.5.2 Transition Behavior of Markovian State Processes

#### 3.5.3 State Equations and State Probabilities

#### 3.5.4 Examples of Transition Probability Densities

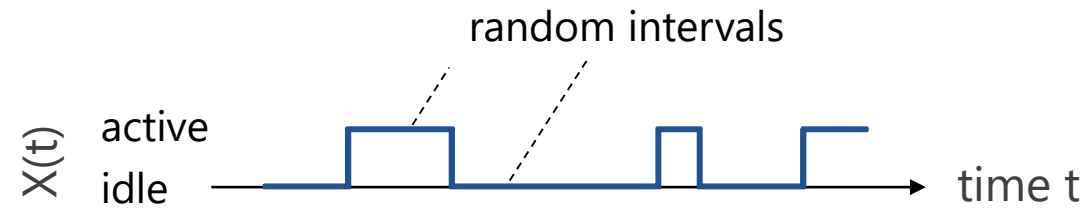
#### 3.5.5 Birth-and-Death Processes

# STOCHASTIC PROCESSES

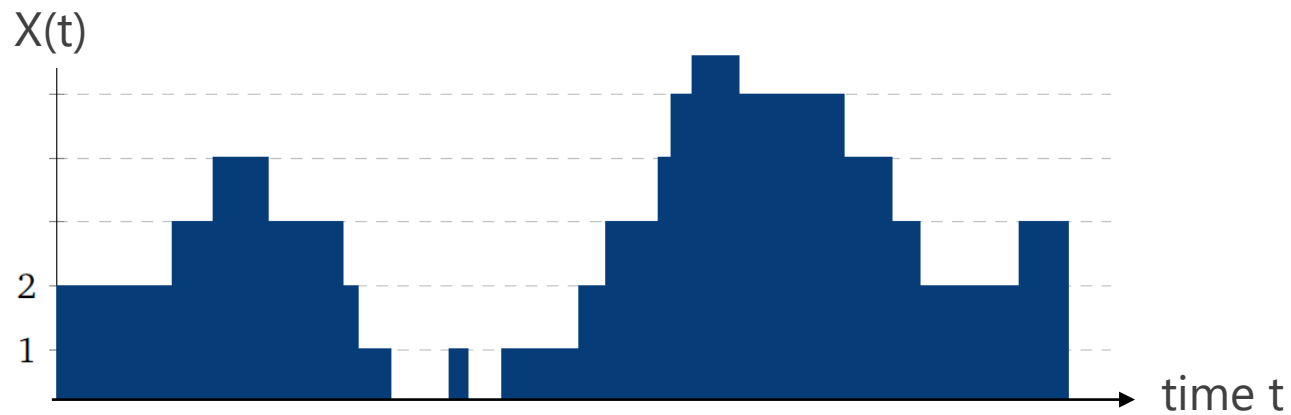
Examples and definition

# Examples

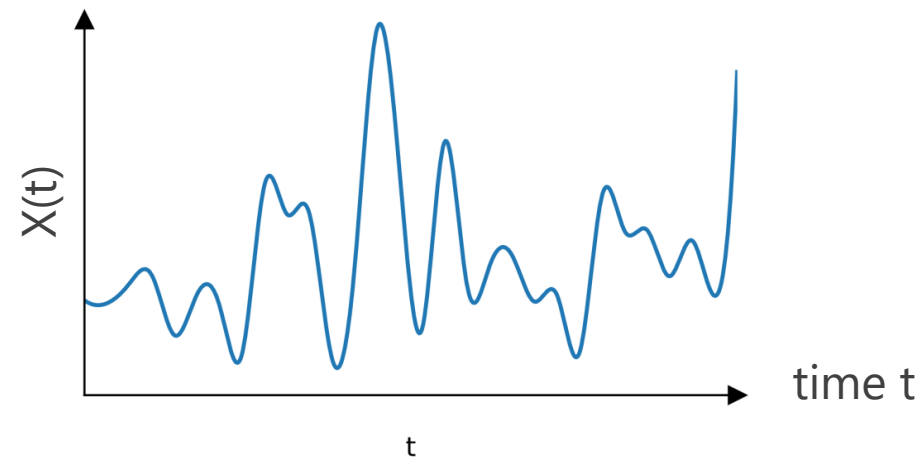
server



buffer state  
of a router

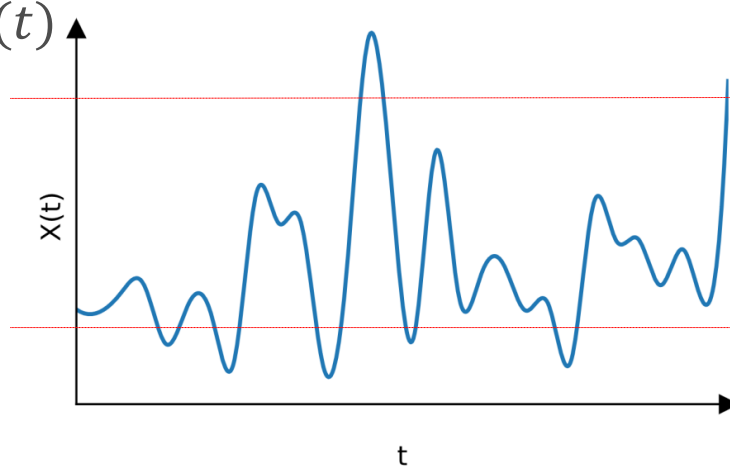


remaining workload  
at a server

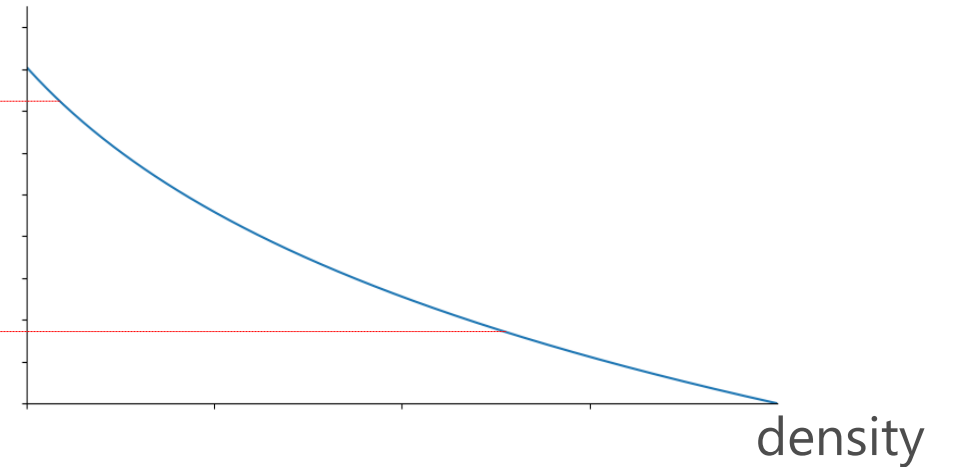


# Objective of Stochastic Modeling

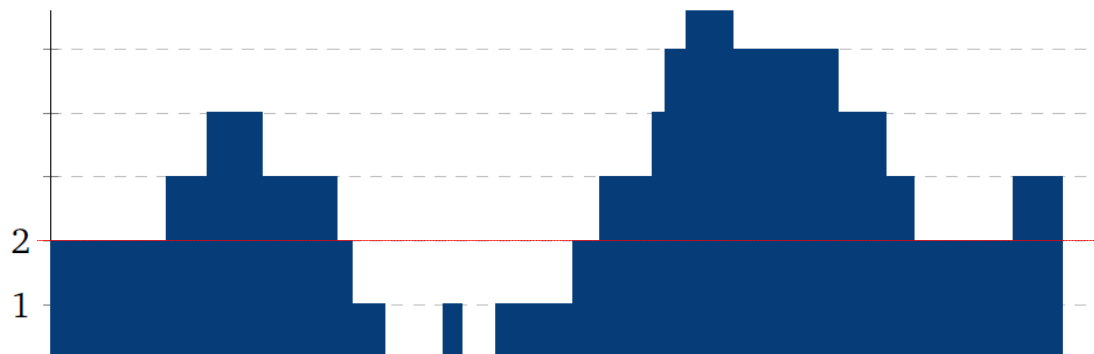
remaining workload  
at a server  $X(t)$



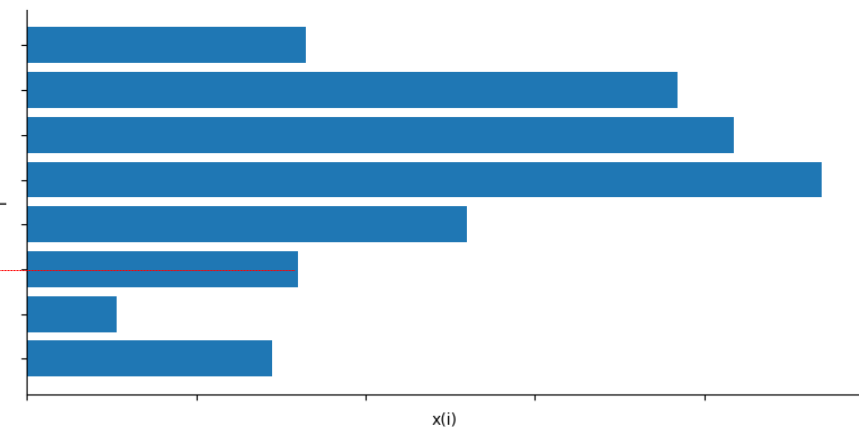
probability density function



system state  
 $X(t)$

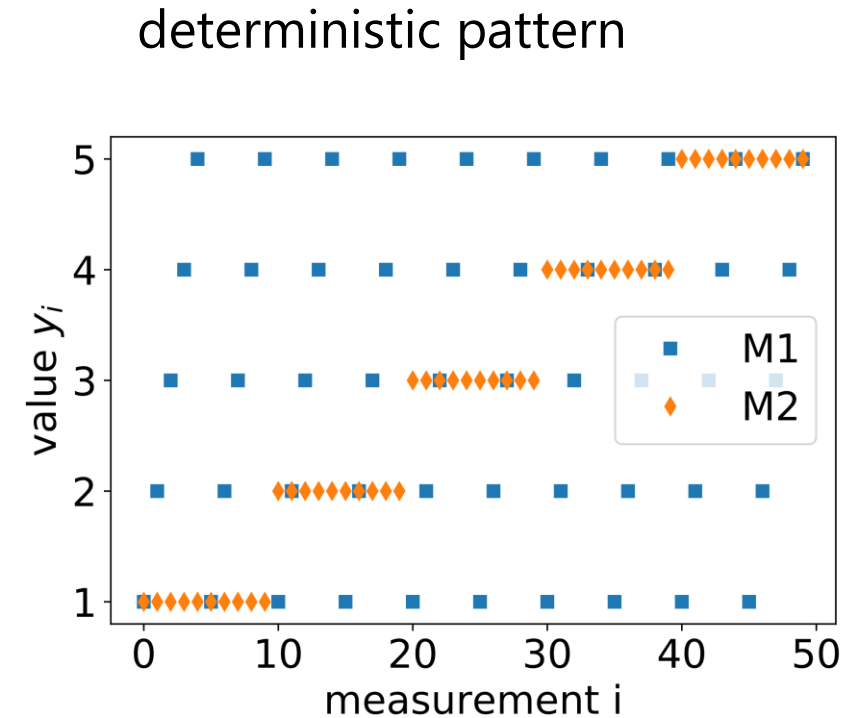
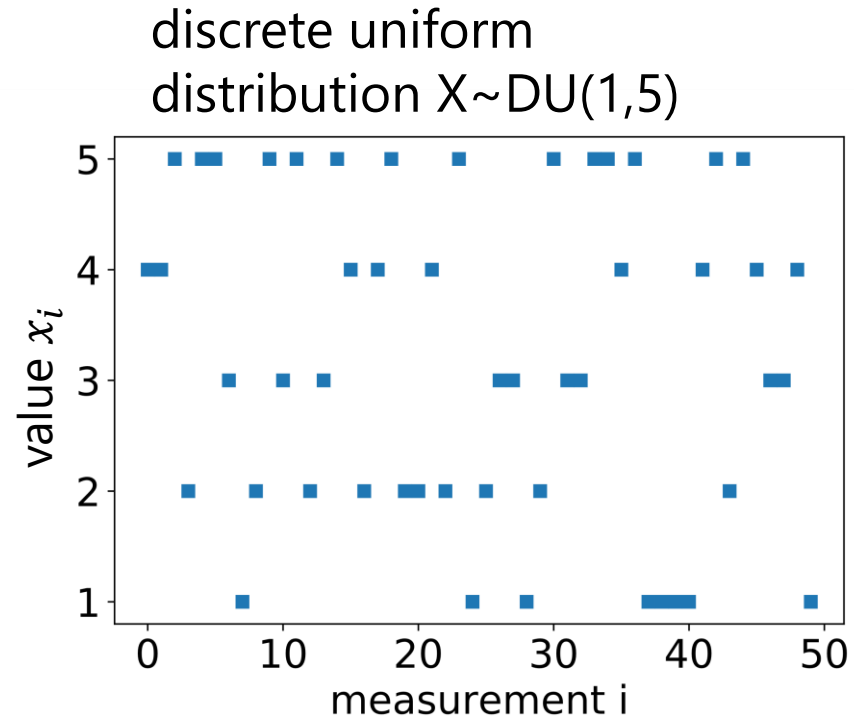


state distribution  $x(i) = P(X = i)$



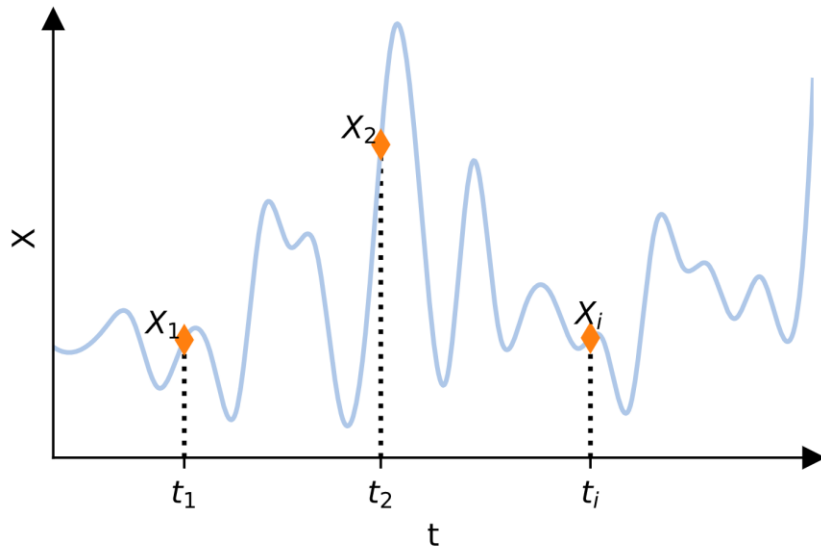
# Stochastic Modeling

- ▶ Same mean values, but different patterns



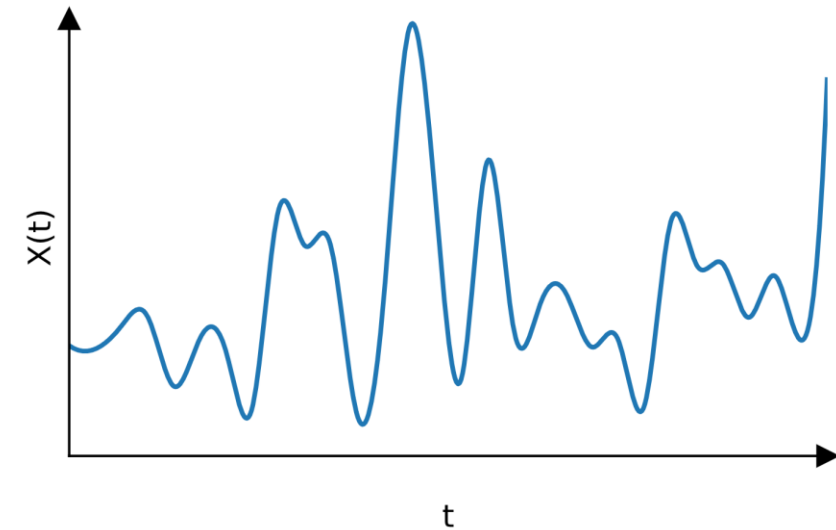
# Definition

- ▶ Random points in time



- ▶  $X_i = X(t_i)$  describes process for  $i = 1, 2, \dots$ 
  - $(X_1, t_1), (X_2, t_2), \dots$
  - $X_i$  is r.v. at time  $t_i$

- ▶ Family of random variables



- ▶ Generalization with family of r.v.s
  - index set or parameter set  $\Gamma$ , e.g., time
  - state space  $\Xi$ : collection of all possible values of the r.v.s
- ▶ **Stochastic process:**  $\{X(t), t\}$  for  $t \in \Gamma$

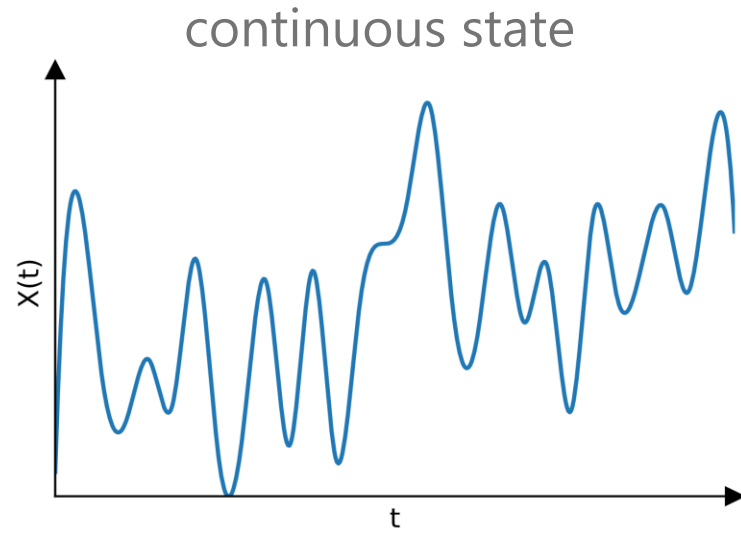


# Example: IoT Gateway

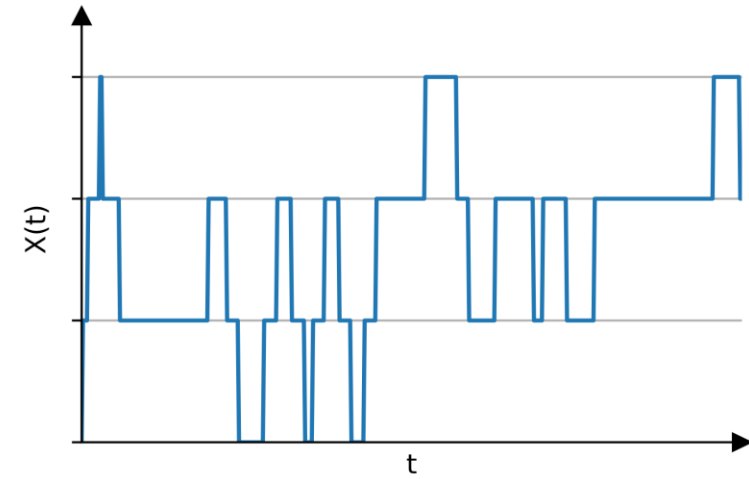
- ▶  $X(t)$  is the number of data packets waiting in a device of a computer network

# Classification of Stochastic State Processes

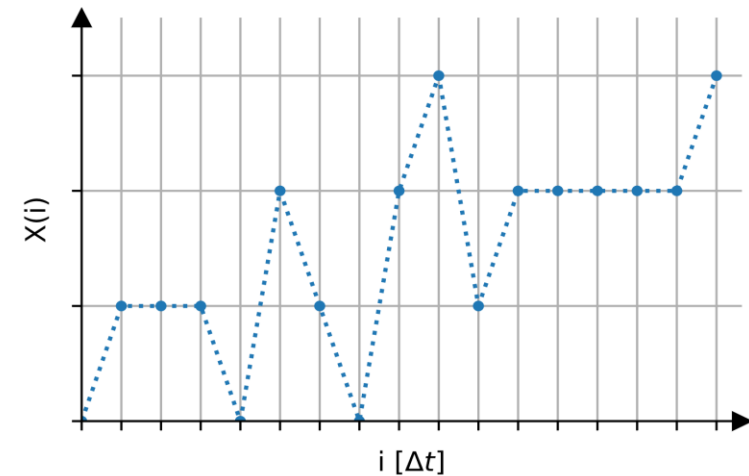
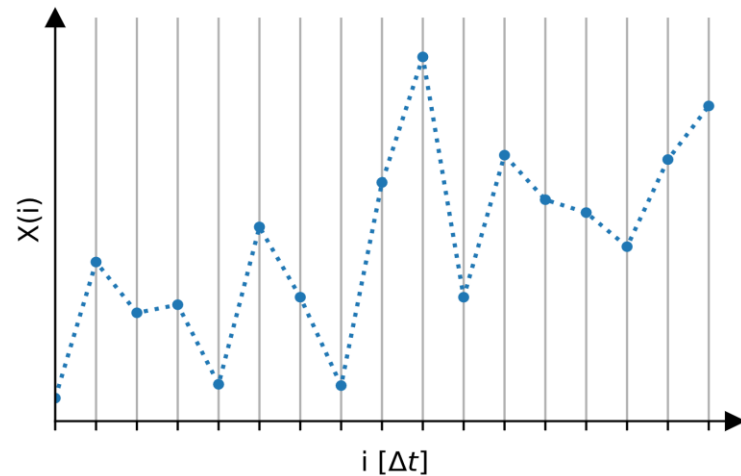
continuous  
time



discrete state



discrete  
time

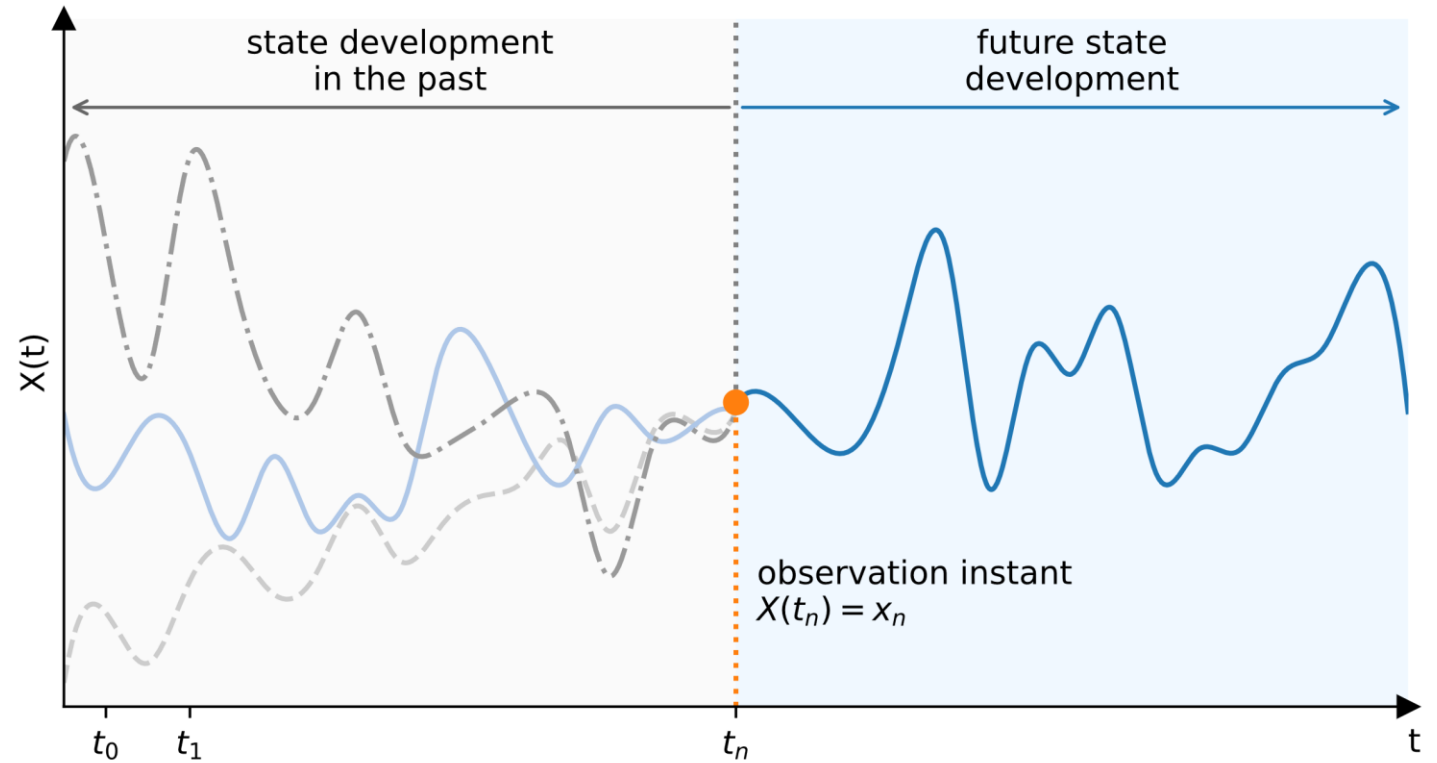


# MARKOV PROCESSES

Markov property

# Markov Process

- ▶ Future development can be calculated from current situation
- ▶ Memoryless process



- ▶ Definition of Markov Process

$$\begin{aligned} P(X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n, \dots, X(t_0) = x_0) \\ = P(X(t_{n+1}) = x_{n+1} \mid X(t_n) = x_n) , \quad t_0 < t_1 < \dots < t_n < t_{n+1} . \end{aligned}$$

# Memorylessness of a Random Variable

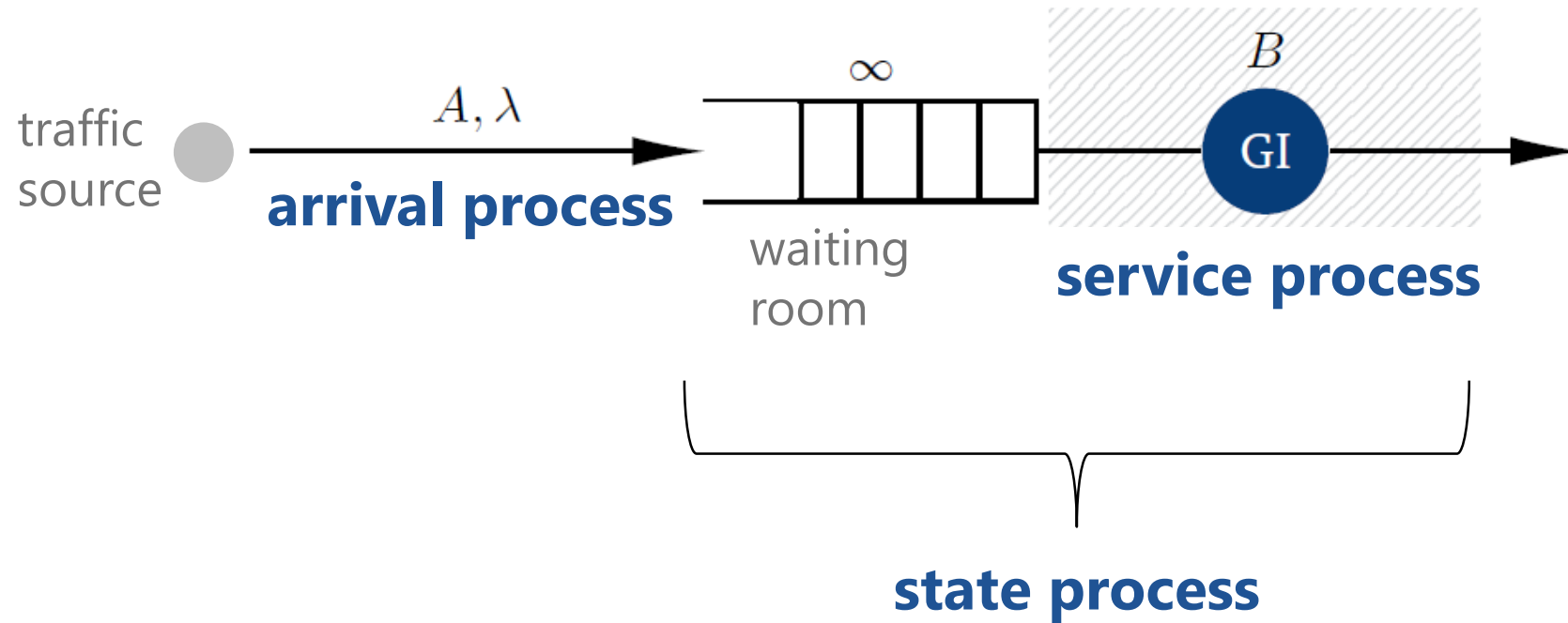




# ELEMENTARY PROCESSES IN PERFORMANCE MODELS

Arrival process, service process, state process

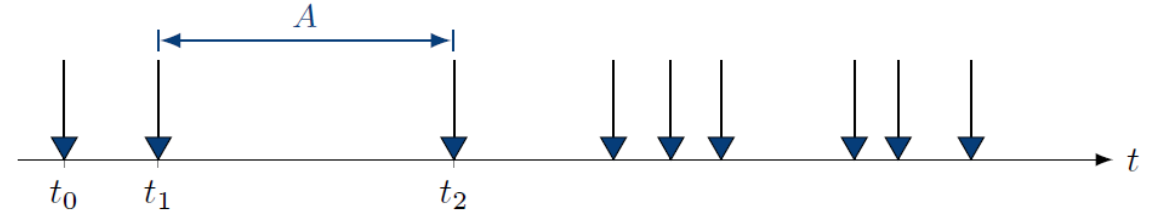
# Random Processes in Traffic Models



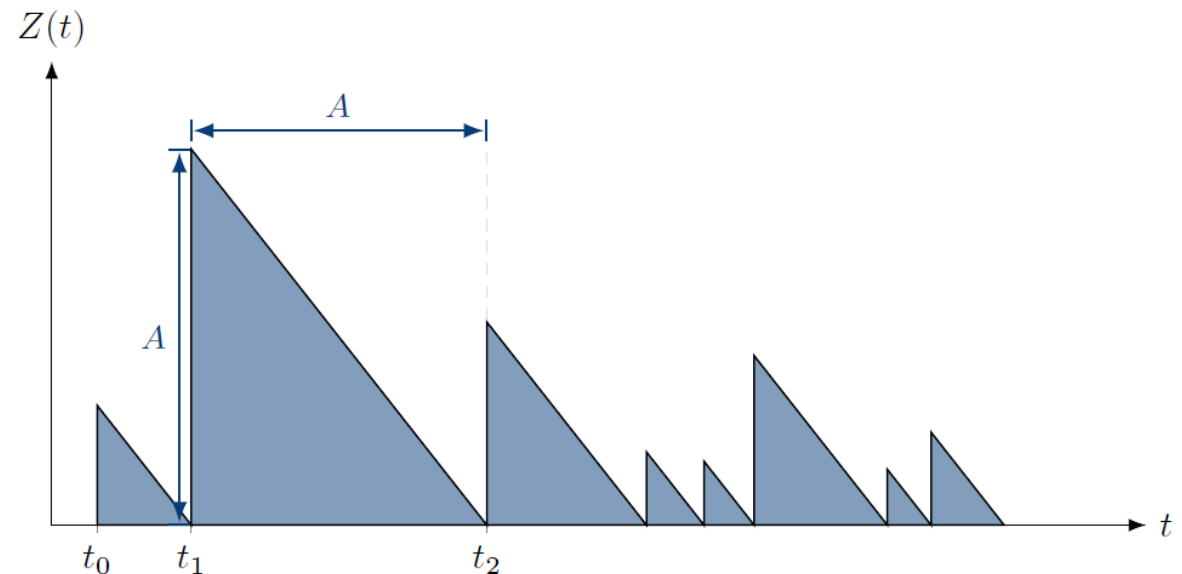


# Characterization of Arrival Process

- ▶ Interarrival times  $A_i$  for  $i$ -th arrival
- ▶ Here:  $A_i$  are iid. and follow same r.v.  $A$  (see renewal process)
- ▶ Remaining time  $Z(t)$  until next arrival
  - $\{Z(t), t\}$  continuous-time and -state
- ▶ **Arrival process with Markov property**
  - r.v.  $Z(t)$  is independent of the past
  - $Z(t)$  is iid. for any time instant  $t$
  - interarrival time  $A$  is memoryless
- ▶ **Poisson process** is the only single arrival process with Markov property



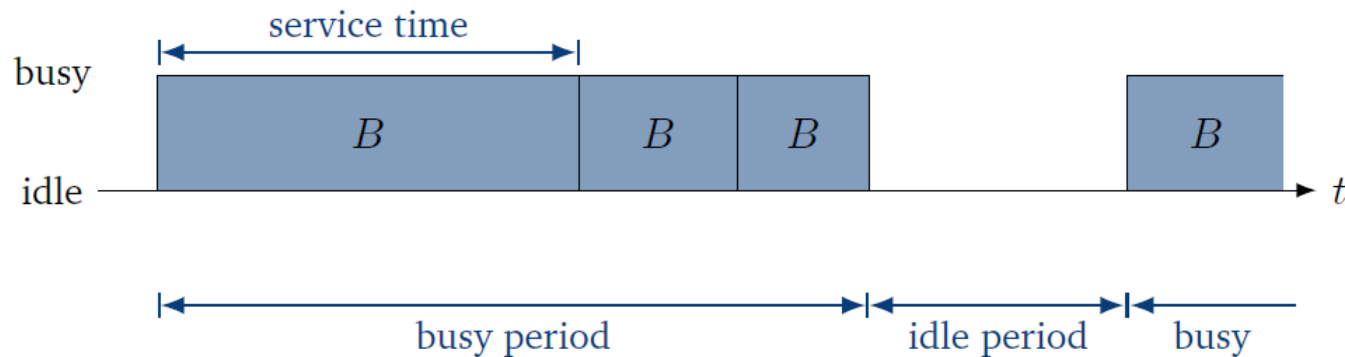
(a) Arrival process.



(b) Arrival process as a state process.

# Characterization of Service Process

- ▶ Service time described with r.v.  $B_i$  for  $i$ -th job
- ▶ Here: service times are iid. for any job and follow r.v.  $B$
- ▶ Several consecutive service times constitute a busy period
- ▶ Between busy periods there is an idle period



## ▶ Service process with Markov property

- memoryless service time
- duration until end of operation independent of the past
- exponential distribution is the only continuous distribution with Markov property

# State Process

- ▶ Depending on traffic model and analysis method, different forms of the state process description are considered
- ▶ Number of customers  $X(t)$ 
  - e.g. for M/GI/1 or GI/M/1
- ▶ Unfinished work  $U(t)$ 
  - e.g. for analysis of GI/GI/1

