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Chapter 4.4

Customer Retrial Model with Finite Number of Sources

Performance Evaluation of the Internet of Things (IoT)

Module Course: Performance Evaluation of Distributed Systems

Prof. Tobias Hoßfeld, Summer Semester 2022

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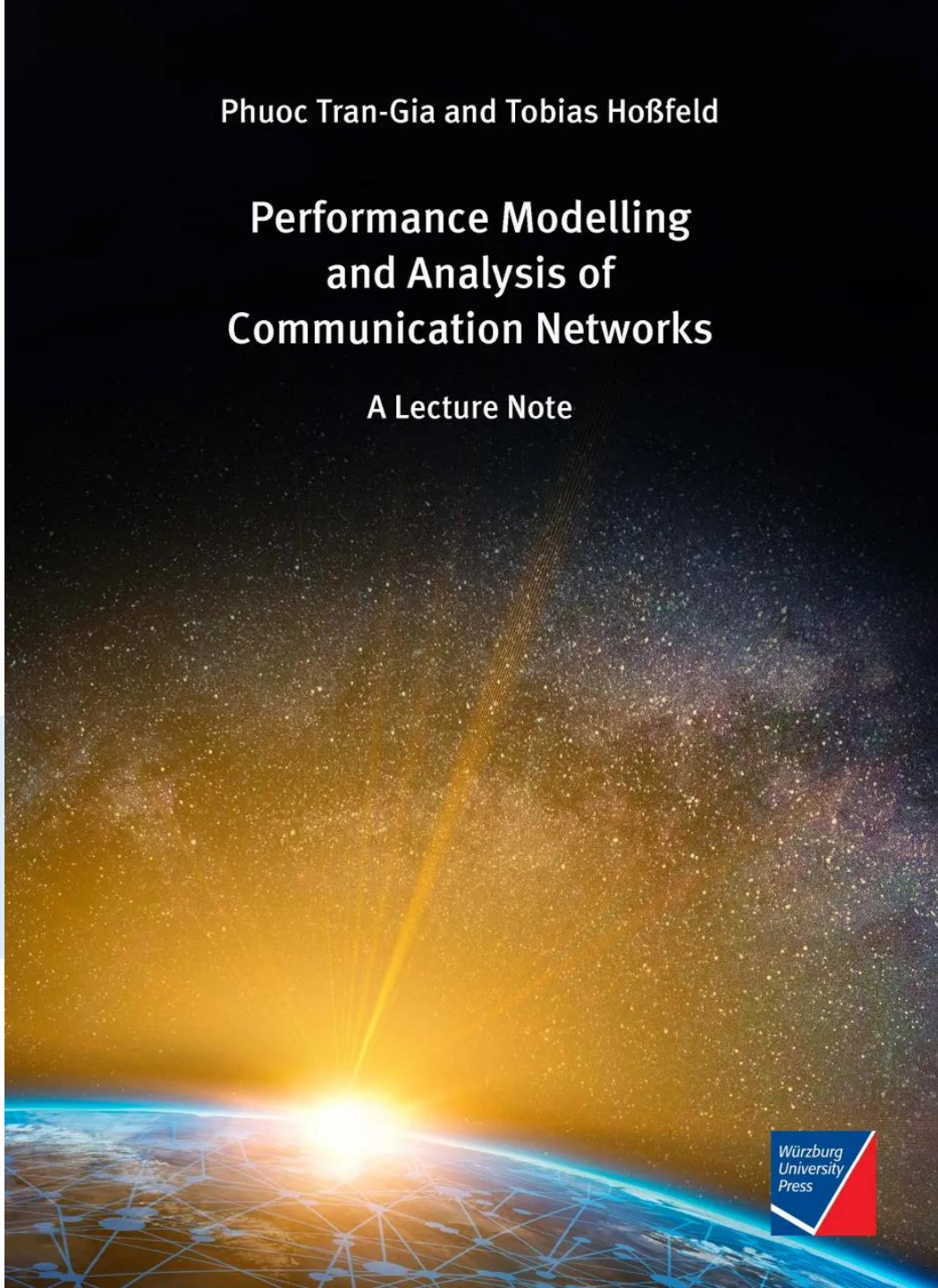
The book must be cited and the disclaimer attached when using lectures slides or scripts.

Tran-Gia, P. & Hossfeld, T. (2021).
Performance Modeling and Analysis of Communication Networks - A Lecture Note. Würzburg University Press.
https://doi.org/10.25972/WUP-978-3-95826-153-2

Website to download book, exercises, slides and scripts:
https://modeling.systems/



Chapter 4

4 Analysis of Markovian Systems

4.1 Loss System M/M/n
- 4.1.1 Model Structure and Parameters
- 4.1.2 State Process and State Probabilities
- 4.1.3 Other System Characteristics
- 4.1.4 Generalization to Loss System M/GI/n
- 4.1.5 Modeling Examples and Applications

4.2 Delay System M/M/n
- 4.2.1 Model Structure and Parameters
- 4.2.2 State Process and State Probabilities
- 4.2.3 Other System Characteristics
- 4.2.4 Delay Distribution
- 4.2.5 Example: Single Server Delay System

4.3 Loss System with Finite Number of Sources
- 4.3.1 Model Structure and Parameters
- 4.3.2 State Process and State Probabilities
- 4.3.3 Example: Mobile Cell with Finite Number of Sources

4.4 Customer Retrial Model with Finite Number of Sources
- 4.4.1 Model Structure and Parameters
- 4.4.2 Recursive Analysis Algorithm
- 4.4.3 Calculation of Traffic Flows
- 4.4.4 Example: Mobile Cell with Customer Retrials

4.5 Processor Sharing Model M/M/1-PS

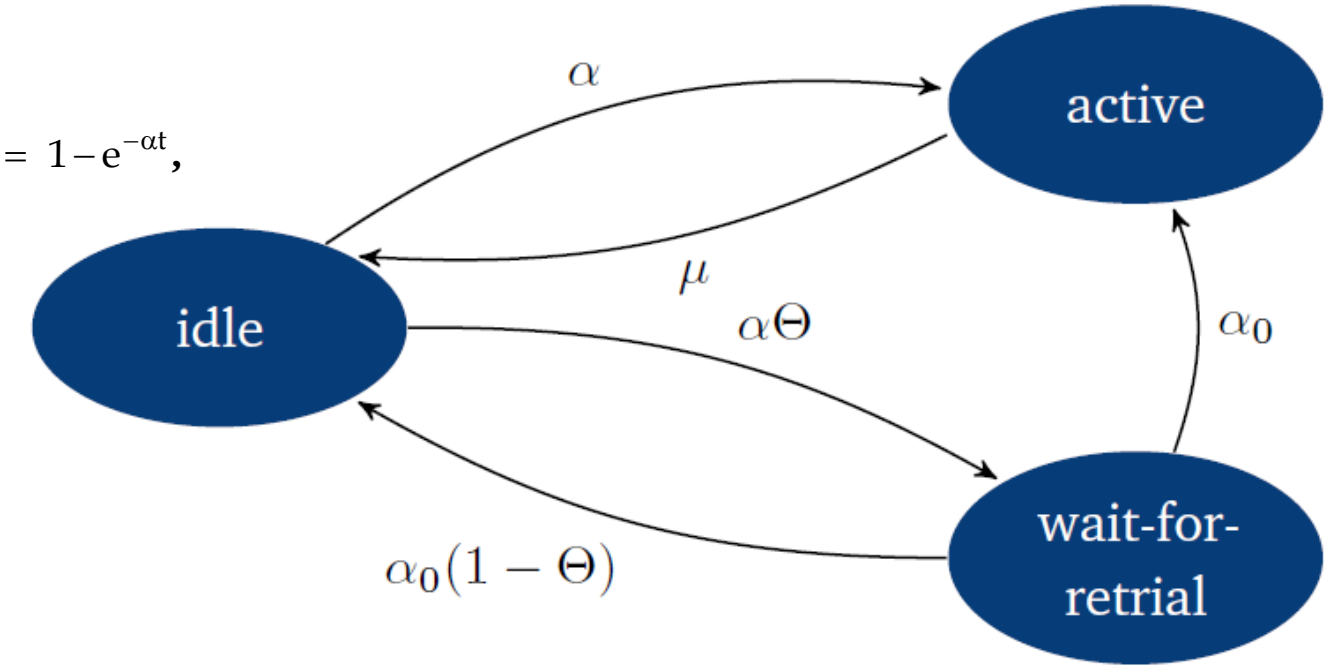
Model of Customer Behavior with Retrial

- Blocked customer
  - will retry with probability Θ : enter wait-for-retrial state
  - will abandon with probability $1 - \Theta$: remain in idle state

idle time I

$$I(t) = P(I \le t) = 1 - e^{-\alpha t},$$

$$E[I] = \frac{1}{\alpha}.$$



call duration B (service time)

$$B(t) = P(B \le t) = 1 - e^{-\mu t},$$

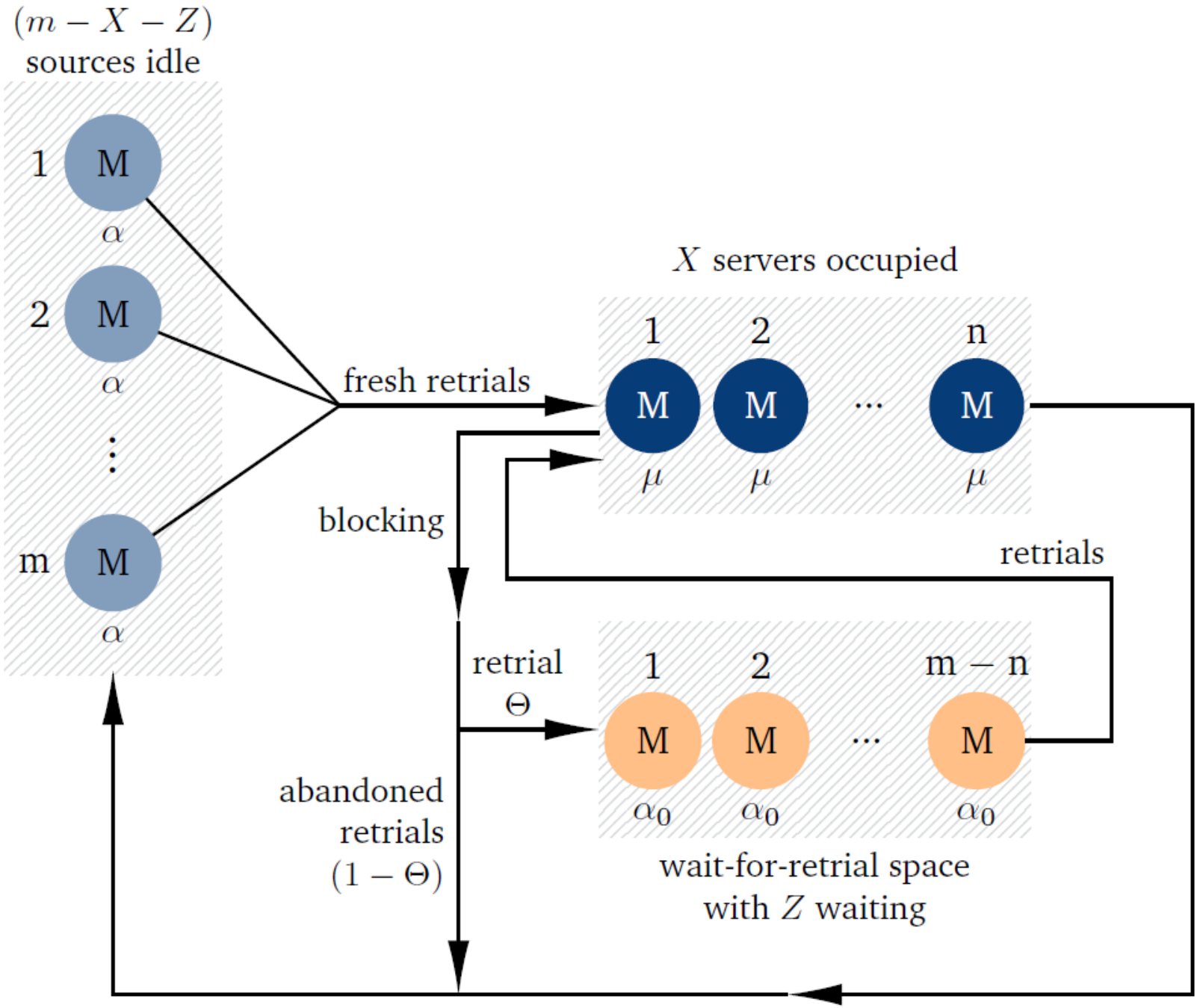
$$E[B] = \frac{1}{\mu}.$$

time in wait-for-retrial state R

$$R(t) = P(R \le t) = 1 - e^{-\alpha_0 t},$$

$$E[R] = \frac{1}{\alpha_0}.$$

Retrial Model



Model Parameters

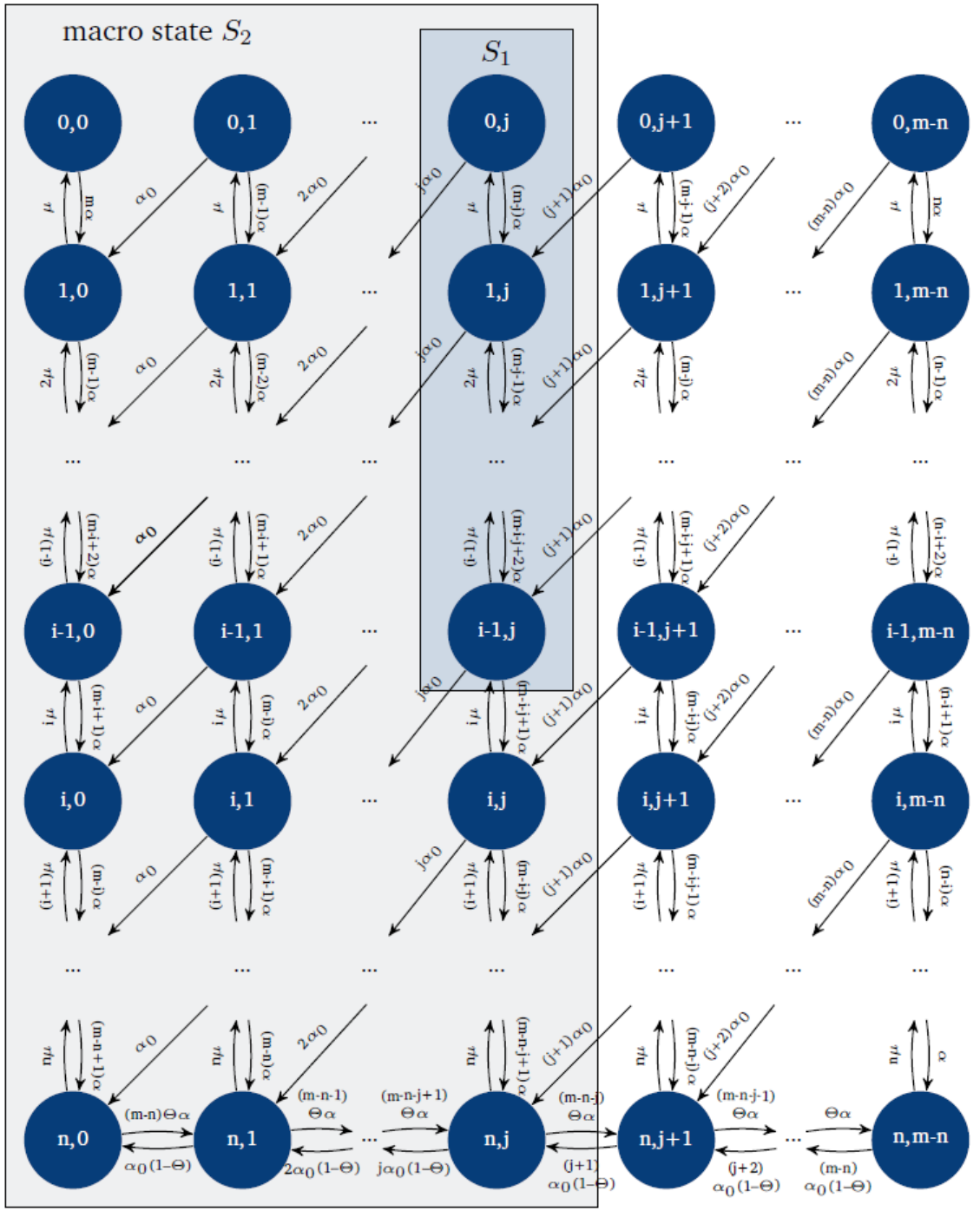
α	call rate of a customer in the idle state
m	number of sources,
μ	service rate of a server in the server stage
n	number of servers
α_0	retrial rate of a customer in the wait-for-retrial state
Θ	retrial probability of customers
X	r.v. for the number of occupied servers
Z	r.v. for the number of customers waiting for retrial
$x(i,j)$	$P(X=i, Z=j) = x(i,j), \quad i=0,1,\ldots,n, \quad j=0,1,\ldots,m-n$ probability that i customers are in server stage and j customers are in the wait-for-retrial stage.

State Transition Diagram

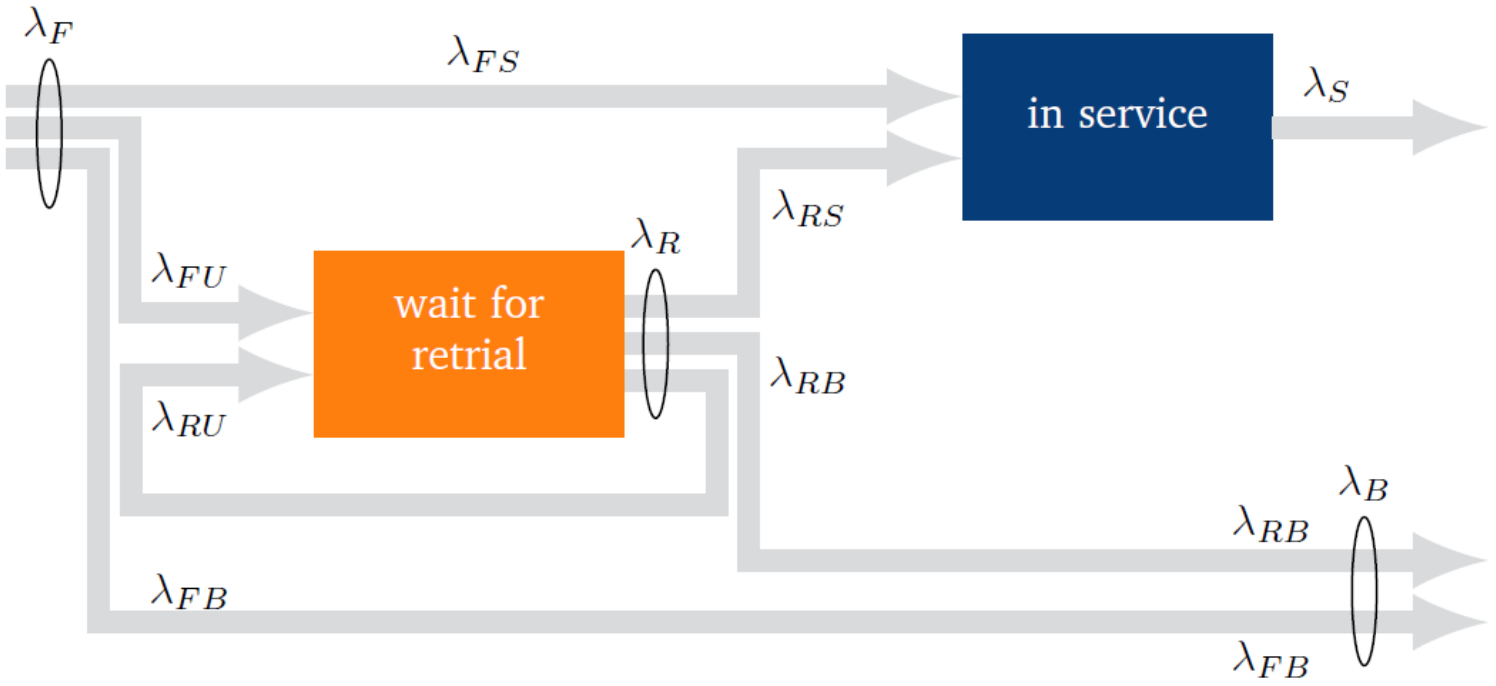
State Transition Diagram (f.)

State Transition Diagram

- Can be solved numerically
- See notebook script at https://modeling.systems/

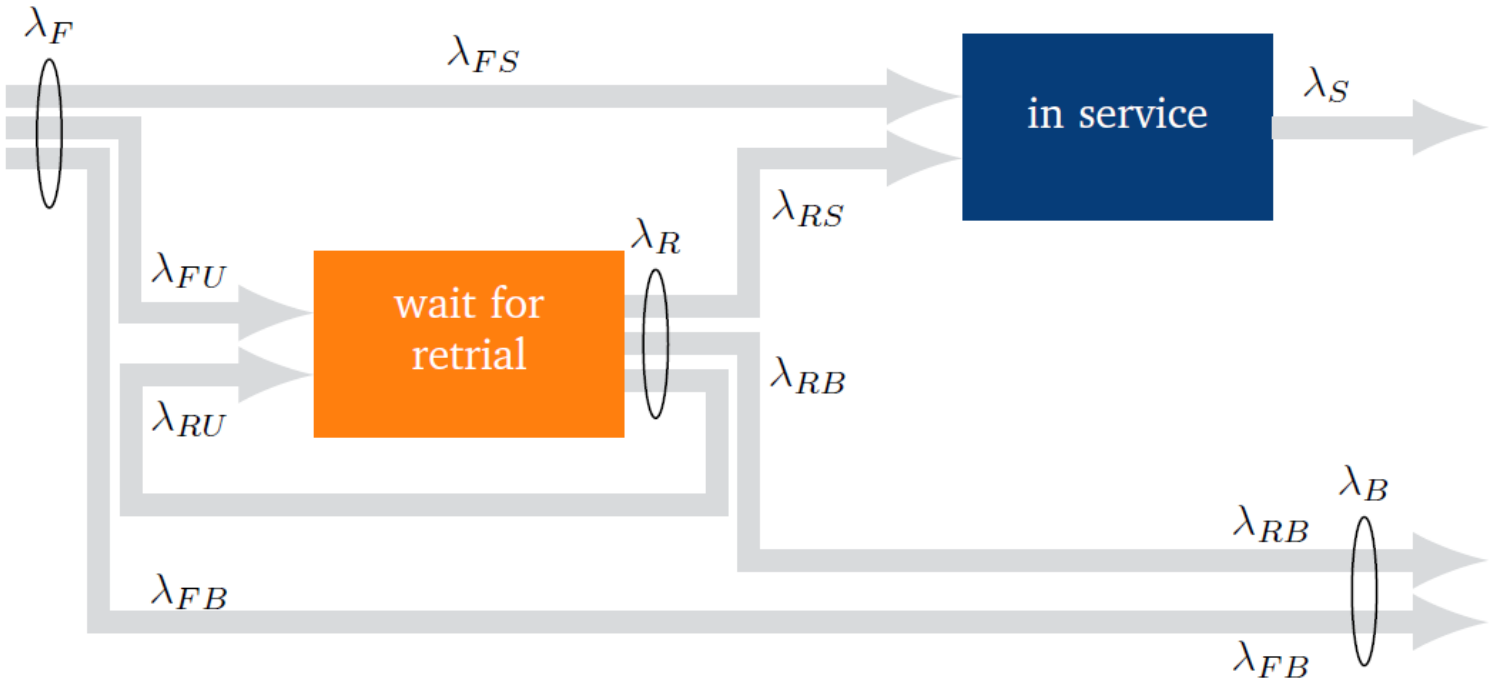


Traffic Flows



F	fresh call, first attempt
R	retrial, repeated call
S	successful call, completed call
U	unsuccessful call or blocked call which will be repeated
B	unsuccessful call or blocked call which is abandoned

Traffic Flows of Fresh Calls



$$\lambda_{FS} = \alpha \sum_{i=0}^{n-1} \sum_{j=0}^{m-n} (m-i-j) \cdot x(i,j),$$

$$\lambda_{FU} = \theta \cdot \alpha \sum_{j=0}^{(m-n)-1} (m-n-j) \cdot x(n,j),$$

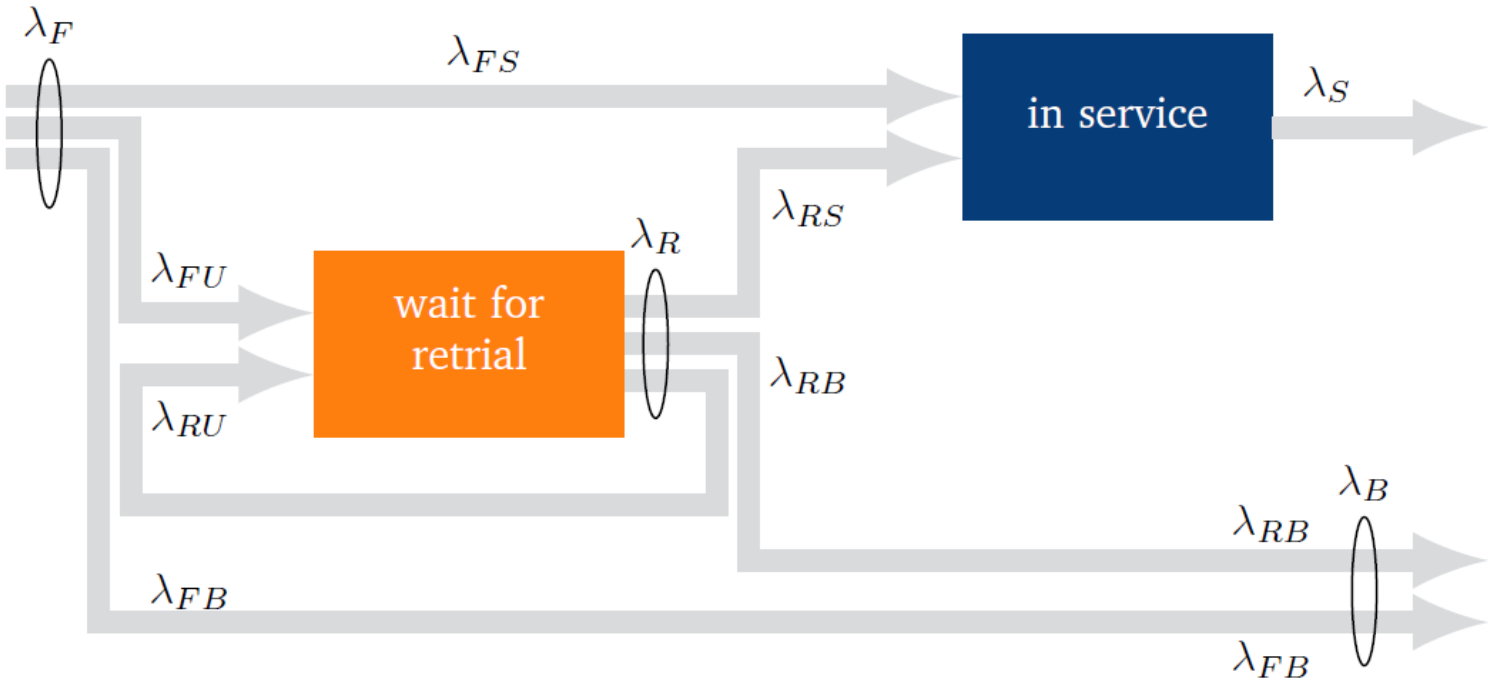
$$\lambda_{FB} = (1-\theta) \cdot \alpha \sum_{j=0}^{(m-n)-1} (m-n-j) \cdot x(n,j).$$

$$\lambda_F = \lambda_{FS} + \lambda_{FU} + \lambda_{FB},$$

blocking probabiltiy for fresh calls

$$p_{B_F} = \frac{\lambda_{FU} + \lambda_{FB}}{\lambda_F}.$$

Mean Number of Attempts for Successful Calls



$$\lambda_{RS} = \alpha_0 \cdot \sum_{i=0}^{n-1} \sum_{j=0}^{m-n} j \cdot x(i,j),$$

$$\lambda_{RU} = \theta \cdot \alpha_0 \sum_{j=0}^{m-n} j \cdot x(n,j),$$

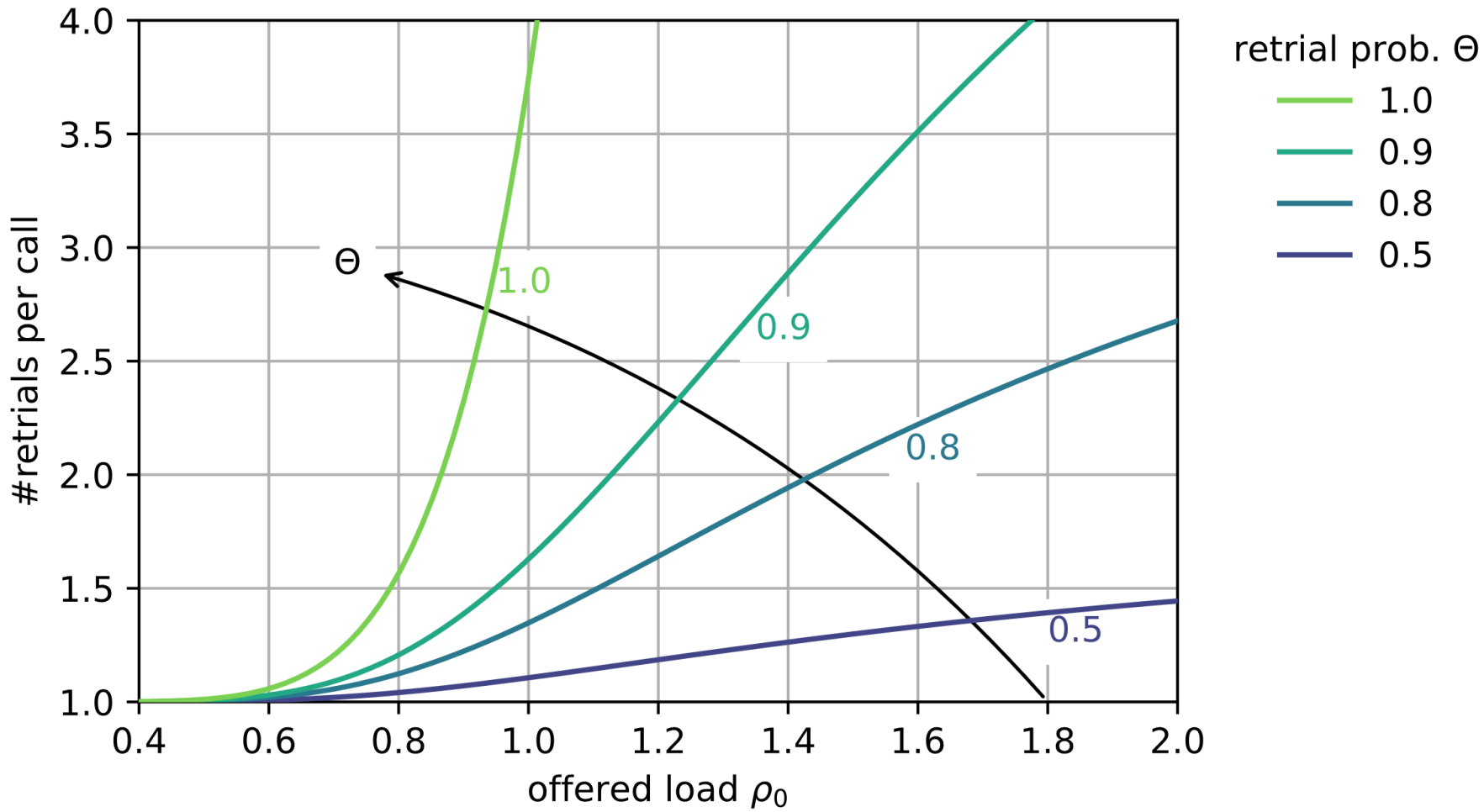
$$\lambda_{RB} = (1-\theta) \cdot \alpha_0 \sum_{j=0}^{m-n} j \cdot x(n,j).$$

$$\lambda_R = \lambda_{RS} + \lambda_{RU} + \lambda_{RB}\ .$$

Mean number of attempts for successful calls

$$\eta_F = \frac{\lambda_{FU} + \lambda_{FS} + \lambda_{RU} + \lambda_{RS}}{\lambda_{FS} + \lambda_{RS}} = \frac{\lambda_F + \lambda_R - \lambda_B}{\lambda_F - \lambda_B} = 1 + \frac{\lambda_R}{\lambda_F - \lambda_B}$$

Results: Retrial Model



Results: Retrial Model