

Chapter 3.4

Superposition of Independent Renewal Processes

Performance Evaluation of the Internet of Things (IoT)

Module Course: Performance Evaluation of Distributed Systems

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*Tran-Gia, P. & Hossfeld, T. (2021).
Performance Modeling and Analysis of Communication
Networks - A Lecture Note. Würzburg University Press.
<https://doi.org/10.25972/WUP-978-3-95826-153-2>*

Website to download book, exercises, slides and scripts:
<https://modeling.systems/>

Chapter 3

3 Elementary Random Processes

3.1 Stochastic Processes

3.1.1 Definition

3.1.2 Markov Processes

3.1.3 Elementary Processes in Performance Models

3.2 Renewal Processes

3.2.1 Definition

3.2.2 Analysis of Recurrence Time

3.3 Poisson Process

3.3.1 Definition of a Poisson Process

3.3.2 Properties of the Poisson Process

3.3.3 Poisson Arrivals during Arbitrarily Distributed Interval

3.4 Superposition of Independent Renewal Processes

3.4.1 Superposition of Poisson Processes

3.4.2 Palm-Khintchine Theorem

3.5 Markov State Process

3.5.1 Definition of Continuous-Time Markov Chain

3.5.2 Transition Behavior of Markovian State Processes

3.5.3 State Equations and State Probabilities

3.5.4 Examples of Transition Probability Densities

3.5.5 Birth-and-Death Processes

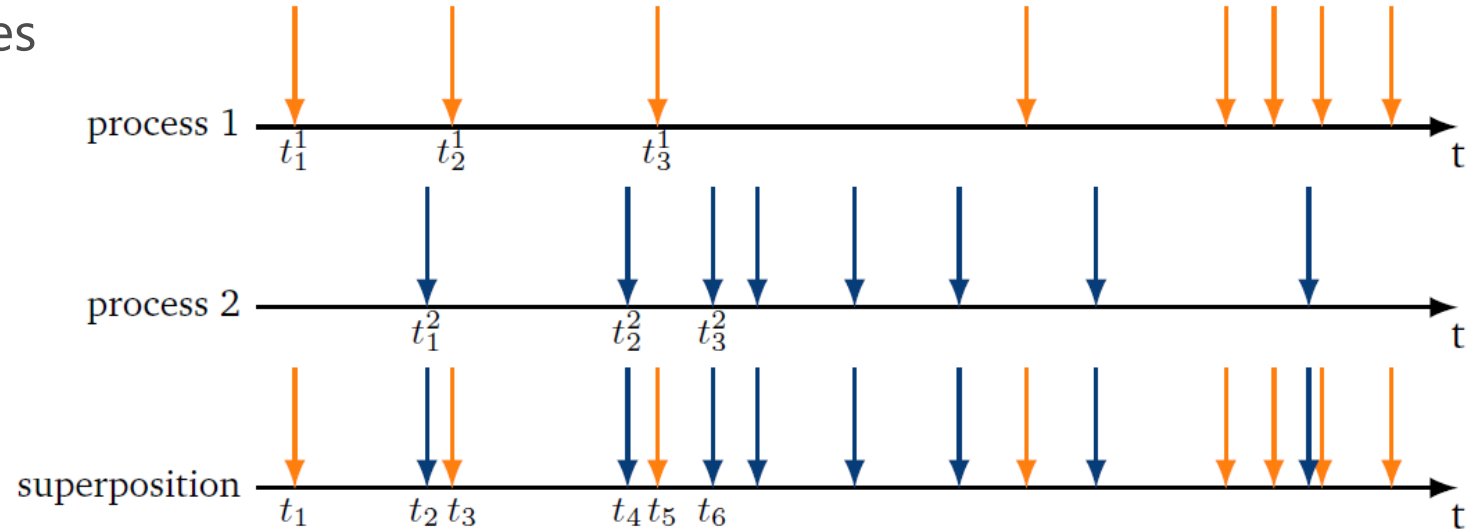
Merging of Poisson Processes



- ▶ Superposition of Poisson processes remains a Poisson process

- Process 1: $A_1 \sim \text{EXP}(\lambda_1)$
- Process 2: $A_2 \sim \text{EXP}(\lambda_2)$

- ▶ Superposition $A \sim \text{EXP}(\lambda_1 + \lambda_2)$



- ▶ Proof: random observer sees minimum of recurrence times of both processes
 - recurrence time is exponentially distributed with same rate
 - $A = \min(A_1, A_2)$ with CDF $A(t) = 1 - (1 - A_1(t)) \cdot (1 - A_2(t)) = 1 - e^{-(\lambda_1 + \lambda_2)t}$
 - $A = \min(A_1, A_2) \sim \text{EXP}(\lambda_1 + \lambda_2)$

- ▶ In general: superposition of n Poisson processes

$$A \sim \text{EXP}\left(\sum_{i=1}^n \lambda_i\right)$$

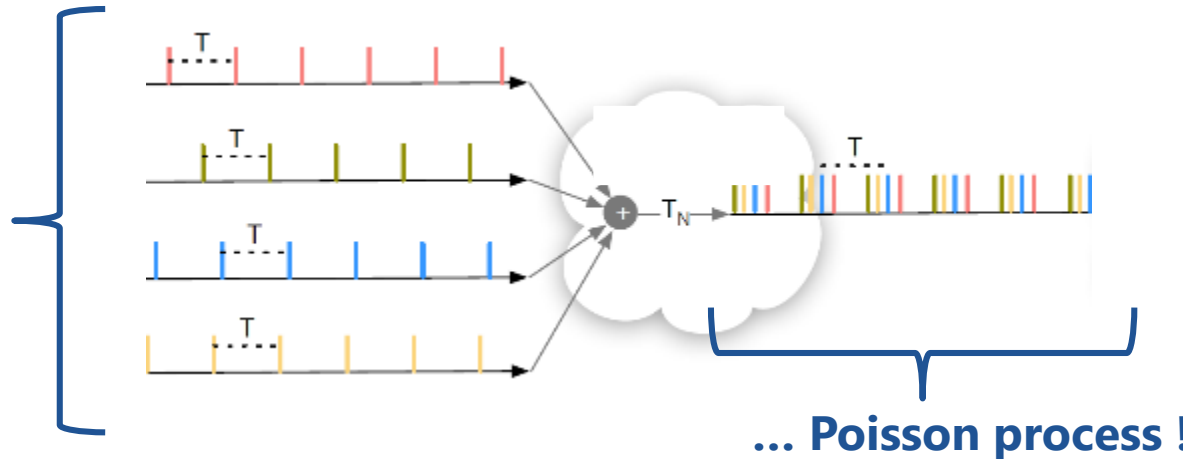
PALM-KHINTCHINE THEOREM

Superposition of renewal processes

Palm-Khintchine Theorem

- ▶ Superposition of a large number n of **independent renewal processes**
 - interarrival times A_i of the renewal processes may follow general distribution
 - each with a finite intensity $\lambda_i = \frac{1}{E[A_i]}$, $i = 1, 2, \dots, n$
 - behaves asymptotically like a Poisson process for $n \rightarrow \infty$
- ▶ Kendall's notation
 - $\sum_i GI_i/\bullet/\bullet$, which must not be confused with the sum of r.v.s
 - $n \cdot GI/\bullet/\bullet$ if the arrival processes follow the same distribution
- ▶ Example: periodic traffic

large number $n \dots$



Formally: Palm-Khintchine Theorem

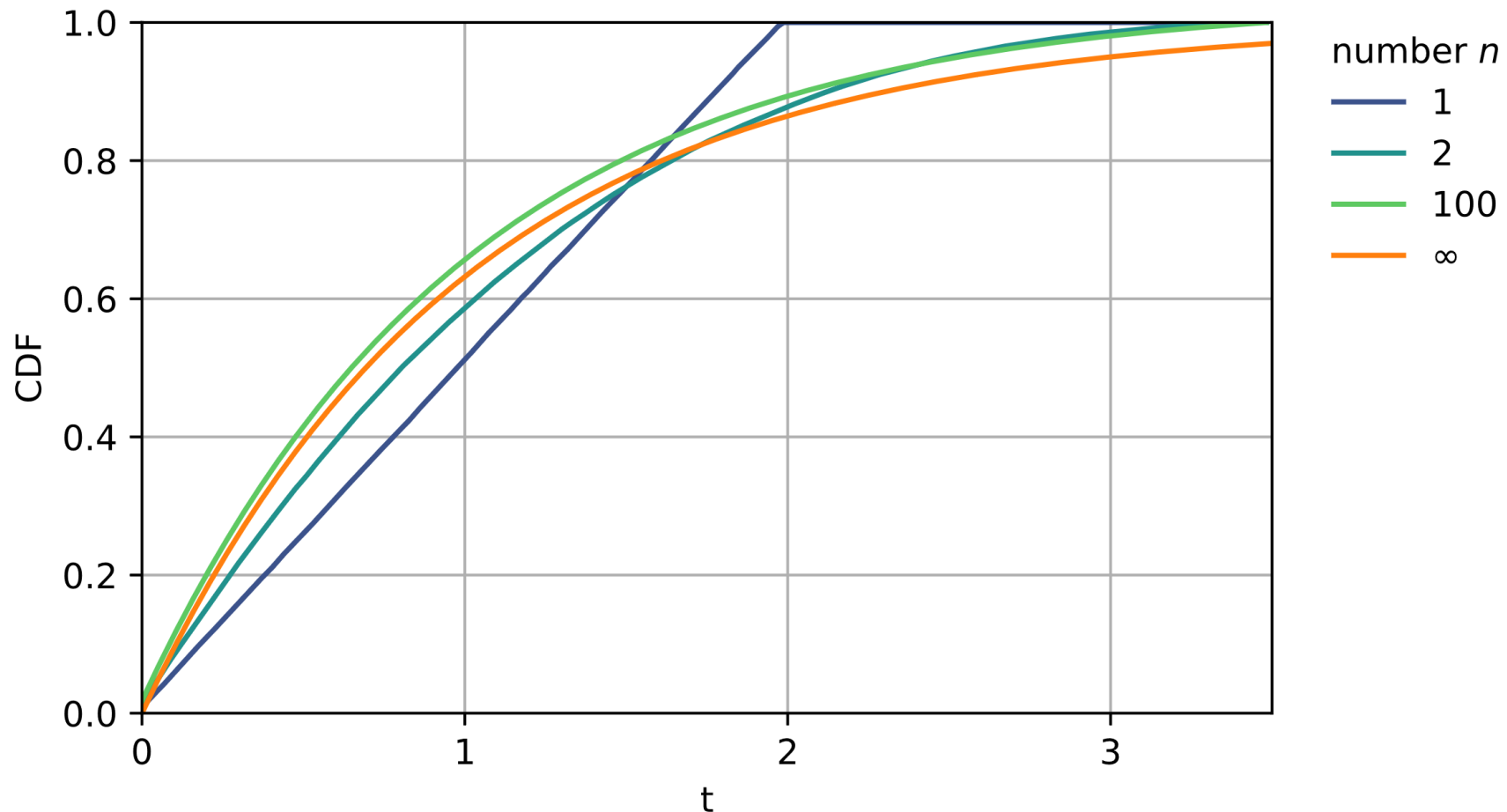
- ▶ The superposition of a large number of independent renewal processes, each with a finite intensity, behaves asymptotically like a Poisson process for $n \rightarrow \infty$:

$$A^* \sim \text{EXP} \left(\sum_i \lambda_i \right)$$

- ▶ if the following two conditions are fulfilled:
 - 1. the overall load is finite, $\sum_i \lambda_i < \infty$
 - 2. no single process dominates the superposition, $\lambda_k \ll \sum_i \lambda_i$ for any k
- ▶ Alternative formulation:
 - We consider renewal process i as a counting process of arrivals $N_i(t)$ in the interval t .
 - Then the superposition of the counting processes $N(t) = N_1(t) + \dots + N_n(t)$ for $n \rightarrow \infty$ tends towards a Poisson process with rate λ .

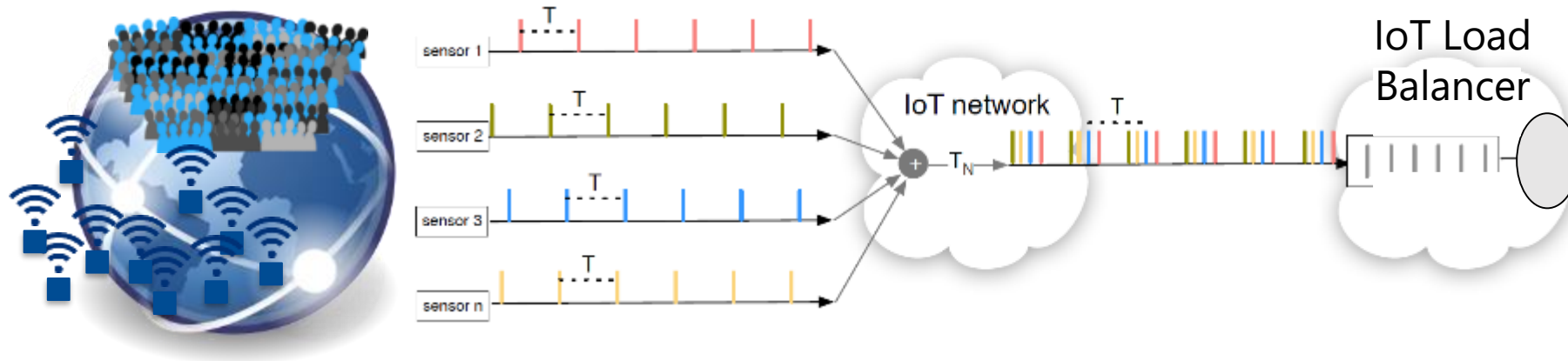
Example: Superposition of Uniformly Distributed IATs

- ▶ n independent processes with IAT $A_i \sim U(0, 2n)$ and $E[A_i] = n$
- ▶ Overall load is $\lambda^* = \sum_{i=1}^n \frac{1}{n} = 1 < \infty$ for any n



Example: IoT Cloud

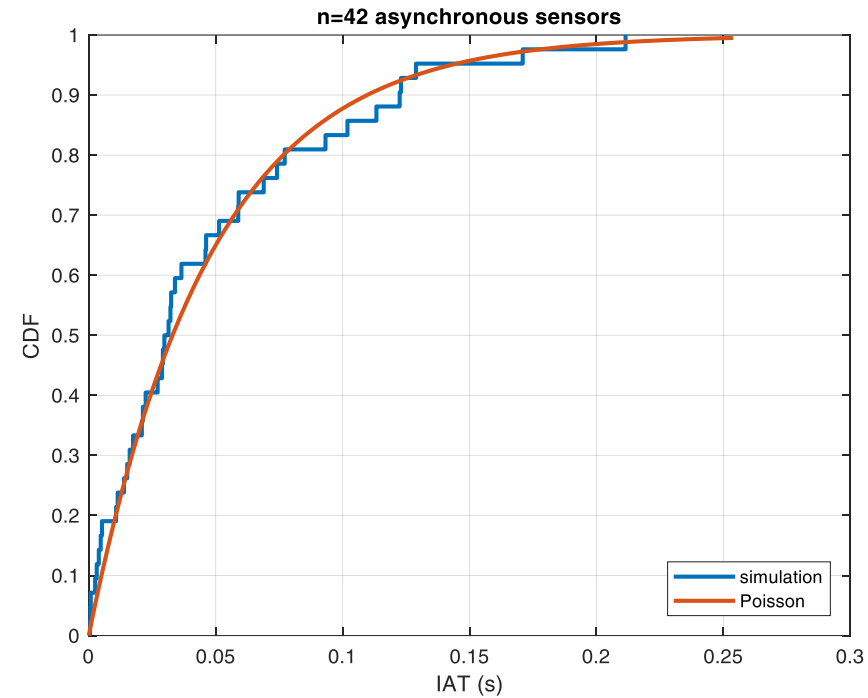
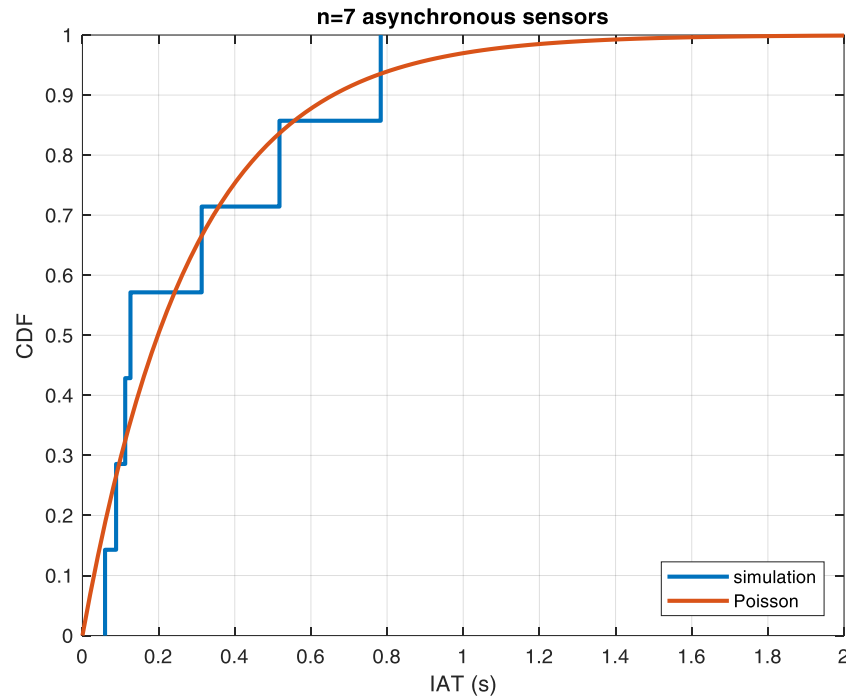
- ▶ Many sensors send data to an IoT cloud
- ▶ IoT Load Balancer distributes messages to servers in the cloud
- ▶ IoT load balancer as a bottleneck
- ▶ Deterministic message processing time



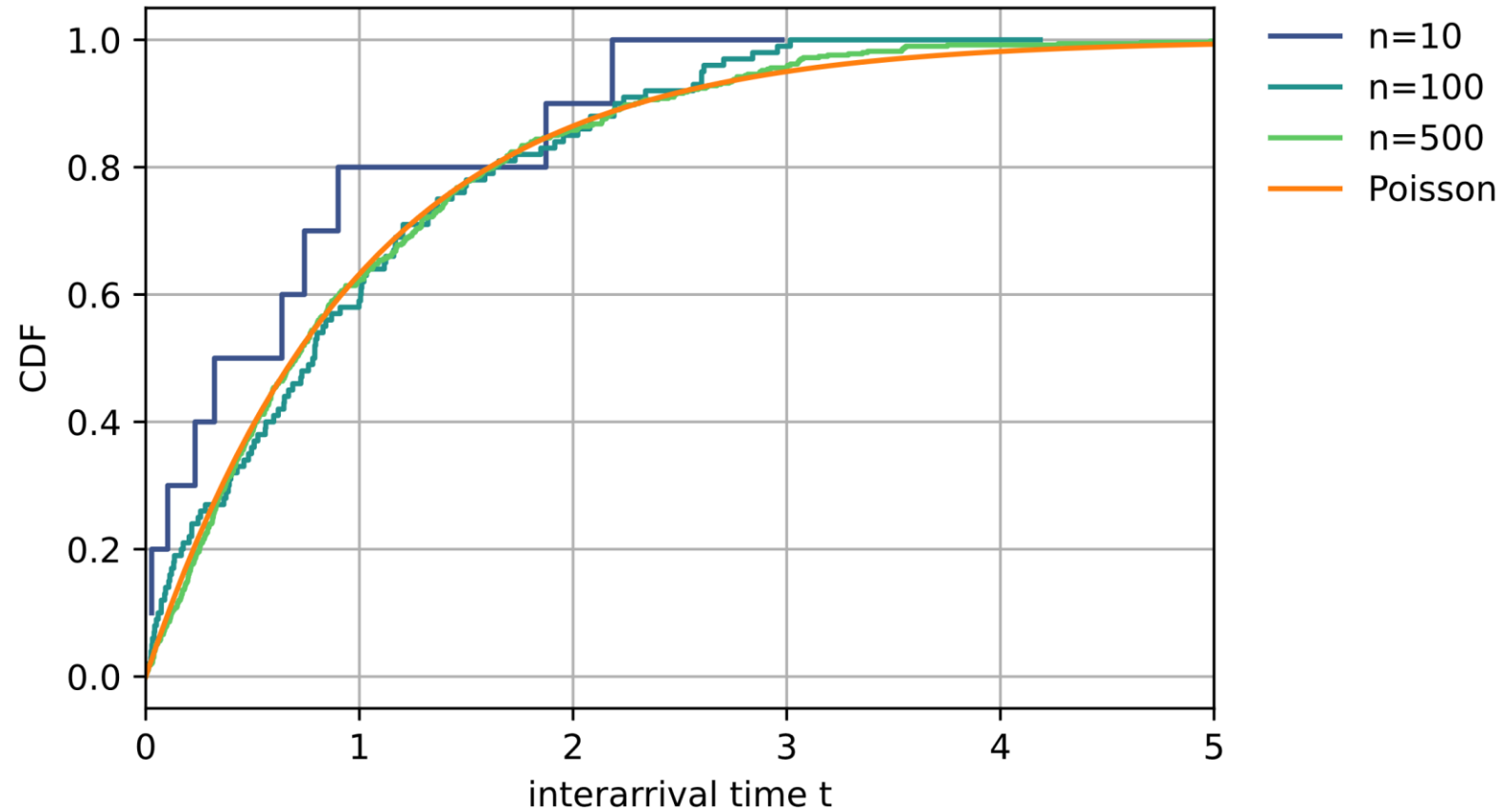
- ▶ How to model the load balancer (Kendall notation)?

Asynchronous Sensors

- ▶ Each node periodically sends every $A = 2\text{s}$
- ▶ Single simulation run is considered



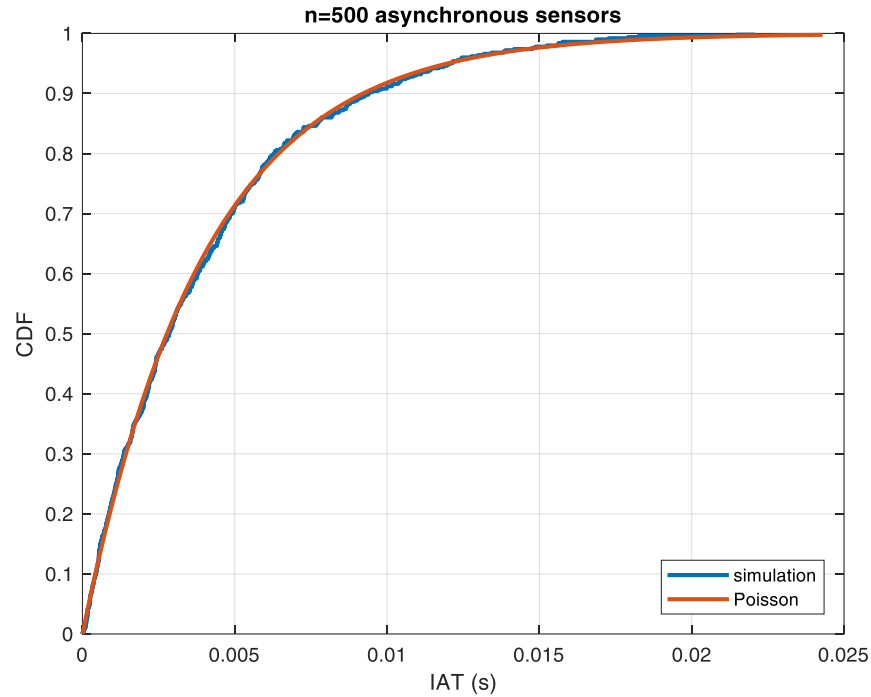
Asynchronous Sensors (f.)



Synchronous Sensors

- How to model the different scenarios?

asynchronous



synchronous

