

Chapter 4.3

Loss System with Finite Number of Sources: Engset Model

Performance Evaluation of the Internet of Things (IoT)

Module Course: Performance Evaluation of Distributed Systems

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*Tran-Gia, P. & Hossfeld, T. (2021).
Performance Modeling and Analysis of Communication
Networks - A Lecture Note. Würzburg University Press.
<https://doi.org/10.25972/WUP-978-3-95826-153-2>*

Website to download book, exercises, slides and scripts:
<https://modeling.systems/>

Chapter 4

4 Analysis of Markovian Systems

4.1 Loss System M/M/n

- 4.1.1 Model Structure and Parameters
- 4.1.2 State Process and State Probabilities
- 4.1.3 Other System Characteristics
- 4.1.4 Generalization to Loss System M/GI/n
- 4.1.5 Modeling Examples and Applications

4.2 Delay System M/M/n

- 4.2.1 Model Structure and Parameters
- 4.2.2 State Process and State Probabilities
- 4.2.3 Other System Characteristics
- 4.2.4 Delay Distribution
- 4.2.5 Example: Single Server Delay System

4.3 Loss System with Finite Number of Sources

- 4.3.1 Model Structure and Parameters
- 4.3.2 State Process and State Probabilities
- 4.3.3 Example: Mobile Cell with Finite Number of Sources

4.4 Customer Retrial Model with Finite Number of Sources

- 4.4.1 Model Structure and Parameters
- 4.4.2 Recursive Analysis Algorithm
- 4.4.3 Calculation of Traffic Flows
- 4.4.4 Example: Mobile Cell with Customer Retrials

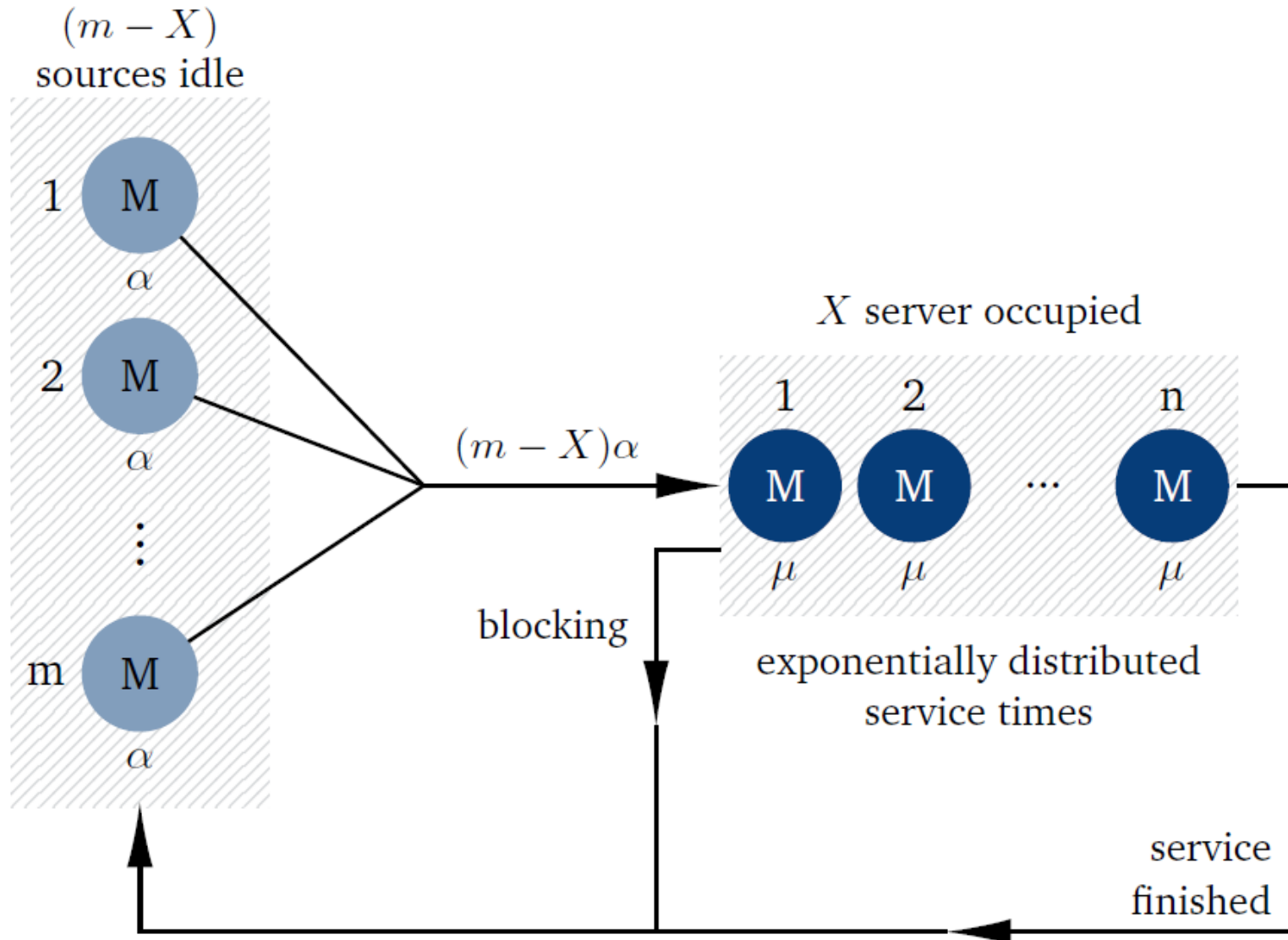
4.5 Processor Sharing Model M/M/1-PS

Poisson Process: Infinite Number of Sources

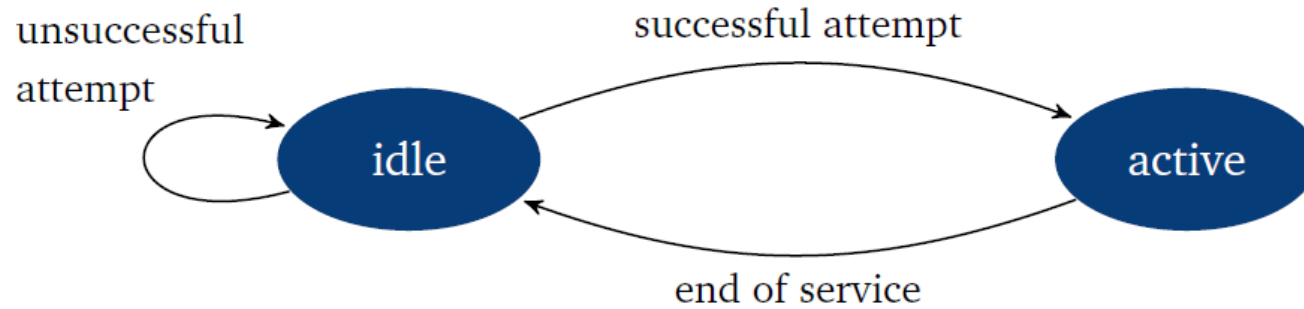
Lecture

MODEL STRUCTURE AND PARAMETERS

Engset Model: Finite Number of Traffic Sources



Model of Customer Behavior



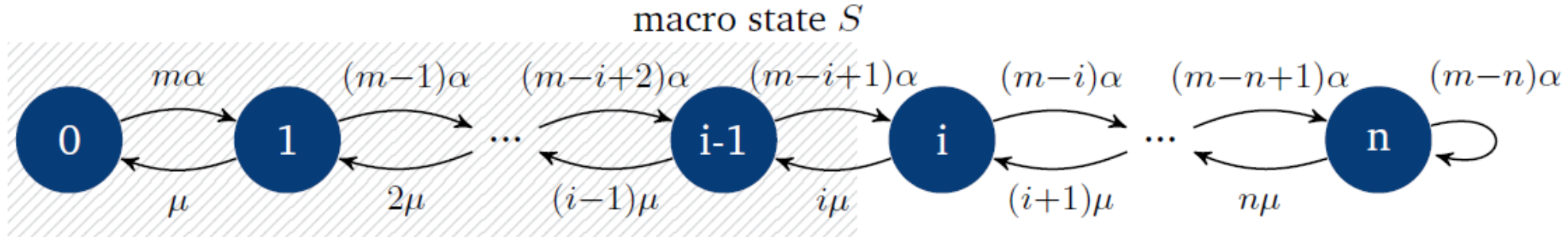
- ▶ Customer in idle mode for time I $I(t) = P(I \leq t) = 1 - e^{-\alpha t}$, $E[I] = \frac{1}{\alpha}$
- ▶ Customer in active mode for time B (service time) $B(t) = P(B \leq t) = 1 - e^{-\mu t}$, $E[B] = \frac{1}{\mu}$
- ▶ Offered traffic of an idle customer $a^* = \frac{\alpha}{\mu}$
- ▶ "On-off" pattern of customer

STATE PROCESS AND STATE PROBABILITIES

State Transition Diagram: Derivation

State Transition Diagram

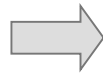
- Birth-and-death process



- State probabilities follow from BD processes (Ch. 3)

$$(m-i+1)\alpha \cdot x(i-1) = i\mu \cdot x(i), \quad i = 1, 2, \dots, n$$

$$\sum_{i=0}^n x(i) = 1$$



$$x(i) = \frac{\binom{m}{i} a^{*i}}{\sum_{k=0}^n \binom{m}{k} a^{*k}}$$

for $i = 0, 1, 2, \dots, n$ and $m > n$

$$a^* = \frac{\alpha}{\mu}$$

State Probabilities: Derivation (f.)

Lecture

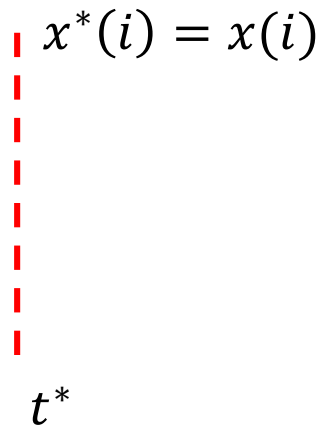
BLOCKING PROBABILITY

Arrival theorem, random observer property

State Probabilities at Arbitrary Time and Arrival Time

- ▶ At arbitrary time

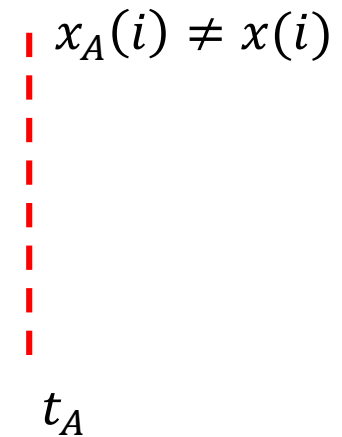
$$x(i) = \frac{\binom{m}{i} a^{*i}}{\sum_{k=0}^n \binom{m}{k} a^{*k}}$$



$x^*(i) = x(i)$

t^*

- ▶ At arrival times
 - PASTA property not valid
 - state-dependent arrival rates
- ▶ Blocking probability
 - at arrival time



$x_A(i) \neq x(i)$

t_A

Arrival Theorem: Random Observer Property

- ▶ An arriving customer observes the system as if in steady state at an arbitrary instant for the system without that customer.
- ▶ **Finite number** of m customers
 - state probabilities $x_A^m(i)$ seen by arriving customer entering a state i are the same as
 - arbitrary-time probabilities $x^{m-1}(i)$ in a system with $m - 1$ customers
 - $x_A^m(i) = x^{m-1}(i)$
- ▶ For **Poisson processes**
 - $m = \infty$ sources
 - Then: $x_A^\infty(i) = x^\infty(i)$
 - PASTA property: state as seen by an outside random observer is the same as the probability of the state seen by an arriving customer

Blocking Probability

- ▶ State probability at arbitrary time

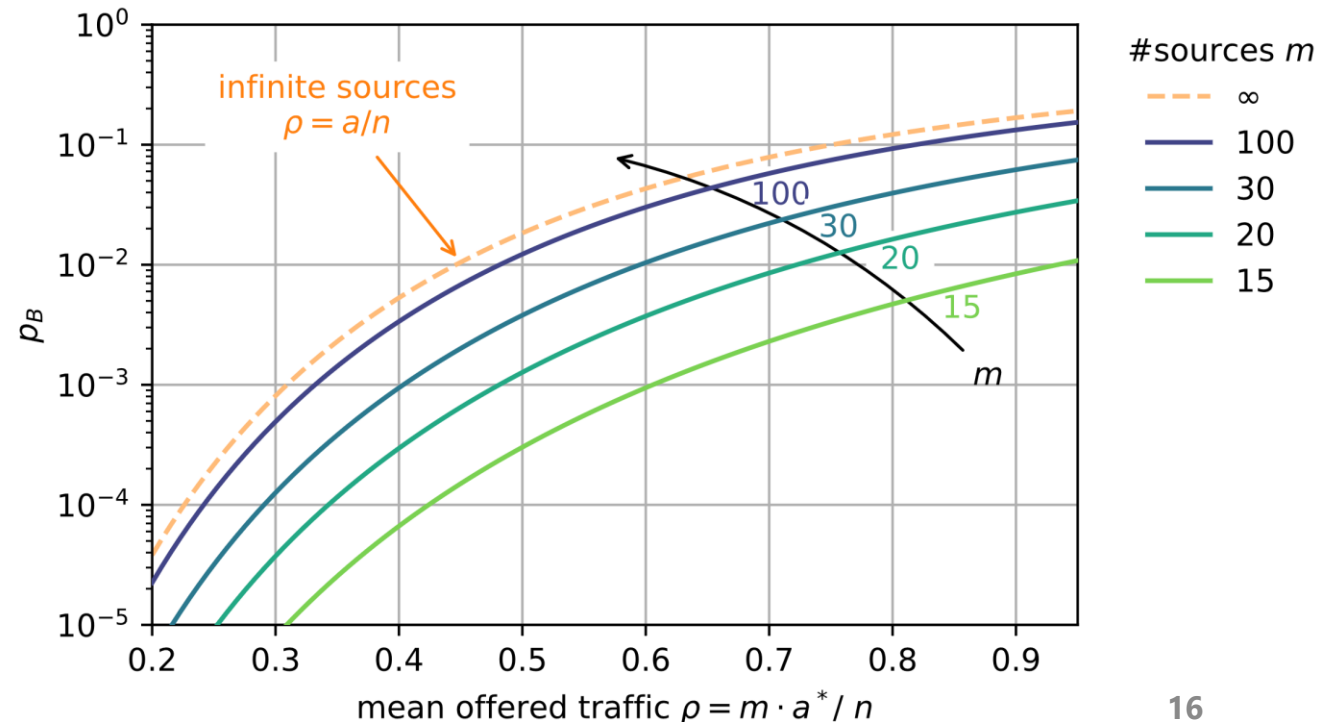
$$x(i) = \frac{\binom{m}{i} a^{*i}}{\sum_{k=0}^n \binom{m}{k} a^{*k}}$$

- ▶ State probability at arrival time

$$x_A(i) = \frac{\binom{m-1}{i} a^{*i}}{\sum_{k=0}^n \binom{m-1}{k} a^{*k}}$$

- ▶ Blocking probability: **Engset formula**

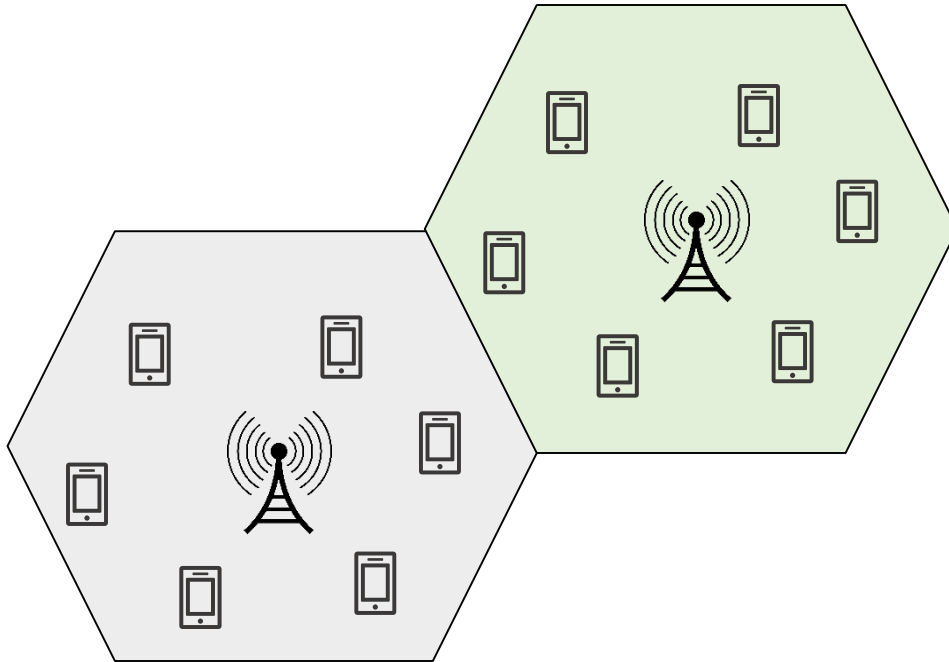
$$p_B = x_A(n) = \frac{\binom{m-1}{n} a^{*n}}{\sum_{k=0}^n \binom{m-1}{k} a^{*k}}$$



EXAMPLE: MOBILE CELL

with finite number of sources

Example: Mobile Cell



How many channels n are required so that the blocking probability is below a given threshold?

- ▶ In a single cell
 - m users
 - rate α of a user
 - offered load of idle user
$$a^* = \alpha / \mu$$
 - mean call duration $E[B]$
- ▶ Number of channels: n
- ▶ If all channels are used, an incoming call is rejected.
- ▶ Engset formula can be applied
- ▶ For $m \rightarrow \infty$: Poisson process with rate $\lambda = \alpha \cdot m$

Blocking Probability

► Parameters

- $E[B] = 2 \text{ min}$
- $a = 12 \text{ Erl}$

► Simplifying assumption of infinite number of sources represents upper limit regarding blocking (Erlang-B for Poisson process)

► Parameters

- $E[B] = 2 \text{ min}$
- $n = 30$

► Blocking probability increases with the granularity of the traffic

