Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Institute of Computer Science
Chair of Communication Networks
Prof. Dr. Tobias Hoßfeld

Chapter 4.3

Loss System with Finite Number of Sources: Engset Model

Performance Evaluation of the Internet of Things (IoT)

Module Course: Performance Evaluation of Distributed Systems

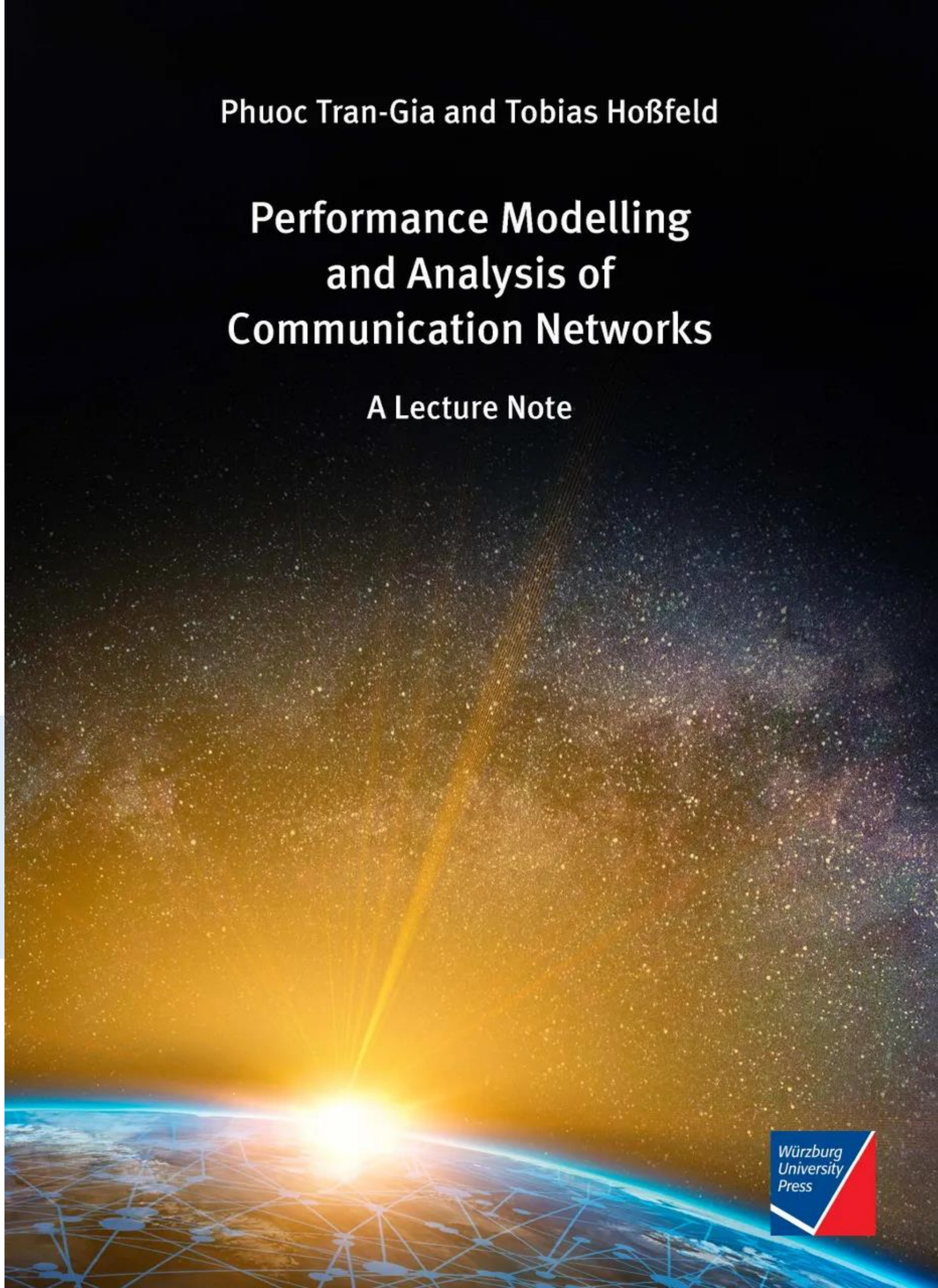
Prof. Tobias Hoßfeld, Summer Semester 2022

Disclaimer and Copyright Notice

Tran-Gia, P. & Hossfeld, T. (2021).
Performance Modeling and Analysis of Communication Networks - A Lecture Note. Würzburg University Press.
https://doi.org/10.25972/WUP-978-3-95826-153-2

Website to download book, exercises, slides and scripts:
https://modeling.systems/

Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG



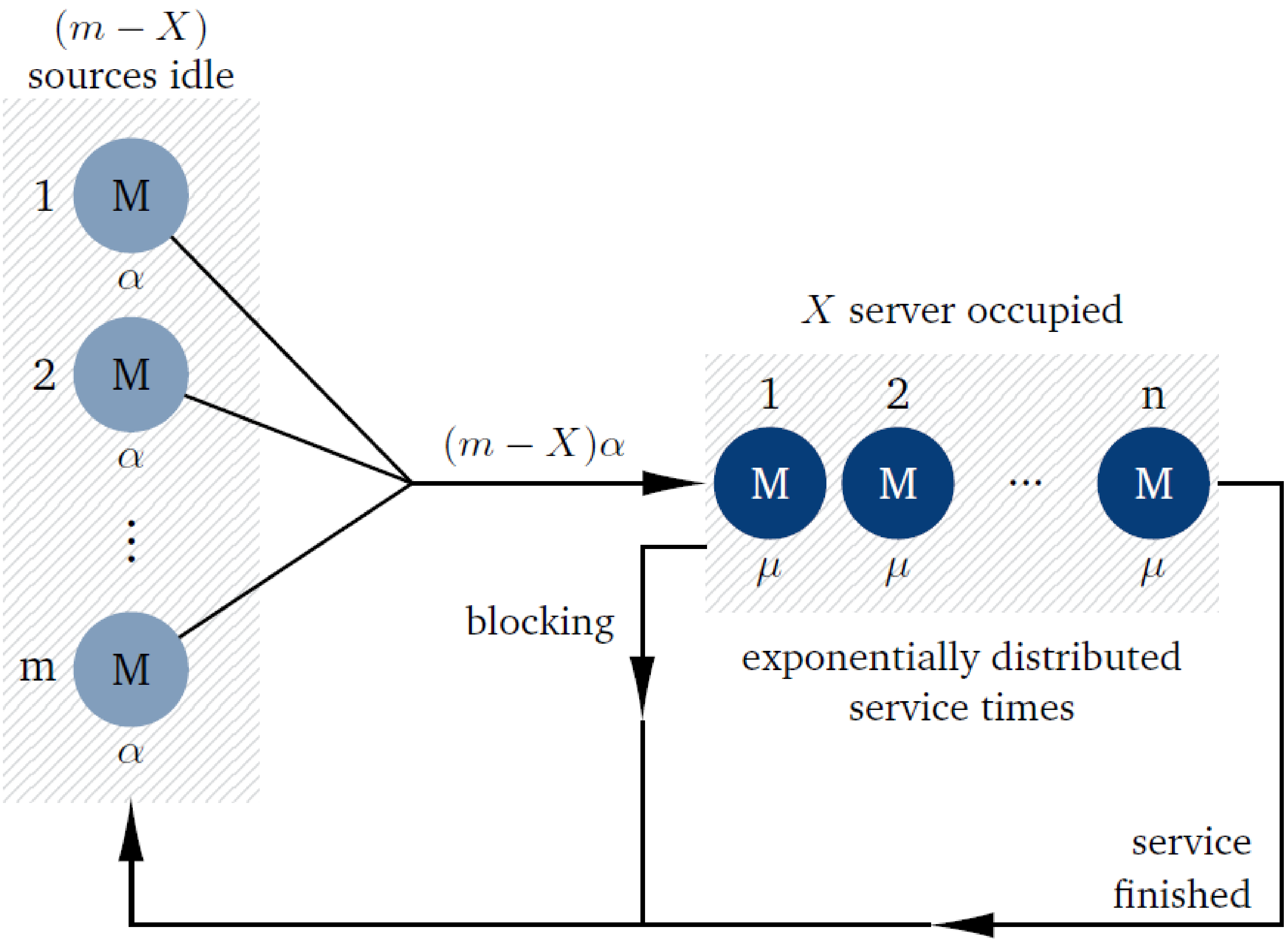
Chapter 4

Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

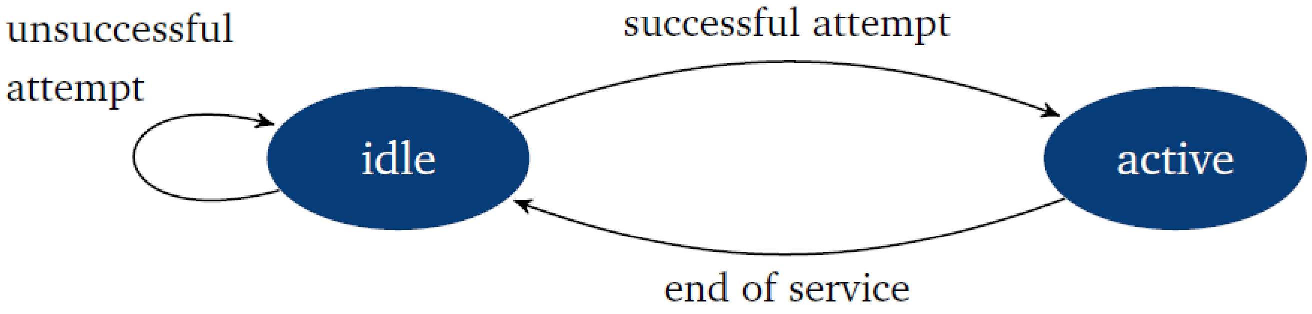
Poisson Process: Infinite Number of Sources

Model Structure and Parameters

Engset Model: Finite Number of Traffic Sources



Model of Customer Behavior



- Customer in idle mode for time I $\quad I(t) = P(I \le t) = 1 - e^{-\alpha t}, \qquad \mathrm{E}[\,I\,] = \frac{1}{\alpha}$
- Customer in active mode for time B (service time) $\quad B(t) = P(B \le t) = 1 - e^{-\mu t}, \qquad \mathrm{E}[\,B\,] = \frac{1}{\mu}$
- Offered traffic of an idle customer $\quad a^* = \frac{\alpha}{\mu}$
- "On-off" pattern of customer

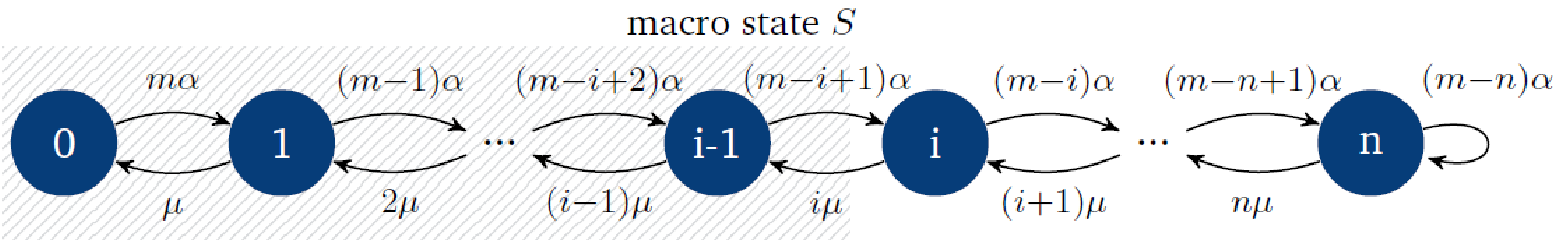
State Process and State Probabilities

State Transition Diagram: Derivation

Lecture

State Transition Diagram

- Birth-and-death process



- State probabilities follow from BD processes (Ch. 3)

$$(m-i+1)\alpha \cdot x(i-1) = i\mu \cdot x(i), \qquad i = 1,2,\dots,n$$

$$\sum_{i=0}^{n} x(i) = 1$$

$$\Rightarrow \quad \boxed{x(i) = \frac{\binom{m}{i} a^{*i}}{\sum_{k=0}^{n} \binom{m}{k} a^{*k}}}$$

for $i = 0,1,2,\dots,n$ and $m > n$

$$a^* = \frac{\alpha}{\mu}$$

State Probabilities: Derivation

Lecture

State Probabilities: Derivation (f.)

Lecture

Blocking Probability

Arrival theorem, random observer property

State Probabilities at Arbitrary Time and Arrival Time

- At arbitrary time

$$x(i) = \frac{\binom{m}{i} a^{*i}}{\sum_{k=0}^{n} \binom{m}{k} a^{*k}}$$

$x^*(i) = x(i)$

t^*

- At arrival times
  - PASTA propery not valid
  - state-dependent arrival rates
- Blocking probability
  - at arrival time

$x_A(i) \neq x(i)$

t_A

Arrival Theorem: Random Observer Property

- An arriving customer observes the system as if in steady state at an arbitrary instant for the system without that customer.

- **Finite number** of m customers
  - state probabilities $x_A^m(i)$ seen by arriving customer entering a state i are the same as
  - arbitrary-time probabilities $x^{m-1}(i)$ in a system with $m-1$ customers
  - $x_A^m(i) = x^{m-1}(i)$

- For **Poisson processes**
  - $m = \infty$ sources
  - Then: $x_A^\infty(i) = x^\infty(i)$
  - PASTA property: state as seen by an outside random observer is the same as the probability of the state seen by an arriving customer

Blocking Probability

- State probability at arbitrary time

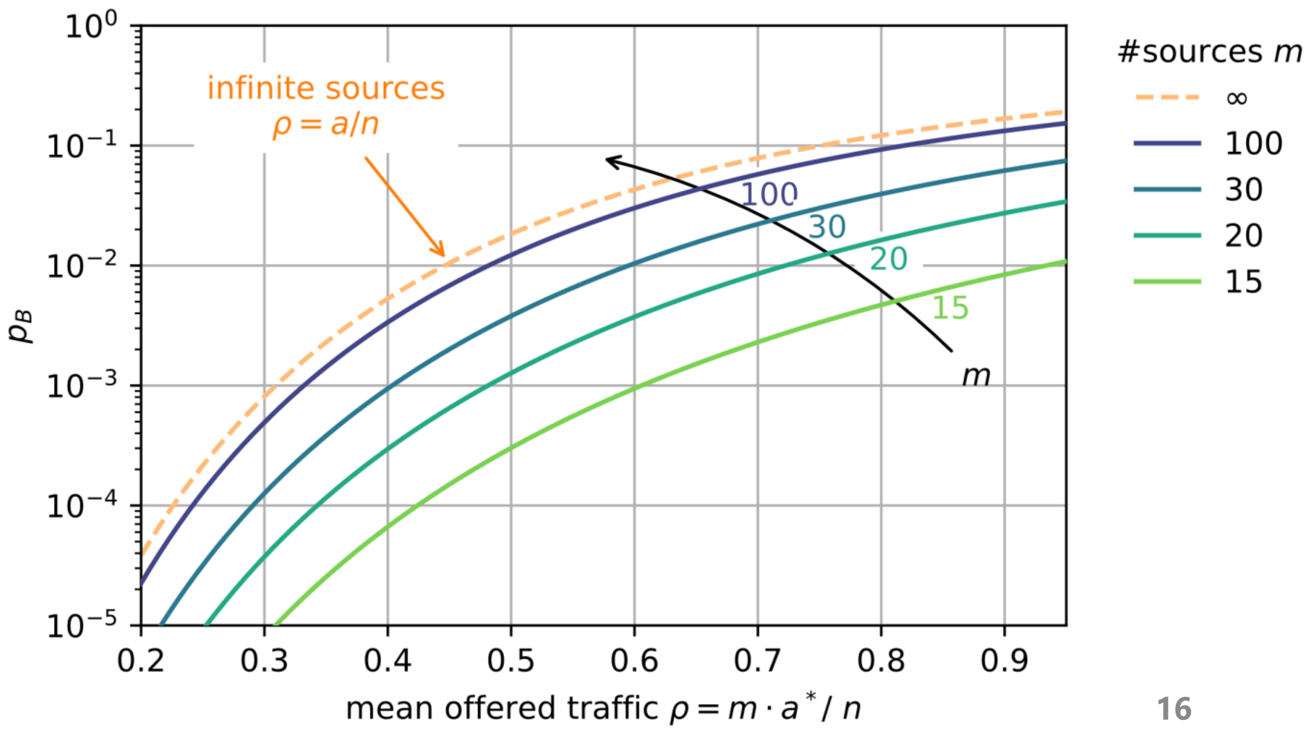
$$x(i) = \frac{\binom{m}{i} a^{*i}}{\sum_{k=0}^{n} \binom{m}{k} a^{*k}}$$

- State probability at arrival time

$$x_A(i) = \frac{\binom{m-1}{i} a^{*i}}{\sum_{k=0}^{n} \binom{m-1}{k} a^{*k}}$$

- Blocking probability: **Engset formula**

$$p_B = x_A(n) = \frac{\binom{m-1}{n} a^{*n}}{\sum_{k=0}^{n} \binom{m-1}{k} a^{*k}}$$

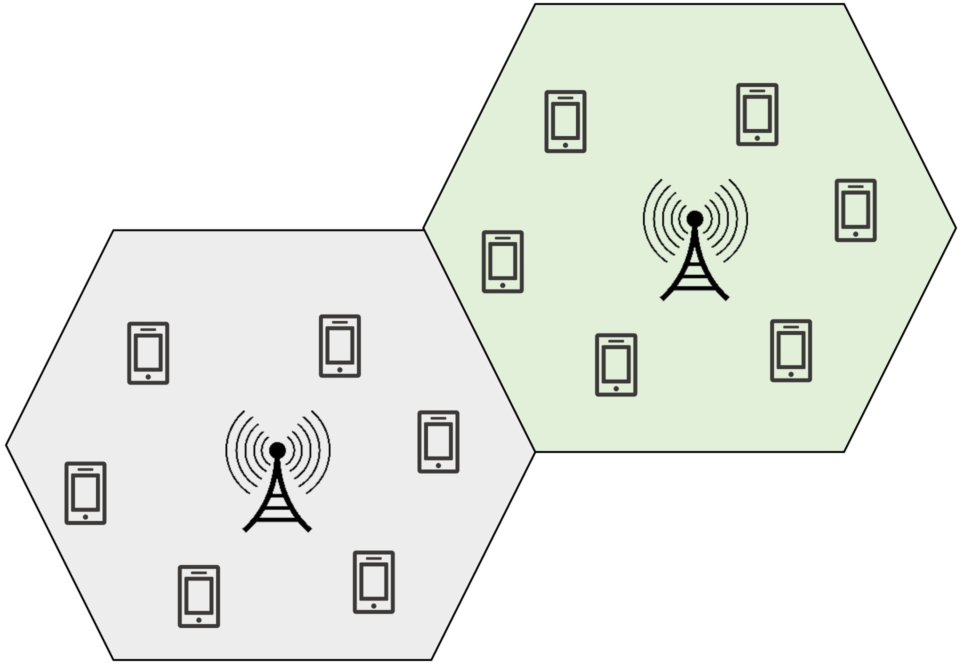


Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Example: Mobile Cell

with finite number of sources

Example: Mobile Cell



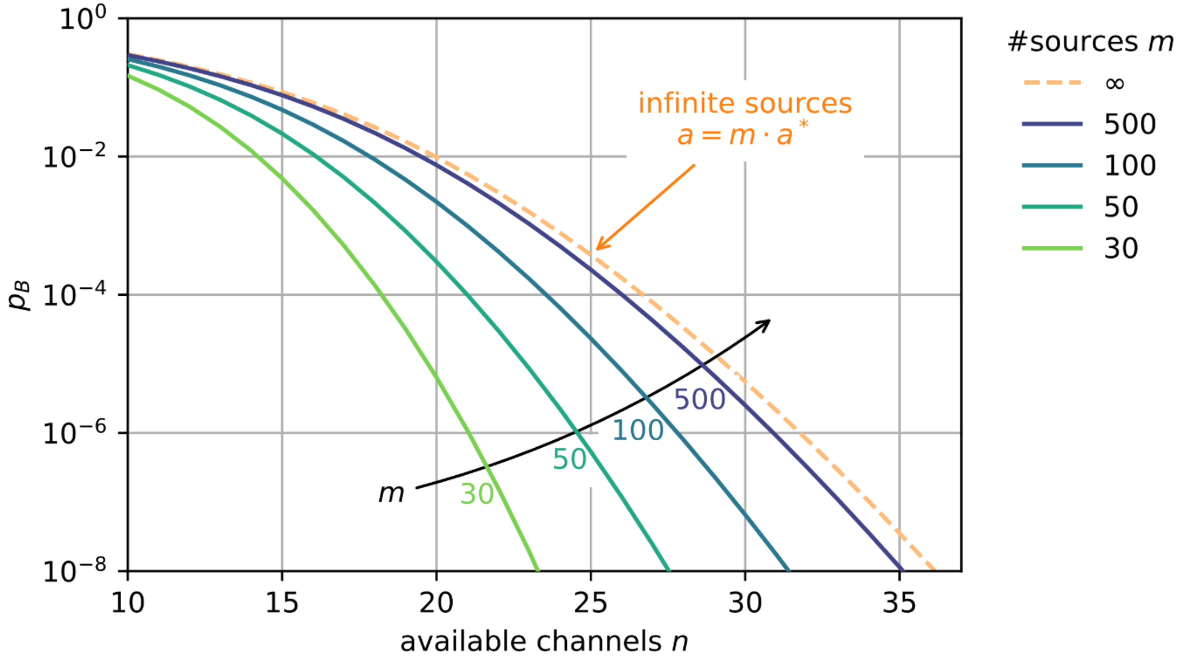
- In a single cell
  - m users
  - rate α of a user
  - offered load of idle user $a^* = \alpha/\mu$
  - mean call duration $E[B]$
- Number of channels: n
- If all channels are used, an incoming call is rejected.

- Engset formula can be applied
- For $m \to \infty$: Poisson process with rate $\lambda = \alpha \cdot \mathrm{m}$

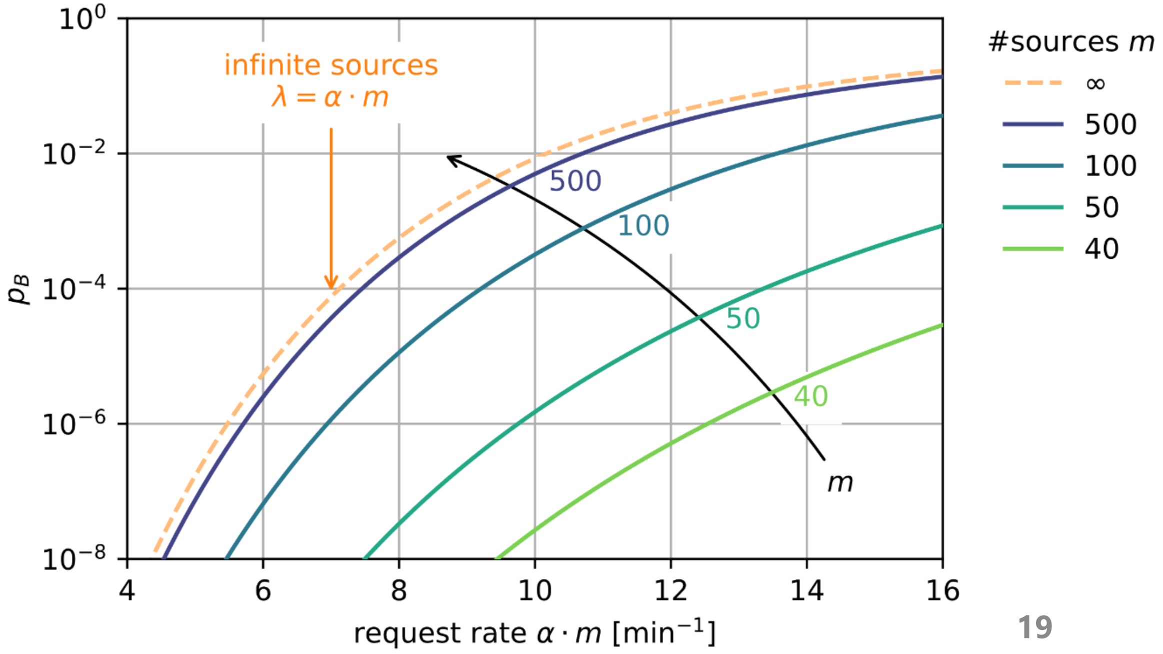
How many channels n are required so that the blocking probability is below a given threshold?

Blocking Probability

- Parameters
  - $E[B] = 2$ min
  - $a = 12$ Erl
- Simplifying assumption of infinite number of sources represents upper limit regarding blocking (Erlang-B for Poisson process)



- Parameters
  - $E[B] = 2$ min
  - $n = 30$
- Blocking probability increases with the granularity of the traffic



Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG