Auctions of Incentive Contracts for Policy Choice

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Abstract

A risk-neutral principal considers hiring one agent to improve a valuable, observable outcome. Who to hire? How to motivate? In this paper, the principal designs an incentive contract that pays according to the realized outcome and sells the contract to an agent through an auction. The paper finds the class of contract-auction pairs that guarantee the principal a non-negative expected payoff. Such pairs, which include linear contracts, are maximin-optimal: they maximize the principal's worst-possible expected payoff. The work is based on two contract-auction-specific assumptions: that the contract induces the contracted agent to weakly improve the outcome, and that the auction satisfies a revenue guarantee. Under these assumptions, the principal pays only for outcome improvements that the contract induces. Therefore, she achieves a non-negative payoff guarantee if her marginal benefit of the outcome exceeds the contract's marginal payment schedule. The principal can design an auction that attains the revenue guarantee if she knows the expected contract payment of the outcome that would occur absent agent activities or if agents know this quantity. The paper extends the analysis to situations where the principal incurs various costs and she or agents have limited commitment power.

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Consider a principal who wishes to hire one agent to improve a valuable outcome. A philanthropist might want to preserve a well-defined section of rainforest for an additional year or reduce the monthly average concentration of greenhouse gases in a given section of the atmosphere; an interest group might want to repeal a piece of legislation by the end of a political cycle. Which agent should the principal hire? How should she motivate the agent she hires? She contracts solely based on the outcome realization, which is observed sometime in the future. How, then, should she design a payment schedule? How should she allocate the resulting contract?

As in McAfee and McMillan (1986), we study a situation where a risk-neutral principal allocates an incentive contract among agents through a competitive bidding process. Our setting differs from this and related procurement settings in that our principal sells the incentive contract as-is rather than having agents bid over the contract terms. So, the principal's payoff increases with the auction's revenue and the outcome improvement induced by the incentive contract and decreases with the incentive contract payment associated with the outcome realization. Moreover, our principal is not Bayesian. To design an incentive contract and choose an auction, she does not rely on her beliefs about the actions agents can take to improve the outcome, the consequences of different actions on the outcome, agents' payoff functions, or the relationship between different agents' cost structures. She observes the realized outcome, knows her benefit from different outcome realizations, and evaluates contract-auction pairs according to their robustness, i.e., their worst-case expected payoff or *payoff guarantee.*

A contract-auction pair that offers the principal a non-negative payoff guarantee is attractive. If the principal issues this contract and sells it using this auction, her expected payoff cannot be negative. At worse, she is left as well-off as she would have been had she not issued or sold the contract. Of course, a contract that pays precisely zero for any outcome realization is worthless and would not induce any agent to improve the outcome, yielding a (non-negative) payoff guarantee of zero. However, are there non-trivial contract-auction pairs that achieve this?

The central result of this paper is the characterization of the class of contract-auction pairs with a non-negative payoff guarantee. To explain this characterization intuitively, consider an incentive contract and an auction format to allocate the contract among several agents. The principal's payoff from this contract-auction pair has three components. First, the auction revenue: agents bid for the contract and pay the principal according to the auction rules. Second, the benefit from the outcome realization: the contract induces the contracted agent to influence the outcome. As a result, the outcome may differ from the counterfactual outcome that would have occurred had the principal not issued or sold the contract, and the principal benefits accordingly. Third, the contract liability: after the outcome is realized, the principal must pay the contracted agent according to the contract's terms and conditions.

Now, the principal's payoff guarantee from the contract-auction pair is the expected payoff she would obtain in a worst-case scenario. Our assumptions are useful because they constrain the kinds of worst-case scenarios that can arise. Consider our first assumption: that the contract induces the contracted agent to improve or at least not worsen the outcome. If we allowed the contracted agent to worsen the outcome, then, in a worst-case scenario, the auction would raise no revenue, the contracted agent would worsen the outcome, and the principal would pay the agent according to the outcome realization. In this scenario, the principal is worse off — she should not have issued the contract and auctioned it off. Next, suppose that the contracted agent weakly improves the outcome but the principal allocates the contract to an agent free of charge. In a worst-case scenario, the contracted agent would do nothing to improve the outcome and the principal would pay the agent for the outcome realization. In this scenario, the principal is worse off because she pays the agent for an outcome that would have occurred anyway, even if she had not issued or sold the contract. We rule out such worst-case scenarios by assuming that the auction satisfies a revenue guarantee. Concretely, our second assumption is that the auction's expected revenue meets or exceeds the expected contract payment of the counterfactual outcome that would have occurred absent agent activities.

If the contract-auction pair satisfies both assumptions, then the principal effectively pays only for outcome improvements that the contract induces. Therefore, the contract-auction pair has a non-negative payoff guarantee if the principal's marginal benefit from the outcome exceeds the contract's marginal payment schedule. Linear contracts, which pay a fixed share of the principal's benefit, naturally satisfy this condition, as do linear contracts that establish a maximum liability for the principal. Such contracts ensure that the principal's benefit exceeds her liability for any possible outcome improvement.

Among contract-auction pairs with non-negative payoff guarantees, are there ones that offer a better guarantee than others? Our answer is negative. The payoff guarantee of every possible contract-auction pair is less than, or equal to, zero, meaning that contract-auction pairs with a non-negative payoff guarantee are also optimal, according to the maximin criterion. The intuition behind this result is straightforward. For every contract-auction pair, our assumptions cannot rule out the possibility that the contracted agent does nothing to improve the outcome and that the auction's expected revenue exactly equals the expected contract payment of the baseline outcome. In this case, the principal's payoff equals zero. Hence, her payoff guarantee is weakly negative.

A drawback of our working assumptions is the *ad-hoc* nature of the revenue guarantee of a contract-auction pair. Why and when should one expect that an auction of an incentive contract will raise revenue that exceeds the contract's payment of the outcome that occurs when the contracted agent does nothing to improve it? This paper shows that this condition is natural if the principal knows the expected contract payment of the baseline outcome, or if agents know this quantity and are risk-neutral. If the principal knows this quantity, an auction will achieve this revenue guarantee if the principal sets it as a reserve price. Instead, suppose the principal does not know this quantity, but it is common knowledge among risk-neutral agents. Here, we show that the first-price auction achieves the revenue guarantee in pure strategy Bayes Nash equilibria across an extensive range of information structures that, in particular, do not require agents (bidders) to have a common prior. Intuitively, any agent who owns the incentive contract can choose not to influence the outcome and obtain the contract payment of the baseline outcome. Because agents are risk-neutral, they are all willing to pay at least the expected contract payment of the baseline outcome to own the contract. Since this fact is common knowledge, agents drive the contract price beyond this quantity in equilibrium.

The theoretical framework that we employ to characterize the set of maximin optimal contract-auction pairs is flexible. The paper adapts it to incorporate various features that may arise in real-world applications. We consider situations where the agent's activities inflict a negative externality on the principal; it is costly for the principal to design the contract and organize the auction; it is costly for the principal to gather the funds necessary to make contract payments; the agent who wins the auction cannot commit to paying for the incentive contract; and the principal cannot commit to honoring the contract's terms and conditions. In particular, we study two ways the principal can eliminate the agent's obligation to pay for the contract: by allocating the contract free of charge or deducting as much of the contract's selling price from her contractual obligation to guarantee the agent a non-negative net transfer.

This paper belongs to the literature on the intersection between contract and auction theory, started by McAfee and McMillan (1986) and Laffont and Tirole (1987). It focuses on the principal's payoff guarantee of contract-auction pairs, thus joining a mechanism design literature that investigates the worst-case properties of contracts and auctions (see Carroll (2019) for a review). In particular, the finding that specific linear contract-auction pairs are maximin optimal resonates with Carroll (2015), who finds that linear contracts are maximin optimal in a contracting environment where the principal is assumed to know about a subset of payoff-relevant actions that the agent can take. We do not require the principal to know about any actions the contracted agent can take. Instead, our optimality results build off the assumption that the contract induces the agent to improve, or at least not worsen, the outcome that the principal cares about and that the auction has a specific revenue guarantee. Our finding that the first-price auction has a revenue guarantee that exceeds a commonly known lower bound on agents' willingness to pay for the contract relates with Bergemann, Brooks, and Morris (2017). They characterize the lowest possible expected revenue from the first-price auction across information structures where bidders' beliefs feature a common (and correctly specified) prior over the joint distribution of valuations. This paper establishes a revenue guarantee of the first-price auction in pure strategy Bayes Nash equilibria without the common prior assumption. Our approach requires that every agent's expected value of the object for sale exceeds a given threshold and that this fact be common knowledge among agents.

A related literature studies auctions of assets where bidders make security bids, i.e., payment commitments that depend on the asset realization (Hansen 1985, DeMarzo, I. Kremer, and Skrzypacz 2005, Bhattacharya, Ordin, and Roberts 2022). Because the principal is liable for the incentive contract payments, having her design the incentive contract and accept security bids is isomorphic to having agents bid over the terms of the incentive contract, as in the traditional procurement settings of McAfee and McMillan (1986) and Laffont and Tirole (1987). In our approach, the principal completely specifies an incentive contract and auctions it off as-is.

Another related literature studies the design and implementation of pull mechanisms such as incentive contracts that pay a fixed price for a good or service, called Advance Market Commitments or AMCs (M. Kremer and Glennerster 2004, M. Kremer, Levin, and Snyder 2022, M. Kremer, Levin, and Snyder 2020). For instance, Levine, M. Kremer, and Albright (2005) designed an AMC that paid pharmaceutical companies to produce vaccines against pneumococcus, a prevalent deadly disease in developing countries. More recently, Frontier Climate launched an Advance Market Commitment that pays participants for every ton of carbon dioxide they capture. In contrast with the design of AMCs, our principal allocates an incentive contract to a single agent. The main reason for this modeling choice is our interest in situations where the principal observes overall output but not agent-specific output. Moreover, AMC participants are not selected through a competitive process. Since participants might have produced output even in the absence of an AMC, the principal's payoff relies on the AMC causing output improvements that outweigh AMC payments for output that occurs even in the absence of an AMC. The point of the auction stage in our design is to reimburse the principal for such payments.

The plan of the paper is the following. Section 1 presents the model and establishes the paper's main results. Section 2 illustrates the framework and the results with several examples. The examples raise issues that merit extensions of the basic model. Section 3 incorporates these extensions. Finally, section 4 concludes.

1 Model

1.1 Notation

A probability space $(\Omega, \Sigma, \mathbb{P})$ describes the situation we will consider, where Ω denotes the sample space, Σ is the Borel σ-algebra on Ω, and $\mathbb{P}$ is a probability measure defined on Σ. We refer to $\mathbb{P}$ as the true probability measure. A generic k-dimensional random vector X is a Borel-measurable function from Ω to the k-dimensional real numbers.

1.2 Setup

A risk-neutral principal considers funding a policy that is implemented during a given time period and targets a scalar, non-negative outcome that lies in a compact set $\mathcal{Y} \subset [0, \infty) \equiv \mathbb{R}_+$, where $\underline{y} = \min \mathcal{Y} < \max \mathcal{Y} = \overline{y}$. She values a realized outcome $y \in \mathcal{Y}$ at $b(y) \geq 0$ dollars, and we assume $b : \mathcal{Y} \mapsto \mathbb{R}_+$ is continuous and strictly increasing. There are $n \in \{1, 2, \dots\}$ agents available to implement policies that target the outcome, and the principal's problem is to hire and motivate one agent. To do so, she issues an incentive contract and sells it to an agent at the beginning of the time period. The incentive contracts we consider belong to the set of continuous and non-decreasing mappings from outcome realizations to monetary transfers, $\mathcal{W}$. So, incentive contract $w : \mathcal{Y} \mapsto \mathbb{R}_+$ has the principal pay the contract owner $w(y) \geq 0$ dollars upon a realization of outcome $y \in \mathcal{Y}$, at the end of the period. A central assumption in this paper is that the principal can measure the realized outcome by the end of the time period.

The principal uses an auction to allocate the incentive contract. A single-object auction consists of a set of bids for each agent, $(\mathcal{B}_1, \dots, \mathcal{B}_n)$, where $0 \in \mathcal{B}_i \subseteq \mathbb{R}_+$ for all $i \in \{1, \dots, n\}$; an allocation rule $q : \times_{i=1}^n \mathcal{B}_i \mapsto [0, 1]^n$, where $\sum_{i=1}^n q_i(b) \leq 1$ for all bid profiles $b \in \times_{i=1}^n \mathcal{B}_i$; and transfers from the agents to the principal $t : \times_{i=1}^n \mathcal{B}_i \mapsto \mathbb{R}_+^n$. We let $\mathcal{A}$ denote the set of single-object auctions, and $\mathcal{M} = \mathcal{W} \times \mathcal{A}$ denote the set of contract-auction pairs.

We treat contract-auction pairs as black boxes and focus exclusively on variables relevant to the principal's payoff. For each contract-auction pair $m = (w, a) \in \mathcal{M}$, we let D_m be the binary random variable that indicates if auction a allocates contract w to any agent. For example, $\{\omega \in \Omega : D_m(\omega) = 0\}$ is the event that the principal does not manage to sell contract w with auction a. We treat agents who do not participate in the auction as auction participants who bid zero. So, the event $\{\omega \in \Omega : D_m(\omega) = 0\}$ occurs if agents do not participate in the auction and the auction allocates the contract only if there is a strictly positive bid. We denote the outcome that is induced by contract-auction pair $m = (w, a) \in \mathcal{M}$ with the random variable Y_m, which takes values in $\mathcal{Y}$. Had the principal's auction not sold contract w, the outcome would be given by Y^0. If, instead, the principal had sold w with auction a, the outcome would be given by Y^1_m. Y^0 and Y^1_m are counterfactual random variables that relate with the outcome Y_m via the standard potential outcomes equation:

$$Y_m = D_m Y^1_m + (1 - D_m) Y^0. \tag{1}$$

Y^1_m differs from Y^0 to the extent that contract w effectively motivates the agent who wins auction a to influence the outcome. We assume that the principal either hires an agent or does not attempt to influence the outcome through other means and that the behavior of other agents is unaffected by the principal's decision to issue and sell an incentive contract. Hence, the outcome is the same if the principal does not sell the contract, or if the contracted agent does not influence it. For each $m = (w, a) \in \mathcal{M}$, we let R_m be the non-negative random variable that measures the principal's revenue from auction a of contract w, and W_m be the principal's payment obligation associated with the outcome realization. Of course, the principal raises revenue and issues contract payments only if she sells the contract so that

$$W_m = D_m \cdot w(Y_m)$$
$$D_m = 0 \text{ implies that } R_m = 0.$$

The timing of the model is the following. First, the principal selects $m = (w, a) \in \mathcal{M}$, issues incentive contract w, and allocates it among agents using auction a. By the end of the auction, R_m is realized. Then, Y_m is realized and observed by the principal. Afterward, all payments occur: the principal pays W_m to the contracted agent and obtains R_m. In this sequence of events, agents delay their auction transfers to the principal until after the outcome realization rather than immediately after the auction. This choice of timing streamlines the presentation but is inessential.

Because the principal is risk-neutral, she obtains an ex-post payoff of $b(Y^0)$ if she chooses not to issue and auction off an incentive contract. In contrast, if she selects contract-auction pair $m \in \mathcal{M}$, she obtains $b(Y_m) - W_m + R_m$ dollars' worth of value.[1] Her ex-post payoff from a contract-auction pair m is then Π_m, where

$$\Pi_m = b(Y_m) - [W_m - R_m] - b(Y^0).$$

Unlike Laffont and Tirole (1986), the principal does not consider any costs of raising $W_m - R_m$,

[1] An implicit assumption is that the principal's other investment decisions are unchanged by her decision to issue and sell a contract. Formally, let K_0 denote the random variable that measures the stock of wealth that the principal would have achieved immediately after the outcome realization had the principal not issued or sold an incentive contract. Similarly, let K_m be the principal's counterfactual wealth had she chosen contract-auction pair m. We assume that $K_m = K_0$ for all $m \in \mathcal{M}$. When the principal chooses contract-auction pair m, she acquires a liability equal to $W_m - R_m$. The rationale for this assumption is that her other investment decisions do not influence this liability and should therefore dismiss it.

the funds needed to pay for m. This omission is relevant if, for example, the principal cares about social welfare and can only raise funds in ways that bring about real resource costs. We introduce costs of raising funds in section 3.2.

The actual expected performance of $m \in \mathcal{M}$ equals $\mathbb{E}_{\mathbb{P}}[\Pi_m]$, where $\mathbb{E}_{\mathbb{P}}$ is the expectation operator with respect to the true probability measure, $\mathbb{P}$. Since the principal is risk-neutral, a contract-auction pair is a sound investment whenever $\mathbb{E}_{\mathbb{P}}[\Pi_m] \geq 0$. Unfortunately, the principal does not know $\mathbb{P}$. Nonetheless, she is willing to make assumptions about the payoff-relevant variables for a subset of contract-auction pairs. These assumptions restrict the set of joint distributions of the payoff-relevant variables, thus ruling out distributions that the principal does not deem possible. Let $\mathcal{P}$ denote the set of probability measures defined on Σ, the σ-algebra on the sample space Ω. The principal considers the following assumptions, which are specific to $P \in \mathcal{P}$ and $m \in \mathcal{M}$:

Assumption I. $\mathbb{E}_P[R_m - D_m \cdot w(Y^0)] \geq 0$.

Assumption II. $P(Y_m^1 - Y^0 \geq 0) = 1$.

A probability measure $P \in \mathcal{P}$ satisfies Assumption I for $m \in \mathcal{M}$ if the principal's expected revenue from contract-auction pair m compensates her for the baseline amount she would have to pay the contracted agent in case the agent did nothing to influence the outcome. $P \in \mathcal{P}$ satisfies Assumption II for $m \in \mathcal{M}$ if the contract induces the agent to improve the outcome with probability one. Notice that $P(Y_m^1 - Y^0 \geq 0) = 1$ implies $P(Y_m - Y^0 \geq 0) = 1$ and $P(b(Y_m) - b(Y_0) \geq 0) = 1$, since b is strictly increasing. Colloquially, $P \in \mathcal{P}$ satisfies Assumptions I and II for $m \in \mathcal{M}$ if the principal does not give away money to agents for free, and contract-auction pair m does not backfire on the principal.

The principal imposes Assumptions I and II for a non-empty subset of contract-auction pairs $\mathcal{M}^* \subseteq \mathcal{M}$ of her choice to rule out probability measures that she does not deem likely. The probability measures that do satisfy the assumptions constitute a subset $\mathcal{P}^* \subseteq \mathcal{P}$, defined as:

$$\mathcal{P}^* \equiv \{P \in \mathcal{P} : P \text{ satisfies Assumptions I and II for all } m \in \mathcal{M}^*\}.$$

$\mathcal{P}^*$ is *well-specified* if the true probability measure, $\mathbb{P}$, belongs to $\mathcal{P}^*$. Otherwise, it is *mis-specified*.

We assume that the principal behaves according to the maximin criterion. She evaluates contract-auction pairs based on their worst-case expected payoff across probability measures $P \in \mathcal{P}^*$. For contract-auction pair $m \in \mathcal{M}$, this payoff guarantee is $\underline{\Pi}_m$, where

$$\underline{\Pi}_m = \begin{cases} \inf_{P \in \mathcal{P}^*} \mathbb{E}_P[\Pi_m] & \text{if } \{\mathbb{E}_P[\Pi_m] : \ P \in \mathcal{P}^*\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

So, the principal's objective is to choose a contract-auction pair that maximizes her payoff guarantee. We now show that such a maximum exists and characterize the optimal contract-auction pairs.

1.3 Analysis

We first inspect the principal's payoff guarantee, $\underline{\Pi}_m$, for all $m \in \mathcal{M}$. Intuitively, payoff guarantees arise from worst-case scenarios constrained by Assumptions I and II. Take, for example, a contract-auction pair $m \in \mathcal{M}^*$, so that any probability measure $P \in \mathcal{P}^*$ satisfies Assumptions I and II for m. In the worst-case scenario that the principal deems possible, the auction raises the lowest revenue consistent with Assumption I, and the potential outcomes Y^0 and Y^1_m are as unfavorable to the principal as Assumption II allows them to be. Here, the auction exactly reimburses the principal for the baseline contract payment, $D_m w(Y^0)$, in expectation. Net of the auction's revenue, the principal only pays for changes in the outcome induced by the incentive contract: the cost of the incentive contract amounts to $w(Y_m) - w(Y^0)$, in expectation. Under Assumption II, the contracted agent improves the outcome. Hence, the lowest expected payoff for the principal is associated with a probability measure under which the contracted agent produces the worst possible outcome improvement, which is given by:

$$\min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}:y_0\leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)],$$

where the minimum exists because $\mathcal{Y} \subset \mathbb{R}_+$ is compact and b and w are continuous functions.

Instead, if the contract-auction pair does not belong to $\mathcal{M}^*$, Assumptions I and II no longer bind. So, in a worst-case scenario, the auction raises no revenue, and the potential outcomes are as adverse as they can be, provided they lie in set $\mathcal{Y}$. Here, the principal's expected cost of the incentive contract is the expectation of $w(Y_m)$, and the smallest possible expected payoff equals

$$\min_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - w(y_1).$$

Lemmas 1 and 2 summarize this discussion. Their formal proofs lie in Appendix A.

Lemma 1. *If $m \in \mathcal{M}^*$, then* $\underline{\Pi}_m = \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}:y_0\leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)].$

Lemma 2. *If $m \in \mathcal{M} \setminus \mathcal{M}^*$, then* $\underline{\Pi}_m = \min_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - w(y_1).$

Taken together, Lemmas 1 and 2 imply that $\Pi_m \leq 0$ if $m \in \mathcal{M}^*$, while $\Pi_m < 0$ if $m \in \mathcal{M} \setminus \mathcal{M}^*$.[2] So, any contract-auction pair m such that $\underline{\Pi}_m = 0$ is optimal. Moreover,

[2] If $y_0 = y_1$, then $b(y_1) - b(y_0) - [w(y_1) - w(y_0)] = b(y_1) - b(y_0) - w(y_1) = 0$. This proves that $\underline{\Pi}_m \leq 0$. To

$\underline{\Pi}_m = 0$ implies that the contract-auction pair lies in $\mathcal{M}^*$. An inspection of Lemma 1 reveals that the principal achieves a zero payoff guarantee when the slope of contract w lies below her benefit b for every pair of possible outcome realizations. In this case, she pays less than her benefit under any outcome improvement. When the principal imposes Assumptions I and II for a set of contract-auction pairs that includes such contracts, this reasoning characterizes the set of maximin optimal contract-auction pairs. Theorem 1 records this result.

Theorem 1. $\underline{\Pi}_m = 0$ *if and only if* $m = (w, a) \in \mathcal{M}^*$ *and, for all* $y_0, y_1 \in \mathcal{Y}$ *such that* $y_0 \leq y_1$*,*

$$b(y_1) - b(y_0) \geq w(y_1) - w(y_0).$$

Proof. Necessity follows immediately because Lemma 1 implies that $\underline{\Pi}_m \leq 0$ and the premise implies that $\underline{\Pi}_m \geq 0$. For sufficiency, notice that, by Lemma 2, $m \in \mathcal{M} \setminus \mathcal{M}^*$ implies $\underline{\Pi}_m < 0$. Hence, $m \in \mathcal{M}^*$. But then,

$$\min_{\{(y_0, y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)] = 0$$

implies that $b(y_1) - b(y_0) - [w(y_1) - w(y_0)] \geq 0$ for all $y_0, y_1 \in \mathcal{Y}$ such that $y_0 \leq y_1$. ■

Theorem 1 serves as a simple and powerful guide for the principal. Maximin optimal contract-auction pairs feature contracts that are less steep than the principal's benefit, i.e., contracts whose marginal payment schedule lies below the principal's marginal benefit of the outcome. Such contracts ensure that the principal pays less than she benefits for any possible outcome improvement. An important subclass of contracts that satisfy this condition is the class of linear contracts, which pay a fixed fraction of the principal's benefit. Thus, Theorem 1 shows that linear contract-auction pairs are optimal.

Corollary 1. $\underline{\Pi}_m = 0$ *for any* $m = (w, a) \in \mathcal{M}^*$ *such that* $w = \alpha \cdot b$ *with* $\alpha \in [0, 1]$.

In fact, linear contract-auction pairs are optimal and offer the principal a non-negative payoff guarantee even under a relaxation of Assumption II that only requires that the contracted agent improve the outcome in expectation, rather than almost surely. Appendix B states the assumption and proves this result.

Contract-auction pairs with zero payoff guarantees are sound investments for the principal. At worse, the principal ends up as well-off as she would have been had she not issued and sold the incentive contract. However, if $\mathcal{P}^*$ is misspecified, then one cannot assert that the

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show that $\underline{\Pi}_m < 0$ for all $m \in \mathcal{M} \setminus \mathcal{M}^*$, consider the following two cases. First, let $w(y) = 0$ for all $y \in \mathcal{Y}$. Since b is strictly increasing, $b(y_1) - b(y_0) - w(y_1) < 0$ for any $(y_0, y_1) \in \mathcal{Y} \times \mathcal{Y}$ such that $y_1 < y_0$. Second, let $w(y) > 0$ for some $y \in \mathcal{Y}$. In this case, $y_0 = y_1 = y$ implies that $b(y_1) - b(y_0) - w(y_1) = -w(y) < 0$. We conclude that $\underline{\Pi}_m < 0$.

principal's expected payoff, $\mathbb{E}_{\mathbb{P}}[\Pi_m]$, exceeds the payoff guarantee, $\underline{\Pi}_m$. In particular, we take issue with Assumption I because it places a seemingly ad-hoc requirement on the auction's revenue performance. Recall that $\mathbb{P}$ satisfies Assumption I for $(w, a) \in \mathcal{M}$ if auction a yields an expected revenue that (weakly) exceeds $\mathbb{E}_{\mathbb{P}}[D_m \cdot w(Y^0)]$. When can the principal expect it to hold? We will show that Assumption I is warranted when either the principal or agents know about the contract's expected baseline payment, $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$.

1.3.1 Reserve Prices

Consider a contract w such that the principal knows $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$. If she sold this contract using a reserve price of $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$, she would secure a selling price that exceeds this price. Formally, if auction $a \in \mathcal{A}$ has a reserve price of $r \geq 0$, then, for all $m = (w, a) \in \mathcal{W} \times \{a\}$, $\mathbb{P}(R_m \geq D_m \cdot r) = 1$. It follows that $\mathbb{P}$ satisfies Assumption I for all contract-auction pairs (w, a) such that the auction has a reserve price of at least $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$. Proposition 1 proves this.

Proposition 1. *$\mathbb{P}$ satisfies Assumption I for all $m = (w, a) \in \mathcal{M}$ such that auction a has a reserve price of $r \geq \mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$.*

Proof. If $\mathbb{P}(D_m = 1) = 0$, the result follows immediately. Otherwise,

$$\begin{aligned} \mathbb{E}_{\mathbb{P}}[R_m - D_m \cdot w(Y^0)] &\geq \mathbb{E}_{\mathbb{P}}[D_m \cdot (r - w(Y^0))] \\ &= \mathbb{P}(D_m = 1)(r - \mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]) \\ &\geq 0. \end{aligned}$$

■

1.3.2 First-price auction with Informed Agents

What if the principal does not know $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$, but agents do? In this section, we assume that agents are risk-neutral, and we consider a principal who knows that $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ is common knowledge among agents. We show in Proposition 2 that the first-price auction achieves a revenue that exceeds $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ in pure strategy Bayes Nash equilibria. To state and prove this result, we first describe the auction as a game of incomplete information.

Suppose the principal sells contract w using the first-price (sealed-bid) auction. Agents are the auction's bidders. They are risk-neutral, maximize expected utility, and have quasilinear preferences over the incentive contract and money. Their values of the incentive contract lie in set $\mathcal{V} = [0, \overline{v}]$. At the start of the auction, agents place bids in set $\mathcal{B} = [0, \overline{b}]$. The agent with the highest bid obtains the incentive contract. If multiple agents place the highest bid, the contract is awarded (uniformly) at random among them.[3] The agent who obtains the contract pays the amount she bid to the principal; others pay zero. This selling price is the auction's revenue.

Agents have private information about the value of the contract. This information is diverse and may include, for example, the beliefs of agents about the relationship between various input combinations and the effect on the outcome or beliefs about input prices. We refer to agents' private information as signals. An information structure is $\mathcal{I} = (\mathcal{S}_1, \ldots, \mathcal{S}_n; \mu_1, \ldots, \mu_n)$, where $\mathcal{S}_i$ is the set of signals that agent i can receive, $\mathcal{S} \equiv \mathcal{S}_1, \times \cdots \times \mathcal{S}_n$ is the set of signal profiles, and $\mu_i \in \Delta(\mathcal{V}^n \times \mathcal{S})$ is agent i's belief about the distribution of value and signal profiles.[4] Every information structure we consider is finite, meaning that $\mathcal{S}_i$ is a finite set for all $i \in \{1, \ldots, n\}$. Notice that we do not require consistency across agents' beliefs in the form of a common prior.

An information structure has *full pooled support* if for every $s \in \mathcal{S}$, there exists $i \in \{1, \ldots, n\}$ such that $\mu_i(\mathcal{V}^n, s) > 0$.

An information structure has a *common low value of* $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ if $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ is common knowledge among agents and constitutes a lower bound of every agent's expected value of the contract: $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1] \in s_i$ for all $s_i \in \mathcal{S}_i$ and $i \in \{1, \ldots, n\}$, and for every $s \in \mathcal{S}$ such that $\mu_i(\mathcal{V}^n, s) > 0$,

$$\int_{\mathcal{V}^n} v_i \frac{\mu_i(dv,\, s)}{\mu_i(\mathcal{V}^n,\, s)} \geq \mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1].$$

Given information structure $\mathcal{I}$, a strategy for agent i is $\sigma_i : \mathcal{S}_i \mapsto \Delta(\mathcal{B})$. If σ_i is a pure strategy, we abuse notation and let $\sigma_i(s_i) \in \mathcal{B}$ denote the bid placed by agent i with type $s_i \in \mathcal{S}_i$. Agent i's ex-ante expected payoff from a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ is:

$$u_i(\sigma; \mathcal{I}) = \int_{v \in \mathcal{V}^n, s \in \mathcal{S}} \int_{b \in \mathcal{B}^n} q_i(b)(v_i - b_i)\sigma_1(db_1 \mid s_1) \cdots \sigma_n(db_n \mid s_n)\mu_i(dv, ds).$$

where $q : \mathcal{B}^n \mapsto \Delta(\{1, \ldots, n\})$ is the first-price auction's allocation rule. That is, $q(b) = (q_1(b), \ldots, q_n(b))$, where

$$q_i(b) = \frac{1\{b_i = \max_j b_j\}}{|\arg\max_j b_j|}.$$

Strategy profile σ is a Bayes Nash Equilibrium under information structure $\mathcal{I}$ if, for every agent i and strategy $\tilde{\sigma}_i$, $u_i(\sigma; \mathcal{I}) \geq u_i(\tilde{\sigma}_i, \sigma_{-i}; \mathcal{I})$.

Armed with this setup, we can state and prove the main result of this section: that pure strategy Bayes Nash equilibria of the first-price auction of an incentive contract w

[3]The specific rule used to allocate the object among many high bidders is inconsequential to our results.

[4]$\Delta(\mathcal{X})$ is the set of probability measures defined over the Borel σ-algebra on set $\mathcal{X}$.

yield revenues that exceed the contract's expected baseline payment, $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$, across information structures that have full pooled support and have a common low value of $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$.

Proposition 2. *Consider an information structure $\mathcal{I}$ that has full pooled support and has a common low value of $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$. If σ is a pure strategy Bayes Nash equilibrium of the first-price auction under $\mathcal{I}$, then $\max_{i\in\{1,\ldots,n\}} \sigma_i(s_i) \geq \mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ for all $s \in \mathcal{S}$.*

Proof. Fix a signal profile $s = (s_1, \ldots, s_n) \in \mathcal{S}$ and an agent $i \in \{1, \ldots, n\}$ such that $\mu_i(\mathcal{V}^n, s) > 0$. This agent exists because $\mathcal{I}$ has full pooled support. Let σ be a strategy profile such that

$$\max_{i\in\{1,\ldots,n\}} \sigma_i(s_i) < \mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1].$$

We will show that σ is not a Bayes Nash equilibrium. Define

$$\mu_i(\tilde{s}_{-i} \mid s_i) \equiv \frac{\mu_i(\mathcal{V}^n, s_i, \tilde{s}_{-i})}{\sum_{s'_{-i}\in\mathcal{S}_{-i}} \mu_i(\mathcal{V}^n, s_i, ds'_{-i})} \quad \text{and} \quad \mu_i(\tilde{\mathcal{V}} \mid s_i, \tilde{s}_{-i}) \equiv \frac{\mu_i(\tilde{\mathcal{V}}, s_i, \tilde{s}_{-i})}{\mu_i(\mathcal{V}^n, s_i, \tilde{s}_{-i})}$$

for all $\tilde{s}_{-i} \in \mathcal{S}_{-i}$ and every Borel set $\tilde{\mathcal{V}} \subseteq \mathcal{V}^n$, and let $\mathcal{S}^m_{-i}(b_i)$ be the set of i's competitor signal profiles such that bid $b_i \in \mathcal{B}$ is the highest bid along with $m \in \{0, \ldots, n-1\}$ others:

$$\mathcal{S}^m_{-i}(b_i) = \left\{ \tilde{s}_{-i} \in \mathcal{S}_{-i} : \; b_i \geq \max_{j\neq i} \sigma_j(\tilde{s}_j) \text{ and } \left|\left\{ j \in \{1, \ldots, n\} \setminus \{i\} : \sigma_j(\tilde{s}_j) = b_i \right\}\right| = m \right\}.$$

This decomposition of competitors' signals allows us to write agent i's expected payoff from bid $b_i \in \mathcal{B}$ conditional on signal s_i as:

$$\sum_{m=0}^{n-1} \frac{1}{m+1} \sum_{\tilde{s}_{-i}\in\mathcal{S}^m_{-i}(b_i)} \mu_i(\tilde{s}_{-i} \mid s_i) \left[\int_{v\in\mathcal{V}^n} (v_i - b_i) \mu_i(dv \mid s_i, \tilde{s}_{-i}) \right].$$

We will devise profitable deviations from $\sigma_i(s_i)$ on the basis of agent i's beliefs about others' signals.

Case 1: $\mu_i(\tilde{s}_{-i} \mid s_i) = 0$ for all $\tilde{s}_{-i} \in \bigcup_{m=0}^{n-1} \mathcal{S}^m_{-i}(\sigma_i(s_i))$. In this case, agent i believes her bid $\sigma_i(s_i)$ never wins the auction. She could do better than that. Suppose that, under signal s_i, she deviates to bid $b_i \in (\max_j \sigma_j(s_j), \mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1])$. If others' signals equal s_{-i}, an event that occurs with probability $\mu_i(s_{-i} \mid s_i) > 0$ according to i, agent i wins the auction, pays less than $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ for the contract and values the contract above $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ in expectation. It follows that this deviation is profitable.

Case 2: $\mu_i(\tilde{s}_{-i} \,|\, s_i) > 0$ for some $\tilde{s}_{-i} \in \bigcup_{m=1}^{n-1} \mathcal{S}_{-i}^m(\sigma_i(s_i))$. In this case, agent i thinks her bid may be a winning bid that ties with her competitors'. If she deviates from $\sigma_i(s_i)$ to $\sigma_i(s_i) + \epsilon$, where $\epsilon > 0$ is small enough that

$$\mathcal{S}_{-i}^0(\sigma_i(s_i) + \epsilon) = \bigcup_{m=0}^{n-1} \mathcal{S}_{-i}^m(\sigma_i(s_i)),$$

the change in agent i's expected payoff equals

$$\begin{aligned}
&\sum_{\tilde{s}_{-i} \in \mathcal{S}_{-i}^0(\sigma_i(s_i)+\epsilon)} \mu_i(\tilde{s}_{-i} \,|\, s_i) \left[\int_{v \in \mathcal{V}^n} (v_i - \sigma_i(s_i) - \epsilon) \mu_i(dv \,|\, s_i, \tilde{s}_{-i}) \right] \\
&\qquad - \sum_{m=0}^{n-1} \frac{1}{m+1} \sum_{\tilde{s}_{-i} \in \mathcal{S}_{-i}^m(\sigma_i(s_i))} \mu_i(\tilde{s}_{-i} \,|\, s_i) \left[\int_{v \in \mathcal{V}^n} (v_i - \sigma_i(s_i)) \mu_i(dv \,|\, s_i, \tilde{s}_{-i}) \right] \\
&= - \sum_{\tilde{s}_{-i} \in \mathcal{S}_{-i}^0(\sigma_i(s_i)+\epsilon)} \epsilon \cdot \mu_i(\tilde{s}_{-i} \,|\, s_i) \\
&\qquad + \sum_{m=1}^{n-1} \frac{m}{m+1} \sum_{\tilde{s}_{-i} \in \mathcal{S}_{-i}^m(\sigma_i(s_i))} \mu_i(\tilde{s}_{-i} \,|\, s_i) \left[\int_{v \in \mathcal{V}^n} (v_i - \sigma_i(s_i)) \mu_i(dv \,|\, s_i, \tilde{s}_{-i}) \right].
\end{aligned}$$

This deviation is profitable for agent i if

$$\epsilon < \frac{\sum_{m=1}^{n-1} \frac{m}{m+1} \sum_{\tilde{s}_{-i} \in \mathcal{S}_{-i}^m(\sigma_i(s_i))} \mu_i(\tilde{s}_{-i} \,|\, s_i) \left[\int_{v \in \mathcal{V}^n} (v_i - \sigma_i(s_i)) \mu_i(dv \,|\, s_i, \tilde{s}_{-i}) \right]}{\sum_{\tilde{s}_{-i} \in \mathcal{S}_{-i}^0(\sigma_i(s_i)+\epsilon)} \mu_i(\tilde{s}_{-i} \,|\, s_i)}.$$

Notice that the expression on the right-hand side is well-defined and strictly positive. Indeed,

$$\int_{v \in \mathcal{V}^n} (v_i - \sigma_i(s_i)) \mu_i(dv \,|\, s_i, \tilde{s}_{-i}) \geq \mathbb{E}_{\mathbb{P}}[w(Y^0) \,|\, D_m = 1] - \sigma_i(s_i) > 0.$$

for all $\tilde{s}_{-i} \in \mathcal{S}_{-i}$.

Case 3 We are left to discuss the case where agent i believes her bid $\sigma_i(s_i)$ can win the auction with a bid that does not tie with anyone else's. Formally, suppose that

$$\begin{aligned}
\mu_i(\tilde{s}_{-i} \,|\, s_i) > 0 \quad &\text{for some} \quad \tilde{s}_{-i} \in \mathcal{S}_{-i}^0(\sigma_i(s_i)) \\
\mu_i(\tilde{s}_{-i} \,|\, s_i) = 0 \quad &\text{for all} \quad \tilde{s}_{-i} \in \bigcup_{m=1}^{n-1} \mathcal{S}_{-i}^m(\sigma_i(s_i))
\end{aligned}$$

In this case, she can deviate to a lower bid that does not affect her likelihood of winning the auction. Consider a bid

$$b_i \in \left(\max_{\tilde{s}_{-i} \in \mathcal{S}^0_{-i}(\sigma_i(s_i))} \max_{j \neq i} \sigma_j(\tilde{s}_j),\ \sigma_i(s_i) \right).$$

Clearly, $\mathcal{S}^m_{-i}(b_i) = \mathcal{S}^m_{-i}(\sigma_i(s_i))$ for all $m \in \{0, 1, \ldots, n-1\}$. Therefore, this deviation is profitable for agent i. ■

Notice it is costless for any contracted agent to induce the counterfactual baseline outcome, Y^0, and obtain an expected profit of $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$. Since agents are risk-neutral, they are all willing to pay at least $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ to own the contract, provided they know this quantity. It follows that any information structure where agents know $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ has a common low value of $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$.

To summarize, the principal's revenue from a first-price auction of contract w exceeds $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ to the extent that agents know this quantity, that the corresponding information structure admits a pure strategy Bayes Nash Equilibrium, and that this equilibrium concept captures agents' actual bidding behavior. Proposition 2 offers two practical lessons for a principal who works under Assumption I. First, the first-price auction is an attractive selling procedure if the principal believes agents know $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$. Second, it is in the principal's interest to have agents share their honest views on $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ prior to the start of the auction. In other words, the principal should strive to make $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ common knowledge among agents.

2 Examples

This section illustrates the model of section 1 with several potential applications.

2.1 Preservation of Natural Habitats

Take the case of a nature conservation organization (the principal) interested in preserving the rainforest in a well-defined area for an additional year. By the end of the year, the rainforest is either preserved or not. A key assumption of our model is that the organization can measure this status. It could do this in various ways, e.g., using image recognition software on satellite images or manually inspecting the area.

Naturally, there are many possible policies to try and preserve the rainforest, and their relative effectiveness depends on the circumstances of the land. If the land is private property and property rights are well-enforced, preservation could be achieved simply by contacting the owner and offering her an incentive to preserve the rainforest on her property. If the land is government-owned, effective attempts at preservation should consider the interests of the relevant government officials. If the land is subject to more elaborate ownership structures, such as community ownership, the interests of all the relevant stakeholders should be considered.

A principal who follows the decision process of section 1 need not know about these circumstances, the set of available policies, or their relative effectiveness at preserving the rainforest. Rather, she auctions off an incentive contract that pays $w \geq 0$ dollars in one year if the rainforest is preserved and 0 dollars otherwise.

There are two reasons why auctioning off the incentive contract is a good idea. One reason relies on the classic argument of Coase (1959) regarding auctions' potential to achieve efficient allocations. Our analysis does not explore this argument, but the idea is the following. If the organization allocated the incentive contract to someone far removed from the area under consideration, her cost of preserving the rainforest would likely be prohibitively high. Instead, if the organization allocated the contract to someone close to this area, this agent would find it less costly to do whatever is needed to preserve the rainforest. The former agent is less likely to preserve the rainforest and receive the incentive contract payment than the latter. Hence, the former agent values the incentive contract less. Allocating the contract to the agent who values it the most means allocating it to whoever thinks they have the most cost-effective intervention to preserve the rainforest.

The second reason to auction off the incentive contract is to secure a good return on the principal's investment. Notice that the rainforest could be preserved for an additional year, with or without an incentive contract. Hence, the principal might pay for preservation that would have happened even if she did not issue an incentive contract. The point of the auction stage in this paper is to rule out this unattractive possibility. Indeed, let p_0 be the probability that the rainforest is preserved absent an incentive contract, and suppose the principal uses an auction format that allocates the contract with certainty. Under Assumption I, the auction has an expected revenue that exceeds $w \cdot p_0$, effectively reimbursing the organization for the contract payments associated with preservation that the contracted agent does not cause. When should we expect the auction to satisfy such a revenue guarantee? If the organization knows p_0, section 1.3.1 shows that she can impose the guarantee through a reserve price of $w \cdot p_0$. Instead, if agents know p_0, section 1.3.2 shows that agents drive the contract price up to at least $w \cdot p_0$ in pure strategy Bayes Nash equilibria of the first-price auction. The latter claim is intuitive: every agent could obtain an expected contract payment of $w \cdot p_0$ without exerting effort to preserve the rainforest. Hence, every agent's valuation of the incentive contract is at least $w \cdot p_0$. Because this fact is common knowledge, competition among agents drives the selling price to at least $w \cdot p_0$.

When the incentive contract induces agents to preserve the rainforest, and the auction has this revenue guarantee, Theorem 1 shows that the principal is guaranteed to benefit from this scheme as long as the contract pays less than her benefit: $w \leq b$. In this case, issuing and auctioning off the incentive contract is a sound investment for the principal. At worse, she is left as well-off as she would have been had she not done this, in expectation.

Across existing preservation interventions, the scheme presented in this paper resembles policies of payments for ecosystem services. Here, the principal — typically a government agency — offers payments to an agent in exchange for concrete preservation efforts. Such programs have been implemented in Costa Rica to pay for the preservation of forests and jungles (Sánchez-Azofeifa, Pfaff, Robalino, and Boomhower 2007); in China, to pay for tree planting along the slopes of river basins (Pan, Xu, Yang, and Yu 2017); and in Mexico, to pay for the design and implementation of sustainable development plans in forest lands (Alix-Garcia, Sims, and Yañez-Pagans 2015). In contrast with these interventions, this paper has the principal pay for observed preservation outcomes instead of requiring the contracted agent to take specific actions.

So far, our discussion ignores other costs the organization might incur in practice. Upon receipt of the contract, the agent might act in ways that go against the organization's interests. There might be spillovers, where the agent works to preserve the rainforest but at the expense of natural habitats in other areas. The agent might use coercion or other violent means to achieve preservation, a real possibility in regions with precarious institutions.

In section 3.1, we extend the model from section 1 to account for such costs. Briefly, the approach augments Assumptions I and II with the assumption that contracted agents profit from their attempts at preserving the rainforest in expectation. This assumption places an upper bound on the agent's costs: in a worst-case scenario, it costs the agent w to preserve the rainforest. If the organization's costs are bounded above by the agent's costs, scaled by some $\gamma > 0$, then at worse, the principal pays w to the agent and incurs $\gamma \cdot w$ as costs, a total cost of $(1+\gamma)w$. So, the principal obtains a non-negative payoff as long as the incentive contract pays $w \leq (1+\gamma)^{-1}b$.

2.2 Reduction of Greenhouse Gas Concentrations

Suppose an environmental organization (the principal) wishes to reduce the monthly average concentration of greenhouse gases (GHG) in a given section of the atmosphere in a given month. By the end of the month, the organization observes the realized concentration of GHG.[5] The principal need not know how to effectively improve the air composition in this area. She may not know the relevant local characteristics to design an effective environmental policy. Reducing the concentration of greenhouse gases above a densely populated area and a cattle ranch are different tasks; the interests of local politicians determine whether a greenhouse gas reduction policy can count on the government's support.

The organization obtains a monetary-equivalent benefit equal to $b(y)$ if the area's average concentration of GHG equals y. For example, third-party estimates of the social cost of carbon could inform this benefit. To follow the policy choice procedure of section 1, the environmental organization issues a linear incentive contract that pays its owner a fraction $\alpha \in (0, 1)$ of the organization's benefit: $w(y) = \alpha \cdot b(y)$.

In this case, Assumption II means that the contracted agent's actions are not counter-productive, leading to a worse air composition than would have been achieved without an incentive contract. Assumption I requires that the auction used by the organization raise enough money to cover the contract liability in the event that the contracted agent does nothing to curb GHG concentrations. Suppose the principal uses an auction that allocates the contract with certainty, that she can accurately assess the concentration of GHG in the absence of an incentive contract, and that she can determine the expected contract payment associated with such outcome realizations. Here, she can auction the linear incentive contract with a reserve price set to this amount. A second approach is for the organization to sell the incentive contract using the first-price auction. Here, the auction's revenue would meet the required guarantee, provided agents knew and agreed about the expected contract payment they would get if they exerted no effort.

Some of the issues raised by the previous example apply here as well. There could be

[5] There are many ways to measure the concentration of greenhouse gases. Specialized tools called spectrometers often produce the raw data for these measurements. Such tools can be mounted on satellites to obtain measurements in various locations along their orbit (e.g., NASA's EMIT mission), or they can be installed in specific locations to obtain repeated local measurements (e.g. the Total Carbon Column Observing Network).

spillovers, meaning that GHG concentration could decrease at the expense of more GHG emissions elsewhere. The environmental organization could care about social welfare and allocate the contract to a government office that reduces emissions at a high social cost.

A different issue arises when the contracted agent has limited commitment. Indeed, at the end of the period, the agent could refuse to pay the environmental organization for the incentive contract, particularly if the contract selling price exceeds the due contract payment. In section 3.3, we explore additional assumptions under which alternative arrangements that eliminate this problem give the organization a non-negative payoff guarantee.

One arrangement has the organization give away the incentive contract for free. Here, the incentive contract needs to incentivize a sizeable reduction in the concentration of GHG for the organization to benefit from it. Another arrangement has the organization deduct the contract's selling price from her contract payment obligation while guaranteeing the agent a non-negative transfer. In this case, we show that the organization still obtains a non-negative payoff guarantee, provided the auction's revenue guarantee holds almost surely, rather than in expectation.

2.3 Lobbying

Consider an interest group that intends to repeal a law by the end of a political cycle. Agents are lobbyists, and, as in example 2.1, the outcome is binary and indicates if the law is repealed by the end of the cycle. It is easy to measure the outcome, unlike examples 2.1 and 2.2 that require dedicated measurement technologies.

To achieve its objective, the interest group issues an incentive contract that pays its owner $w \geq 0$ dollars if the law is repealed by the end of the period and auctions it off. Assumption II requires that the contract induce the contracted lobbyist to improve the probability that the law is repealed. Notice that the law could be repealed by the end of the cycle even without an incentive contract or if the contracted lobbyist did nothing to repeal the law. Assumption I requires that lobbyists bid the contract price up to the expected contract payment in this baseline course of events.

3 Extensions

In this section, we extend the model of section 1 in several directions. We propose weak additional assumptions that allow the principal to design maximin optimal contract-auction pairs that offer her a non-negative payoff guarantee in the face of various costs categories. We consider alternative arrangements that eliminate the agent's obligation to pay for the incentive contract. One arrangement has the principal give away the incentive contract for free. In this case, we construct contract-auction pairs that give the principal a non-negative payoff guarantee under a refined version of Assumption II that requires the agents' effect on the outcome to exceed a positive threshold. Another arrangement has the principal deduct the price of the incentive contract from her contract payment obligation while giving the agent a money-back guarantee. Here, we characterize maximin optimal contract-auction pairs under a more robust version of Assumption I that requires the auction's revenue to exceed the contract payment in the baseline outcome Y^0 almost surely, rather than in expectation. Finally, we show how to design optimal contract-auction pairs that limit the principal's maximum payment obligation.

3.1 Miscellaneous Costs for the Principal

This section incorporates costs that the principal incurs following his decision to auction off an incentive contract. For each $m \in \mathcal{M}$, let C_m^p be the non-negative random variable that denotes the principal's costs from contract-auction pair m. Costs naturally determine the principal's payoff, Π_m^{nx}, which is now given by

$$\Pi_m^{nx} = b(Y_m) - b(Y^0) - [W_m + C_m^p - R_m].$$

Depending on the application, several cost categories may add up to C_m^p. It is costly to design an incentive contract and implement an auction. Example 2.1 illustrates that the contracted agent's actions can be socially costly or go against the principal's interests. Moreover, the incentive contract could be cost-plus, i.e., reimburse a fraction of the contracted agent's costs to produce outcome Y_m.

Notice that assumptions I and II do not restrict the principal's costs C_m^p. Hence, they are insufficient to produce finite payoff guarantees: the set $\{\mathbb{E}_P[\Pi_m^{nx}] : \ P \in \mathcal{P}^*\}$ does not have a lower bound. To make progress, we need more assumptions. In this section, we characterize the set of maximin optimal contract-auction pairs (which have a non-negative payoff guarantee) under two additional assumptions. The assumptions require that the contracted agent profit, or at least not lose money, from the actions she takes to improve the outcome and profit from the incentive contract and that the agent's costs exceed the principal's costs, scaled by some $\gamma > 0$. Intuitively, if the agent makes a non-negative expected profit from the incentive contract, her costs cannot exceed the additional incentive contract payment brought about by her actions, in expectation. So, if the agent's costs place an upper bound on the principal's costs, the principal can design the incentive contract to control the costs she would incur in a worst-case scenario.

Concretely, fix $\gamma > 0$ and let C_m^a be the non-negative random variable that denotes the costs incurred by the contracted agent to produce outcome Y_m, under contract-auction pair $m \in \mathcal{M}$. The principal and the agents incur costs only if the principal actually sells the contract, so $D_m = 0$ implies that $C_m^a = C_m^p = 0$. Consider the following assumptions.

Assumption III. $\mathbb{E}_P[W_m - D_m w(Y^0) - C_m^a] \geq 0$.

Assumption IV. $\mathbb{E}_P[\gamma \cdot C_m^a - C_m^p] \geq 0$.

$P \in \mathcal{P}$ satisfies Assumption III for $m \in \mathcal{M}$ if the contracted agent profits from her efforts to influence the outcome and obtain a better contract payment, in expectation. Alternatively, P satisfies Assumption III for m if the contracted agent knows enough about actions detrimental to the outcome to not make counterproductive choices, in expectation. In turn, $P \in \mathcal{P}$ satisfies Assumption IV for $m \in \mathcal{M}$ if the principal's expected cost is bounded above by that of the agent, scaled by γ.

The principal imposes assumptions I, II, III and IV for contract-auction pairs in $\mathcal{M}^*$, so that the set of admissible probability measures is $\mathcal{P}^{nx}$, where

$$\mathcal{P}^{nx} \equiv \{P \in \mathcal{P} : \ P \text{ satisfies assumptions I, II, III and IV for all } m \in \mathcal{M}^*\}.$$

Her payoff guarantee is then $\underline{\Pi}_m^{nx}$, where

$$\underline{\Pi}_m^{nx} = \begin{cases} \inf_{P \in \mathcal{P}^{nx}} \mathbb{E}_P[\Pi_m^{nx}] & \text{if } \{\mathbb{E}_P[\Pi_m^{nx}] : \ P \in \mathcal{P}^{nx}\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

Proposition 3 characterizes the set of contract-auction pairs that offer the principal a non-negative payoff guarantee and shows that such pairs are also maximin optimal. Appendix A collects the proof, which proceeds as in Lemmas 1 and 2 and Theorem 1.

Proposition 3.

$$\underline{\Pi}_m^{nx} = \begin{cases} \min\limits_{\{(y_0, y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\gamma)[w(y_1) - w(y_0)] & \textit{if } m \in \mathcal{M}^* \\ -\infty & \textit{otherwise,} \end{cases}$$

and $\underline{\Pi}_m^{nx} = 0$ *if and only if* $m = (w, a) \in \mathcal{M}^*$ *and, for all* $y_0, y_1 \in \mathcal{Y}$ *such that* $y_0 \leq y_1$*,*

$$(1 + \gamma)[w(y_1) - w(y_0)] \leq b(y_1) - b(y_0).$$

When the agent's costs bound those of the principal, contract-auction pairs that offer the principal a non-negative payoff guarantee feature a contract payment schedule whose slope is uniformly lower than that of the principal's benefit, scaled down by $(1 + \gamma)^{-1}$. The idea behind this result is the following. Take a contract-auction pair $m \in \mathcal{M}^*$. Under Assumption I, the worst-case scenario that the principal deems possible is one where the auction exactly reimburses the principal for the baseline contract payment, $\mathbb{E}_P[D_m w(Y^0)]$. In this case, the principal's expected payoff amounts to her benefit, $\mathbb{E}_P[b(Y_m) - b(Y^0)]$, minus the contract payments net of the auction's revenue, $\mathbb{E}_P[w(Y_m) - w(Y^0)]$, minus her additional costs, $\mathbb{E}_P[C_m^p]$. By Assumption IV, these additional costs are bounded above by the agent's costs, scaled by γ. Moreover, by Assumption III, the agent's costs are bounded above by her benefit from the incentive contract. So, the principal's expected costs are bounded above by $\mathbb{E}[\gamma \cdot (w(Y_m) - w(Y^0))]$ and, in a worst possible scenario, the principal incurs total costs equal to $(1+\gamma) \cdot \mathbb{E}[w(Y_m) - w(Y^0)]$. Under Assumption II, the incentive contract induces the contracted agent to improve the outcome. Thus, the worst possible payoff for the principal corresponds with the lowest payoff obtained by looking across all outcome improvements, which equals

$$\min_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - (1 + \gamma)[w(y_1) - w(y_0)].$$

So, the principal obtains a non-negative payoff guarantee if she obtains a non-negative payoff for every possible outcome improvement. She achieves this when the slope of the contract payment schedule is always below that of the principal's benefit from the outcome, scaled down by $(1 + \gamma)^{-1}$.

3.2 Cost of Funds

Suppose the principal finds that raising funds to make contract payments is costly. In Laffont and Tirole (1987) and Laffont and Tirole (1986), these costs arise because the principal is a social planner who can only obtain funds in ways that bring about real resource costs. Let $\lambda > 0$ denote the principal's cost of raising a dollar. We assume that the principal knows this cost. Her ex-post payoff from contract-auction pair $m \in \mathcal{M}$ is then Π_m^{cf}, where

$$\Pi_m^{cf} = b(Y_m) - b(Y^0) - (1 + \lambda)[W_m - R_m].$$

Under Assumptions I and II, the principal's payoff guarantee is $\underline{\Pi}_m^{cf}$, where

$$\underline{\Pi}_m^{cf} = \begin{cases} \inf_{P \in \mathcal{P}^*} \mathbb{E}_P[\Pi_m^{cf}] & \text{if } \{\mathbb{E}_P[\Pi_m^{cf}] : \; P \in \mathcal{P}^*\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

Proposition 4, proven in Appendix A, mirrors Lemmas 1 and 2 and Theorem 1 and adapts them to this situation.

Proposition 4.

$$\underline{\Pi}_m^{cf} = \begin{cases} \min\limits_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\lambda)[w(y_1) - w(y_0)] & \textit{if } m \in \mathcal{M}^* \\ \min\limits_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - (1+\lambda) w(y_1) & \textit{otherwise,} \end{cases}$$

and $\underline{\Pi}_m^{cf} = 0$ *if and only if* $m = (w, a) \in \mathcal{M}^*$ *and, for all* $y_0, y_1 \in \mathcal{Y}$ *such that* $y_0 \leq y_1$*,*

$$(1+\lambda)[w(y_1) - w(y_0)] \leq b(y_1) - b(y_0).$$

In particular, a contract-auction pair $m = (w, a) \in \mathcal{M}^*$ such that $w = \alpha \cdot b$ with $\alpha \in [0, (1+\lambda)^{-1}]$ is maximin optimal and offers the principal a non-negative payoff guarantee. This result is intuitive: for every dollar worth of benefit, the principal pays α dollars, scaled up by her costs of raising the money, a total of $\alpha(1+\lambda)$. If $\mathcal{P}^*$ is well-specified, she can obtain a non-negative payoff provided that her remaining benefit, $1-\alpha(1+\lambda)$, is non-negative, which occurs whenever $\alpha \leq (1+\lambda)^{-1}$.

3.3 Limited Commitment

Recall that, at the end of the time period, the agent who won the auction pays the principal for the incentive contract, and the principal pays the agent according to the contract terms and the outcome realization. So far, our discussion has ignored potential commitment problems: after the outcome realization, each party could attempt to change the contract terms, or downright refuse to pay. Naturally, the relevance of such problems is context-specific. Decarolis (2014) studies a procurement setting where such commitment problems are empirically relevant.

This section studies two arrangements that eliminate the agent's commitment problem and inspects the consequences of the principal's commitment problem. In the first arrangement, the principal gives the incentive contract to an agent free of charge. In the second arrangement, the principal deducts the selling price of the incentive contract from her contract payment obligation but guarantees the agent a non-negative net transfer. For each arrangement, we propose additional assumptions that the principal can make, and we characterize the set of maximin optimal contract-auction pairs, which feature non-negative payoff guarantees. When agents believe the principal cannot commit to pay for the incentive contract, they may expect to obtain a payment below $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ if they were to exert no effort. Hence, the first-price auction no longer has a revenue guarantee equal to $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$, even if agents know this quantity.

3.3.1 Free Contracts

A simple way to eliminate the agent's commitment problem is to dispel the commitment. To do this, the principal could give the contract away for free. Although extreme, handing out incentive contracts for free is a common practice. Pharmaceutical companies did not pay to participate in the advance market commitment that targeted the production of a pneumococcal conjugate vaccine (M. Kremer, Levin, and Snyder 2020), and procurement contracts are often awarded to interested parties for free (Bajari and Tadelis 2001).

Suppose the principal is interested in contract-auction pairs such that the agent who wins the auction does not pay for the contract. Of course, such contract-auction pairs are inconsistent with Assumption I. So, in this case, the principal only imposes Assumption II for a subset of contract-auction pairs $\mathcal{M}^{fc} \subset \mathcal{M}$ with the property that $m \in \mathcal{M}^{fc}$ implies that $P(R_m = 0) = 1$ for all $P \in \mathcal{P}$. The set of probability measures she considers is then $\mathcal{P}^{fc}$, where

$$\mathcal{P}^{fc} \equiv \{P \in \mathcal{P} : P \text{ satisfies Assumption II for all } m \in \mathcal{M}^{fc}\},$$

and her payoff guarantee from a given contract-auction pair $m \in \mathcal{M}$ is $\underline{\Pi}_m^{fc}$, where

$$\underline{\Pi}_m^{fc} = \begin{cases} \inf_{P \in \mathcal{P}^{fc}} \mathbb{E}_P[\Pi_m] & \text{if } \{\mathbb{E}_P[\Pi_m] : \ P \in \mathcal{P}^{fc}\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

The following lemma characterizes the principal's payoff guarantee. Its proof is in Appendix A and proceeds as in Lemmas 1 and 2.

Lemma 3.

$$\underline{\Pi}_m^{fc} = \begin{cases} \min\limits_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - w(y_1) & \textit{if } m \in \mathcal{M}^{fc} \\ \min\limits_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1) & \textit{otherwise.} \end{cases}$$

It follows from Lemma 3 that for all $m \in \mathcal{M}^{fc}$, $\underline{\Pi}_m^{fc} \leq 0$, and $\underline{\Pi}_m^{fc} = 0$ if and only if $w(y) = 0$ for all $y \in \mathcal{Y}$.[6] Intuitively, because Assumption I is absent, the auction's revenue

[6] Fix $m = (w, a) \in \mathcal{M}^{fc}$. If $y_0 = y_1$, then $b(y_1) - b(y_0) - w(y_1) = -w(y_1) \leq 0$. Hence, $\underline{\Pi}_m^{fc} \leq 0$. Next,

is only constrained to be non-negative. But then, for any given contract-auction pair, a probability measure such that $Y_m = Y^0$, $R_m = 0$, and $Y^0 > 0$ almost surely has the principal pay for an outcome realization that would have occurred had she not issued or sold the contract. Such a probability measure gives the principal a strictly negative payoff, except when w is the zero contract. Hence, the principal's revenue guarantee is strictly negative for all contracts that pay positive amounts.

The principal must make different assumptions to obtain a non-negative payoff guarantee for such contracts. In this section, we explore a stronger version of Assumption II, requiring that the causal effect of contract-auction pairs in $\mathcal{M}^{fc}$ on the principal's benefit from the outcome be bounded below by some $c > 0$. This assumption gives rise to the set of probability measures $\mathcal{P}^{fc}(c)$, given by

$$\mathcal{P}^{fc}(c) \equiv \{P \in \mathcal{P} : P(b(Y_m^1) - b(Y^0) \geq c) = 1 \text{ for all } m \in \mathcal{M}^{fc}\},$$

and a payoff guarantee $\underline{\Pi}_m^{fc}(c)$ for each $m \in \mathcal{M}$, where

$$\underline{\Pi}_m^{fc}(c) = \begin{cases} \inf_{P \in \mathcal{P}^{fc}(c)} \mathbb{E}_P[\Pi_m] & \text{if } \{\mathbb{E}_P[\Pi_m] :\ P \in \mathcal{P}^{fc}(c)\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

Lemma 4 characterizes the principal's payoff guarantee in these circumstances.

Lemma 4.

$$\underline{\Pi}_m^{fc}(c) = \begin{cases} \min\left\{0, \min\limits_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: b(y_1)-b(y_0)\geq c\}} b(y_1) - b(y_0) - w(y_1)\right\} & \textit{if } m \in \mathcal{M}^{fc} \\ \min\limits_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - w(y_1) & \textit{otherwise.} \end{cases}$$

With the tractable description of the payoff guarantee from Lemma 4, we can characterize the class of contracts that yield a non-negative payoff guarantee.

Proposition 5. $\underline{\Pi}_m^{fc}(c) \geq 0$ *if and only if* $m \in \mathcal{M}^{fc}$ *and* $w(y) \leq c$ *for all* $y \in \mathcal{Y}$ *such that* $y \geq b^{-1}(c + b(\underline{y}))$.

Proposition 5, proven in Appendix A, conveys a simple message. If the principal does not charge money for the incentive contract, then her contract payment obligation must not exceed the benefit she gets from the outcome improvement that the contract induces. A contract-auction pair $m = (w, a)$ is a sound investment if $\mathbb{P}(b(Y_m^1) - b(Y^0) \geq c) = 1$ and the contract pays less than c for any outcome realization.

$w = 0$ implies that $\underline{\Pi}_m^{fc} = 0$, since b is strictly increasing. Finally, $\underline{\Pi}_m^{fc} = 0$ implies that, for all $y \in \mathcal{Y}$, $0 = \underline{\Pi}_m^{fc} \leq b(y) - b(y) - w(y) = -w(y)$ so that $w(y) = 0$, since w is non-negative.

3.3.2 Money-back Guarantee for Agents

Another way that the principal can address the agent's commitment problem is by offering her a non-negative net transfer. Recall that the arrangement we have studied thus far involves two transactions. The contracted agent purchases the contract from the principal, and the principal issues contract payments to the agent after the outcome realization. In this section, we study an alternative arrangement whereby the agent who wins the auction does not transfer R_m dollars to the principal, and the principal transfers $W_m - R_m$ dollars to the agent after the outcome realization if $W_m - R_m > 0$, and does not transfer any money otherwise. As in section 3.3.1, the principal eliminates the agent's commitment problem by removing the agent's payment obligation. Unlike section 3.3.1, however, the principal does charge the agent for the incentive contract: she deducts the selling price of the incentive contract from her contract payments. Because the principal never deducts so much as to have the agent owe her money, she guarantees the agent a non-negative net transfer.[7] So, the principal's ex-post benefit from contract-auction pair m is Π_m^{mb}, given by

$$\Pi_m^{mb} = b(Y_m) - b(Y^0) - \max\{0,\ W_m - R_m\}.$$

Assumptions I and II cannot guarantee the principal a non-negative payoff. To see this, consider the following.

Example For a given contract-auction pair, consider the following unfavorable scenario. The contracted agent does not influence the outcome, and the auction satisfies the expected revenue guarantee: with equal probability, the contract sells for double the contract's expected baseline payment, or it sells for a price of zero. To achieve an expected payoff of zero, the principal would have to collect all of the contract's selling price when the contract sells for a positive price. But under the current arrangement, she can only collect half of this price. Hence, her expected payoff is negative even though Assumptions I and II hold.

Formally, fix a contract-auction pair $m = (w, a) \in \mathcal{M}$ such that contract w is not identically equal to zero. Now consider a probability measure $P \in \mathcal{P}$, where $P(Y_m^1 = Y^0, D_m = 1) = 1$, $P(R_m = 0) = P(R_m = 2 \cdot w(Y^0)) = 0.5$, and $\mathbb{E}_P[w(Y^0)] > 0$. Suppose that Y^0 and

[7] Strictly speaking, contract-auction-specific variables differ in this alternative arrangement relative to the setting where agents commit to pay for the contract. For example, the contracted agent might produce different outcomes under each arrangement, even if she held the same contract. To keep the notation simple, we reinterpret the model's variables. Before, Y_m referred to the outcome induced by contract-auction pair m when agents committed to pay for the contract. Now, it refers to the induced outcome in a situation where the principal offers the contracted agent a non-negative net transfer.

R_m are independent, so that

$$P(Y^0 \in \mathcal{Y}^0, R_m \in \mathcal{R}) = P(Y^0 \in \mathcal{Y}^0) \cdot P(R_m \in \mathcal{R})$$

for all Borel sets $\mathcal{Y}^0 \subseteq \mathcal{Y}$ and $\mathcal{R} \subseteq \mathbb{R}_+$. P clearly satisfies Assumption II for m. It also satisfies Assumption I for m, since

$$\mathbb{E}_P[R_m - D_m w(Y^0)] = \frac{1}{2}\mathbb{E}_P[R_m \mid R_m = 2 \cdot w(Y^0)] - \mathbb{E}_P[w(Y^0)] = \mathbb{E}_P[w(Y^0)] - \mathbb{E}_P[w(Y^0)] = 0.$$

However, the principal's expected payoff is strictly negative:

$$\begin{aligned}
&\mathbb{E}_P[\Pi_m^{mb}] \\
&= -\mathbb{E}_P[\max\{0,\ W_m - R_m\}] \\
&= -\frac{1}{2}\mathbb{E}_P[\max\{0,\ w(Y^0) - R_m\} \mid R_m = 0] - \frac{1}{2}\mathbb{E}_P[\max\{0,\ w(Y^0) - R_m\} \mid R_m = 2 \cdot w(Y^0)] \\
&= -\frac{1}{2}\mathbb{E}_P[w(Y^0)] - \frac{1}{2}\mathbb{E}_P[\max\{0,\ -w(Y^0)\}] \\
&= -\frac{1}{2}\mathbb{E}_P[w(Y^0)] \\
&< 0.
\end{aligned}$$

To obtain a non-negative payoff guarantee, the principal must make stronger assumptions. We could construct the previous example because, under Assumption I, the auction's revenue exceeds the contract's baseline payment, but only in expectation. We now consider the following refinement of Assumption I:

Assumption I'. $P(R_m - D_m \cdot w(Y^0) \geq 0) = 1$.

Assumption I' strengthens Assumption I by requiring that the auction's revenue exceed the contract's baseline payment almost surely, rather than in expectation. The principal imposes Assumptions I' and II for contract-auction pairs that lie in $\mathcal{M}^* \subset \mathcal{M}$. Thus, she considers probability measures that lie in $\mathcal{P}^{mb}$, where

$$\mathcal{P}^{mb} \equiv \{P \in \mathcal{P} : P \text{ satisfies Assumptions I' and II for all } m \in \mathcal{M}^*\}.$$

Her payoff guarantee from a given contract-auction pair $m \in \mathcal{M}$ is $\underline{\Pi}_m^{mb}$, where

$$\underline{\Pi}_m^{mb} = \begin{cases} \inf_{P \in \mathcal{P}^{mb}} \mathbb{E}_P[\Pi_m^{mb}] & \text{if } \{\mathbb{E}_P[\Pi_m^{mb}] :\ P \in \mathcal{P}^{mb}\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

The following lemma shows that the refinement of Assumption I allows the principal to recover the same payoff guarantee she obtained when agents committed to pay for the contract. We prove it in Appendix A.

Lemma 5.

$$\underline{\Pi}_m^{mb} = \begin{cases} \min\limits_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}:y_0\le y_1\}} b(y_1)-b(y_0)-[w(y_1)-w(y_0)] & \text{if } m\in\mathcal{M}^* \\ \min\limits_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1)-b(y_0)-w(y_1) & \text{otherwise.} \end{cases}$$

Under Assumption I', the auction's revenue equals the contract's baseline payment in a worst-case scenario. On the other hand, Assumption II requires that the contract induces an outcome improvement. Because contracts' payment schedules are non-decreasing, the principal's contract payment obligation must exceed the auction's revenue. Hence, the principal effectively appropriates the total selling price of the contract in a worst-case scenario, even if agents cannot commit to paying for it. For this reason, the principal achieves the same revenue guarantee as in our baseline case with full commitment.

As in Theorem 1, contract-auction pairs that give the principal a non-negative payoff guarantee feature contracts whose marginal payment schedule is below the principal's marginal benefit from the outcome. Proposition 6 establishes this result.

Proposition 6. $\underline{\Pi}_m^{mb} = 0$ *if and only if* $m=(w,a)\in\mathcal{M}^*$ *and, for all* $y_0,y_1\in\mathcal{Y}$ *such that* $y_0\le y_1$*,*

$$b(y_1)-b(y_0)\ge w(y_1)-w(y_0).$$

The arguments in favor of Assumption I from sections 1.3.1 and 1.3.2 also support Assumption I', even though Assumption I' is stronger than Assumption I. Namely, if the principal knows the contract's expected baseline payment, $\mathbb{E}_{\mathbb{P}}[w(Y^0)\,|\,D_m=1]$, then she can design an auction whose revenue meets or exceeds this quantity almost surely if she sets it as a reserve price. Instead, suppose the contract's expected baseline payment is common knowledge among agents. Every agent can obtain an expected contract payment equal to $\mathbb{E}_{\mathbb{P}}[w(Y^0)\,|\,D_m=1]$, so that this quantity is a commonly known lower bound of all agents' willingness to pay for the contract. Here, Proposition 2 shows that the first-price auction's revenue exceeds the contract's expected baseline payment in pure strategy Bayes-Nash equilibria.

3.3.3 Principal Commitment Problems

What if the principal herself cannot commit to making contract payments? It is helpful to distinguish between the principal's contract payment obligation and the actual contract payment. Denote the principal's actual contract payment under contract-auction pair m with the non-negative random variable W_m^{pc}. Since the principal may not fully pay her obligation, $W_m^{pc} \leq W_m$.

To simplify the notation, we do not redefine the principal's payoff-relevant random variables, (Y_m^1, D_m, R_m). Section 1 defined these contract-auction-specific outcomes in a context where the principal fully committed to the contract terms. Here, they differ to the extent that agents detect the principal's lack of commitment. So, the principal's ex-post payoff from contract-auction pair $m \in \mathcal{M}$ is Π_m^{pc}, given by

$$\Pi_m^{pc} = b(Y_m) - b(Y^0) - [W_m^{pc} - R_m],$$

and her payoff guarantee under Assumptions I and II is $\underline{\Pi}_m^{pc}$, where

$$\underline{\Pi}_m^{pc} = \begin{cases} \inf_{P \in \mathcal{P}^*} \mathbb{E}_P[\Pi_m^{pc}] & \text{if } \{\mathbb{E}_P[\Pi_m^{pc}] :\ P \in \mathcal{P}^*\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

Since $W_m^{pc} \leq W_m$ for all $m \in \mathcal{M}$, it is straightforward to show that Lemmas 1 and 2 and Theorem 1 extend to this situation. Proposition 7 summarizes.

Proposition 7.

$$\underline{\Pi}_m^{pc} = \begin{cases} \min\limits_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)] & \textit{if } m \in \mathcal{M}^* \\ \min\limits_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1) & \textit{otherwise,} \end{cases}$$

and $\underline{\Pi}_m^{pc} = 0$ *if and only if* $m = (w, a) \in \mathcal{M}^*$ *and, for all* $y_0, y_1 \in \mathcal{Y}$ *such that* $y_0 \leq y_1$,

$$w(y_1) - w(y_0) \leq b(y_1) - b(y_0).$$

The difficulty that the principal's limited commitment raises is that Assumption I is harder to justify. Concretely, the first-price auction need not induce a selling price that matches or exceeds the contract's expected baseline payment, $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$, even if agents know this quantity. Before, every agent could obtain an expected payment of $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$, even if they did not attempt to influence the outcome. For this reason, equilibrium behavior had agents bid the contract price up to or beyond this quantity. Now, the agents' expected baseline payment is lower, because agents believe the principal may not honor her obligations. Hence, $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$ is no longer a commonly known lower bound on agents' willingness to pay for the contract, and the contract's equilibrium selling price could be lower than $\mathbb{E}_{\mathbb{P}}[w(Y^0) \mid D_m = 1]$. In an extreme case where agents believe the principal does not make any contract payments, the contract is worthless, does not sell for a positive price, and fails to improve the outcome.

3.4 Limited Liability

A practical drawback of the simple, linear contracts of the form $w(y) = \alpha b(y)$ is that the principal cannot control how much she will owe at the end of the period. In this section, we assume that the principal is willing or able to spend at most $\overline{w} \geq 0$ dollars in contract payments.

Theorem 1 immediately applies to this circumstance. Indeed, for any contract $w \in \mathcal{W}$ such that $w(y_1) - w(y_0) \leq b(y_1) - b(y_0)$ for all $y_0 \leq y_1$, the contract w^{ll} defined as $w^{ll}(y) = \min\{w(y), \overline{w}\}$ features a maximum liability of $\overline{w}$, and satisfies $w^{ll}(y_1) - w^{ll}(y_0) \leq b(y_1) - b(y_0)$ for all $y_0 \leq y_1$. So, any contract-auction pair $(w^{ll}, a) \in \mathcal{M}^*$ features a non-negative payoff guarantee and is maximin optimal. The following Corollary of Theorem 1 summarizes.

Corollary 2. *Let $w \in \mathcal{W}$ be such that $w(y_1) - w(y_0) \leq b(y_1) - b(y_0)$ for all $y_0, y_1 \in \mathcal{Y}$ where $y_0 \leq y_1$ and define a limited liability contract $w^{ll}(y) = \min\{w(y), \overline{w}\}$. $\underline{\Pi}_m = 0$ for all $a \in \mathcal{A}$ such that $m = (w^{ll}, a) \in \mathcal{M}^*$.*

4 Conclusion

In this paper, a decision-maker called the principal goes about improving a valuable, measurable outcome by selling a contract that pays its holder according to the outcome realization. I showed that this scheme is guaranteed to benefit her, provided the contract induces the contractor to improve the outcome and the selling price is high enough to cover the contract payment for baseline outcomes that would have happened in case the contractor did nothing to improve the outcome. To secure this selling price, the principal can set a reserve price if she knows the expected contract payment in the baseline outcome. Instead, if potential contractors know this quantity and are risk-neutral, the principal can achieve this price if she sells the contract using a first-price auction.

The principal can implement this scheme even if she faces severe information constraints. She may ignore the relative cost-effectiveness of different policies or interventions. Indeed, she may even ignore which policies or interventions are available. She need not know the potential contractors' preferences or informational constraints beyond that the incentive contract induces them to improve the outcome.

Moreover, I proposed weak additional assumptions under which the principal continues to obtain a guaranteed benefit from the scheme in the face of complications that may arise in practice. In these situations, the principal incurs various additional costs, and contractors cannot be trusted to pay for the incentive contract.

Overall, the scheme is flexible enough to be useful for real-world applications. I took a special interest in those concerned with environmental objectives, e.g., the preservation of natural habitats or the reduction in the concentration of greenhouse gases in the atmosphere.

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A Omitted Proofs

PROOF OF LEMMA 1 Let $m \in \mathcal{M}^*$. For any $P \in \mathcal{P}^*$, it follows that:

$$\begin{aligned}\mathbb{E}_P[\Pi_m] &= \mathbb{E}_P[b(Y_m) - b(Y^0) - [W_m - R_m]] \\ &\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m[w(Y_m) - w(Y^0)]],\end{aligned}$$

since P satisfies Assumption I for m.

By the potential outcomes model (1), if $P(D_m = 1) = 0$, then

$$\mathbb{E}_P[\Pi_m] \geq \mathbb{E}_P[b(Y_m) - b(Y^0) \mid D_m = 0] = \mathbb{E}_P[b(Y^0) - b(Y^0) \mid D_m = 0] = 0.$$

Otherwise,

$$\begin{aligned}\mathbb{E}_P[\Pi_m] &\geq P(D_m = 1)\mathbb{E}_P[b(Y_m^1) - b(Y^0) - [w(Y_m^1) - w(Y^0)] \mid D_m = 1] \\ &\geq P(D_m = 1) \min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)],\end{aligned}$$

where the second inequality holds because P satisfies Assumption II and b is strictly increasing, and the minimum exists because $\mathcal{Y}$ is compact and b and w are continuous.

Since

$$\min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)] \leq 0,$$

we conclude that

$$\mathbb{E}_P[\Pi_m] \geq \min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)].$$

Next, consider $\tilde{P} \in \mathcal{P}$ such that $\tilde{P}(D_{\tilde{m}} = 1, R_{\tilde{m}} = D_{\tilde{m}}w(Y^0), Y_{\tilde{m}}^1 = \underline{y}_1, Y^0 = \underline{y}_0) = 1$ for all $\tilde{m} \in \mathcal{M}$, where

$$\left(\underline{y}_1, \underline{y}_0\right) \in \operatorname*{arg\,min}_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)].$$

Clearly, $\tilde{P}$ satisfies Assumptions I and II for all contract-auction pairs in $\mathcal{M}^*$, so $\tilde{P} \in \mathcal{P}^*$. Moreover,

$$\mathbb{E}_{\tilde{P}}[\Pi_m] = b(\underline{y}_1) - b(\underline{y}_0) - [w(\underline{y}_1) - w(\underline{y}_0)].$$

It follows that

$$\underline{\Pi}_m = \min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)].$$

■

PROOF OF LEMMA 2 Let $m \in \mathcal{M} \setminus \mathcal{M}^*$. For any $P \in \mathcal{P}^*$, we have that

$$\begin{aligned}\mathbb{E}_P[\Pi_m] &= \mathbb{E}_P[b(Y_m) - b(Y^0) - [W_m - R_m]] \\ &\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m w(Y_m)],\end{aligned}$$

since R_m is non-negative, so that $\mathbb{E}_P[R_m] \geq 0$. By the potential outcomes model (1), $\mathbb{E}_P[\Pi_m] \geq 0$ if $P(D_m = 1) = 0$. If instead $P(D = 1) > 0$, then

$$\begin{aligned}\mathbb{E}_P[\Pi_m] &\geq P(D_m = 1)\mathbb{E}_P[b(Y_m^1) - b(Y^0) - w(Y_m^1) \mid D_m = 1] \\ &\geq P(D_m = 1) \min_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1).\end{aligned}$$

The minimum exists because $\mathcal{Y}$ is compact, and b and w are continuous. Since w is non-negative, $\min_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1) \leq 0$. We conclude that

$$\mathbb{E}_P[\Pi_m] \geq \min_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1).$$

Next, consider $\tilde{P} \in \mathcal{P}^*$ such that $\tilde{P}\left(D_m = 1, R_m = 0, Y_m = \tilde{y}^1, Y^0 = \underline{y}_0\right) = 1$, where

$$\left(\tilde{y}^1, \underline{y}_0\right) \in \arg\min_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1).$$

Such $\tilde{P}$ exists because $m \notin \mathcal{M}^*$, so that $\tilde{P}$ need not satisfy assumptions I or II for m. Since

$$\mathbb{E}_{\tilde{P}}[\Pi_m] = \min_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1),$$

we have that $\underline{\Pi}_m = \min_{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y}} b(y_1) - b(y_0) - w(y_1)$. ■

PROOF OF PROPOSITION 3 First, let $m \in \mathcal{M}^*$, fix $P \in \mathcal{P}^{nx}$ and notice that

$$\begin{aligned}\mathbb{E}_P[\Pi_m^{nx}] &\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m(w(Y_m) + C_m^p - w(Y^0))] \\ &\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m(w(Y_m) + \gamma \cdot C_m^a - w(Y^0))] \\ &\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - (1+\gamma)D_m(w(Y_m) - w(Y^0))],\end{aligned}$$

where the inequalities follow because P satisfies assumptions I, IV and III for m, respectively. If $P(D_m = 1) = 0$, then $\mathbb{E}_P[\Pi_m^{nx}] \geq 0$. Otherwise,

$$\begin{aligned}
\mathbb{E}_P[\Pi_m^{nx}] &\geq P(D_m = 1)\mathbb{E}_P[b(Y_m) - b(Y^0) - (1+\gamma)(w(Y_m) - w(Y^0)) \mid D_m = 1] \\
&\geq P(D_m = 1) \min_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\gamma)[w(y_1) - w(y_0)] \\
&\geq \min_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\gamma)[w(y_1) - w(y_0)],
\end{aligned}$$

since P satisfies Assumption II for m. Next, consider a probability measure $\tilde{P} \in \mathcal{P}$ such that

$$\tilde{P}\left(D_{\tilde{m}} = 1, R_{\tilde{m}} = D_{\tilde{m}} w(Y^0), C_{\tilde{m}} = D_{\tilde{m}}(w(Y_{\tilde{m}}^1) - w(Y^0)), Y_{\tilde{m}}^1 = \underline{y}_1, Y^0 = \underline{y}_0\right) = 1,$$

where

$$(\underline{y}_0, \underline{y}_1) \in \underset{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1}{\arg\min} b(y_1) - b(y_0) - (1+\gamma)[w(y_1) - w(y_0)]$$

for all $\tilde{m} \in \mathcal{M}$. Clearly, $\tilde{P}$ satisfies assumptions I, II and III for all $\tilde{m} \in \mathcal{M}^*$, so that $\tilde{P} \in \mathcal{P}^{nx}$. Hence,

$$\underline{\Pi}_m^{nx} = \min_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\gamma)[w(y_1) - w(y_0)].$$

Next, consider some $m \in \mathcal{M} \setminus \mathcal{M}^*$. We will show that $\{\mathbb{E}_P[\Pi_m^{nx}] : P \in \mathcal{P}^*\}$ does not have a lower bound, i.e. for all $c \in \mathbb{R}$, there exists $P \in \mathcal{P}^*$ such that $\mathbb{E}_P[\Pi_m^{nx}] < c$. Given $c \in \mathbb{R}$, let $P_c \in \mathcal{P}$ be such that

$$P_c\left(D_{\tilde{m}} = 1, R_{\tilde{m}} = D_{\tilde{m}} w(Y^0), C_{\tilde{m}} = D_{\tilde{m}}(w(Y_{\tilde{m}}^1) - w(Y^0)), Y_{\tilde{m}}^1 = \underline{y}_1, Y^0 = \underline{y}_0\right) = 1$$

for all $\tilde{m} \in \mathcal{M}^*$, and

$$P_c\left(D_{\tilde{m}} = 1, R_{\tilde{m}} = D_{\tilde{m}} w(Y^0), C_{\tilde{m}} = D_{\tilde{m}} \cdot k_c, Y_{\tilde{m}}^1 = \underline{y}_1, Y^0 = \underline{y}_0\right) = 1$$

for all $\tilde{m} \in \mathcal{M} \setminus \mathcal{M}^*$, where $k_c > \gamma^{-1}[b(\underline{y}_1) - b(\underline{y}_0) - (w(\underline{y}_1) - w(\underline{y}_0))]$. Clearly, $\tilde{P} \in \mathcal{P}^{nx}$, yet $\mathbb{E}_{\tilde{P}}[\Pi_m^{nx}] < c$.

To prove the second claim, notice that necessity follows immediately because $\underline{\Pi}_m^{nx} \leq 0$ and the premise implies that $\underline{\Pi}_m^{nx} \geq 0$. For sufficiency, notice that, $m \in \mathcal{M} \setminus \mathcal{M}^*$ implies $\underline{\Pi}_m^{nx} < 0$. Hence, $m \in \mathcal{M}^*$. But then,

$$\min_{\{(y_0,y_1) \in \mathcal{Y} \times \mathcal{Y} : y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\gamma)[w(y_1) - w(y_0)] = 0$$

implies that $b(y_1) - b(y_0) - (1+\gamma)[w(y_1) - w(y_0)] \geq 0$ for all $y_0, y_1 \in \mathcal{Y}$ such that $y_0 \leq y_1$. ■

PROOF OF PROPOSITION 4 The proof mirrors that of Lemmas 1 and 2 and Theorem 1. First, let $m \in \mathcal{M}^*$. Following the proof of Lemma 1, one can obtain that, for all $P \in \mathcal{P}^*$,

$$\begin{aligned}\mathbb{E}_P[\Pi_m^{cf}] &\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - (1+\lambda)D_m[w(Y_m) - w(Y^0)]] \\ &\geq \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\lambda)[w(y_1) - w(y_0)]\end{aligned}$$

and that $\mathbb{E}_{\tilde{P}}[\Pi_m^{cf}] = \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\lambda)[w(y_1) - w(y_0)]$ for some $\tilde{P} \in \mathcal{P}^*$, so that

$$\underline{\Pi}_m^{cf} = \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\lambda)[w(y_1) - w(y_0)].$$

Similarly, one can follow the proof of Lemma 2 to find that for $m \in \mathcal{M} \setminus \mathcal{M}^*$,

$$\underline{\Pi}_m^{cf} = \min_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - (1+\lambda)w(y_1).$$

To prove the second claim, notice that necessity follows immediately because $\underline{\Pi}_m^{cf} \leq 0$ and the premise implies that $\underline{\Pi}_m^{cf} \geq 0$. For sufficiency, notice that, $m \in \mathcal{M} \setminus \mathcal{M}^*$ implies $\underline{\Pi^{cf}}_m < 0$. Hence, $m \in \mathcal{M}^*$. But then,

$$\min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - (1+\lambda)[w(y_1) - w(y_0)] = 0$$

implies that $b(y_1) - b(y_0) - (1+\lambda)[w(y_1) - w(y_0)] \geq 0$ for all $y_0, y_1 \in \mathcal{Y}$ such that $y_0 \leq y_1$. ■

PROOF OF LEMMA 3 If $m \in \mathcal{M} \setminus \mathcal{M}^{fc}$, one can proceed as in Lemma 2 to find that

$$\underline{\Pi}_m^{fc} = \min_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - w(y_1).$$

Otherwise, for any $P \in \mathcal{P}^{fc}$ we have that:

$$\begin{aligned}\mathbb{E}_P[\Pi_m] &= \mathbb{E}_P[b(Y_m) - b(Y^0) - [W_m - R_m]] \\ &= \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m w(Y_m)],\end{aligned}$$

since $P(R_m = 0) = 1$.

By the potential outcomes model (1), if $P(D_m = 1) = 0$, then

$$\mathbb{E}_P[\Pi_m] = \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m w(Y_m) \mid D_m = 0] = \mathbb{E}_P[b(Y^0) - b(Y^0) \mid D_m = 0] = 0.$$

Otherwise,

$$
\begin{aligned}
\mathbb{E}_P[\Pi_m] &= P(D_m = 1)\mathbb{E}_P[b(Y^1_m) - b(Y^0) - w(Y^1_m) \mid D_m = 1] \\
&\geq P(D_m = 1) \min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - w(y_1)],
\end{aligned}
$$

where the inequality holds because P satisfies Assumption II and b is strictly increasing, and the minimum exists because $\mathcal{Y}$ is compact and b and w are continuous.

Since

$$
\min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - w(y_1) < 0,
$$

we conclude that

$$
\mathbb{E}_P[\Pi_m] \geq \min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)].
$$

Next, consider $\tilde{P} \in \mathcal{P}$ such that $\tilde{P}(D_{\tilde{m}} = 1, R_{\tilde{m}} = D_{\tilde{m}} w(Y^0), Y^1_{\tilde{m}} = \underline{y}_1, Y^0 = \underline{y}_0) = 1$ for all $\tilde{m} \in \mathcal{M}$, where

$$
\left(\underline{y}_1, \underline{y}_0\right) \in \operatorname*{arg\,min}_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - w(y_1).
$$

Clearly, $\tilde{P}$ satisfies Assumption II for all contract-auction pairs in $\mathcal{M}^{fc}$, so $\tilde{P} \in \mathcal{P}^{fc}$. Moreover,

$$
\mathbb{E}_{\tilde{P}}[\Pi_m] = b(\underline{y}_1) - b(\underline{y}_0) - w(\underline{y}_1).
$$

It follows that

$$
\underline{\Pi}^{fc}_m = \min_{\{(y_0,y_1)\in\mathcal{Y}^2 : y_0 \leq y_1\}} b(y_1) - b(y_0) - w(y_1).
$$

■

PROOF OF LEMMA 4 The proof mirrors that of Lemma 3. If $m \in \mathcal{M} \setminus \mathcal{M}^{fc}$, one can proceed as in Lemma 2 to find that

$$
\underline{\Pi}^{fc}_m(c) = \min_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - w(y_1).
$$

Otherwise, for any $P \in \mathcal{P}^{fc}(c)$ we have that:

$$
\mathbb{E}_P[\Pi_m] = \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m w(Y_m)],
$$

since $P(R_m = 0) = 1$.

By the potential outcomes model (1), if $P(D_m = 1) = 0$, then

$$\mathbb{E}_P[\Pi_m] = \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m w(Y_m) \mid D_m = 0] = \mathbb{E}_P[b(Y^0) - b(Y^0) \mid D_m = 0] = 0.$$

Otherwise,

$$\begin{aligned}\mathbb{E}_P[\Pi_m] &= P(D_m = 1)\mathbb{E}_P[b(Y_m^1) - b(Y^0) - w(Y_m^1) \mid D_m = 1] \\ &\geq P(D_m = 1) \min_{\{(y_0,y_1)\in\mathcal{Y}^2 : b(y_1)-b(y_0)\geq c\}} b(y_1) - b(y_0) - w(y_1),\end{aligned}$$

where the inequality holds because $P(b(Y_m^1) - b(Y^0) \geq c) = 1$ and the minimum exists because $\mathcal{Y}$ is compact and b and w are continuous. Hence,

$$\mathbb{E}_P[\Pi_m] \geq \min\left\{0, \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y} : b(y_1)-b(y_0)\geq c\}} b(y_1) - b(y_0) - w(y_1)\right\}.$$

Moreover, there exists $P' \in \mathcal{P}$ such that $P'(D_{\tilde{m}} = 0, b(Y_{\tilde{m}}^1) - b(Y^0) \geq c, R_{\tilde{m}} = 0) = 1$ for all $\tilde{m} \in \mathcal{M}$. Clearly, $P' \in \mathcal{P}^{cf}(c)$ and $\mathbb{E}_P'[\Pi_m] = 0$.

Finally, consider $\tilde{P} \in \mathcal{P}$ such that $\tilde{P}(D_{\tilde{m}} = 1, R_{\tilde{m}} = 0, Y_{\tilde{m}}^1 = \underline{y}_1, Y^0 = \underline{y}_0) = 1$ for all $\tilde{m} \in \mathcal{M}$, where

$$\left(\underline{y}_1, \underline{y}_0\right) \in \operatorname*{arg\,min}_{\{(y_0,y_1)\in\mathcal{Y}^2 : b(y_1)-b(y_0)\geq c\}} b(y_1) - b(y_0) - w(y_1).$$

Clearly, $\mathbb{E}_{\tilde{P}}[b(Y_m^1) - b(Y^0)] \geq c$, so $\tilde{P} \in \mathcal{P}^{fc}$. Moreover,

$$\mathbb{E}_{\tilde{P}}[\Pi_m] = b(\underline{y}_1) - b(\underline{y}_0) - w(\underline{y}_1).$$

We conclude that

$$\underline{\Pi}_m^{fc}(c) = \min\left\{0, \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y} : b(y_1)-b(y_0)\geq c\}} b(y_1) - b(y_0) - w(y_1)\right\}.$$

■

PROOF OF PROPOSITION 5 Notice that, because b is strictly increasing,

$$\begin{aligned}&\min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y} : b(y_1)-b(y_0)\geq c\}} b(y_1) - b(y_0) - w(y_1) \\ =& \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y} : b(y_1)-b(y_0)= c\}} b(y_1) - b(y_0) - w(y_1) \\ =& \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y} : b(y_1)-b(y_0)= c\}} c - w(y_1).\end{aligned}$$

Therefore, by Lemma 4,

$$
\begin{aligned}
\underline{\Pi}_m^{fc}(c) = 0 &\iff \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: b(y_1)-b(y_0)\geq c\}} b(y_1) - b(y_0) - w(y_1) \geq 0 \\
&\iff \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: b(y_1)-b(y_0)= c\}} c - w(y_1) \geq 0 \\
&\iff w(y) \leq c \text{ for all } y \in \mathcal{Y} \text{ such that } b(y) \geq c + b(\underline{y}).
\end{aligned}
$$

PROOF OF LEMMA 5 Let $m \in \mathcal{M}^{mb}$. For any $P \in \mathcal{P}^{mb}$, it follows that:

$$
\begin{aligned}
\mathbb{E}_P[\Pi_m] &= \mathbb{E}_P[b(Y_m) - b(Y^0) - \max\{0,\ W_m - R_m\}] \\
&= \mathbb{E}_P[b(Y_m) - b(Y^0) - \max\{0,\ D_m w(Y_m) - R_m\}] \\
&\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - \max\{0,\ D_m(w(Y_m) - w(Y^0))\}] \\
&= \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m[w(Y_m) - w(Y^0)]],
\end{aligned}
$$

where the inequality holds because P satisfies Assumption I' for m, and the last equality holds because P satisfies Assumption II and w is non-decreasing. At this stage, we can proceed as in Lemma 1 to find that

$$
\underline{\Pi}_m = \min_{\{(y_0,y_1)\in\mathcal{Y}^2: y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)].
$$

Now consider $m \in \mathcal{M} \setminus \mathcal{M}^{mb}$. For any $P \in \mathcal{P}^*$, we have that

$$
\begin{aligned}
\mathbb{E}_P[\Pi_m] &= \mathbb{E}_P[b(Y_m) - b(Y^0) - \max\{0,\ W_m - R_m\}] \\
&\geq \mathbb{E}_P[b(Y_m) - b(Y^0) - \max\{0,\ W_m\}] \\
&= \mathbb{E}_P[b(Y_m) - b(Y^0) - D_m w(Y_m)],
\end{aligned}
$$

where the inequality holds because R_m is non-negative, and the last equality follows because w is non-negative. Here, we can proceed as in Lemma 2 to find that

$$
\underline{\Pi}_m = \min_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - w(y_1).
$$

$\blacksquare$

PROOF OF PROPOSITION 6 The proof mirrors that of Theorem 1.

PROOF OF PROPOSITION 7 First, let $m \in \mathcal{M}^*$. Following the proof of Lemma 1, one can obtain that, for all $P \in \mathcal{P}^*$,

$$\begin{aligned}\mathbb{E}_P[\Pi_m^{pc}] &= b(Y_m) - b(Y^0) - [W_m^{pc} - R_m] \\ &\geq b(Y_m) - b(Y^0) - [W_m - R_m] \\ &\geq \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)]\end{aligned}$$

and that $\mathbb{E}_{\tilde{P}}[\Pi_m^{pc}] = \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)]$ for some $\tilde{P} \in \mathcal{P}^*$, so that

$$\underline{\Pi}_m^{pc} = \min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)].$$

Similarly, one can follow the proof of Lemma 2 to find that for $m \in \mathcal{M} \setminus \mathcal{M}^*$,

$$\underline{\Pi}_m^{pc} = \min_{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}} b(y_1) - b(y_0) - w(y_1).$$

To prove the second claim, notice that necessity follows immediately because $\underline{\Pi}_m^{pc} \leq 0$ and the premise implies that $\underline{\Pi}_m^{pc} \geq 0$. For sufficiency, notice that, $m \in \mathcal{M} \setminus \mathcal{M}^*$ implies $\underline{\Pi}_m^{pc} < 0$. Hence, $m \in \mathcal{M}^*$. But then,

$$\min_{\{(y_0,y_1)\in\mathcal{Y}\times\mathcal{Y}: y_0 \leq y_1\}} b(y_1) - b(y_0) - [w(y_1) - w(y_0)] = 0$$

implies that $b(y_1) - b(y_0) - [w(y_1) - w(y_0)] \geq 0$ for all $y_0, y_1 \in \mathcal{Y}$ such that $y_0 \leq y_1$. ■

B Optimality of Linear Contracts

In this section, we show that linear contract-auction pairs are maximin optimal under the following relaxation of Assumption II:

Assumption V. $\mathbb{E}_P[b(Y_m) - b(Y^0)] \geq 0$.

Assumption V requires that the principal's benefit from the outcome under m exceed her benefit from the counterfactual, baseline outcome in expectation, rather than almost surely. It is a weaker assumption than Assumption II: if $P \in \mathcal{P}$ satisfies Assumption II for $m \in \mathcal{M}$, then it satisfies Assumption V for m, but not vice versa.

Thus, the principal imposes Assumptions I and V for contract-auction pairs in $\mathcal{M}^*$. The resulting set of admissible probability measures is $\mathcal{P}'$, where

$$\mathcal{P}' \equiv \{P \in \mathcal{P} : P \text{ satisfies Assumptions I and V for all } m \in \mathcal{M}^*\},$$

and her payoff guarantee from contract-auction pair m equals $\underline{\Pi}'_m$, given by

$$\underline{\Pi}'_m = \begin{cases} \inf_{P \in \mathcal{P}'} \mathbb{E}_P[\Pi_m] & \text{if } \{\mathbb{E}_P[\Pi_m] : \; P \in \mathcal{P}'\} \text{ has a lower bound} \\ -\infty & \text{otherwise.} \end{cases}$$

Proposition 8 establishes that under the relaxation of Assumption II, linear contract-auction pairs are still maximin optimal.

Proposition 8. *Suppose there is some contract-auction pair $m' = (w', a') \in \mathcal{M}^*$ such that $w'(y) = \alpha \cdot b(y)$, where $\alpha \in [0, 1]$. Then, the maximum of $\{\underline{\Pi}'_m : \; m \in \mathcal{M}\}$ exists, equals zero and is attained by m'.*

Proof. The proof involves two simple steps. First, we show that $\underline{\Pi}_m \leq 0$ for all $m \in \mathcal{M}$. Then, we prove that $\mathbb{E}_P[\Pi_{m'}] \geq 0$ for all $P \in \mathcal{P}'$ with equality for some P. These findings imply that $\max_{m \in \mathcal{M}} \underline{\Pi}_m$ exists, equals zero, and is attained by m'.

Consider any contract-auction pair $m \in \mathcal{M}$ and let $P \in \mathcal{P}$ be such that $P(D_{\tilde{m}} = 0, R_{\tilde{m}} = 0, Y^1_{\tilde{m}} - Y^0 > 0) = 1$ for all $\tilde{m} \in \mathcal{M}$. By the potential outcomes model (1), $P(\Pi_m = 0) = 1$, so that $\mathbb{E}_P[\Pi_m] = 0$. Since $\mathcal{M}^* \subseteq \mathcal{M}$, P satisfies Assumptions I and V for all contract-auction pairs in $\mathcal{M}^*$. Hence, $P \in \mathcal{P}'$ and we conclude that $\underline{\Pi}_m \leq 0$.

Next, consider contract-auction pair m'. For any $P \in \mathcal{P}'$, it follows that:

$$\begin{aligned} \mathbb{E}_P[\Pi_{m'}] &= \mathbb{E}_P[b(Y_{m'}) - b(Y^0) - [W_{m'} - R_{m'}]] \\ &\geq \mathbb{E}_P[b(Y_{m'}) - b(Y^0) - D_{m'}[w(Y_{m'}) - w(Y^0)]] \\ &= \mathbb{E}_P[(1 - \alpha D_{m'})(b(Y_{m'}) - b(Y^0))]. \end{aligned}$$

The inequality holds because P satisfies Assumption I. If $P(D_{m'} = 1) = 0$, it follows that:

$$\begin{aligned} \mathbb{E}_P[\Pi_{m'}] &\geq \mathbb{E}_P[b(Y_{m'}) - b(Y^0) \,|\, D_{m'} = 0] \\ &= \mathbb{E}_P[b(Y^0) - b(Y^0) \,|\, D_{m'} = 0] \\ &= 0, \end{aligned}$$

by the potential outcomes model (1). Otherwise, we obtain

$$\begin{aligned} \mathbb{E}_P[\Pi_{m'}] &\geq (1 - \alpha) P(D_{m'} = 1) \mathbb{E}_P[b(Y_{m'}) - b(Y^0) \,|\, D_{m'} = 1] \\ &= (1 - \alpha) \mathbb{E}_P[b(Y_{m'}) - b(Y^0)] \\ &\geq 0, \end{aligned}$$

where the last inequality holds because P satisfies Assumption V and because $\alpha \in [0, 1]$. Finally, as in our previous argument, there exists $\hat{P} \in \mathcal{P}'$ such that $\hat{P}(Y_{m'} = Y^0) = 1$ and

$\mathbb{E}_P[R_{m'} - D_{m'} \cdot w(Y^0)] = 0$, so that $\mathbb{E}_{\hat{P}}[\Pi_{m'}] = 0$. ■

Proposition 8 is useful for the principal in situations where she considers Assumption II too stringent but is willing to assume that contract-auction pairs induce outcome changes that are beneficial to her, on average.