Exam Practice

Answer Set Solving in Practice

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Date: ...

You have ... minutes to do the exam.
To pass the exam you must get at least $\boxed{\ldots}$ out of ... possible points.

All **non electronic** support material can be used.

Name: **Matriculation Number:**

Points: (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) **Total:**

Important: All extra pages must have your name

Exercise 1 (5 + 5 = 10 Points)

Find the stable models of the following logic programs P.

1-a) $P = \left\{ \begin{array}{l} a \leftarrow b \\ a \leftarrow \neg b, c \\ b \leftarrow \neg c \\ c \leftarrow \neg b, d \\ d \leftarrow a, c \\ d \leftarrow \neg a \end{array} \right\}$

1-b) $P = \left\{ \begin{array}{l} a \leftarrow \neg c \\ b \leftarrow a, \neg d \\ b \leftarrow \neg a, \neg c \\ c \leftarrow \neg b, a \\ c \leftarrow \neg d \\ d \leftarrow \neg c, a \end{array} \right\}$

Exercise 2 (5 + 5 = 10 Points)

Find the stable models of the following logic programs with variables P.

2-a) $$P = \left\{ \begin{array}{r@{\;}l} p(a) & \leftarrow \\ p(b) & \leftarrow \\ p(c) & \leftarrow \\ q(c) & \leftarrow \\ r(X) & \leftarrow p(X), \neg q(X) \\ s(X,Y) & \leftarrow r(X), r(Y) \\ t(X) & \leftarrow s(X,Y), \neg t(Y), \neg eq(X,Y) \\ t(X) & \leftarrow s(Y,X), \neg t(Y), \neg eq(Y,X) \\ eq(X,X) & \leftarrow \end{array} \right\}$$

2-b) $$P = \left\{ \begin{array}{r@{\;}l} p(a,b,c) & \leftarrow \\ p(X,X,X) & \leftarrow \\ q(X,Y) & \leftarrow p(X,Y,Z), \neg p(Y,Y,Y) \\ s(X,X) & \leftarrow \neg q(X,X) \end{array} \right\}$$

Exercise 3 (5 + 5 = 10 Points)

Consider the following logic program P:

$$P = \left\{ \begin{array}{r} b \leftarrow \neg a \\ \{c\} \leftarrow b \\ \{d\} \leftarrow \neg c \\ e \leftarrow 1\{\neg c, \neg d\} \\ \leftarrow \neg e \end{array} \right\}$$

3-a) Compile P into a normal logic program P' using the translations from the lecture slides. In particular, use the $x(i,j)$ construction to translate the cardinality rule of the program.

3-b) Determine the stable models of P and the corresponding stable models of P'.

Exercise 4 (4 + 4 + 2 = 10 Points)

Consider the following cardinality constraint in the head of a rule: $1\,\{a, \neg b, c\}\,1$.

4-a) Compile the cardinality constraint into cardinality rules of the form

$$a_0 \leftarrow l\,\{a_1, \ldots, a_m, \neg a_{m+1}, \ldots, \neg a_n\}.$$

along with normal and choice rules as well as integrity constraints.

4-b) Compile the logic program P resulting from the previous subtask into a program P' with normal and choice rules as well as integrity constraints only, using the $x(i, j)$ construction from the lecture slides.

4-c) Determine the stable models of P and the corresponding stable models of P'.

Exercise 5 (5 + 10 = 15 Points)

For the following logic program P and set of facts I:

- Find the dependency graph G_P, the positive dependency graph G_P^+, and one topological order L_P. Moreover, given L_P, determine the set of atoms R_r for every rule $r \in P$.
- Find the ground instantiation of P and I. Start by initializing the sets of true atoms F and possible atoms D to I. Then, ground successively the components of L_P applying on-the-fly simplifications. For this, use the sets F, D and R_r, and update F and D after grounding each component.

$$P = \left\{ \begin{array}{rl} \mathit{holds}(F,V) & \leftarrow \mathit{fluent}(F), \mathit{opp}(V,W), \neg \mathit{holds}(F,W) \\ \{\mathit{occ}(A)\} & \leftarrow \mathit{action}(A) \\ \mathit{next}(F,V) & \leftarrow \mathit{occ}(A), \mathit{eff}(A,F,V) \\ & \leftarrow \mathit{occ}(A), \mathit{pre}(A,F,V), \neg \mathit{holds}(F,V) \\ & \leftarrow \mathit{goal}(F,V), \neg \mathit{next}(F,V) \end{array} \right\}$$

$$I = \left\{ \begin{array}{rlrl} \mathit{opp}(t,f) & \leftarrow & \mathit{opp}(f,t) & \leftarrow \\ \mathit{fluent}(g) & \leftarrow & \mathit{fluent}(h) & \leftarrow \\ \mathit{action}(a) & \leftarrow & \mathit{action}(b) & \leftarrow \\ \mathit{pre}(a,g,t) & \leftarrow & \mathit{pre}(b,h,t) & \leftarrow \\ \mathit{eff}(a,h,f) & \leftarrow & \mathit{eff}(b,g,f) & \leftarrow \\ \mathit{goal}(g,f) & \leftarrow & \mathit{goal}(h,f) & \leftarrow \\ \mathit{occ}(a) & \leftarrow & & \end{array} \right\}$$

Exercise 6 (10 Points)

Determine the stable models of the following normal logic program P using the simplistic solving algorithm of the lecture, and specify the sets L and U that are generated at each iteration inside the $expand_P$ procedure.

6-a) To be added.

Exercise 7 (5 + 5 = 10 Points)

Let P be the following normal logic program:

$$P = \left\{ \begin{array}{lll} a \leftarrow \neg g, b & b \leftarrow a, \neg d & b \leftarrow a, e \\ c \leftarrow \neg a, \neg g & d \leftarrow c, \neg f & e \leftarrow a, b \\ e \leftarrow & f \leftarrow e, \neg g & f \leftarrow a, \neg f \end{array} \right\}$$

7-a) Find the Fitting Semantics of P.

7-b) Find the Wellfounded Semantics of P.

Exercise 8 (5 + 5 = 10 Points)

Find all the unfounded sets of the following normal programs P with respect to the partial interpretation $\langle \{c\}, \{d\} \rangle$.

8-a) $$P = \left\{ \begin{array}{l} a \leftarrow b, \neg d \\ b \leftarrow a, c \\ b \leftarrow \neg c \\ c \leftarrow d \\ c \leftarrow b \\ d \leftarrow \neg a \end{array} \right\}$$

8-b) $$P = \left\{ \begin{array}{l} a \leftarrow b, \neg c \\ a \leftarrow c \\ b \leftarrow c \\ c \leftarrow \neg a, b \\ d \leftarrow \neg a, \neg b \\ d \leftarrow \neg b, c \end{array} \right\}$$

Exercise 9 (8 + 4 + 3 = 15 Points)

Let P be the following normal logic program:

$$P = \left\{ \begin{array}{lll} a \leftarrow c & b \leftarrow c, a & c \leftarrow b, \neg d \\ a \leftarrow \neg d & b \leftarrow \neg e & c \leftarrow b, f \\ d \leftarrow \neg e, b & e \leftarrow f, \neg d & f \leftarrow a \end{array} \right\}$$

9-a) Find all models of $CF(P)$.

9-b) Write the loop formulas in $LF(P)$.

9-c) Find all the stable models of P.

Exercise 10 (10 + 5 = 15 Points)

Let P be the following logic program:

$$P = \left\{ \begin{array}{lll} a \leftarrow \neg b & b \leftarrow \neg a, c & c \leftarrow \neg a, \neg e \\ c \leftarrow \neg b, g & d \leftarrow a, c & e \leftarrow \neg b, f \\ f \leftarrow \neg d, \neg e & g \leftarrow c, \neg f & g \leftarrow \neg d, \neg e \end{array} \right\}$$

The following table represents an assignment A with the decision level dl of every literal. Decision literals are placed under σ_d, and literals under $\overline{\sigma}$ are implied by a noogod $\delta \in \Delta_P \cup \Lambda_P$:

dl	σ_d	$\overline{\sigma}$	δ
1	$\mathbf{F}\{\neg b, f\}$		
		$\mathbf{F}e$	$\{\mathbf{T}e, \mathbf{F}\{\neg b, f\}\}$
2	$\mathbf{T}g$		
3	$\mathbf{T}\{\neg b\}$		

10-a) Find a conflict nogood ε with $\mathsf{NogoodPropagation}(P,\emptyset,A)$.

10-b) Derive a conflict nogood δ with the FirstUIP method starting from ε.

Solution 1

1-a) $X_1 = \{a, b\}$

1-b) $X_1 = \{c\}$

Solution 2

2-a) $X_1 = \{p(a), p(b), p(c), q(c), r(a), r(b), s(a, a), s(b, a), s(a, b), s(b, b), eq(a, a), eq(b, b), eq(c, c), t(b)\}$
$X_2 = \{p(a), p(b), p(c), q(c), r(a), r(b), s(a, a), s(b, a), s(a, b), s(b, b), eq(a, a), eq(b, b), eq(c, c), t(a)\}$

2-b) $X_1 = \{p(a, b, c), p(a, a, a), p(b, b, b), p(c, c, c), s(a, a), s(b, b), s(c, c)\}$

Solution 3

To be added.

Solution 4

To be added.

Solution 5

To be added.

Solution 6

To be added.

Solution 7

7-a) $\langle\{e, f\}, \{d, g\}\rangle$

7-b) $\langle\{c, e, f\}, \{a, b, d, g\}\rangle$

Solution 8

8-a) $U_1 = \emptyset$ $\quad U_2 = \{a, b\}$ $\quad U_2 = \{b, c\}$ $\quad U_3 = \{a, b, c\}$

8-b) $U_1 = \emptyset$ $\quad U_3 = \{b, c\}$ $\quad U_4 = \{a, b, c\}$

Solution 9

9-a) $X_1 = \{a, e, f\}$ $\quad X_2 = \{b, d\}$ $\quad X_3 = \{a, b, c, d, f\}$ $\quad X_4 = \{a, b, c, e, f\}$

9-b) $$LF(P) = \left\{ \begin{array}{c} a \vee b \vee c \rightarrow \neg d \vee \neg e \\ a \vee c \vee f \rightarrow \neg d \vee (b \wedge \neg d) \\ b \vee c \rightarrow \neg e \\ a \vee b \vee c \vee f \rightarrow \neg d \vee \neg e \end{array} \right\}$$

9-c) $X_1 = \{a, e, f\}$ $\quad X_2 = \{b, d\}$

Solution 10

10-a)

dl	σ_d	$\overline{\sigma}$	δ
1	$\mathbf{F}\{\neg b, f\}$		
		$\mathbf{F}e$	$\{\mathbf{T}e, \mathbf{F}\{\neg b, f\}\}$
2	$\mathbf{T}g$		
3	$\mathbf{T}\{\neg b\}$		
		$\mathbf{F}b$	$\{\mathbf{T}b, \mathbf{T}\{\neg b\}\}$
		$\mathbf{T}a$	$\{\mathbf{F}a, \mathbf{T}\{\neg b\}\}$
		$\mathbf{T}\{\neg b, g\}$	$\{\mathbf{F}\{\neg b, g\}, \mathbf{F}b, \mathbf{T}g\}$
		$\mathbf{T}c$	$\{\mathbf{F}c, \mathbf{T}\{\neg b, g\}\}$
		$\mathbf{T}\{a, c\}$	$\{\mathbf{F}\{a, c\}, \mathbf{T}a, \mathbf{T}c\}$
		$\mathbf{T}d$	$\{\mathbf{F}d, \mathbf{T}\{a, c\}\}$
		$\mathbf{F}\{\neg d, \neg e\}$	$\{\mathbf{T}\{\neg d, \neg e\}, \mathbf{T}d\}$
		$\mathbf{F}f$	$\{\mathbf{T}f, \mathbf{F}\{\neg d, \neg e\}\}$
		$\mathbf{F}\{\neg a, c\}$	$\{\mathbf{T}\{\neg a, c\}, \mathbf{F}b\}$
		$\mathbf{F}\{\neg a, \neg e\}$	$\{\mathbf{T}\{\neg a, \neg e\}, \mathbf{T}a\}$
		$\mathbf{T}\{c, \neg f\}$	$\{\mathbf{F}\{c, \neg f\}, \mathbf{T}c, \mathbf{F}f\}$
			$\{\mathbf{T}g, \mathbf{F}\{\neg d, \neg e\}, \mathbf{F}\{\neg a, \neg e\}\}$

10-b)

δ	ε	δ'
$\{\mathbf{T}g, \underline{\mathbf{F}\{\neg d, \neg e\}}, \underline{\mathbf{F}\{\neg a, \neg e\}}\}$	$\{\underline{\mathbf{T}\{\neg a, \neg e\}}, \underline{\mathbf{T}a}\}$	$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{F}\{\neg d, \neg e\}}\}$
$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{F}\{\neg d, \neg e\}}\}$	$\{\underline{\mathbf{T}\{\neg d, \neg e\}}, \underline{\mathbf{T}d}\}$	$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}d}\}$
$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}d}\}$	$\{\underline{\mathbf{F}d}, \underline{\mathbf{T}\{a, c\}}\}$	$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}\{a, c\}}\}$
$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}\{a, c\}}\}$	$\{\underline{\mathbf{F}\{a, c\}}, \underline{\mathbf{T}a}, \underline{\mathbf{T}c}\}$	$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}c}\}$
$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}c}\}$	$\{\underline{\mathbf{F}c}, \underline{\mathbf{T}\{\neg b, g\}}\}$	$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}\{\neg b, g\}}\}$
$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{T}\{\neg b, g\}}\}$	$\{\underline{\mathbf{F}\{\neg b, g\}}, \underline{\mathbf{F}b}, \mathbf{T}g\}$	$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{F}b}\}$
$\{\mathbf{T}g, \underline{\mathbf{T}a}, \underline{\mathbf{F}b}\}$	$\{\underline{\mathbf{F}a}, \underline{\mathbf{T}\{\neg b\}}\}$	$\{\mathbf{T}g, \underline{\mathbf{T}\{\neg b\}}, \underline{\mathbf{F}b}\}$
$\{\mathbf{T}g, \underline{\mathbf{T}\{\neg b\}}, \underline{\mathbf{F}b}\}$	$\{\underline{\mathbf{T}b}, \underline{\mathbf{T}\{\neg b\}}\}$	$\{\mathbf{T}g, \underline{\mathbf{T}\{\neg b\}}\}$
$\underline{\underline{\{\mathbf{T}g, \underline{\mathbf{T}\{\neg b\}}\}}}$		