Exam

Answer Set Solving in Practice

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You have 90 minutes to do the exam.
To pass the exam you must get at least $\boxed{28}$ out of 90 possible points.

All **non-electronic** (and non-alive) support materials can be used.

First Name: **Last Name:**

Points: (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) **Total:**

EXAM NUMBER: ....

Exercise 1 (8 Points)

Determine the <u>models</u> of the following logic program P and decide which models are <u>stable</u>:

$$P = \left\{ \begin{array}{l} a \leftarrow c, d \\ a \leftarrow g, f \\ b \leftarrow \\ c \leftarrow a, f \\ e \leftarrow b \\ d \leftarrow a, g \\ d \leftarrow e, b \\ g \leftarrow b, d \\ g \leftarrow c, b \end{array} \right\}$$

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Exercise 2 (8 Points)

Determine the stable models of the following logic program P:

$$P = \left\{ \begin{array}{r@{\;}l} \mathit{coffee} \leftarrow & \sim \mathit{icecream} \\ \mathit{tee} \leftarrow & \sim \mathit{coffee} \\ \mathit{icecream} \leftarrow & \sim \mathit{tee}, \mathit{sunny} \\ \mathit{rainy} \leftarrow & \sim \mathit{sunny} \\ \mathit{sunny} \leftarrow & \sim \mathit{rainy} \end{array} \right\}$$

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Exercise 3 (6 Points)

Determine the stable models of the following extended logic program P:

$$P = \left\{ \ 1\,\{a, b\}. \qquad \{a, b, c\}\,2 \leftarrow 2\,\{a, b, d\}. \qquad d \leftarrow 1\,\{a, c\}\,1. \ \right\}$$

Exercise 4 (8 Points)

Determine the stable models of the following logic program P:

$$P = \left\{ \begin{array}{l} a(0,1) \leftarrow \\ a(0,2) \leftarrow \\ a(1,2) \leftarrow \\ b(X) \leftarrow a(X,Y), \sim r(X,Y) \\ b(Y) \leftarrow r(X,Y) \\ r(X,Y) \leftarrow a(X,Y), \sim b(X) \\ r(X,Y) \leftarrow a(X,Y), a(Y,Z), b(Z) \end{array} \right\}$$

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Exercise 5 (10 Points)

Given an undirected graph, the Unblocked Set problem is to select a subset of vertices such that each selected vertex is adjacent to some remaining (unselected) vertex, where the number of selected vertices must be equal or greater than a given threshold. For example, the graph $(\{1,2,3,4,5,6\},\{\{1,2\},\{1,3\},\{2,4\},\{3,5\},\{4,5\},\{4,6\}\})$ has two unblocked sets with 4 vertices (at least): $\{1,2,5,6\}$ and $\{2,3,5,6\}$. We represent such a problem instance by facts as follows:

```
vertex(1). vertex(2). vertex(3). vertex(4). vertex(5). vertex(6).
edge(1,2). edge(1,3). edge(2,4). edge(3,5). edge(4,5). edge(4,6).
threshold(4).

% Solutions: {select(1),select(2),select(5),select(6)},
%            {select(2),select(3),select(5),select(6)}
```

Specify a uniform problem encoding such that atoms over the predicate `select/1` within stable models correspond to unblocked sets for arbitrary instances.

Exercise 6 (6 Points)

Consider the following logic program P:

$$P = \left\{ \begin{array}{r@{}l} above(a,b) \leftarrow & \\ above(a,c) \leftarrow & \\ above(b,c) \leftarrow & \\ on(X) \leftarrow & \ above(X,Y) \\ switch(X) \leftarrow & \ {\sim}\, on(X) \\ next(X,Y) \leftarrow & \ switch(X), X \neq Y \\ next(X,Y) \leftarrow & \ above(X,Y), {\sim}\, switch(Y) \end{array} \right\}$$

Decide whether P is <u>safe</u>, and make appropriate modifications in case P is unsafe.

Exercise 7 ($5 + 3 + 2 = 10$ Points)

Consider the following logic program P:

$$P = \left\{ \begin{array}{l} a \leftarrow {\sim}d \\ b \leftarrow a, {\sim}d \\ c \leftarrow e \\ d \leftarrow {\sim}a, c \\ d \leftarrow {\sim}a, e \\ e \leftarrow {\sim}b \\ e \leftarrow {\sim}d, f \\ f \leftarrow b, c \\ f \leftarrow {\sim}a, d \end{array} \right\}$$

7–a Determine the <u>supported models</u> of P.

7–b Write the <u>loop formulas</u> in $LF(P)$.

7–c Determine the <u>stable models</u> of P.

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Exercise 8 (8 Points)

Determine the <u>unfounded sets</u> of the following logic program P w.r.t. the interpretation $\langle \{a\}, \{b\} \rangle$:

$$P = \left\{ \begin{array}{c} a \leftarrow e, \sim c \\ a \leftarrow d, b \\ b \leftarrow \sim d \\ c \leftarrow d, e \\ d \leftarrow \sim a \\ d \leftarrow b \\ e \leftarrow d \\ e \leftarrow a, c \end{array} \right\}$$

Exercise 9 (4 + 4 = 8 Points)

Consider the following logic program P:

$$P = \left\{ \begin{array}{l} a \leftarrow c, \sim f \\ a \leftarrow e, \sim d \\ b \leftarrow \\ c \leftarrow a, \sim e \\ c \leftarrow d, g \\ d \leftarrow e, f \\ e \leftarrow b, \sim g \\ f \leftarrow d, \sim a \\ f \leftarrow \sim b, \sim c \end{array} \right\}$$

9–a Determine the least fixpoint of Fitting's operator.

9–b Determine the least fixpoint of the Well-founded operator.

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Exercise 10 (6 + 4 = 10 Points)

Consider the following logic program P:

$$P = \left\{ \begin{array}{l} a \leftarrow \sim b, \sim d \\ a \leftarrow \sim c, e \\ b \leftarrow \sim c, g \\ c \leftarrow a, \sim d \\ d \leftarrow \sim c \\ e \leftarrow a, \sim g \\ e \leftarrow \sim b, \sim f \\ f \leftarrow a, d \\ g \leftarrow \sim b, \sim f \end{array} \right\}$$

10–a Apply <u>nogood propagation</u> on $\Delta_P \cup \Lambda_P$ to the assignment given below until some nogood becomes violated.

dl	σ_d	$\overline{\sigma}$	δ
1	$\mathbf{F}b$		
		$\mathbf{F}\{\sim c, g\}$	$\{\mathbf{T}\{\sim c, g\}, \mathbf{F}b\}$
2	$\mathbf{T}e$		
3	$\mathbf{T}\{\sim c\}$		

10–b Resolve the conflict using the <u>First-UIP scheme</u> and determine the <u>decision level</u> at which an <u>assertion</u> is obtained after backjumping.

Exercise 11 (8 Points)

1. Complete now the following proof that the equivalence $((F \wedge G) \leftarrow H) \leftrightarrow ((F \leftarrow H) \wedge (G \leftarrow H))$ is probable in intuitionistic logic.

1	$\quad (F \wedge G) \leftarrow H$	
2	$\quad\quad H$	
3	$\quad\quad F \wedge G$	....
4	$\quad\quad$	$\wedge$E, 3
5	$\quad$	....
6	$\quad\quad$	
7	$\quad\quad$	....
8	$\quad\quad$	$\wedge$E, 7
9	$\quad G \leftarrow H$	....
10	$\quad$	$\wedge$I, 5, 9
11	$((F \leftarrow H) \wedge (G \leftarrow H)) \leftarrow ((F \wedge G) \leftarrow H)$	....
12	$\quad$	
13	$\quad$	$\wedge$E, 12
14	$\quad G \leftarrow H$	....
15	$\quad\quad$	
16	$\quad\quad F$	....
17	$\quad\quad G$	....
18	$\quad\quad$	....
19	$\quad (F \wedge G) \leftarrow H$	....
20	$((F \wedge G) \leftarrow H) \leftarrow ((F \leftarrow H) \wedge (G \leftarrow H))$	....
21	$((F \wedge G) \leftarrow H) \leftrightarrow ((F \leftarrow H) \wedge (G \leftarrow H))$	....

Rules of natural deduction for intuitionistic logic

Implication

$$\frac{F \leftarrow G \qquad G}{F} \qquad\qquad \frac{\begin{array}{r|l} 1 & G \\ \cline{2-2} 2 & \vdots \\ 3 & F \end{array}}{F \leftarrow G}$$

Conjunction

$$\frac{F \wedge G}{F} \qquad\qquad \frac{F \wedge G}{G} \qquad\qquad \frac{F \quad G}{F \wedge G}$$

Disjunction

$$\frac{F \vee G \quad \begin{array}{r|l} 1 & F \\ \cline{2-2} 2 & \vdots \\ 3 & H \end{array} \quad \begin{array}{r|l} 1 & G \\ \cline{2-2} 2 & \vdots \\ 3 & H \end{array}}{H} \qquad \frac{F}{F \vee G} \qquad \frac{G}{F \vee G}$$

Negation and $\bot$

$$\frac{\bot}{F} \qquad\qquad \frac{F \qquad \neg F}{\bot} \qquad\qquad \frac{\begin{array}{r|l} 1 & F \\ \cline{2-2} 2 & \vdots \\ 3 & \bot \end{array}}{\neg F}$$

Equivalence

$$\frac{F \leftarrow G \qquad G \leftarrow F}{F \leftrightarrow G} \qquad\qquad \frac{F \leftrightarrow G}{F \leftarrow G} \qquad\qquad \frac{F \leftrightarrow G}{G \leftarrow F}$$

Substitution of equivalents

$$\frac{F \leftrightarrow G}{H \leftrightarrow H[F/G]}$$

with $H[F/G]$ being the result of replacing the subformula F oin H by G

Hosoi rule for here-and-there logic

$$\frac{}{F \vee ((G \leftarrow F) \vee \neg G)}$$

Solution 1.1

$\underline{\{b, d, e, g\}}$, $\{a, b, d, e, g\}$, $\{a, b, c, d, e, g\}$, $\{a, b, c, d, e, f, g\}$

Solution 1.2

- $\{icecream, sunny\}$
- $\{rainy, umbrella\}$

Solution 1.3

$\{a, b, d\}$, $\{a, d\}$, $\{b\}$

Solution 1.4

$\{a(0,1), a(0,2), a(1,2), b(0), b(1)\}$, $\{a(0,1), a(0,2), a(1,2), b(1), b(2), r(0,1), r(0,2)\}$

Solution 1.5

```
N { select(X) : vertex(X) } :- threshold(N).

unblocked(X) :- edge(X,Y), not select(Y).
unblocked(X) :- edge(Y,X), not select(Y).
:- select(X), not unblocked(X).
```

Solution 1.6

$$P = \left\{ \begin{array}{rl} above(a,b) & \leftarrow \\ above(a,c) & \leftarrow \\ above(b,c) & \leftarrow \\ on(X) & \leftarrow above(X,Y) \\ dom(X) & \leftarrow on(X) \\ dom(Y) & \leftarrow above(X,Y) \\ switch(X) & \leftarrow dom(X), \sim on(X) \\ next(X,Y) & \leftarrow switch(X), dom(Y), X \neq Y \\ next(X,Y) & \leftarrow above(X,Y), \sim switch(Y) \end{array} \right\}$$

Solution 1.7

7–a $\{b, a\}$, $\{c, d, e, f\}$, $\{b, a, c, e, f\}$

7–b $c \vee e \vee f \rightarrow (\neg b) \vee (\neg a \wedge d)$, $d \vee e \vee f \rightarrow (\neg a \wedge c) \vee (\neg b) \vee (b \wedge c)$, $c \vee d \vee e \vee f \rightarrow (\neg b)$

7–c $\{a, b\}$, $\{c, d, e, f\}$

Solution 1.8

$\emptyset$, $\{d\}$, $\{c, d\}$, $\{a, d, e\}$, $\{c, d, e\}$, $\{a, c, d, e\}$

Solution 1.9

9–a $\bigsqcup_{i\geq 0} \Phi_P^i \langle \emptyset, \emptyset \rangle = \{b, \sim g, e, \sim c\}$

9–b $\bigsqcup_{i\geq 0} \Omega_P^i \langle \emptyset, \emptyset \rangle = \{b, \sim g, e, \sim c, \sim d, \sim f, a\}$

Solution 1.10