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A Semi-supervised Gaussian Mixture Model for Image Segmentation*

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Abstract

In this paper, the results of a semi-supervised approach based on the Expectation-Maximisation algorithm for model-based clustering are presented. We show in this work that, if the appropriate generative model is chosen, the classification accuracy on clustering for image segmentation can be significantly improved by the combination of a reduced set of labelled data and a large set of unlabelled data. This technique has been tested on real images as well as on medical images from a dermatology application. The preliminary results are quite promising. Not only the unsupervised accuracies have been improved as expected but the segmentation results obtained are considerably better than the results obtained by other powerful and well-known unsupervised image segmentation techniques.

1. Introduction

Semi-supervised learning has received an increasing attention for the last years and has been widely extended to many fields [3]. Semi-supervised approaches arise from the idea of using together a large amount of unlabelled data, which is often cheap and easy, and labelled data, which is hard to obtain since it requires human experts or special devices. Thus, a semi-supervised methodology provides a learning environment from both labelled and unlabelled data. For instance, in semi-supervised classification we start from a training data with s labelled instances and u unlabelled samples, often $u \gg s$. The important point here is to manage a better classifier (or clustering result) than from the unlabelled data alone [10].

The standard Expectation-Maximisation (EM) algorithm [5] has been widely used and studied by the machine learning community. This algorithm estimates the maximum likelihood (or maximum *a posteriori*) parameters for generative models optimising the bound of the data log-likelihood in an iterative way. Similarly to the methodology presented in [3] for semi-supervised text classification, we define in this work an alternative framework where the supervised information is embedded in a probabilistic generative model, which can also deal with a mixture where each class can have multiple components.

Summarising, this work mainly contributes with:

- A step forward from the standard EM algorithm by means of introducing supervised information to the EM procedure. A fully explanation of the derived framework will be given in section 2.1.
- We propose a Gaussian mixture with multiple modes per class. This methodology allows the model to suit better to the different data distributions (Sect. 2.2).
- We also offer preliminary experimental evidences that our method improves the results obtained by other unsupervised techniques of the literature. The usefulness of this technique will be shown for a particular application in Dermatology (Sect. 3).

2. Semi-supervised EM Algorithm

In this section, we discuss a standard Gaussian mixture model in which the supervised dependencies have been explicitly introduced. We firstly present in section 2.1 the notation and probabilistic framework used throughout the paper. The proposed semi-supervised EM algorithm is explained in section 2.2.

2.1. Mathematical formulation

Let Ω be a set of N observed values $y_j \in \mathbb{R}^D$. We define the joint density function $p(\Theta \mid \Omega) =$

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$\prod_{j=1}^{N} p(y_j, x \mid \Theta)$, where Θ stands for the parameters of the distribution.

Let us also suppose that Ω is a Gaussian mixture with M modes, $x_1, \ldots, x_M$. For each mode k, $k = 1, \ldots, M$, we assume a Gaussian density conditional model, i.e., $p(y|x_k, \Theta) = \mathcal{N}(\mu_k, R_k)$; which is parametrised by the mean μ_k and covariance R_k of the mode. The unknown parameters of the observation model for all components are collectively denoted by $\Theta = \{\mu_k, R_k, \rho_k\}_{k=1}^{M}$, where ρ_k stands for the prior probability of the mode k.

Our goal is to find the semi-supervised parameters of Θ that maximise the expected log-likelihood $\mathcal{Q}$ of all observations with respect to the hidden variables x,

$$\mathcal{Q} = \sum_{j=1}^{N}\sum_{k=1}^{M} p(x_k|y_j, \Theta)\ \ ln\left[p(y_j|x_k, \Theta)\, p(x_k, \Theta)\right]$$

The Ω set is divided into a labelled set S and an unlabelled set U, $\Omega = S \cup U$. The supervised instances are considered as pairs of sample-label $S = \left\{(y_1^s, x_1^s), \ldots, (y_{|S|}^s, x_{|S|}^s)\right\}$ whereas $U = \left\{y_1^u, \ldots, y_{|U|}^u\right\}$ with $N = |S| + |U|$. A superscript s or u indicates if the instance belongs to the supervised or to the unsupervised samples respectively.

The unknown parameters of the observation model have been derived by means of applying Lagrange multipliers to $\mathcal{L}$. The Lagrangian equation $\mathcal{L}$ in table 1 is obtained from $\mathcal{Q}$ under the following conditions:

- General constraint for mode priors $\sum_{k=1}^{M} \rho(x_k, \Theta) = 1$.
- In the supervised part, it is assumed that $p(x_k|y_j, \Theta_t) = \delta(x_k, x_j^s)$. The δ function returns 1 if y_j belongs to the mode x_k or 0 otherwise.

The maximisation of $\mathcal{L}$ results in the following semi-supervised update equations 1, 2 and 3, where the parameter t states for the iteration step.

$$\mu_k = \frac{\sum_j^{S(k)} y_j^s \ + \ \sum_j^U p(x_k|y_j^u, \Theta_t)\ y_j^u}{|S(k)| \ + \ \sum_j^U p(x_k|y_j^u, \Theta_t)} \quad (1)$$

$$\rho(x_k|\Theta) = \frac{|S(k)| \ + \ \sum_j^U p(x_k|y_j^u, \Theta_t)}{|S| \ + \ \sum_j^U \sum_k^M p(x_k|y_j^u, \Theta_t)} \quad (2)$$

$$R_k = \frac{\sum_j^{S(k)} \mathcal{D}^s \ + \ \sum_j^U p(x_k|y_j^u, \Theta_t)\ \mathcal{D}^u}{|S(k)| \ + \ \sum_j^U p(x_k|y_j^u, \Theta_t)} \quad (3)$$

In these equations, the subset of supervised instances for a particular mode k are denoted as $S(k)$. We have also simplified the notation in expression 3 introducing $\mathcal{D} = (y_j - \mu_k)(y_j - \mu_k)^T$, therefore, $\mathcal{D}$ will be a D-dimensional matrix. Finally, the cardinality of a set is denoted by $|\cdot|$.

2.2. Algorithm

Briefly explained, the algorithm here presented has three input sources: the number of classes, the supervised data set S and the unsupervised data set U. In addition, the algorithm will consider that each class can have multiple modes. The number of modes of each class are estimated in the initialisation stage.

Initialisation: The EM algorithm is initialised for each class $c_i \in C$ estimating the number of Gaussian modes used per class. The whole set of unsupervised data is joined together with the supervised samples of each class c_i in a temporary set P_i. A standard unsupervised EM procedure for modelling Gaussian mixtures is applied on P_i [1]. Thus, P_i is divided into several clusters in which there will only be k_i clusters containing samples of supervised samples from class c_i. Therefore, class c_i will be initialised with k_i modes, re-labelling the samples of class c_i to the mode in k_i the sample belongs, estimating the mean and covariance of each mode from the labelled samples. Eventually, all modes from each class are then merged in order to initialise the mixture.

Semi-supervised EM: The iterations for the proposed semi-supervised algorithm are started using the update equations described in 1, 2 and 3. As it can be seen from the previous section, all the update equations have a supervised and an unsupervised part that are integrated in the same expressions.

E-step : Estimation of the Gaussian mode posteriors $p(x_i|y_j, \Theta_t)$ using the last maximisation step.

M-step : Update parameters μ, R and ρ using the posteriors estimated in the E step in expressions 1, 2 and 3.

The objective of the semi-supervised EM algorithm is to iteratively optimise the Θ parameters. The process stops when the change for the log-likelihood between two consecutive steps is less than a certain ϵ. The threshold value used is set as proposed in [1], $\epsilon = \frac{1}{100}\left(1 + D + \frac{(D+1)D}{2}\right) log(ND)$, where N is the number of samples and D is the dimension of each sample.

$$\mathcal{L}(\Theta) = \left[\sum_j^{S(k)} \sum_k^M \mathcal{T}^s + \sum_j^U \sum_k^M p(x_k|y_j^u, \Theta_t)\ \mathcal{T}^u\right] - \lambda\left(\sum_k^M p(x_k|\Theta) - 1\right)$$
$$\text{where } \mathcal{T} = \frac{-D}{2} ln(2\pi) - \frac{1}{2} ln\left(det(R_k)\right) - \frac{1}{2}(y_j - \mu_k)^T R_k^{-1}(y_j - \mu_k) + ln\left(p(x_k \mid \Theta)\right)$$

Table 1. Lagrangian Equation ($\mathcal{L}$) for the semi-supervised Gaussian mixture.

3. Experimental results

The experimental part of this work is focused on segmenting colour images. From each image, the number of classes is known and a small number of labelled pixels is available. The supervised data S represent 1% of total samples from each class, and the rest of pixels form the unsupervised subset U. Each sample is a $2D$-vector representing the chroma of a pixel in the $L^*a^*b^*$ colour space, that is, the a^*b^* dimensions.

Figure 1 shows some colour segmentation results on the well-known images *house* and *toys*. Note that there are some isolated pixels that are bad labelled. This is due to the fact that there is no spatial constraint in the process, but some spatial regularisation could be applied as in [2, 7].

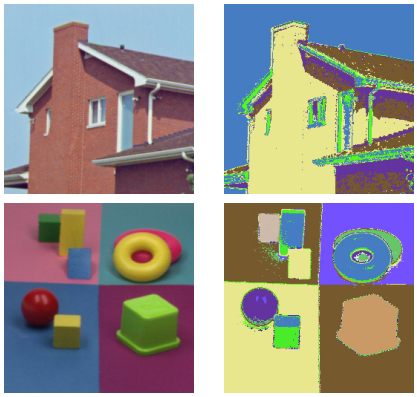

Figure 1. Image segmentation results of the semi-supervised algorithm. Left column: original images. Right column: results of the proposed method.

Preliminary results are also provided for images from a skin diagnosis application, where several skin pathologies have to be analysed. The proposed technique has been used to automatically isolate skin regions, in order to segment unhealthy areas out of healthy skin. Currently, this unhealthy skin is drawn manually, which is time-consuming as well as prone to errors due to manual mistakes and tiredness. Hence, adding very few labelled data to the clustering process by means of an expert, for instance, just clicking on few image points, the semi-supervised process here proposed can improve this task in a quick and practical way, introducing some interactivity with the user.

Figure 2 shows the results of the proposed technique compared with other three well-known unsupervised image segmentation algorithms. Firstly, **MS** is an effective algorithm that can be used to obtain the dominant colours of an image. It was proposed by Comaniciu and Meer in [4] and it is based on the *mean shift* algorithm applied in the spatial domain. Secondly, **SRM** algorithm was proposed by Nock and Nielsen in [9] based on the idea of using perceptual grouping and region merging for image segmentation. Finally, **JSEG** algorithm proposed by Deng and Manjunath provides colour-texture homogeneous regions which are useful for salient region detection [6]. It is important to point out that these algorithms have been applied using the same parameters their authors provide. It is also worth saying that the algorithm for image segmentation from Felzenszwalb and Huttenlocher [8] was also tested but, for this particular application, the results were too poor and they are not included due to the limited space.

From the results shown in figure 2, it can be noticed that, on the one hand, the proposed semi-supervised method is significantly better than the **MS** and **SRM** algorithms. On the other hand, the results of the **JSEG** algorithm are comparable to the proposed technique. The healthy and unhealthy skin have been well separated by this algorithm, allowing an easy and precise identification of each part. Moreover, the problem of the regions detected in the external part of the image could be easily overcome. However, as it can be seen, there exist some particular areas in the inner part of the unhealthy skin in which the proposed algorithm provides better results.

The proposed algorithm has also been compared with a standard unsupervised EM algorithm for Gaussian mixtures [1]. The ground-truth references (manually labelled) were available for these images and, therefore, the classification rates for the example images *p02* and *p34* are provided. Table 2 shows the percentages with regard to the number of well-classified pixels when the clustering results are compared with the ground-truth images. Notice that the result obtained on the *p02* image is significantly better for the proposed algorithm.

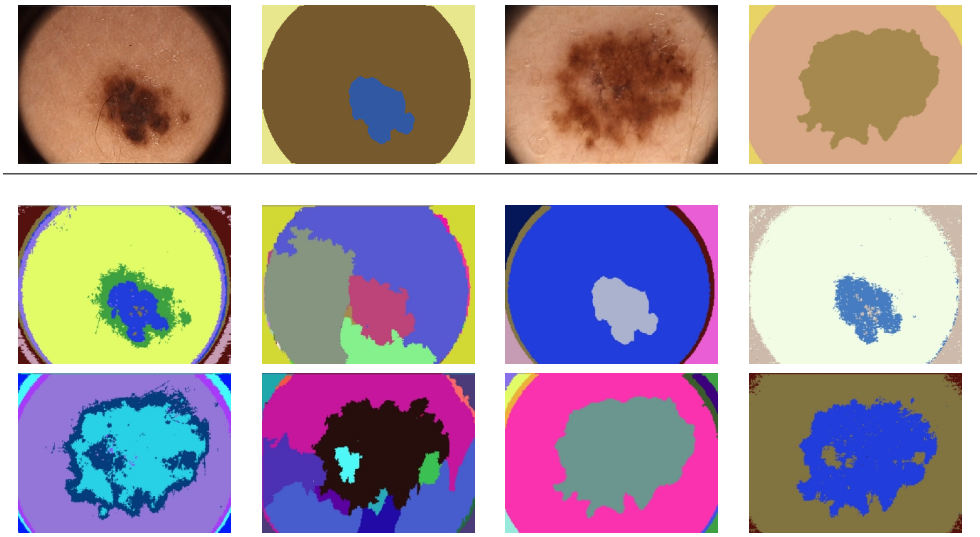

Figure 2. Application in Dermatology. Top row: original images - *p02* (left), *p34* (right) - and their ground-truth reference. Bottom rows show 4 image segmentation results corresponding to (left to right) MS, SRM, JSEG and the proposed method respectively.

	Semi-supervised EM	Unsupervised EM
p02	**95.63%**	78.76%
p34	**91.87%**	90.94%

Table 2. Classification rates for the images of the dermatology application.

4. Conclusions

In this paper, a semi-supervised algorithm using Gaussian mixtures for image segmentation has been proposed. Supervised information has been included in the optimisation process and the update functions of a semi-supervised EM algorithm have been derived. In addition, the proposed method allows mixtures with multiple modes per class. These modifications of the standard EM algorithm contributes to a general improvement of classification rates when only few labelled samples are available. The proposed algorithm has been tested on real images, obtaining promising results, which have demonstrated an improvement with respect to other well-known unsupervised algorithms for image segmentation.

Further work includes testing the algorithm in databases with a higher dimensionality. Future improvements are also directed to introduce a procedure to give more or less relevance to the supervised or unsupervised part in the process, expressing somehow the level of confidence in the supervised data.

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